Radiation fundamentals II

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Outline

• Introduction to spectral lines
• Recombination lines
• Radiative transfer
• Molecular lines
• HI 21 cm line
• Polarization
• Pulsar emission and polarization

"Essential Radio Astronomy" textbook:
https://science.nrao.edu/opportunities/courses/era
Introduction to spectral lines

- Spectral lines are narrow emission or absorption features in the spectra of gaseous and ionized sources.
- Spectral lines are intrinsically quantum phenomena.
- Spectral lines are narrow $\Delta \nu \ll \nu$.
Introduction to spectral lines

• Spectral lines are powerful tools to study the physical and chemical conditions in astronomical objects.
  - The rest frequency of the line identifies the atoms/molecules in the source.
  - Doppler shift of the line frequency determines the velocity of the source along the line-of-sight, leading to the redshift and Hubble distance of the source.
  - Doppler shift can also give the rotation curve and mass distribution for resolved galaxies.
  - Width of the line gives the motion of gas within the source.
  - Height of the line gives the temperature of the gas.
  - Area under the line gives the density of the gas (and gas mass).
Introduction to spectral lines

- Radio waves penetrate dust in our Galaxy and in other galaxies, which allows to detect line emission from dusty molecular clouds, protostars, and molecular disks orbiting AGNs.

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- Radio waves penetrate dust in our Galaxy and in other galaxies, which allows to detect line emission from dusty molecular clouds, protostars, and molecular disks orbiting AGNs.

- Most of the ISM exists in several distinct phases with similar pressure, but with different temperature.
  - Cold (10s of K) dense molecular clouds
  - Cool (~$10^2$ K) neutral HI gas
  - Warm (~$5 \times 10^3$ K) neutral HI gas
  - Hot (~$10^6$ K) low-density ionized gas (e.g. bubbles formed by expanding supernova remnants)
Recombination lines

- Recombination line is a spectral line formed by an electron transition from one energy state to another energy state in an atom.

- Energy levels are quantized, so the spectral lines have definite frequencies.
- The line frequency depends on the energy of the quantum levels.

(Murchikova et al. 2019)
Recombination lines

Recombination line frequency

\[ \nu = R_M c \left[ \frac{1}{n^2} - \frac{1}{(n + \Delta n)^2} \right]; R_M \equiv R_\infty \left(1 + \frac{m_e}{M}\right)^{-1} \]

Rydberg constant \( R_\infty \equiv \left(\frac{2\pi^2 m_e e^4}{\hbar^3 c}\right) = 1.09737312\ldots \times 10^5 \text{ cm}^{-1} \)

- \( M \) — nuclear mass
- \( m_e \) — electron mass
- \( n \) — quantum number
- \( \Delta n \) — change in the quantum number \( n \)
Recombination lines

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M — nuclear mass
\( m_e \) — electron mass
n — quantum number
\( \Delta n \) — change in the quantum number n

For H atom — \( M(H) \approx 1836.1 m_e \)

\[ \nu = 3.288 \times 10^{15} \left( \frac{1}{n^2} - \frac{1}{(n + \Delta n)^2} \right) \text{ Hz} \]
Recombination lines

Recombination line frequency

For H atom

\[
\nu = \frac{\Delta E}{h} = 3.288 \times 10^{15} \left( \frac{1}{n^2} - \frac{1}{(n + \Delta n)^2} \right) \text{Hz}
\]

In order to have spectral lines in radio waves (say \( \nu < 1000 \text{ GHz} \)):

\[
\left( \frac{1}{n^2} - \frac{1}{(n + \Delta n)^2} \right) < 3 \times 10^{-4}
\]

This satisfies only when \( n > 19 \)
Recombination lines

Recombination line frequency

\[ \nu = 3.288 \times 10^{15} \left( \frac{1}{n^2} - \frac{1}{(n + \Delta n)^2} \right) \text{ Hz} \]

What is the frequency of the $110 \rightarrow 109$ transition of H?

