Radiation fundamentals II

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Outline

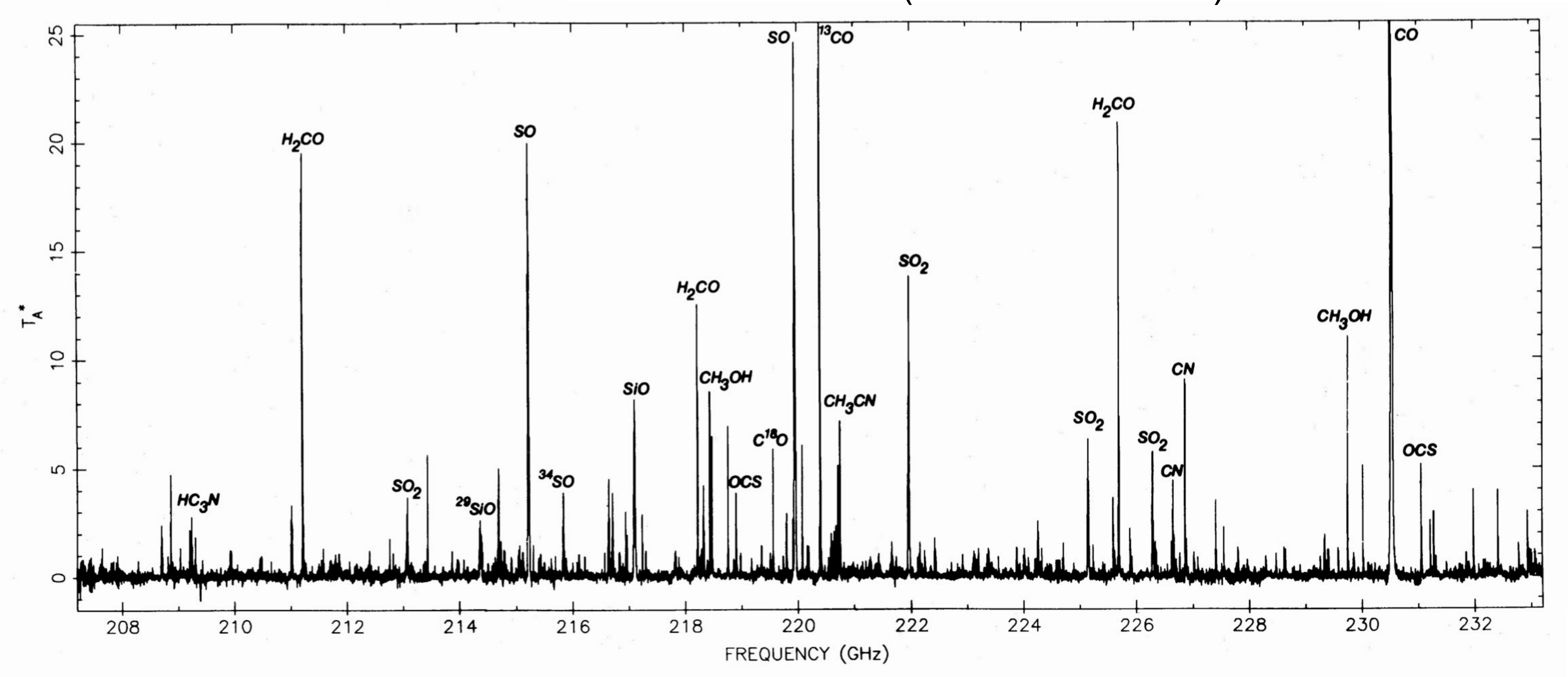
- Introduction to spectral lines
- Recombination lines
- Radiative transfer
- Molecular lines
- HI 21 cm line
- Polarization
- Pulsar emission and polarization

"Essential Radio Astronomy" textbook:
https://science.nrao.edu/opportunities/courses/era

Introduction to spectral lines

- Spectral lines are narrow emission or absorption features in the spectra of gaseous and ionized sources.
- Spectral lines are intrinsically quantum phenomena.
- Spectral lines are narrow $\Delta\nu \ll \nu$.

Orion molecular cloud OMC-1 (Blake et al. 1987)

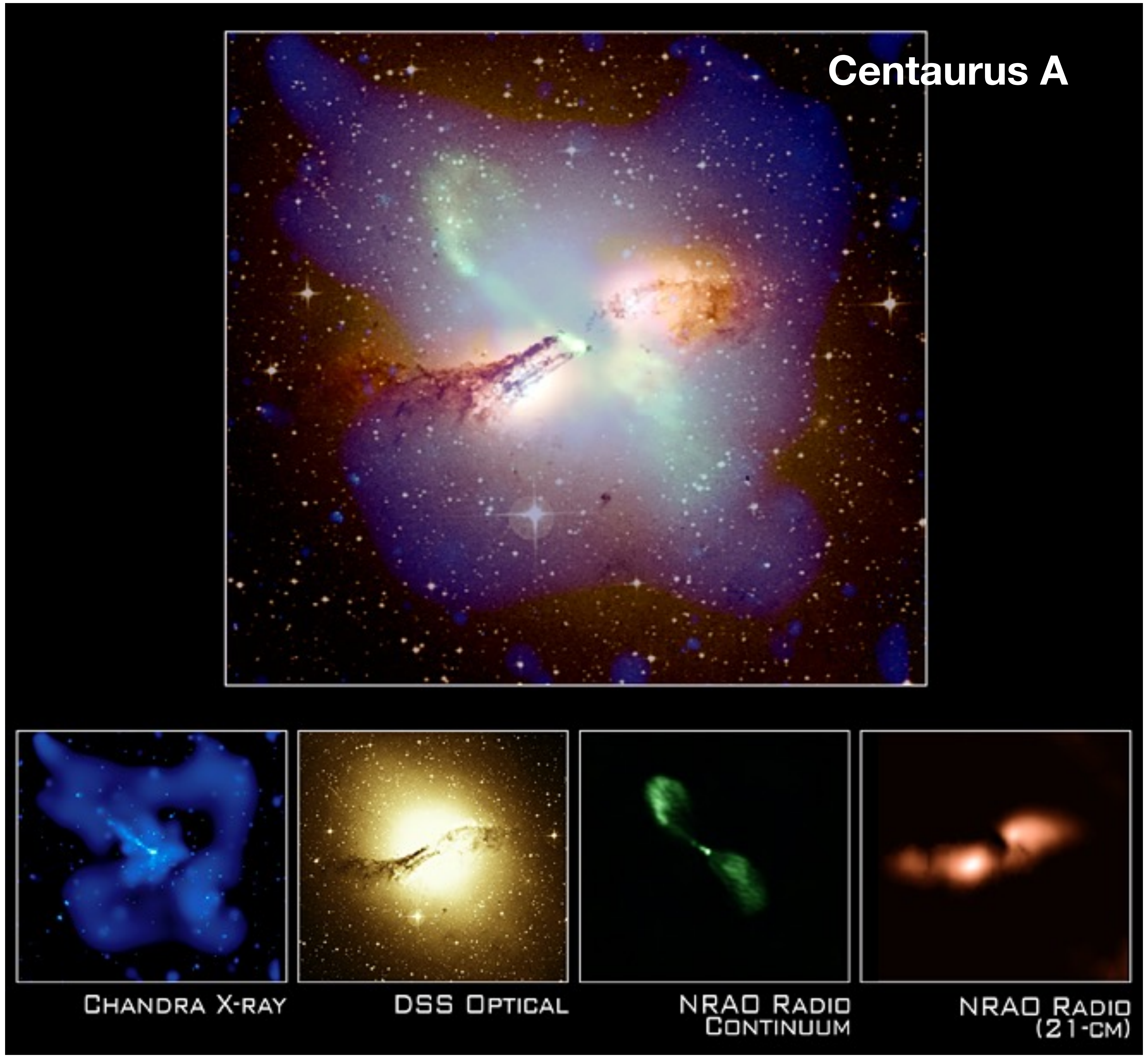


Introduction to spectral lines

- Spectral lines are powerful tools to study the physical and chemical conditions in astronomical objects.
  - The rest frequency of the line identifies the atoms/molecules in the source.
  - Doppler shift of the line frequency determines the velocity of the source along the line-of-sight, leading to the redshift and Hubble distance of the source.
  - Doppler shift can also give the rotation curve and mass distribution for resolved galaxies.
  - Width of the line gives the motion of gas within the source.
  - Height of the line gives the temperature of the gas.
  - Area under the line gives the density of the gas (and gas mass).

Introduction to spectral lines

- Radio waves penetrate dust in our Galaxy and in other galaxies, which allows to detect line emission from dusty molecular clouds, protostars, and molecular disks orbiting AGNs.

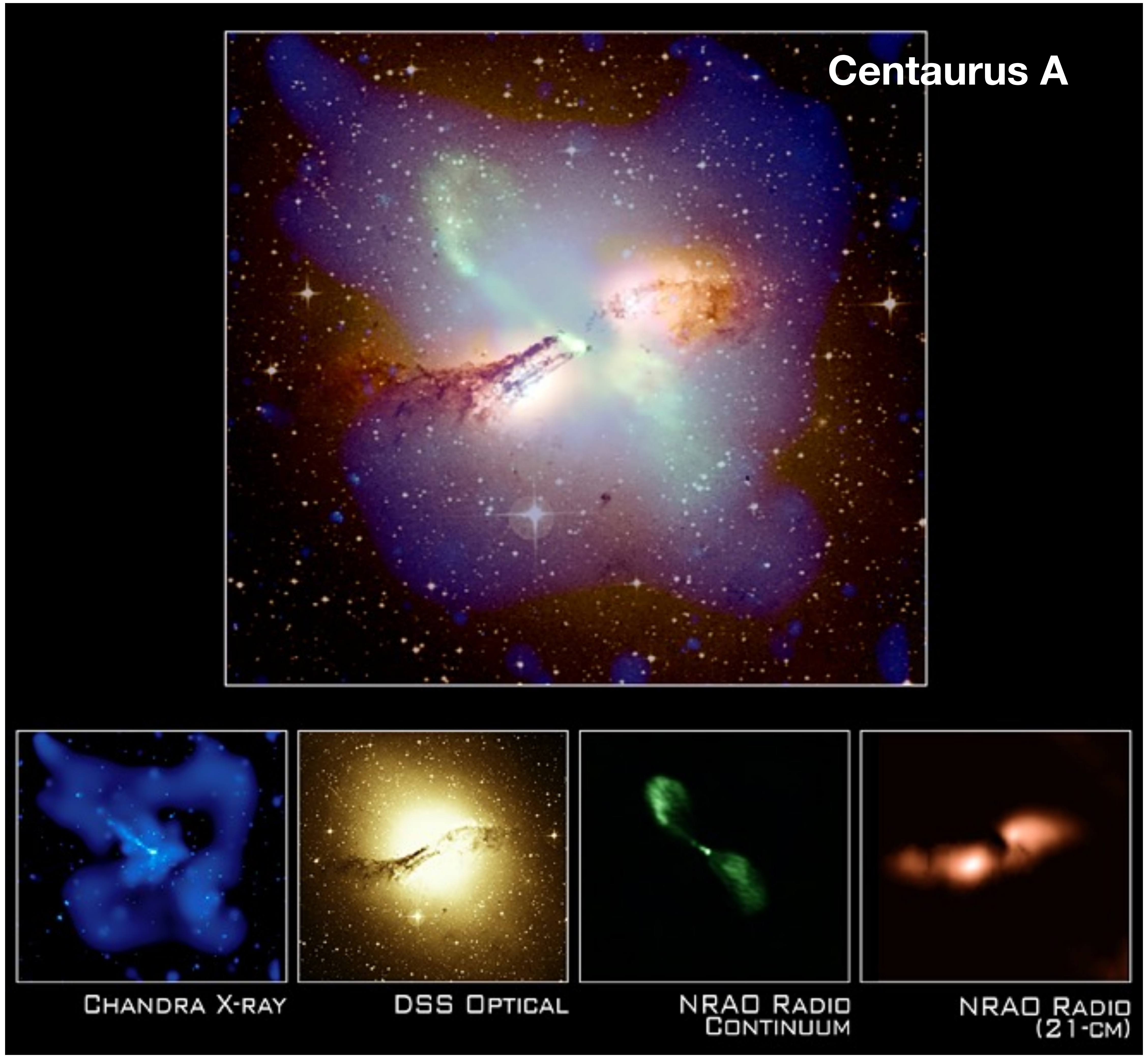


Credit: X-ray (NASA/CXC/M. Karovska et al.); Radio 21-cm image (NRAO/VLA/J.Van Gorkom/Schminovich et al.), Radio continuum image (NRAO/VLA/J. Condon et al.); Optical (Digitized Sky Survey U.K. Schmidt Image/STScI)

Introduction to spectral lines

- Radio waves penetrate dust in our Galaxy and in other galaxies, which allows to detect line emission from dusty molecular clouds, protostars, and molecular disks orbiting AGNs.

- Most of the ISM exists in several distinct phases with similar pressure, but with different temperature.
  - Cold (10s of K) dense molecular clouds
  - Cool (~10^2 K) neutral HI gas
  - Warm (~5×10^3 K) neutral HI gas
  - Hot (~10^6 K) low-density ionized gas (e.g. bubbles formed by expanding supernova remnants)



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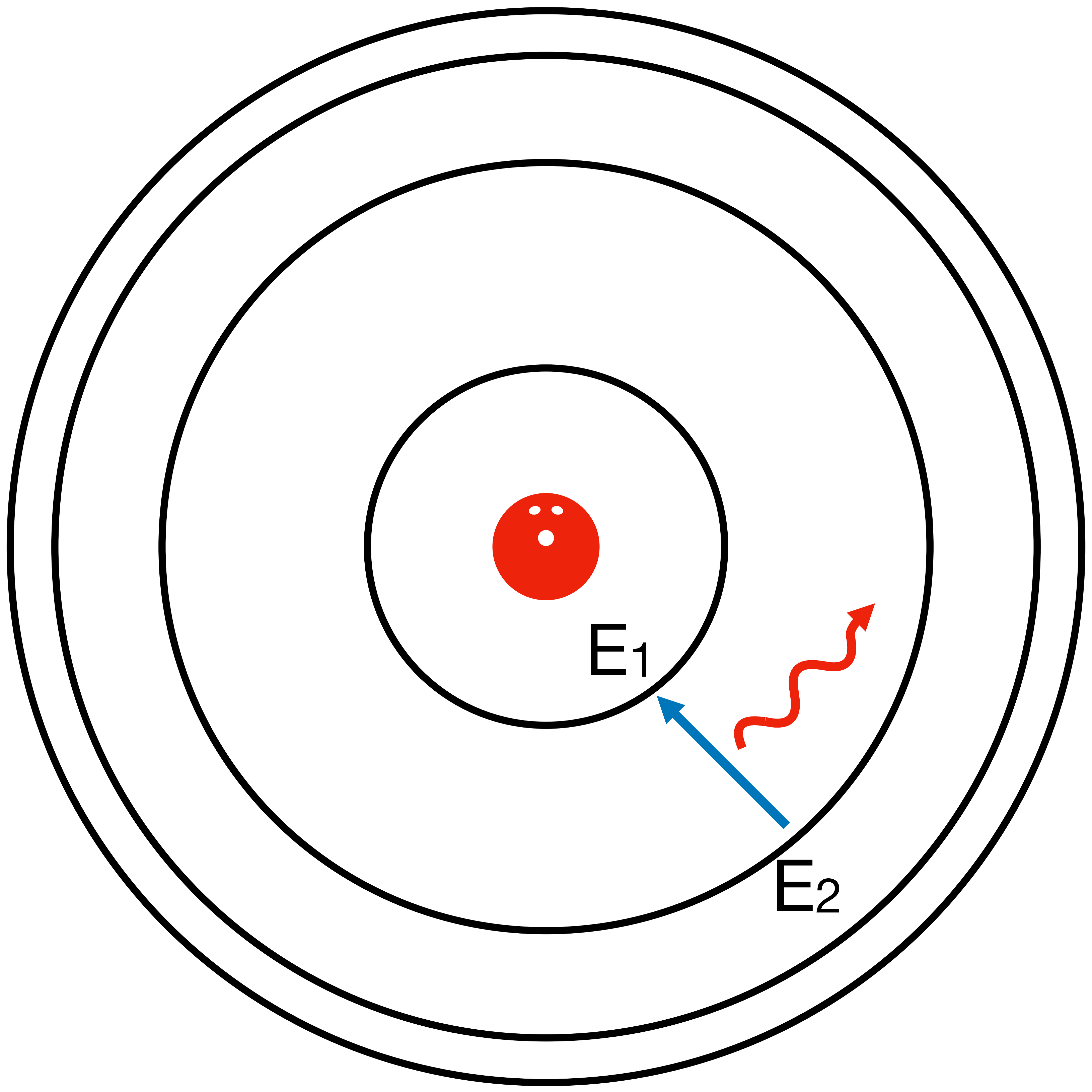
Recombination lines

- Recombination line is a spectral line formed by an electron transition from one energy state to another energy state in an atom.



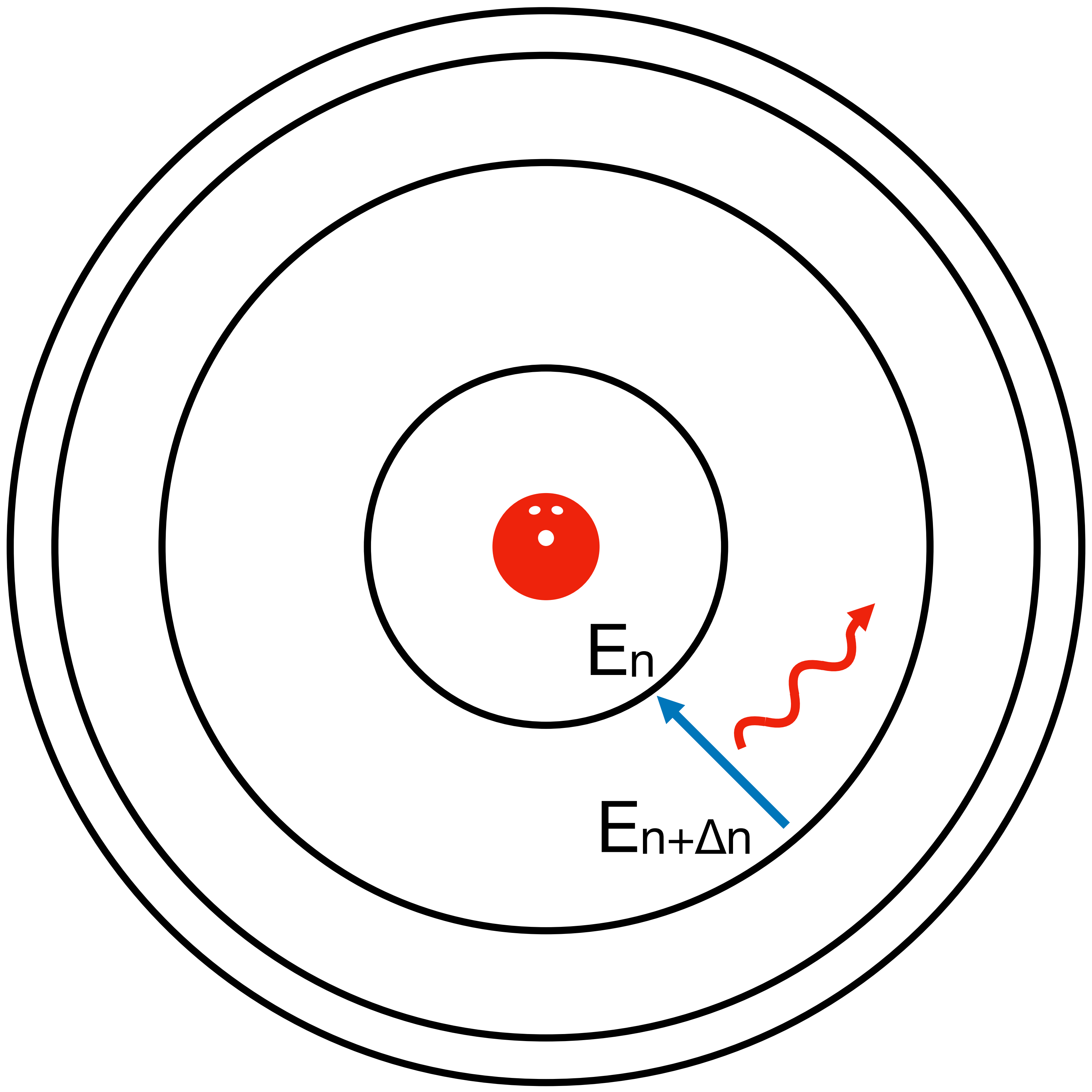
- Energy levels are quantized, so the spectral lines have definite frequencies.
- The line frequency depends on the energy of the quantum levels.

(Murchikova et al. 2019)

Recombination lines

Recombination line frequency

$$\nu = R_M c \left[\frac{1}{n^2} - \frac{1}{(n+\Delta n)^2}\right]; R_M \equiv R_\infty \left(1 + \frac{m_e}{M}\right)^{-1}$$



Rydberg constant $R_\infty \equiv \left(\frac{2\pi^2 m_e e^4}{h^3 c}\right) = 1.09737312....\times 10^5\ cm^{-1}$

M — nuclear mass

m_e — electron mass

n — quantum number

Δn — change in the quantum number n

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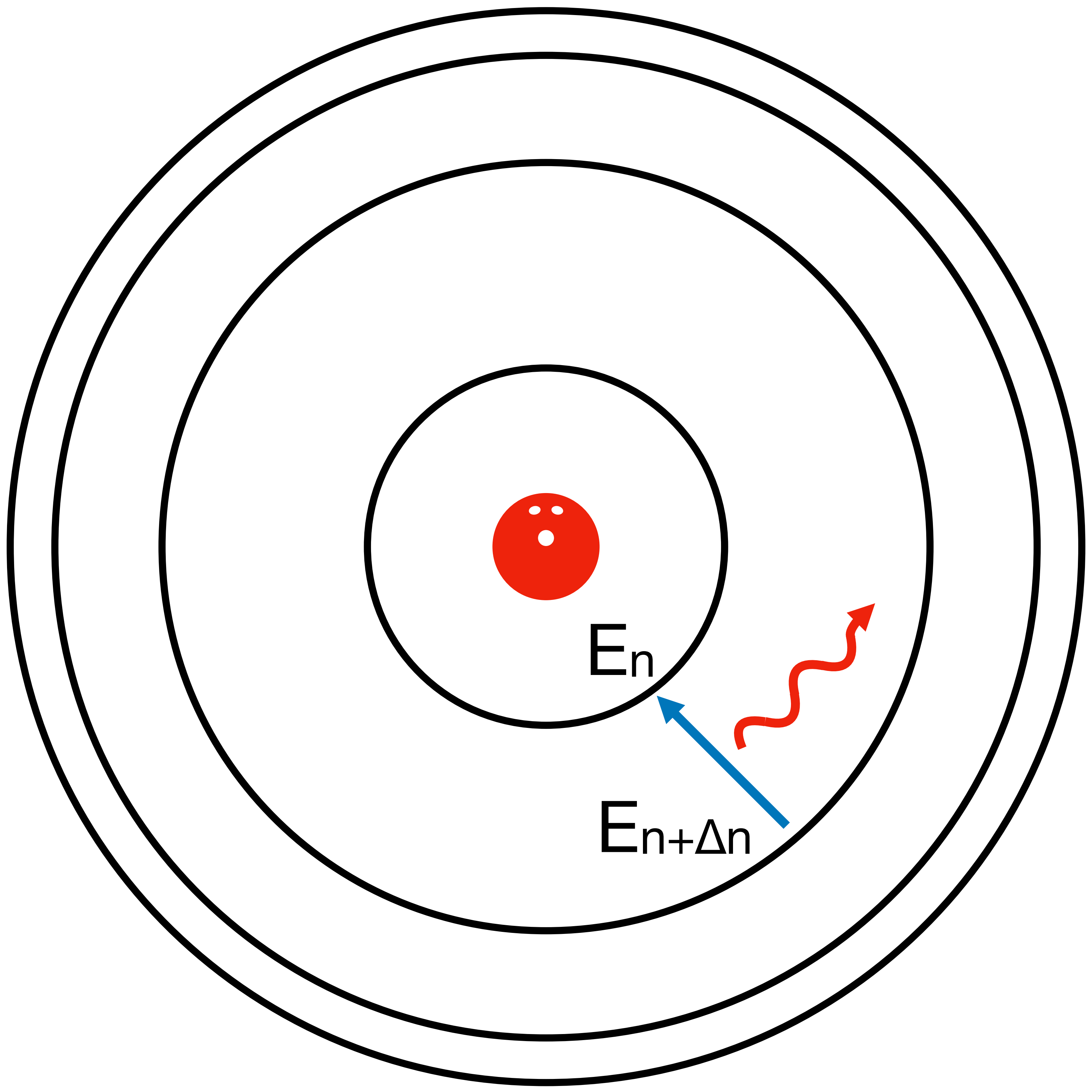

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n — quantum number

Δn — change in the quantum number n

For H atom — $M(H) \approx 1836.1 m_e$

$$\nu = 3.288\times 10^{15} \left(\frac{1}{n^2} - \frac{1}{(n+\Delta n)^2}\right) Hz$$

Recombination lines

Recombination line frequency

For H atom $$\nu = \frac{\Delta E}{h} = 3.288{\times}10^{15}\left(\frac{1}{n^2} - \frac{1}{(n+\Delta n)^2}\right)\ Hz$$

In order to have spectral lines in radio waves (say $\nu < 1000\ GHz$):

$$\left(\frac{1}{n^2} - \frac{1}{(n+\Delta n)^2}\right) < 3{\times}10^{-4}$$

This satisfies only when n > 19

Recombination lines

Recombination line frequency

$$\nu = 3.288 \times 10^{15} \left(\frac{1}{n^2} - \frac{1}{(n + \Delta n)^2} \right) \; Hz$$

What is the frequency of the 110 → 109 transition of H?

This is known as $H109\alpha$

$$\nu = 3.288 \times 10^{15} \left(\frac{1}{109^2} - \frac{1}{110^2} \right) \approx 5.0089 \times 10^{9} \; Hz \approx 5.01 \; GHz$$

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Recombination lines of heavier atoms have a slightly higher frequencies.

$$\nu = R_M c \left[\frac{1}{n^2} - \frac{1}{(n + \Delta n)^2} \right]; R_M \equiv R_\infty \left(1 + \frac{m_e}{M} \right)^{-1}$$

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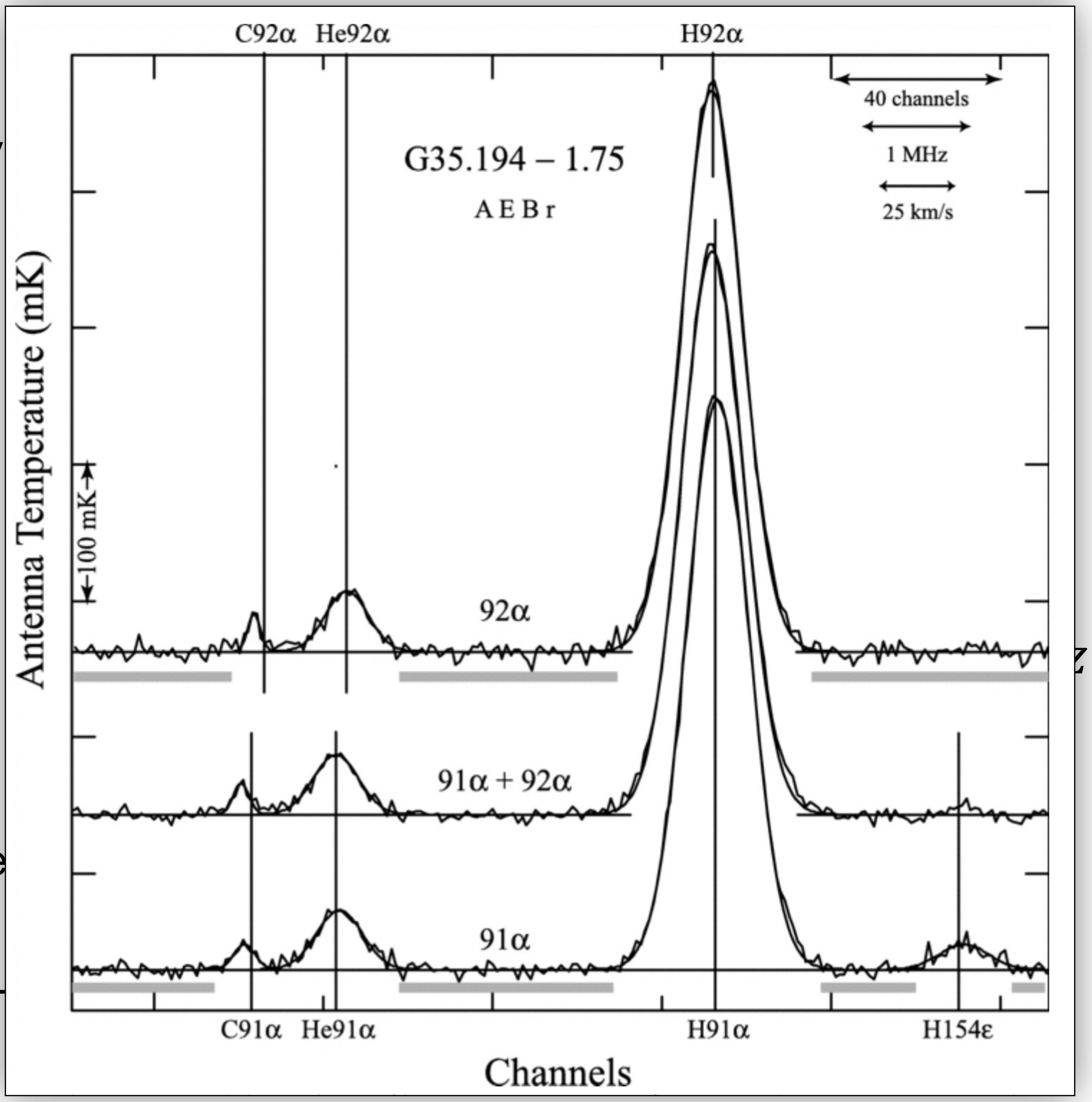