This is known as $H109\alpha$

\[ \nu = 3.288 \times 10^{15} \left( \frac{1}{109^2} - \frac{1}{110^2} \right) \approx 5.0089 \times 10^9 \text{ Hz} \approx 5.01 \text{ GHz} \]
Recombination lines

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Recombination lines of heavier atoms have a slightly higher frequencies.

\[ \nu = R_M c \left[ \frac{1}{n^2} - \frac{1}{(n + \Delta n)^2} \right]; R_M \equiv R_{\infty} \left( 1 + \frac{m_e}{M} \right)^{-1} \]
Recombination lines

Recombination line frequency

\[ \nu = R \frac{1}{n^2} - \left( \frac{1}{n} \right) \]

What is the frequency of the \(110 \rightarrow 109\) transition of \(H\)?

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\[ \nu = 3.288 \times 10^\frac{1}{109} - 1 \approx 5.0089 \times 10^\frac{1}{110} \approx 5.01 \text{ GHz} \]

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\[ \nu = R \frac{1}{n^2} - \left( \frac{1}{n} \right) \]
Recombination lines

Recombination line frequency

\[ \nu = 3.288 \times 10^3 \nu_{109} - 1 \]

What is the frequency of the 110 → 109 transition of H?

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\[ \nu \approx 5.01 \text{ GHz} \]

Recombination lines of heavier atoms have a slightly higher frequencies.

\[ \nu = R_m c \left( \frac{n^2}{n} \right) - R_h \]

\[ R_m \equiv R_h (1 + m \frac{M}{M}) \]

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Radiative transfer

- Three types of radiative transfer

Spontaneous emission

Higher energy state

Lower energy state

Absorption

Stimulated emission
Spontaneous emission rate for the transition from \( n \) to \( (n-1) \) is equal the average power emitted by an atom divided by the energy of a photon.

\[
A_{n,n-1} = \frac{\langle P \rangle}{h\nu}
\]

From Larmor's formula

\[
\langle P \rangle = \frac{2e^2}{3c^3} (2\pi\nu)^4 \frac{a_n^2}{2}; a_n \approx \frac{n^2 h^2}{4\pi^2 m_e e^2}
\]

\[
A_{n,n-1} \approx \left( \frac{64\pi^6 m_e e^{10}}{3c^3 h^6} \right) \frac{1}{n^5}
\]
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\]

For a hydrogen atom \( - A_{n,n-1} \approx 5.3 \times 10^9 \left( \frac{1}{n^5} \right) \text{s}^{-1} \)

For previous example; transition of H from energy level 110 to 109 \( \rightarrow A_{110,109} \approx 0.3 \text{ s}^{-1} \)

i.e. emit one photon every 3 seconds
Radiative transfer

Absorption and stimulated emission

• Absorption coefficient $\kappa_{\text{abs}}$ is the fraction of spectral brightness removed per unit length by absorption from the lower level to the upper level.

• Stimulated emission is treated as negative absorption $\kappa_{\text{stim}}$.

• The net absorption coefficient is the sum of the above two.

• The stimulated emission nearly cancels pure absorption and significantly reduces the net line opacity at radio frequencies.

• The brightness of an optically thin ($\tau \ll 1$) radio emission line is proportional to the column density of emitting gas, but can be nearly independent of the gas temperature.
Molecular lines

• Symmetric molecules have no permanent electric dipole moment (e.g. \( \text{H}_2 \)).

• Most asymmetric molecules have asymmetric charge distribution, leading to a permanent dipole moment, and they are polar (e.g. \( \text{CO} \)).

• Polar molecules radiate energy at their rotation frequency.