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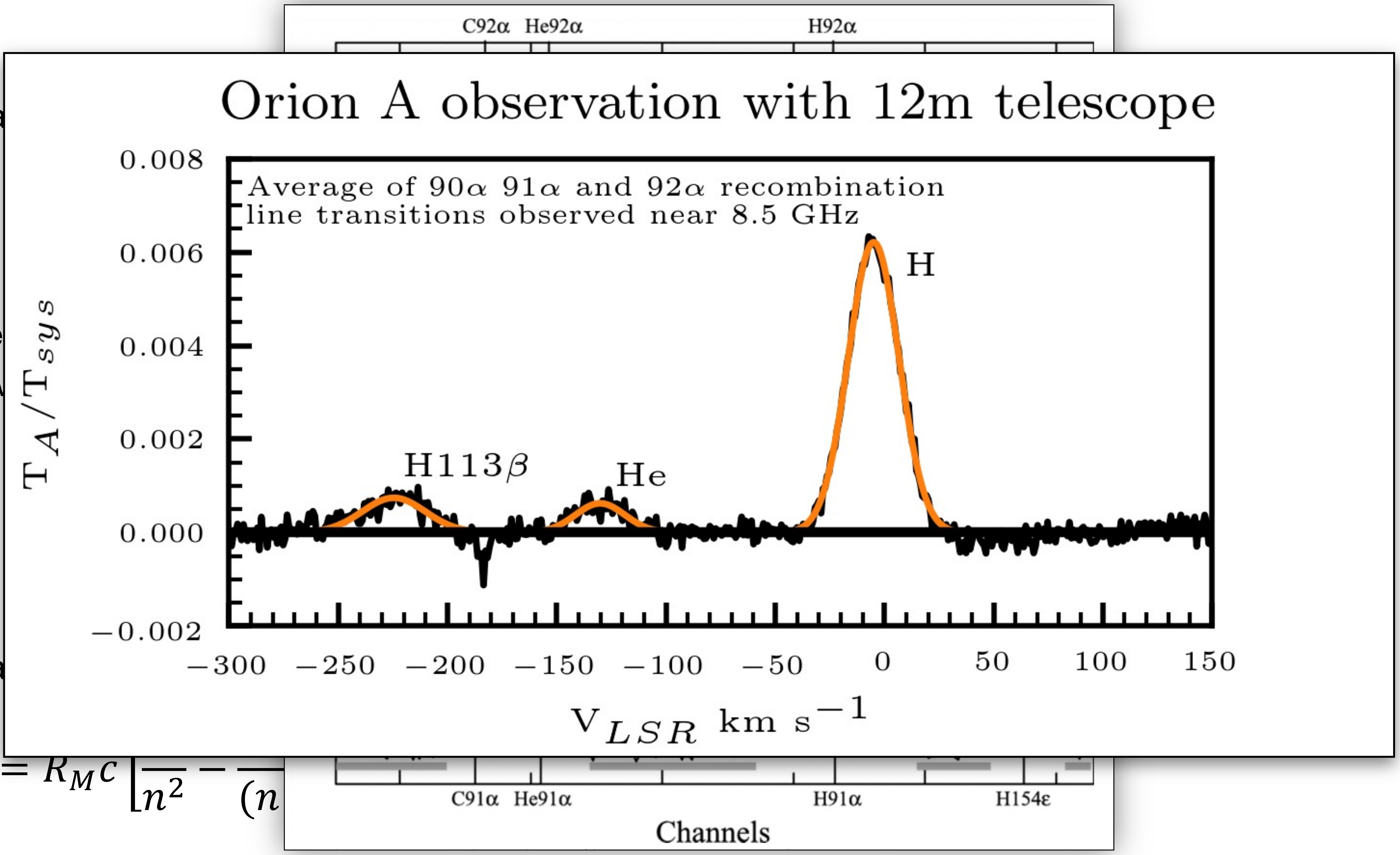
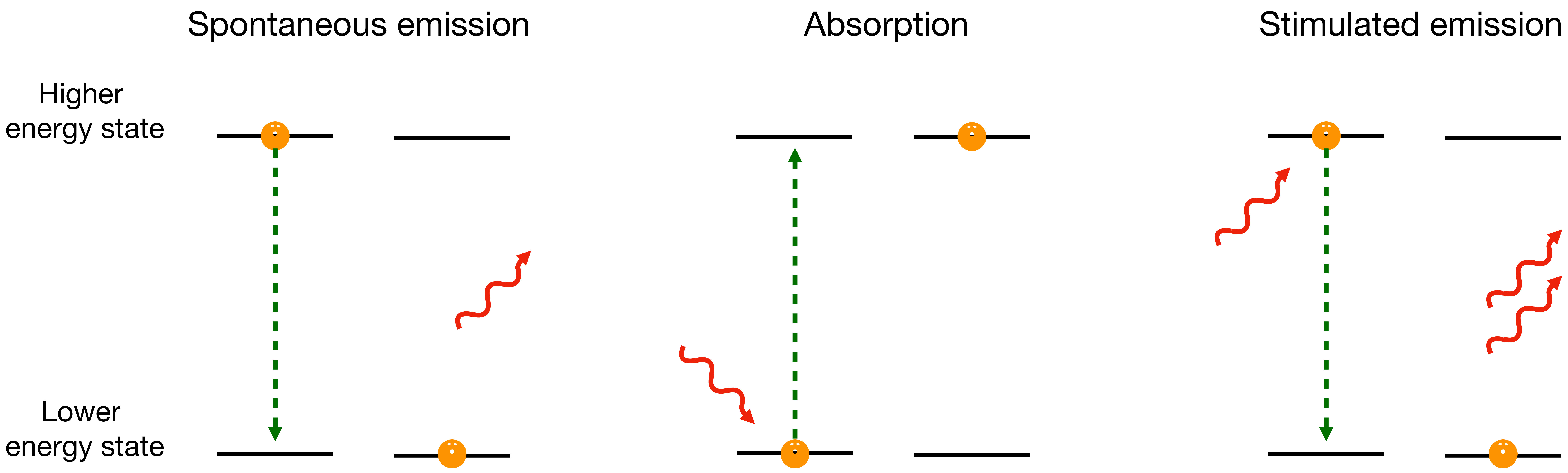


Radiative transfer

- Three types of radiative transfer

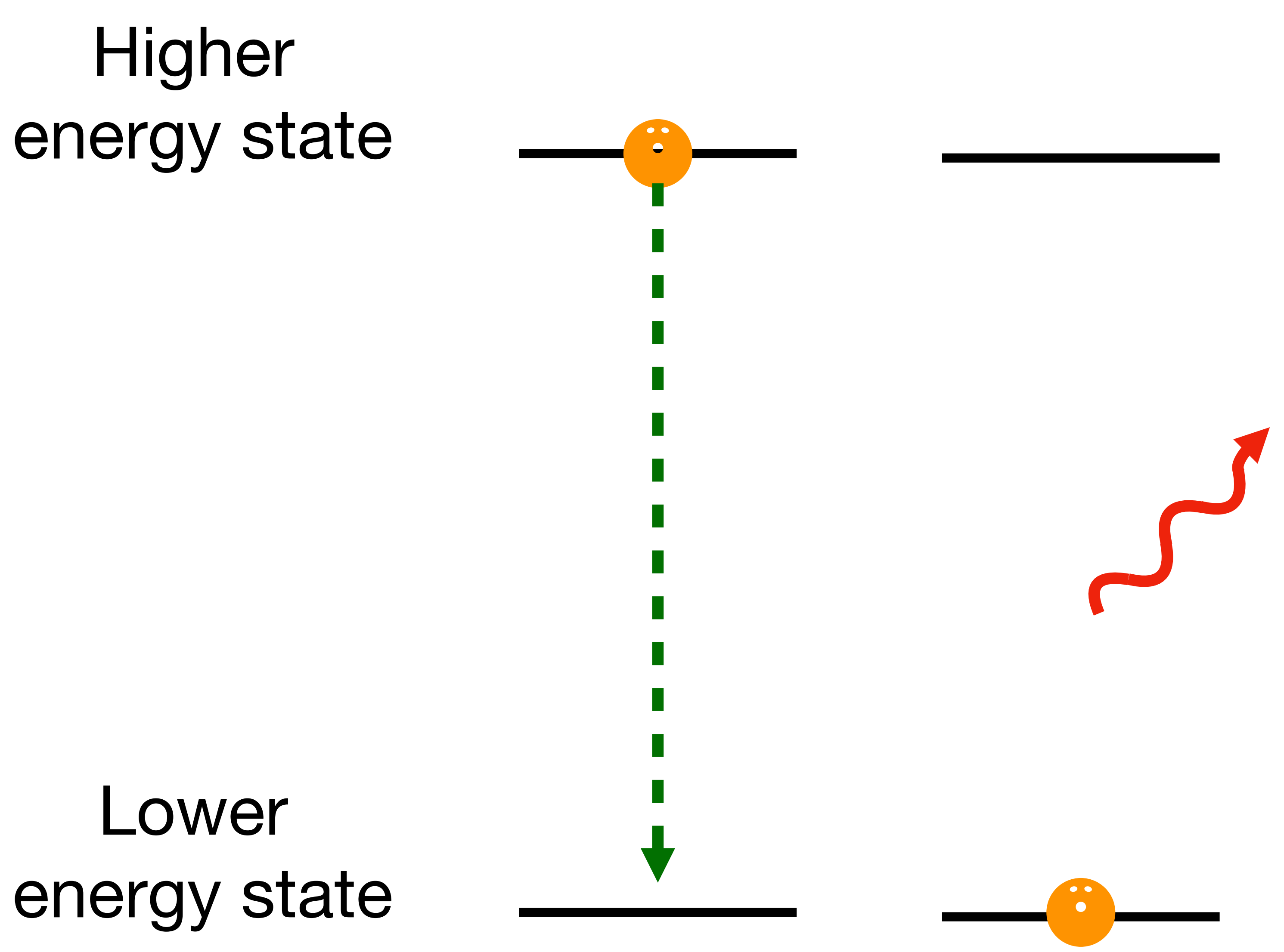


Radiative transfer

Spontaneous emission rate for the transition from n to (n-1) is equal the average power emitted by an atom divided by the energy of a photon.

$$A_{n,n-1} = \frac{<P>}{h\nu}$$

Spontaneous emission



From Larmor's formula $<P> = \frac{2e^2}{3c^3}(2\pi\nu)^4 \frac{a_n^2}{2}$; $a_n \approx \frac{n^2h^2}{4\pi^2 m_e e^2}$

$$A_{n,n-1} \approx \left(\frac{64\pi^6 m_e e^{10}}{3c^3h^6}\right)\frac{1}{n^5}$$

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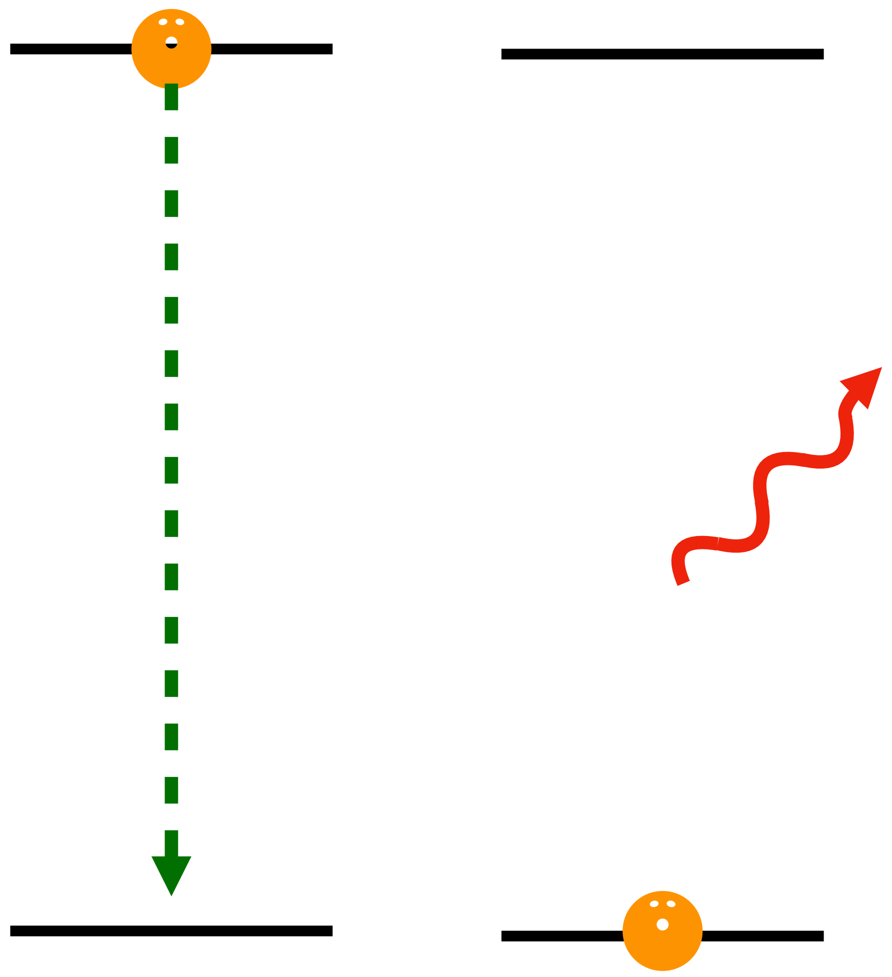

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$$A_{n,n-1} \approx \left(\frac{64\pi^6 m_e e^{10}}{3c^3h^6}\right)\frac{1}{n^5}$$

For a hydrogen atom $- A_{n,n-1} \approx 5.3\times10^9\left(\frac{1}{n^5}\right)\, s^{-1}$

For previous example; transition of H from energy level 110 to 109 $\rightarrow A_{110,109} \approx 0.3\, s^{-1}$

i.e. emit one photon every 3 seconds

Radiative transfer

Absorption and stimulated emission

- Absorption coefficient κ_{abs} is the fraction of spectral brightness removed per unit length by absorption from the lower level to the upper level.

- Stimulated emission is treated as negative absorption κ_{stim}.

- The net absorption coefficient is the sum of the above two.

- The stimulated emission nearly cancels pure absorption and significantly reduces the net line opacity at radio frequencies.

- The brightness of an optically thin ($\tau \ll 1$) radio emission line is proportional to the column density of emitting gas, but can be nearly independent of the gas temperature.

Molecular lines

- Symmetric molecular have no permanent electric dipole moment (e.g. H_2).
- Most asymmetric molecules have asymmetric charge distribution, leading to a permanent dipole moment, and they are polar (e.g. CO).
- Polar molecules radiate energy at their rotation frequency.
- Rotation energy of a polar molecule is quantized:

$$E_{rot} = \frac{\hbar^2}{2I} J(J+1); J = 0,1,2,.....$$

- Energy change is restricted by quantum mechanical selection rule: $\Delta J = \pm 1$
- Energy released during the rotational transition from J to $(J-1)$:

$$\Delta E = [J(J+1) - (J-1)J] \frac{\hbar}{2I} = \frac{\hbar^2 J}{I}; J = 1,2,....$$

Molecular lines

- Frequency of the line due to the rotational transition from J to $(J-1)$:

$$\Delta E = [J(J+1) - (J-1)J]\frac{\hbar}{2I} = \frac{\hbar^2 J}{I} = h\nu; \;\; J = 1,2,\ldots.$$

$$\nu = \frac{hJ}{4\pi^2 m r_e^2}$$

Upper energy level $J = 1,2,\ldots.$; reduced mass $m \equiv \left(\frac{m_A m_B}{m_A + m_B}\right)$; equilibrium distance $r_e = \frac{1}{2\pi}\left(\frac{hJ}{m\nu}\right)^{1/2}$

- Most abundant element in molecular clouds is H_2.
- H_2 is extremely difficult to observe:
  - Symmetric molecule, no dipole moment —> inefficient radiator.
  - Small moment of inertial, widely-spaced energy levels, so lowest energy transition is equivalent to T ~ 500 K.
  - However, MC are cold T ~ 20 K, so transition happens rarely.

Molecular lines

- CO is the second most abundant molecule in molecular clouds, which is a proxy for H_2.
- What is the frequency of $J = 1 \rightarrow 0$ line of the $^{12}C^{16}O$ molecule?

$$\nu = \frac{hJ}{4\pi^2 m r_e^2}; \quad J = 1,2,3, \ldots\ldots$$

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$$m \equiv \left(\frac{m_C m_O}{m_C + m_O}\right) \approx m_H \left(\frac{12 \cdot 16}{12 + 16}\right) \approx 1.15\times10^{-23}\ g$$

$$r_e = 1.12675\times10^{-8}\ cm$$

Frequency of the fundamental transition $\nu_{J=1-0} = 115.2712\ GHz$

Frequency of the second transition $\nu_{J=2-1} = 2 \cdot \nu_{J=1-0} = 230.5424\ GHz$

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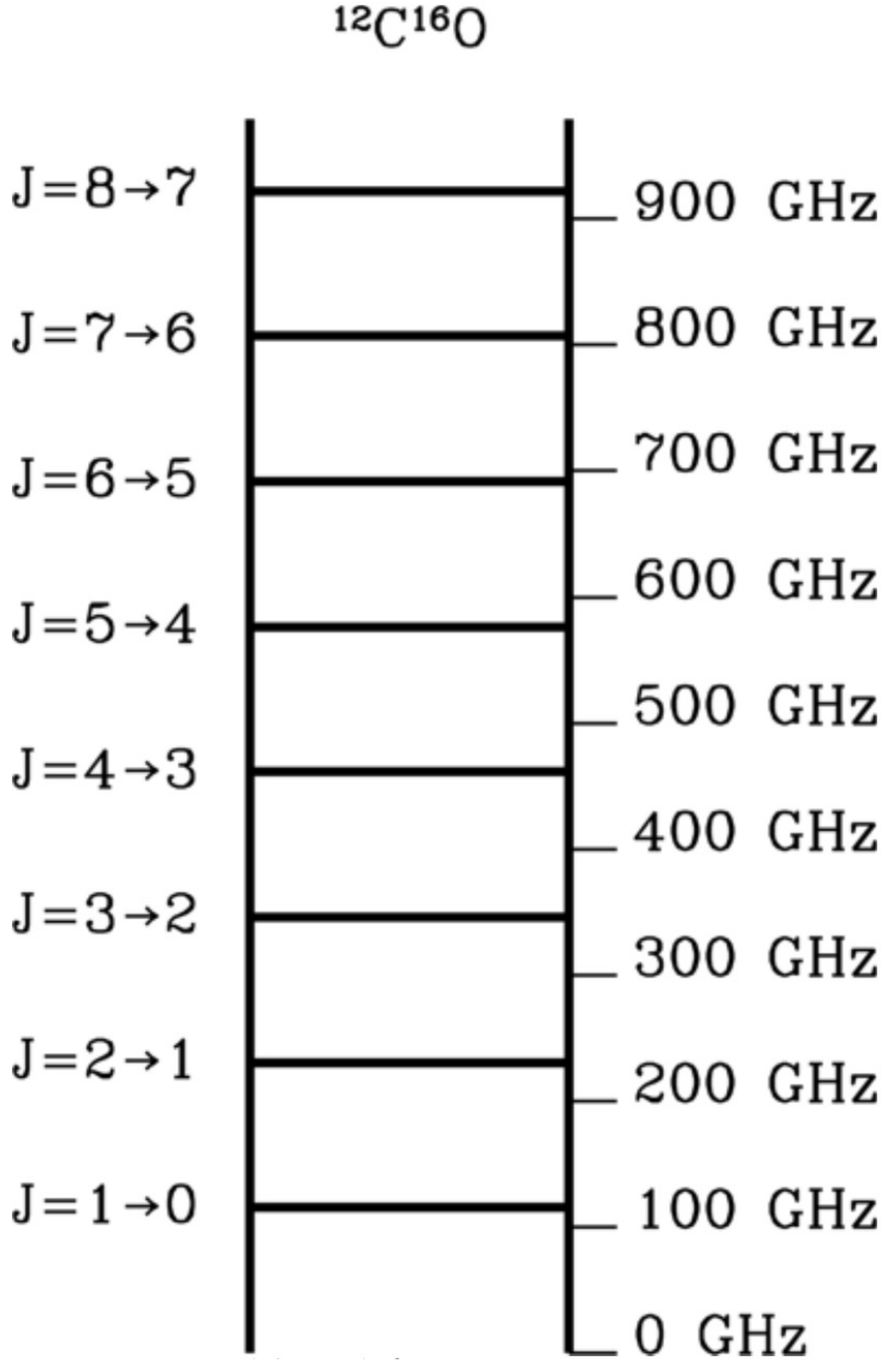
- Frequencies are harmonics of $J = 1 \rightarrow 0$ line.



CO rotational ladder (Condon & Ransom)

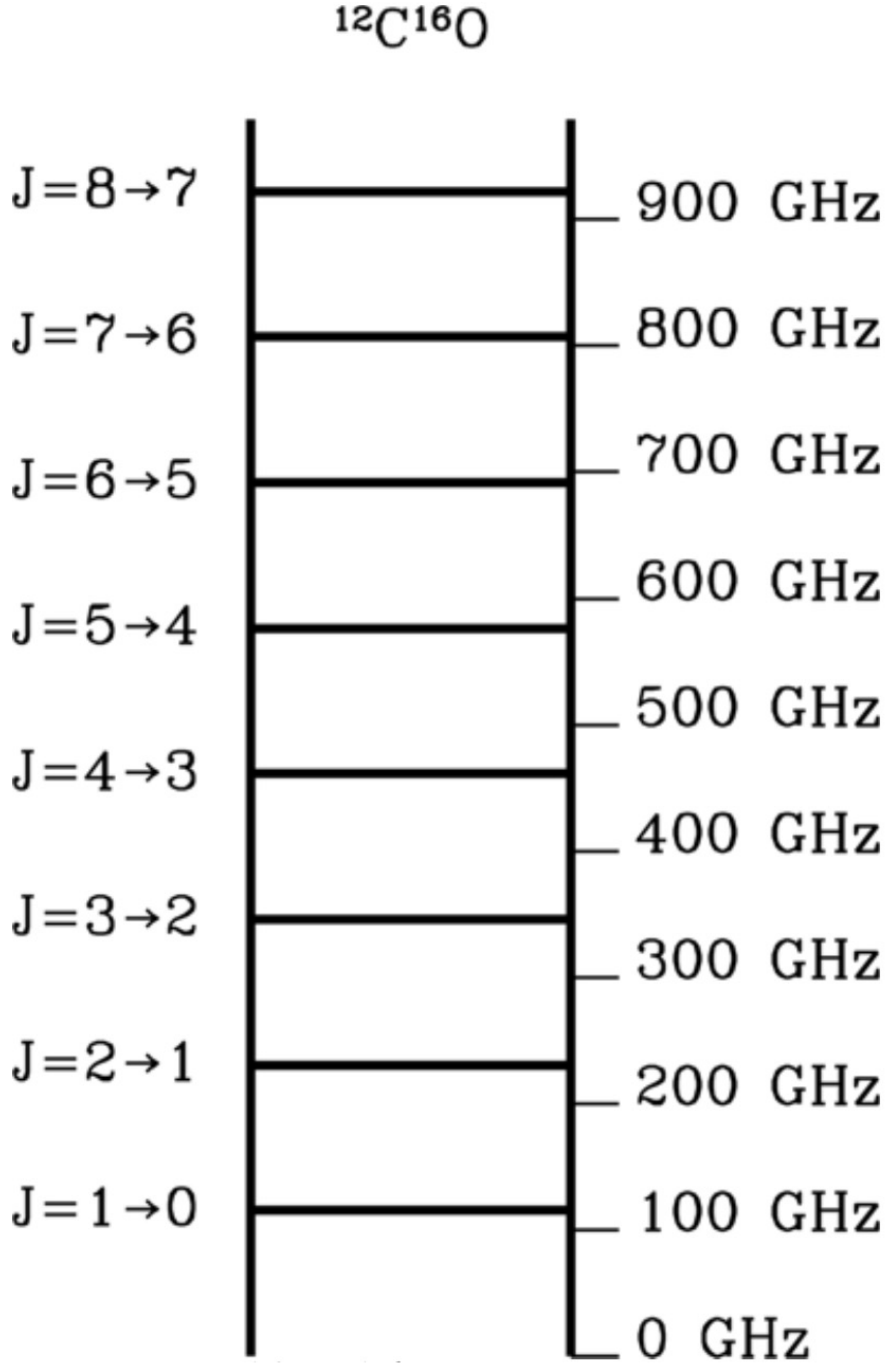
Molecular lines

$$\nu = \frac{hJ}{4\pi^2 m r_e^2}; \quad J = 1,2,3\ldots..$$

- Frequency of the second transition $\nu_{J=2-1} = 2 \cdot \nu_{J=1-0} = 230.5424\,GHz$

- Centrifugal force increases as the molecule spins rapidly (i.e. large J), leading to increase the molecular separation r_e ,make the line frequencies slightly lower than the harmonics.

- The actual $J = 2 \to 1$ line frequency is $\nu_{J=2-1} = 230.538\,GHz$



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Molecular lines

Molecular excitation:

- Molecules excited into higher energy states by ambient radiation and collisions in dense gas.

- Minimum temperature needed for excitation $J \rightarrow J-1$ is

$$T_{min} \approx \frac{E_{rot}}{k_B} = \frac{\nu h}{2k_B} J(J+1)$$

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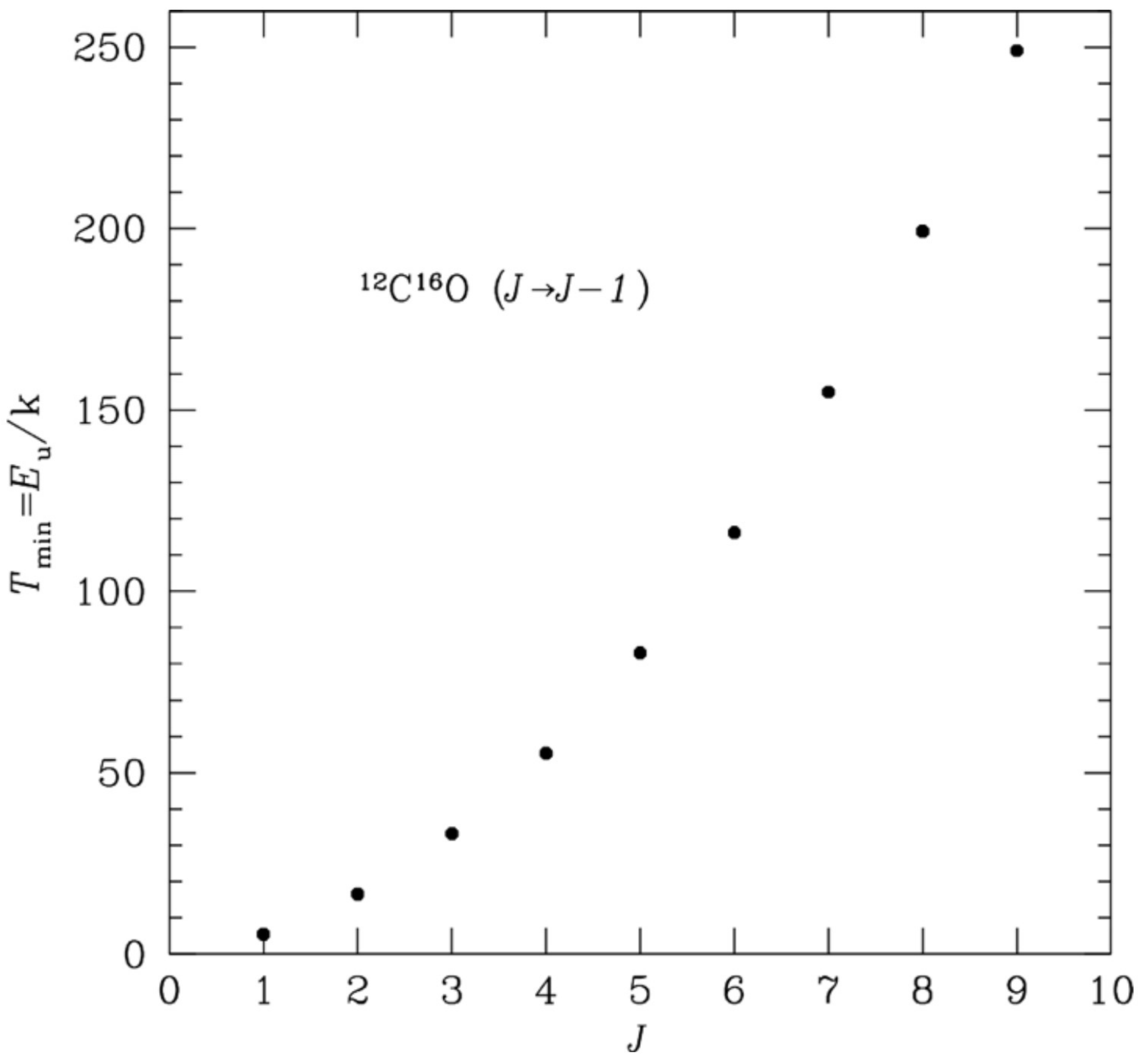
- Minimum temperature to excite the $J = 2-1$ line at 230.5 GHz of $^{12}C^{16}O$ is $T_{min} \approx 16.6\ K$.