• Rotation energy of a polar molecule is quantized:

\[
E_{\text{rot}} = \frac{\hbar^2}{2I} J(J + 1); J = 0, 1, 2, \ldots
\]

• Energy change is restricted by quantum mechanical selection rule: \( \Delta J = \pm 1 \)

• Energy released during the rotational transition from \( J \) to \( (J - 1) \):

\[
\Delta E = [J(J + 1) - (J - 1)J] \frac{\hbar}{2I} = \frac{\hbar^2 J}{I}; J = 1, 2, \ldots
\]
Molecular lines

- Frequency of the line due to the rotational transition from $J$ to $(J - 1)$:

$$\Delta E = \left[ J(J + 1) - (J - 1)J \right] \frac{\hbar}{2l} = \frac{\hbar^2 J}{l} = \hbar \nu; \; J = 1, 2, \ldots$$

$$\nu = \frac{hJ}{4\pi^2 m r_e^2}$$

Upper energy level $J = 1, 2, \ldots$; reduced mass $m \equiv \left( \frac{m_A m_B}{m_A + m_B} \right)$; equilibrium distance $r_e = \frac{1}{2\pi} \left( \frac{hJ}{m
u} \right)^{1/2}$

- Most abundant element in molecular clouds is H$_2$.
- H$_2$ is extremely difficult to observe:
  - Symmetric molecule, no dipole moment $\rightarrow$ inefficient radiator.
  - Small moment of inertial, widely-spaced energy levels, so lowest energy transition is equivalent to $T \sim 500$ K.
  - However, MC are cold $T \sim 20$ K, so transition happens rarely.
Molecular lines

• CO is the second most abundant molecule in molecular clouds, which is a proxy for H$_2$.

• What is the frequency of $J = 1 \rightarrow 0$ line of the $^{12}$C$^{16}$O molecule?

$$\nu = \frac{hJ}{4\pi^2mr_e^2}; \quad J = 1,2,3,\ldots$$
Molecular lines

• CO is the second most abundant molecule in molecular clouds, which is a proxy for H₂.

• What is the frequency of $J = 1 \rightarrow 0$ line of the $^{12}$C$^{16}$O molecule?

$$\nu = \frac{hJ}{4\pi^2mr_e^2}; \quad J = 1,2,3,…..$$

$$m \equiv \left( \frac{m_cm_0}{m_c + m_0} \right) \approx m_H \left( \frac{12 \cdot 16}{12 + 16} \right) \approx 1.15 \times 10^{-23} \text{ g}$$

$$r_e = 1.12675 \times 10^{-8} \text{ cm}$$

Frequency of the fundamental transition $\nu_{J=1-0} = 115.2712 \text{ GHz}$

Frequency of the second transition $\nu_{J=2-1} = 2 \cdot \nu_{J=1-0} = 230.5424 \text{ GHz}$
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- Frequencies are harmonics of $J = 1 \rightarrow 0$ line.
Molecular lines

\[ \nu = \frac{hJ}{4\pi^2mr_e^2}; \quad J = 1,2,3,\ldots \]

- Frequency of the second transition \( \nu_{J=2-1} = 2 \cdot \nu_{J=1-0} = 230.5424 \, GHz \)

- Centrifugal force increases as the molecule spins rapidly (i.e. large \( J \)), leading to increase the molecular separation \( r_e \), make the line frequencies slightly lower than the harmonics.

- The actual \( J = 2 \rightarrow 1 \) line frequency is \( \nu_{J=2-1} = 230.538 \, GHz \)
Molecular lines

Molecular excitation:

• Molecules excited into higher energy states by ambient radiation and collisions in dense gas.

• Minimum temperature needed for excitation $J \rightarrow J - 1$ is

$$T_{\text{min}} \approx \frac{E_{\text{rot}}}{k_B} = \frac{v h}{2 k_B} J (J + 1)$$
Molecular lines

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$$T_{\text{min}} \approx \frac{E_{\text{rot}}}{k_B} = \frac{\nu \hbar}{2k_B} J(J + 1)$$

- Minimum temperature to excite the $J = 2 - 1$ line at 230.5 GHz of $^{12}\text{C}^{16}\text{O}$ is $T_{\text{min}} \approx 16.6 \, K$.

$T_{\text{min}}$ required for collisions to excite the molecules is proportional to $J(J + 1)$.