T_{min} required for collisions to excite the molecules is proportional to $J(J+1)$.

High-J lines are weak in cold molecular clouds.

Molecular lines

Spontaneous emission coefficient of $J \to J-1$ molecular line:

$$A_{J\to J-1} \approx 1.165\times10^{-11}|\mu|^2 \left(\frac{J}{2J+1}\right)\left(\frac{\nu}{GHz}\right) s^{-1}$$

; where $|\mu|$ is mean electric dipole moment

Spontaneous emission coefficient of CO $J = 1 \to 0$ line at $\nu \approx 115GHz$ is $A_{10} \approx 7.1\times10^{-8}\ s^{-1}$.

Typical time for a CO molecule to emit a photon is $\left(\frac{1}{A_{10}}\right) \approx 10^7\ s$.

Critical density of CO $J = 1 \to 0$ transition is $n^* \approx \frac{A_{10}}{\sigma v} \approx 1.4\times10^3\ cm^{-3}$.

Many Galactic molecular clouds have higher densities than this, so Galactic CO $J = 1 \to 0$ emission is strong and widespread.

Spontaneous emission coefficient c

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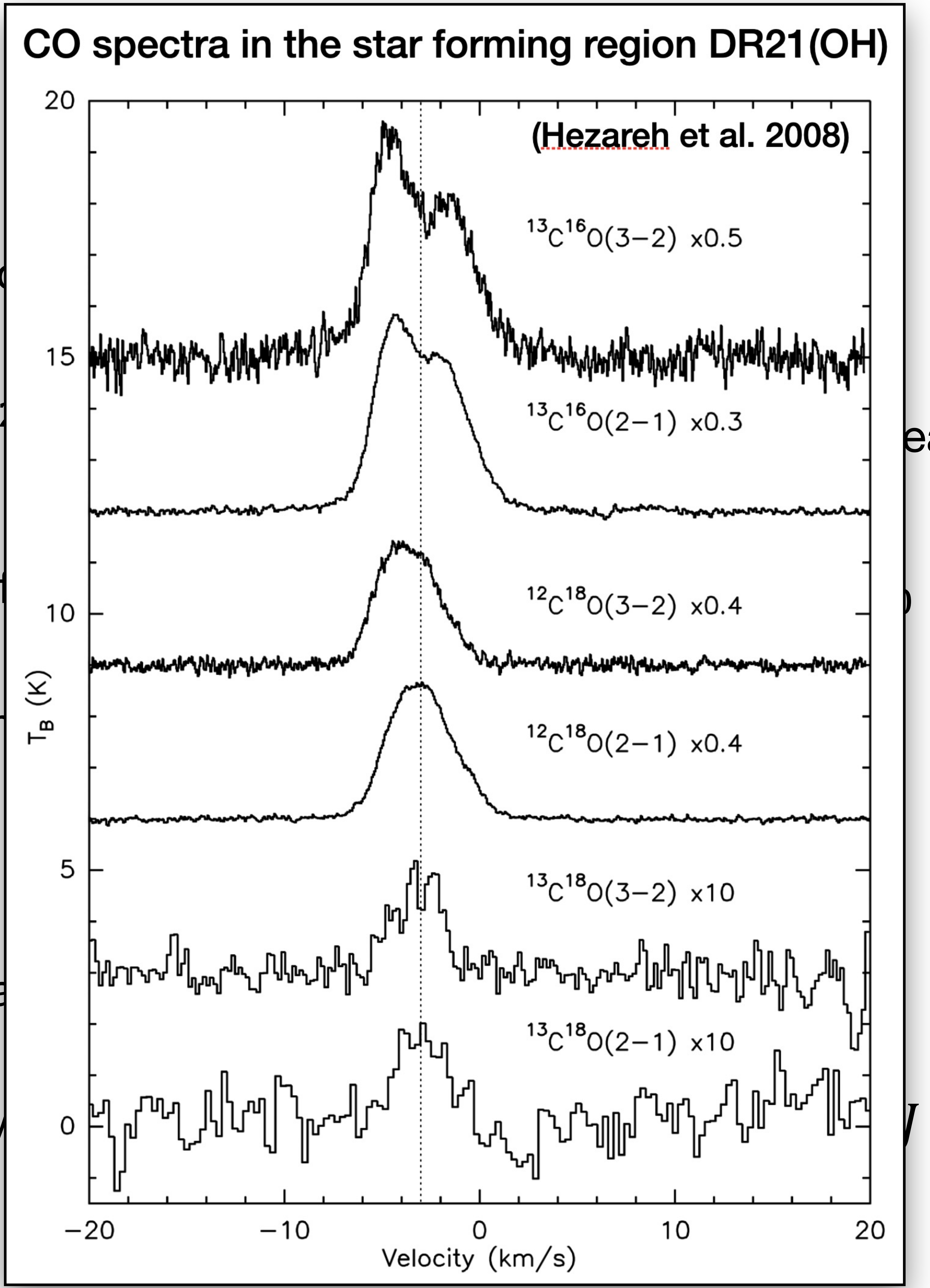
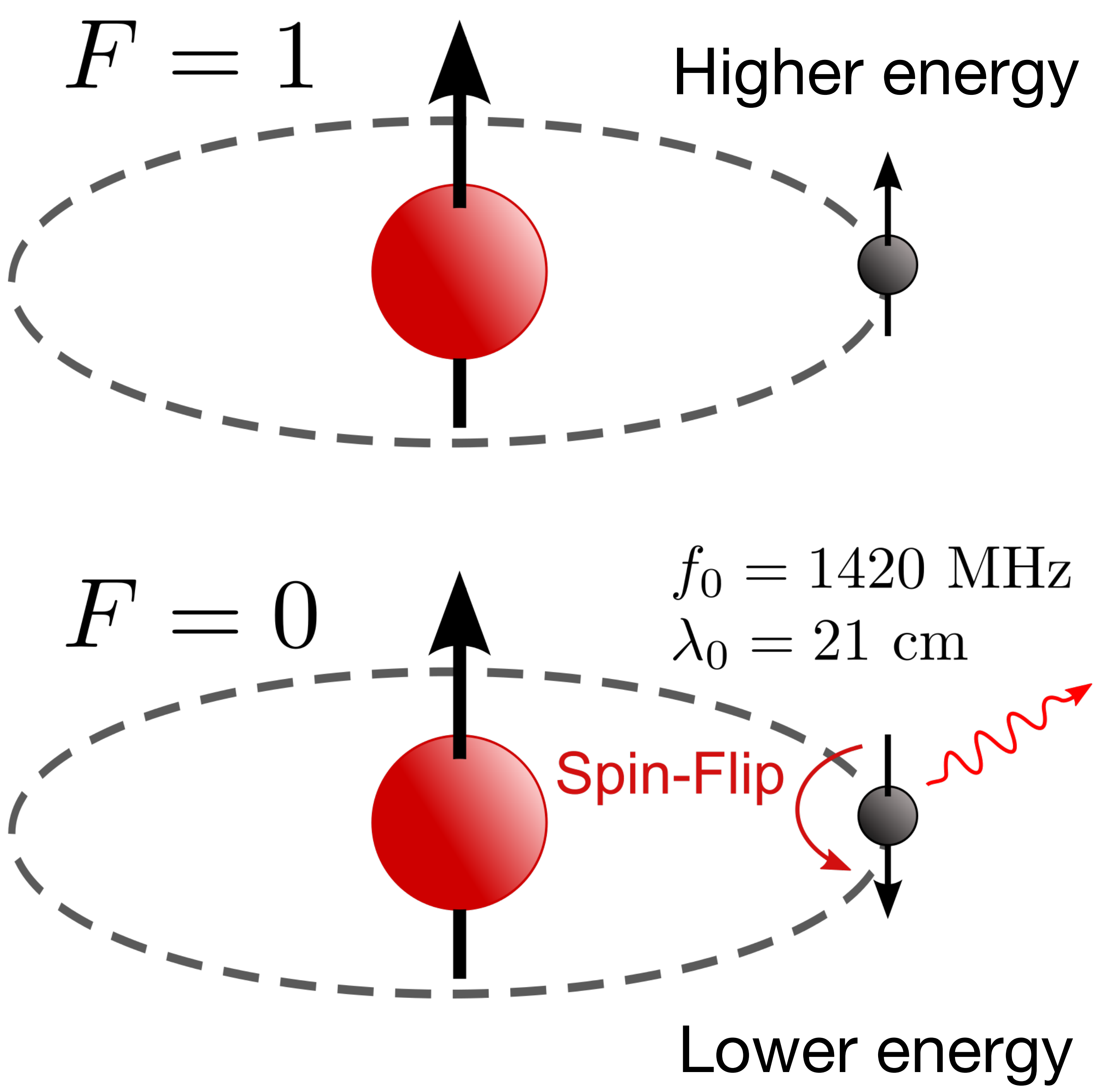


HI 21 cm hyperfine line

- Hydrogen is the most abundant element in the ISM. H_2 molecules are extremely difficult to observe.
- Neutral hydrogen (HI) is abundant and detected through its hyperfine line.
- HI line is intrinsically a quantum phenomenon.
- The spin of proton and electron in a hydrogen atom can be parallel or anti-parallel.
- The parallel state has more energy than the anti-parallel state.
- The HI hyperfine line is created by flipping the spin of the two states.



Credits: Wikimedia Commons

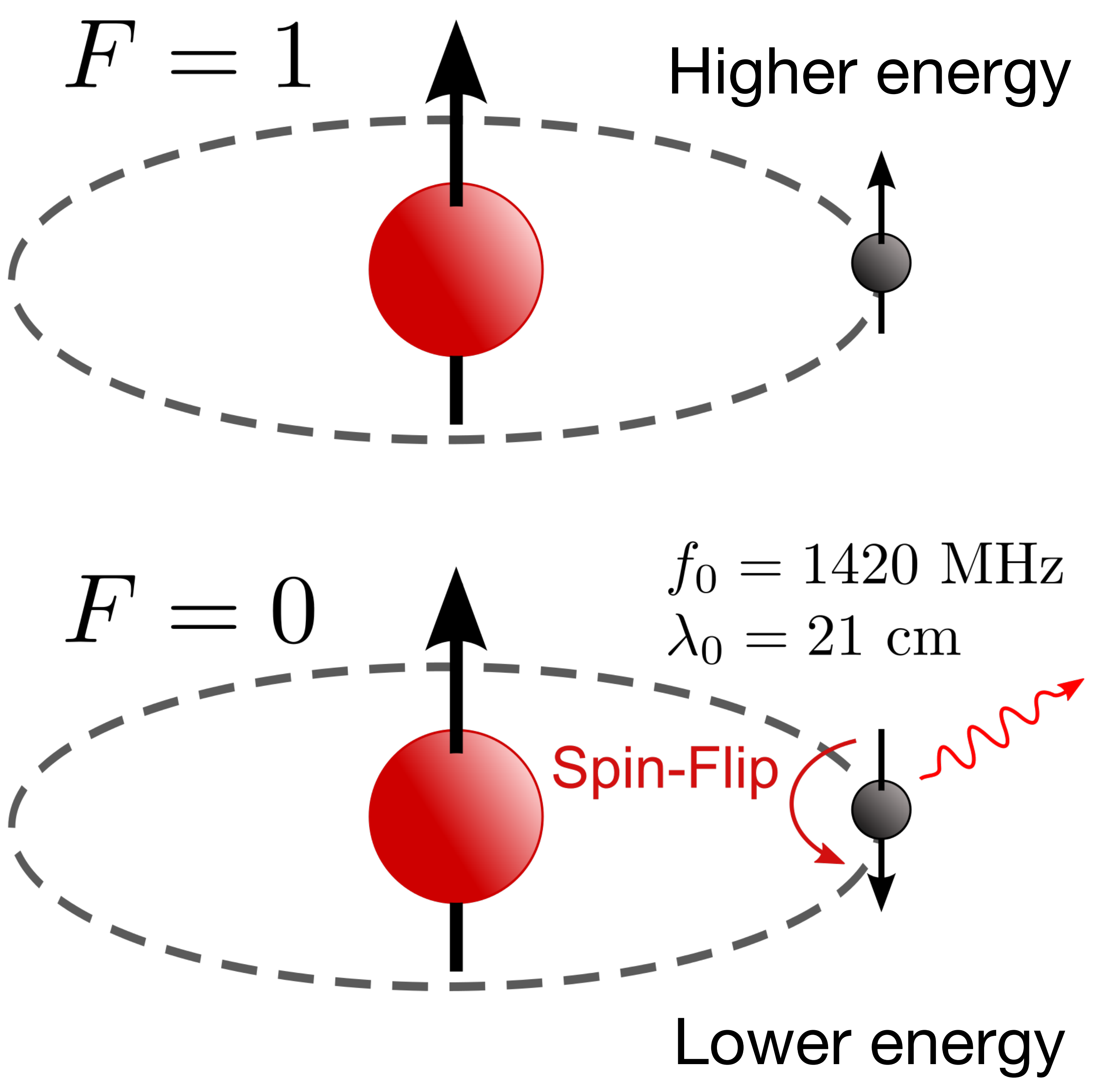
HI 21 cm hyperfine line

HI line center frequency is

$$\nu_{10} = \frac{8}{3} g_I \left(\frac{m_e}{m_p}\right) \alpha^2 (R_M c); \quad R_M \equiv R_\infty \left(1 + \frac{m_e}{m_p}\right)^{-1}$$