High-J lines are weak in cold molecular clouds.
Molecular lines

Spontaneous emission coefficient of $J \rightarrow J - 1$ molecular line:

$$A_{J\rightarrow J-1} \approx 1.165 \times 10^{-11} |\mu|^2 \left( \frac{J}{2J+1} \right) \left( \frac{\nu}{GHz} \right) s^{-1}$$

; where $|\mu|$ is mean electric dipole moment

Spontaneous emission coefficient of CO $J = 1 \rightarrow 0$ line at $\nu \approx 115GHz$ is $A_{10} \approx 7.1 \times 10^{-8} s^{-1}$.

Typical time for a CO molecule to emit a photon is $\left( \frac{1}{A_{10}} \right) \approx 10^7 s$.

Critical density of CO $J = 1 \rightarrow 0$ transition is $n^* \approx \frac{A_{10}}{\sigma v} \approx 1.4 \times 10^3 cm^{-3}$.

Many Galactic molecular clouds have higher densities than this, so Galactic CO $J = 1 \rightarrow 0$ emission is strong and widespread.
Spontaneous emission coefficient of \( J \rightarrow J-1 \) molecular line:

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Critical density of CO \( J = 1 \rightarrow 0 \) transition is \( n^* \approx 1.4 \times 10^4 \) cm\(^{-3}\). Many Galactic molecular clouds have higher densities than this, so Galactic CO \( J = 1 \rightarrow 0 \) emission is strong and widespread.
Hydrogen is the most abundant element in the ISM. H$_2$ molecules are extremely difficult to observe.

Neutral hydrogen (HI) is abundant and detected through its hyperfine line.

HI line is intrinsically a quantum phenomenon.

The spin of proton and electron in a hydrogen atom can be parallel or anti-parallel.

The parallel state has more energy than the anti-parallel state.

The HI hyperfine line is created by flipping the spin of the two states.

Credits: Wikimedia Commons

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HI 21 cm hyperfine line

HI line center frequency is

\[ \nu_{10} = \frac{8}{3} g_I \left( \frac{m_e}{m_p} \right) \alpha^2 (R_M c); \quad R_M \equiv R_\infty \left( 1 + \frac{m_e}{m_p} \right)^{-1} \]

Rydberg constant \( R_\infty \equiv \left( \frac{2\pi^2 m_e e^4}{\hbar^3 c} \right) = 1.09737312\ldots \times 10^5 \text{ cm}^{-1} \)

Nuclear g-factor \( g_I \approx 5.58569 \), fine-structure constant \( \alpha \approx 1/137.036 \), and \( m_p \approx 1836.1 \times m_e \)

\[ \nu_{10} \approx 1420.405751 \text{ MHz} \quad \Rightarrow \lambda = \frac{c}{\nu} \approx 21 \text{ cm} \]
HI 21 cm hyperfine line

- Emission coefficient of the HI 21 cm line is $A_{10} \approx \frac{64\pi^4}{3hc^3} v_{10}^3 |\mu_B|^2$

  Bohr magneton is $|\mu_B| = \frac{e\hbar}{2m_e c} \approx 9.27401 \times 10^{-21} \text{ erg/G}$

- Emission coefficient of HI line is small $A_{10} \approx 2.85 \times 10^{-15} \text{ s}^{-1}$

- Radiative half-life of this transition is very long:
  
  $\tau_{1/2} = A_{10}^{-1} \approx 3.5 \times 10^{14} \text{ s} \approx 11 \text{ million years}$

- H atom takes 11 Myr to flip the spin states and emit a 1420 MHz photon.
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H atom takes 11 Myr to flip the spin states and emit a 1420 MHz photon.

However, there are $\sim 10^{67}$ H atoms in our Galaxy $\rightarrow 10^{52}$ atoms emit 1420 MHz photons per second!
HI 21 cm hyperfine line

- Radio image shows more detailed structure than the optical image.
- The regions that optically obscured due to dust in the galaxy can be seen at radio.
HI 21 cm hyperfine line

- Radio image shows more detailed structure than the optical image.
- The regions that optically obscured due to dust in the galaxy can be seen at radio.

Radio shows tidal bridges between the Galaxies, but not in optical.
HI 21 cm hyperfine line

Example:

• HI line Frequency is not at 1420 MHz ??

• Line is redshifted!
HI 21 cm hyperfine line

Example:

- HI line Frequency is not at 1420 MHz ??
- Line is redshifted!
- Galaxy is moving away from us.

\[
\frac{\nu_r}{c} \approx \frac{\nu_e - \nu_o}{\nu_e} ; (\nu_r << c)
\]

HI line spectrum of Galaxy UGC 11707

(Haynes et al. 1998)
**Example:**

- HI line Frequency is not at 1420 MHz ?
- Line is redshifted!
- Galaxy is moving away from us.

\[
\frac{v_r}{c} \approx \frac{v_e - v_0}{v_e}; (v_r << c)
\]

Using the observed frequency 1416.2 MHz → \(v_r \approx 890 \text{ km/s}\)

Using Hubble’s law \(d \approx \frac{v_r}{H_0} \approx 13 \text{ Mpc}\)
HI 21 cm hyperfine line

Example:

- Galaxy is rotating, so
  - Half of the galaxy is approaching us
  - Half of the galaxy is moving away

- Doppler shift within the galaxy broaden the line, and the profile is two-horned.

- The total HI mass of the galaxy:

\[
\left( \frac{M_H}{M_\odot} \right) \approx 2.36 \times 10^5 \left( \frac{d}{Mpc} \right)^2 \int \left( \frac{S_v}{Jy} \right) \left( \frac{dv}{km/s} \right) 
\]

\[
\int s(v)dv \approx 0.35 Jy \cdot 200 \frac{km}{s} \approx 70 Jy km/s 
\]

\[
M_H \approx 2.8 \times 10^9 M_\odot 
\]
HI 21 cm hyperfine line

Galactic HI line emission:

• Galaxy rotates differentially.
• Each cloud along the line-of-sight is in its own orbit, leading to different radial velocities.
• Therefore, HI spectrum is split into several components.
HI 21 cm hyperfine line

Galactic HI line emission:

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• Maximum velocity is reached when the cloud is closest to the GC along the LoS.
HI 21 cm hyperfine line

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• Maximum velocity is reached when the cloud is closest to the GC along the LoS.

• By sampling many lines-of-sight across the Galaxy, we can measure circular velocities as a function of R, and then generate the rotation curve.
Polarization

- The electric field of an electromagnetic wave traveling in the z-direction can be projected onto orthogonal x- and y-directions.

\[ \vec{E} = [\hat{x}E_x \exp(i\phi_x) + \hat{y}E_y \exp(i\phi_y)] \exp[i(\vec{k} \cdot \vec{z} - \omega t)] \]
Polarization

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Horizontal linear polarization

Vertical linear polarization

Credit: Applied Photophysics
Polarization

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\]

• Superposition of two orthogonally polarized components create different polarization states.

When \(\delta = \phi_x - \phi_y = 0\), then we get linear polarization.

If \(E_x = E_y\), then the linear polarization position angle \(\psi_{PPA}\) = 45°.
Polarization

• The electric field of an electromagnetic wave traveling in the z-direction can be projected onto orthogonal x- and y-directions.

\[ \vec{E} = [\hat{x}E_x \exp(\phi_x) + \hat{y}E_y \exp(\phi_y)] \exp[i(k \cdot \vec{z} - \omega t)] \]

• If \( E_x = E_y \), and \( \delta = \phi_x - \phi_y = 90^\circ \), then we get circular polarization.

• If \( \delta = \phi_x - \phi_y = 90^\circ \), but \( E_x \neq E_y \), then we get elliptical polarization.