Rydberg constant $R_\infty \equiv \left(\frac{2\pi^2 m_e e^4}{h^3 c}\right) = 1.09737312\ldots \times 10^5\ cm^{-1}$

Nuclear g-factor $g_I \approx 5.58569$, fine-structure constant $\alpha \approx 1/137.036$, and $m_p \approx 1836.1 \times m_e$



$$\boxed{\nu_{10} \approx 1420.405751 MHz} \rightarrow \lambda = \frac{c}{\nu} \approx 21 cm$$

Credits: Wikimedia Commons

HI 21 cm hyperfine line

- Emission coefficient of the HI 21 cm line is $A_{10} \approx \frac{64\pi^4}{3hc^3}\nu_{10}^3|\mu_B|^2$

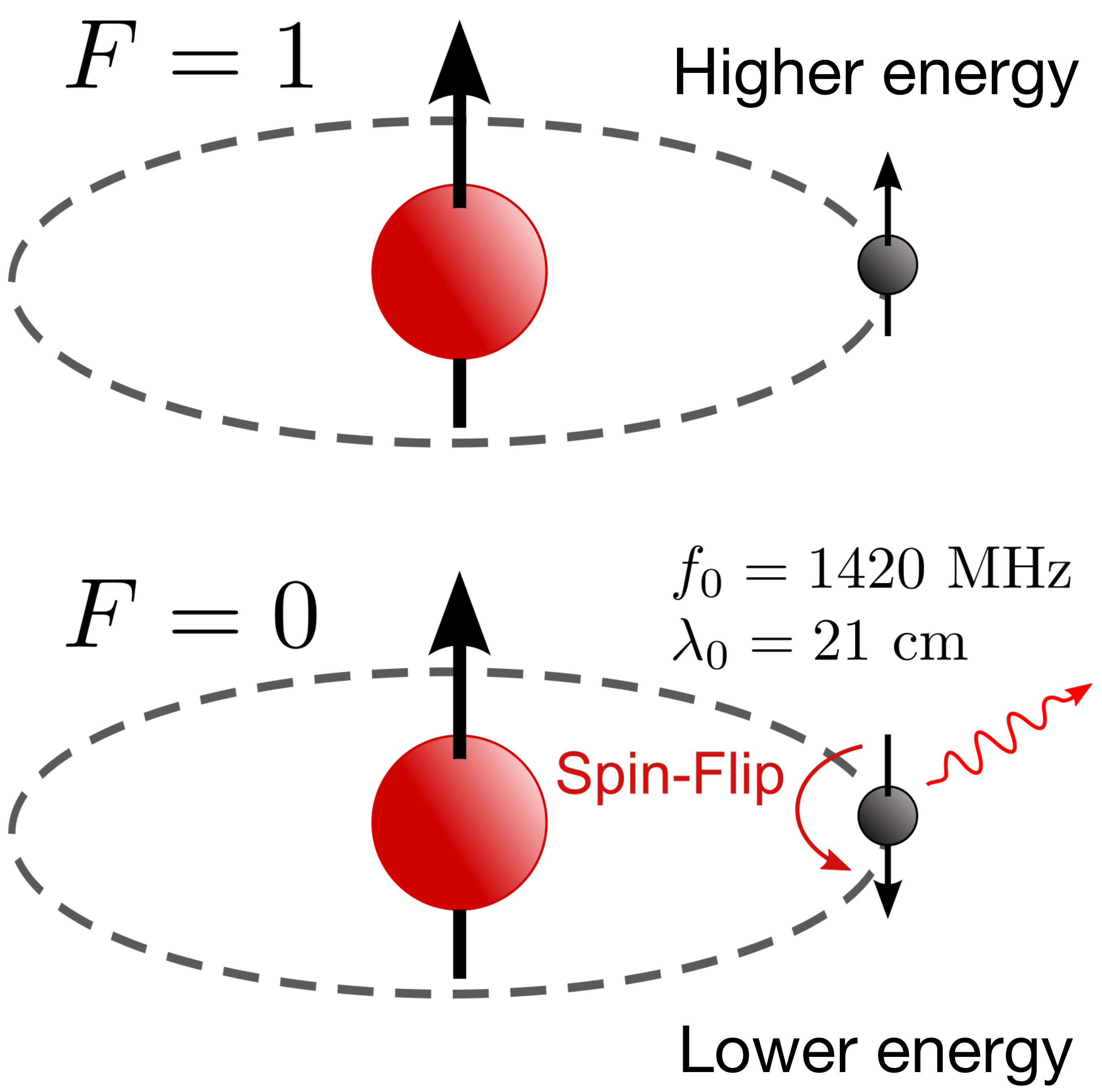
Bohr magneton is $|\mu_B| = \frac{e\hbar}{2m_e c} \approx 9.27401\times10^{-21}\ erg/G$

- Emission coefficient of HI line is small $A_{10} \approx 2.85\times10^{-15}\ s^{-1}$

- Radiative half-life of this transition is very long:

$$\tau_{1/2} = A_{10}^{-1} \approx 3.5\times10^{14}\ s \approx 11\ million\ years$$

- H atom takes 11 Myr to flip the spin states and emit a 1420 MHz photon.



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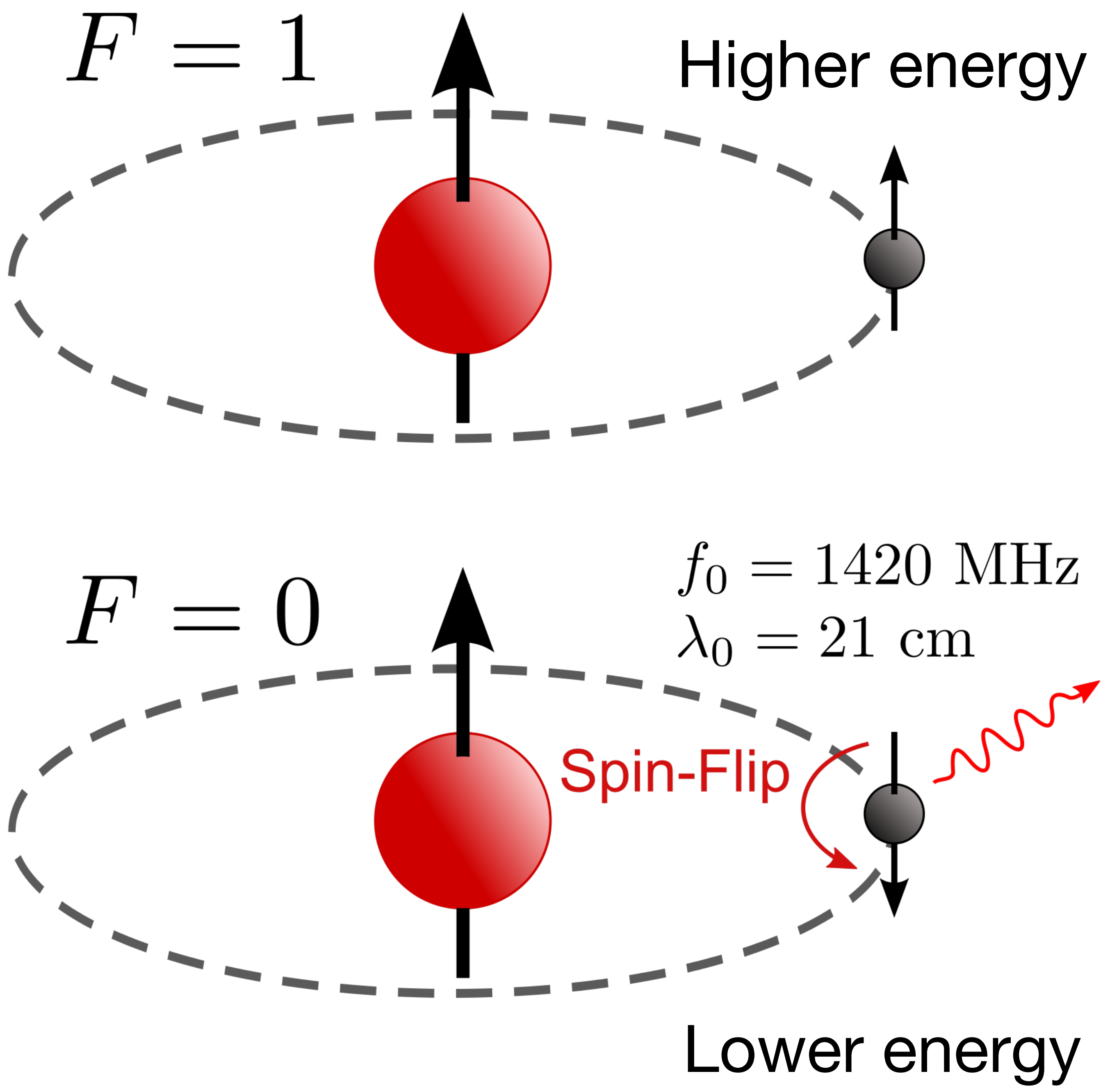

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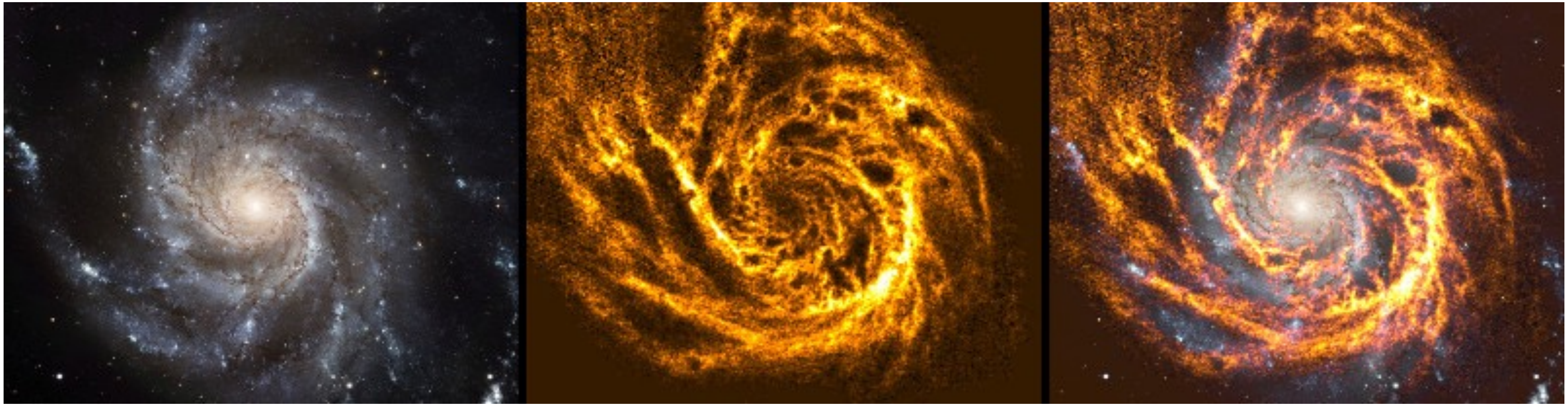
- However, there are ~10^{67} H atoms in our Galaxy → 10^{52} atoms emit 1420 MHz photons per second!

HI 21 cm hyperfine line

Optical image (HST) | **HI image (VLA)** | **Radio + optical**



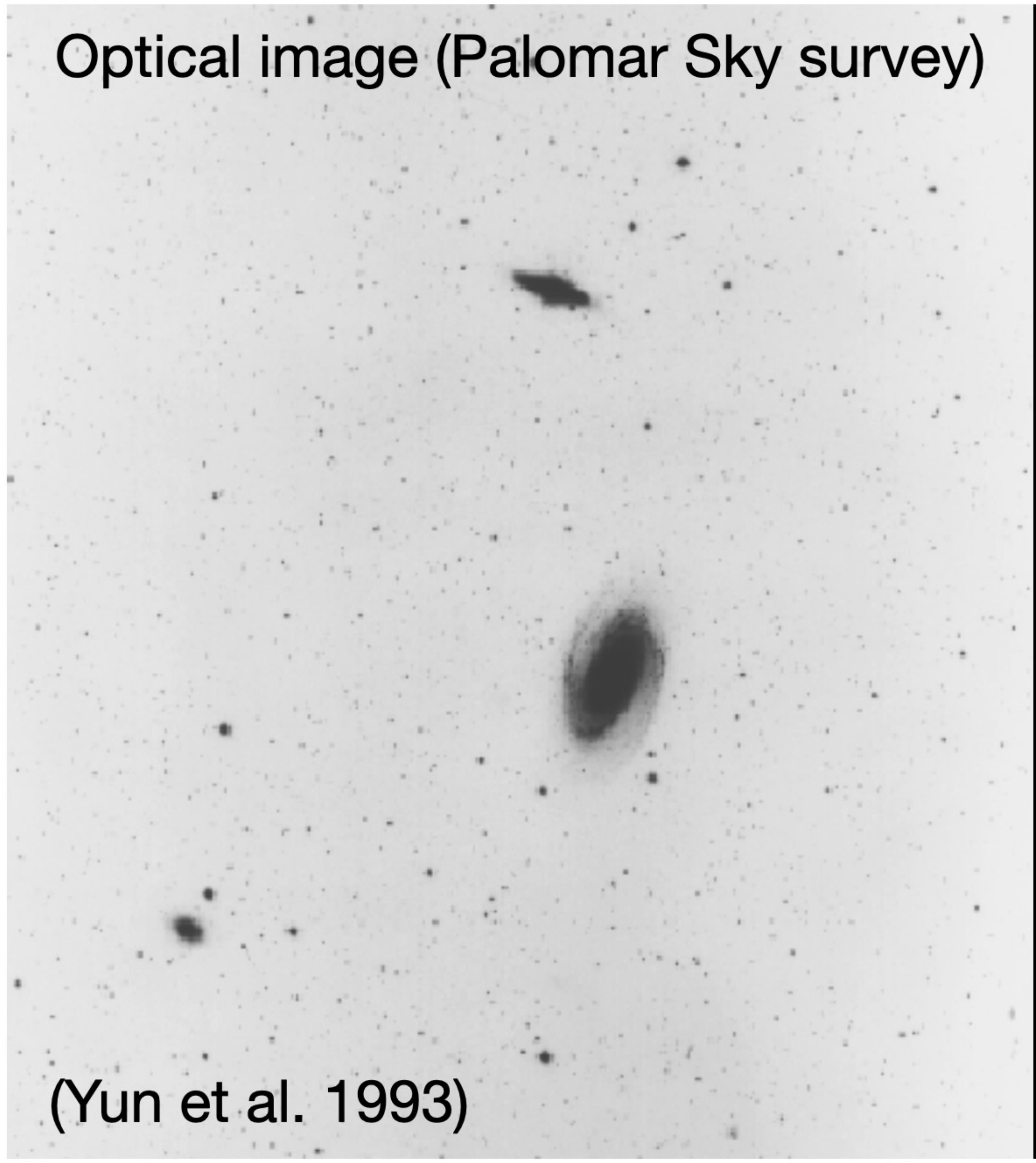
Pinwheel Galaxy (Messier 101)