• If \( E_x = E_y \), but the phases are completely uncorrelated, then the light is unpolarized.
Polarization

\[ \vec{E} = [x E_x \exp(i\phi_x) + y E_y \exp(i\phi_y)] \exp[i(\vec{k} \cdot \vec{z} - \omega t)] \]

- Polarization of a source can be characterized by Stokes parameters (for a “linear feed” with a cross dipole)

\[
I = < E_x^2 + E_y^2 > / R_0 \\
Q = < E_x^2 - E_y^2 > / R_0 \\
U = < 2E_x E_y \cos(\delta) > / R_0 \\
V = < 2E_x E_y \sin(\delta) > / R_0
\]

\( R_0 = 4.19 \times 10^{-10} \text{ s cm}^{-1} \) is the radiation resistance of free space.
Polarization

\[ \vec{E} = [\hat{x}E_x \exp(i\phi_x) + \hat{y}E_y \exp(i\phi_y)] \ \exp[i(\vec{k} \cdot \hat{z} - \omega t)] \]

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\[ R_0 = 4.19 \times 10^{-10} \ s \ cm^{-1} \] is the radiation resistance of free space.

- Polarized flux density is \( I_p = \sqrt{Q^2 + U^2 + V^2} \)
- Degree of polarization is \( p = \frac{I_p}{I} \)
- If the light is unpolarized, then \( Q = U = V = 0; I_p = 0; p = 0 \).
Polarization

\[ \vec{E} = [\hat{x}E_x \exp(i\phi_x) + \hat{y}E_y \exp(i\phi_y)] \exp[i(\vec{k} \cdot \vec{z} - \omega t)] \]

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\[ R_0 = 4.19 \times 10^{-10} \text{ s cm}^{-1} \text{ is the radiation resistance of free space.} \]

- Linear polarization \( L = \sqrt{Q^2 + U^2} \)
- Linear polarization position angle \( \psi = \frac{1}{2} \tan^{-1} \left( \frac{U}{Q} \right) \)
- Circular polarized flux is \( |V| \)

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Polarization

• Faraday rotation: When the linearly polarized light propagates through ISM, the polarization position angle can be rotated.

\[ \Delta \psi_{PPA} = \lambda^2 \times RM \]

Rotation measure

\[ RM \approx 8.1 \times 10^5 \int_{los} \left( \frac{n_e}{cm^{-3}} \right) \left( \frac{B_\parallel}{gauss} \right) \left( \frac{dl}{pc} \right) \text{rad/m}^2 \]

• Change in \( \psi_{PPA} \) can be measured with multi-frequency observations.
• We can constrain \( RM \).
• \( \langle B_\parallel \rangle \) can be estimate.
Pulsar emission

- Pulsars are excellent rotators.
- Pulsars are powered by loss of rotational kinetic energy.
- Energy is dissipated as dipole radiation: 
  \[
  \dot{E} = -\frac{dE_{rot}}{dt} \approx 3.95 \times 10^{31} \left( \frac{\dot{P}}{10^{-15}} \right) \left( \frac{P}{s} \right) \text{ erg/s}
  \]

Pulsar period increases over time and spin-down gradually!

Spin-down rate of J2215+1538 is \( \dot{P} = (2.3672 \pm 2) \times 10^{-15} \text{ s/s} \).
Pulsar polarization

- Pulsars are highly magnetized dipoles ($\approx 10^8 - 10^{12} G$).
- Particle accelerate along magnetic field lines (curvature radiation) $\rightarrow$ photons emit along the plane of the magnetic field.
- Pulsar emission is highly linearly polarized.
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- Emission beam across our line of sight results in a S-shaped curve in the polarization position angle sweep $\psi_{PPA}$. 
Pulsar polarization

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• Pulsar emission is highly linearly polarized.

• Emission beam across our line of sight results in a S-shaped curve in the polarization position angle sweep $\psi$.

Ben Perera — Arecibo Observatory

Dipole field lines

J0659+1414

J1718-3825

(Weltevrede et al. 2010)

GBO/AO single dish summer school — 13 September 2021
Thank you!

The derivations of the equation and more details can be found in the textbook “Essential Radio Astronomy”

https://science.nrao.edu/opportunities/courses/era