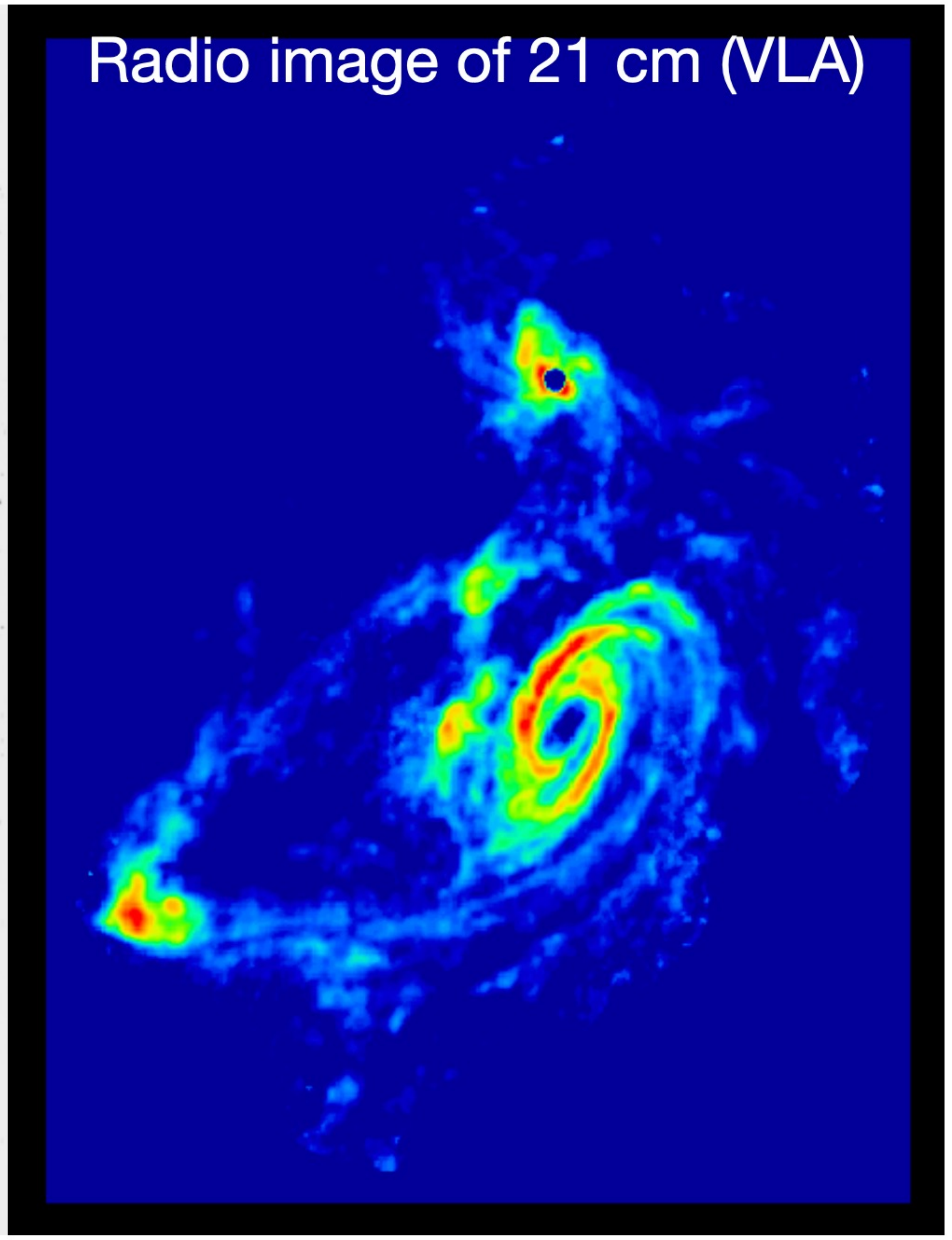
- Radio image shows more detailed structure than the optical image.
- The regions that optically obscured due to dust in the galaxy can be seen at radio.

HI 21 cm hyperfine line

Optical i

M81 group of Galaxies (M81, M82, NGC 3077)





Radio shows tidal bridges between the Galaxies, but not in optical

optical

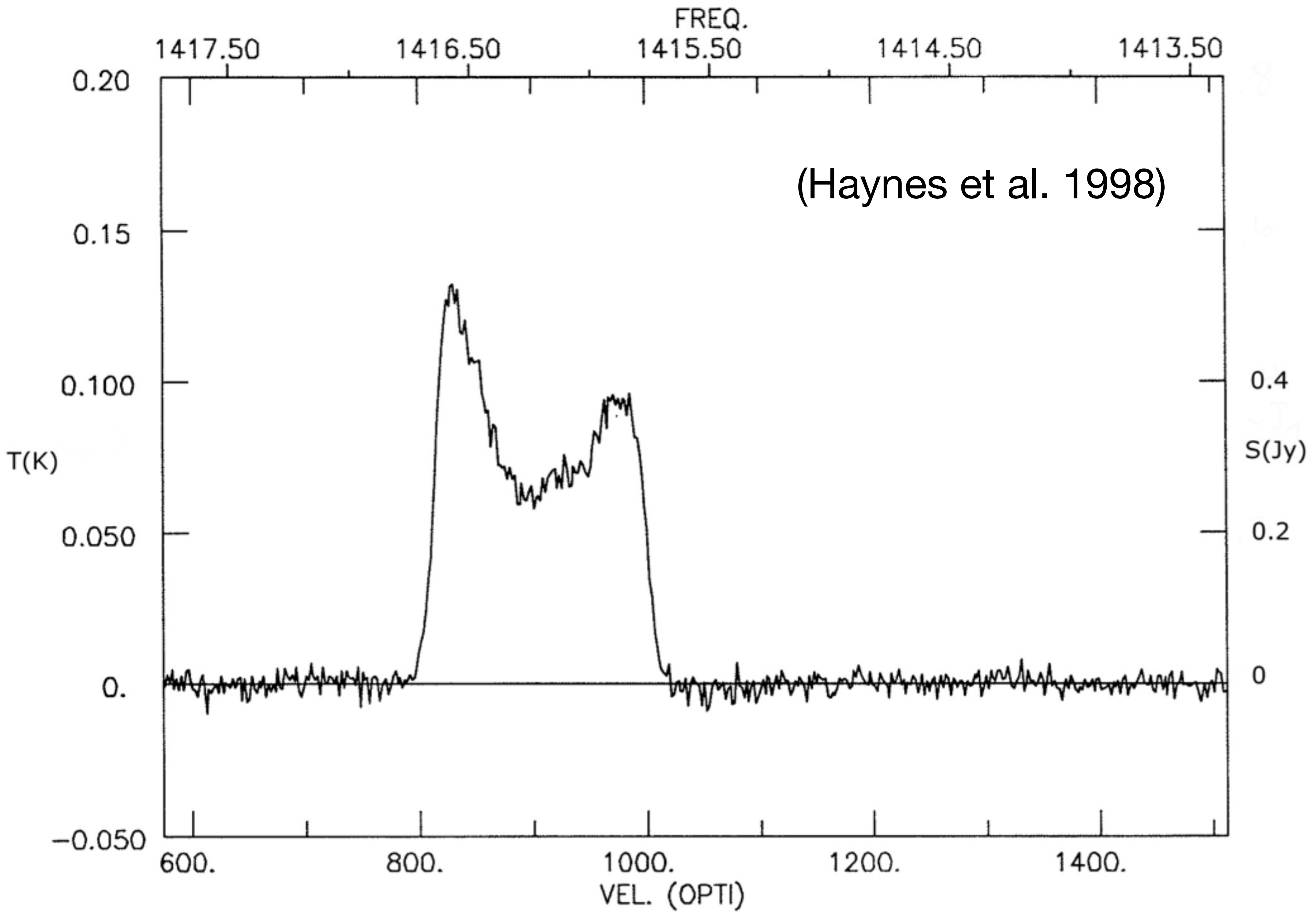
- Radi
- The r dio.

HI 21 cm hyperfine line

<u>Example:</u>

- HI line Frequency is not at 1420 MHz ??
- Line is redshifted!

HI line spectrum of Galaxy UGC 11707



HI 21 cm hyperfine line

Example:

- HI line Frequency is not at 1420 MHz ??
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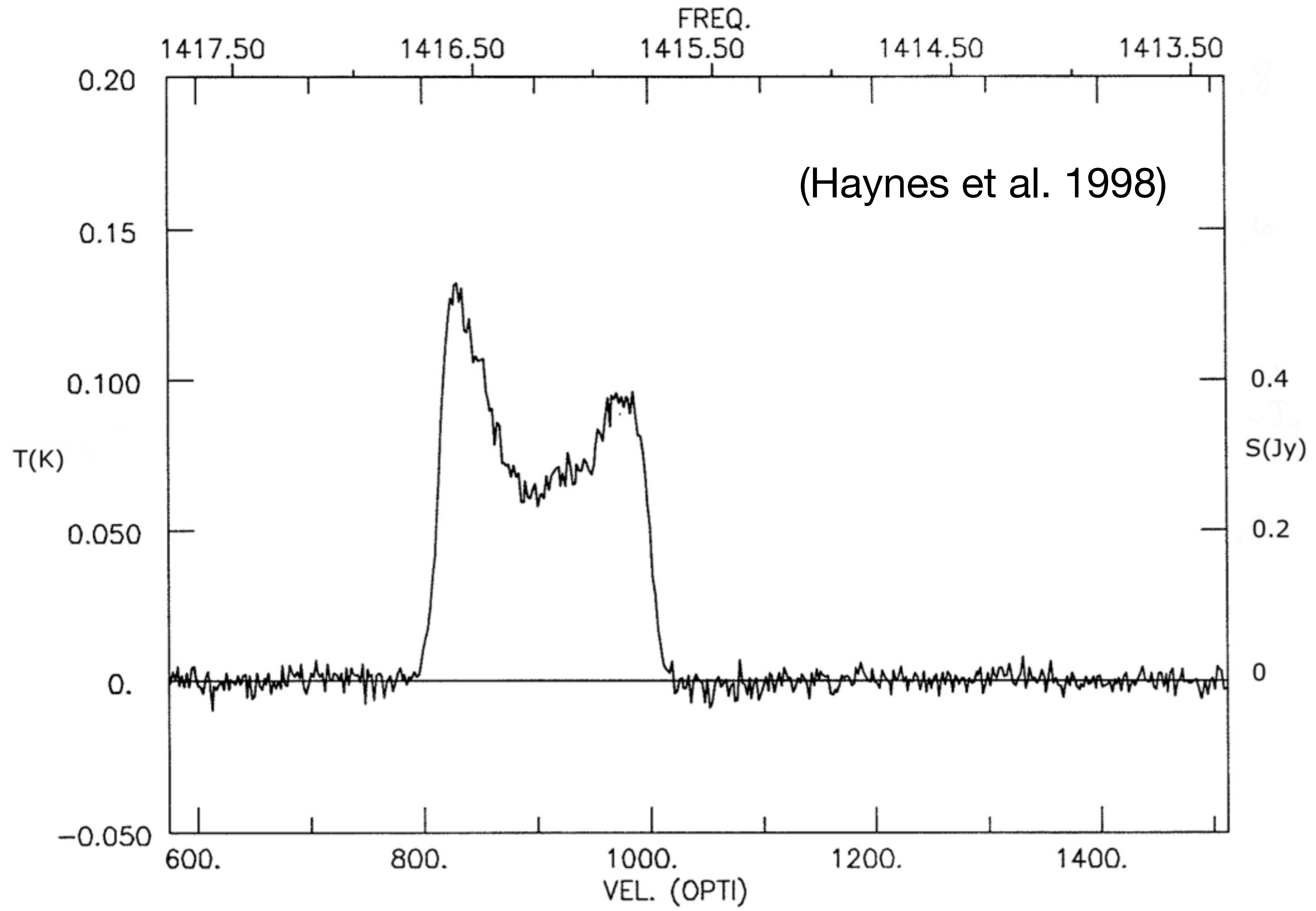
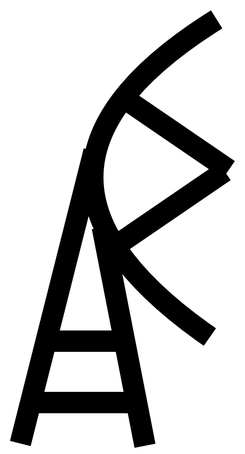
- Galaxy is moving away from us.

$$\frac{v_r}{c} \approx \frac{\nu_e - \nu_o}{\nu_e}; (v_r << c)$$

HI line spectrum of Galaxy UGC 11707





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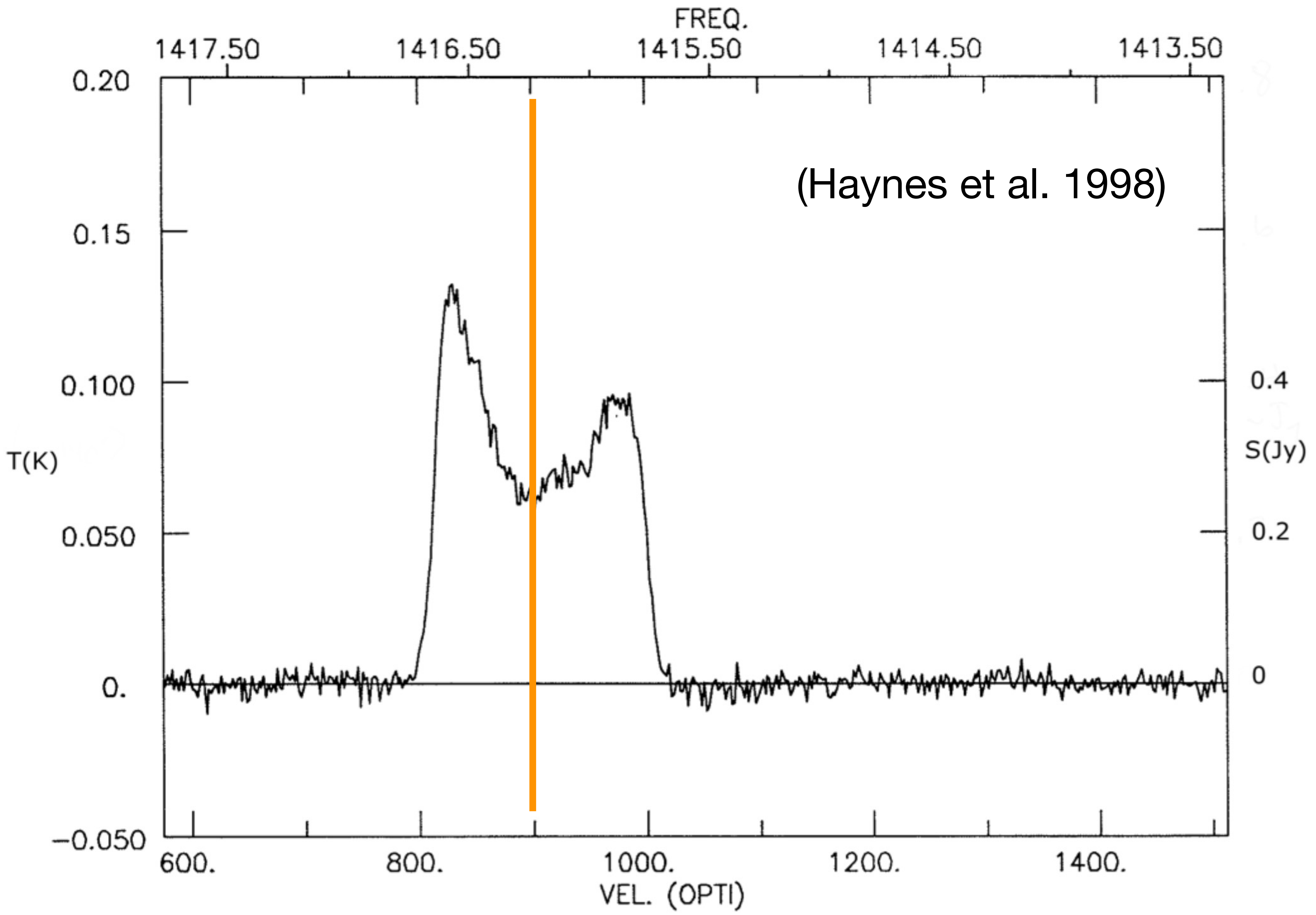


Using the observed frequency 1416.2 MHz → $v_r \approx 890\ km/s$

Using Hubble's law $d \approx \frac{v_r}{H_0} \approx 13\ Mpc$

HI 21 cm hyperfine line

<u>Example:</u>

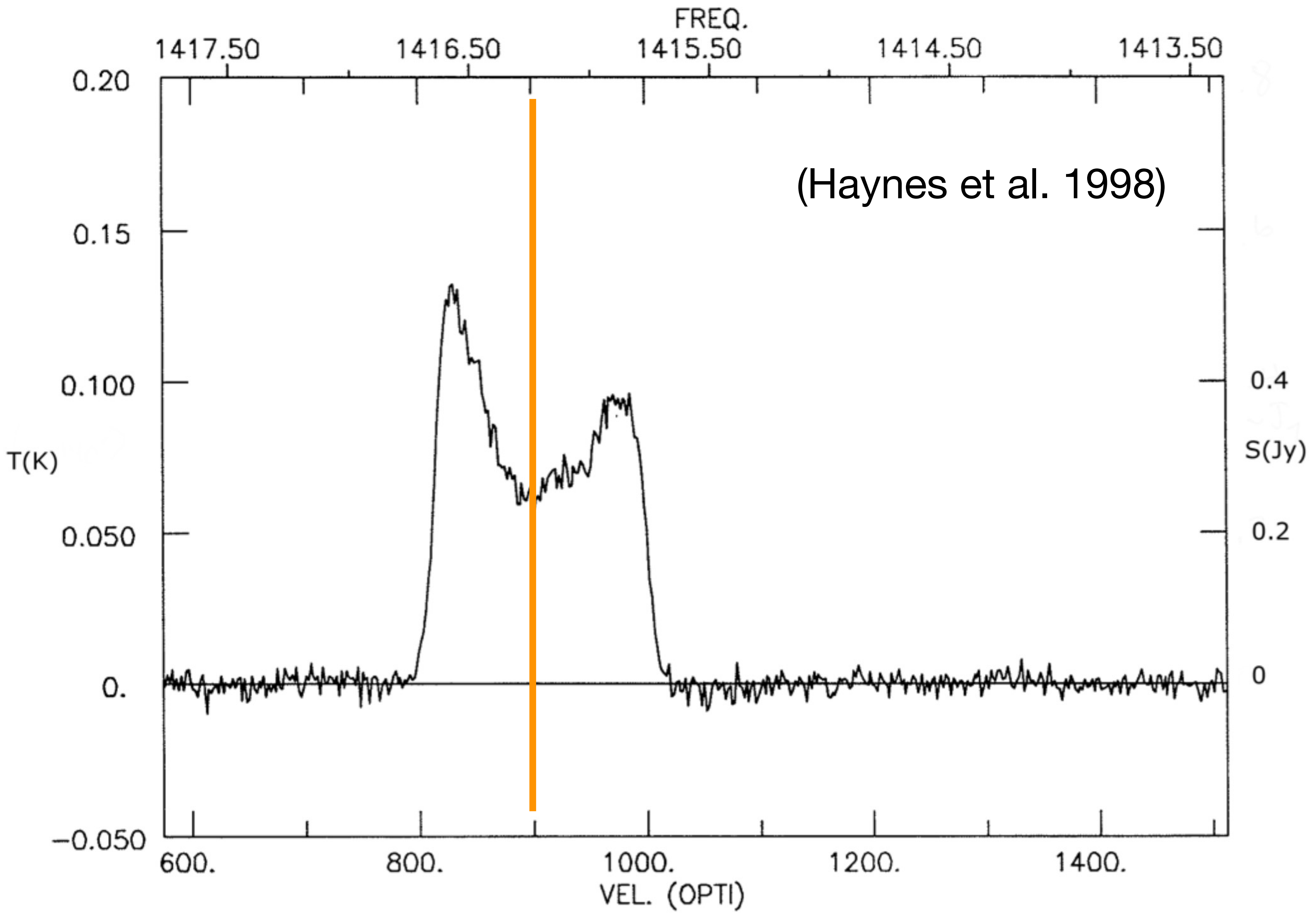
- Galaxy is rotating, so
  - Half of the galaxy is approaching us
  - Half of the galaxy is moving away
- Doppler shift within the galaxy broaden the line, and the profile is two-horned.
- The total HI mass of the galaxy:

$$\left(\frac{M_H}{M_\odot}\right) \approx 2.36\times10^5 \left(\frac{d}{Mpc}\right)^2 \int \left(\frac{S_\nu}{Jy}\right)\left(\frac{dv}{km/s}\right)$$

$$\int s(v)dv \approx 0.35 Jy \cdot 200\frac{km}{s} \approx 70\ Jy\ km/s$$

$$M_H \approx 2.8\times10^9\ M_\odot$$

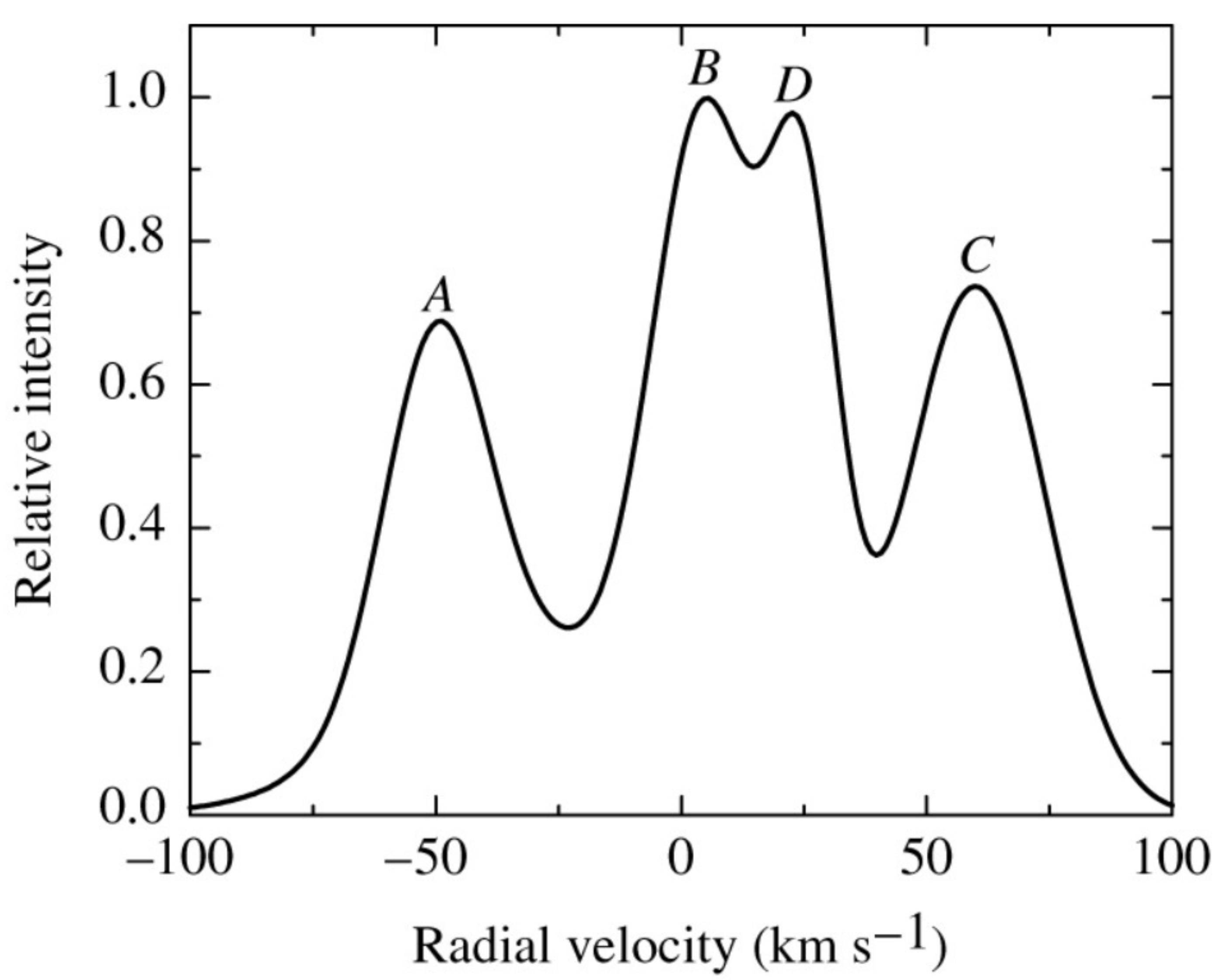
HI line spectrum of Galaxy UGC 11707

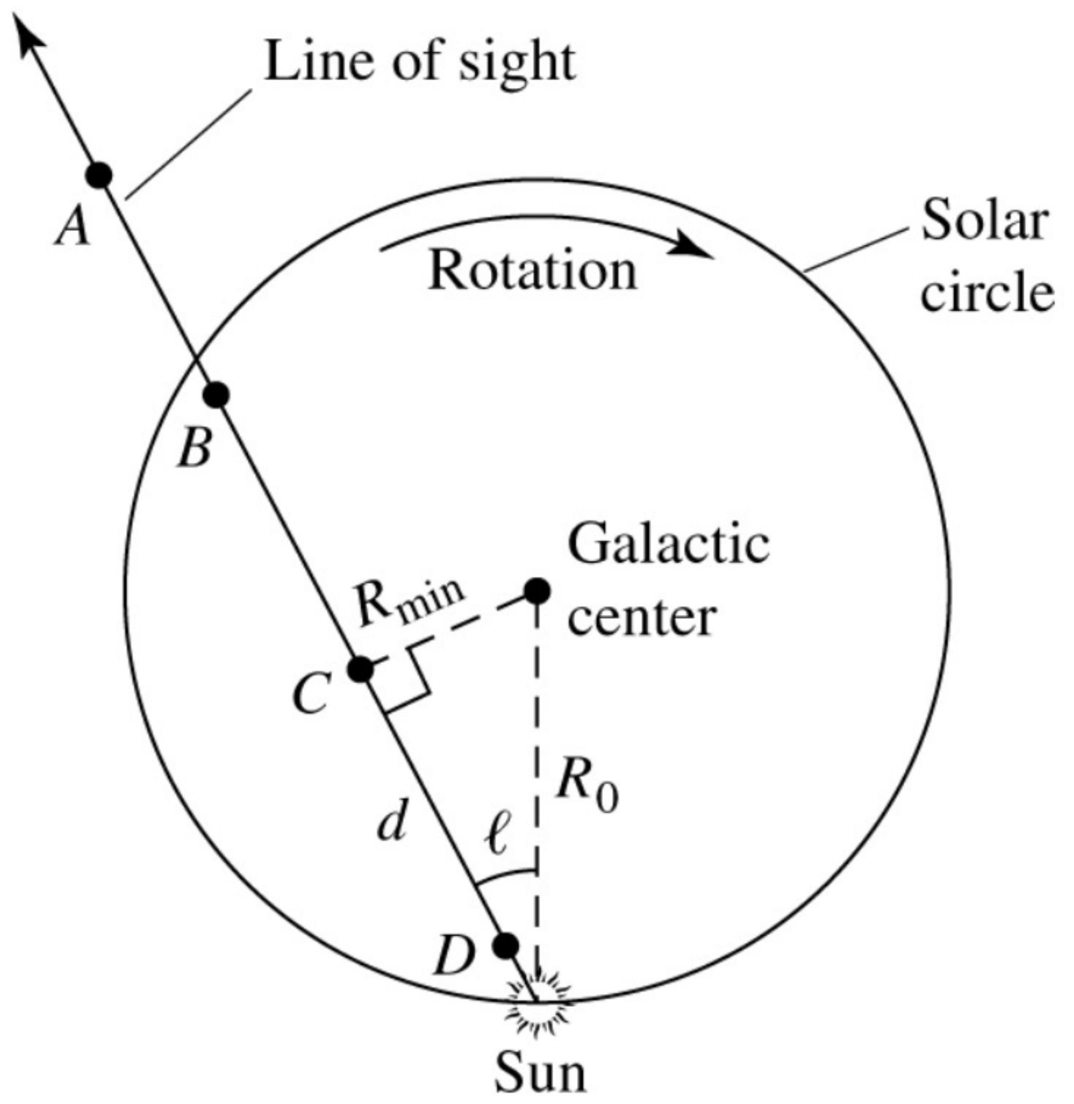


HI 21 cm hyperfine line

Galactic HI line emission:

- Galaxy rotates differentially.
- Each cloud along the line-of-sight is in its own orbit, leading to different radial velocities.
- Therefore, HI spectrum is split into several components.





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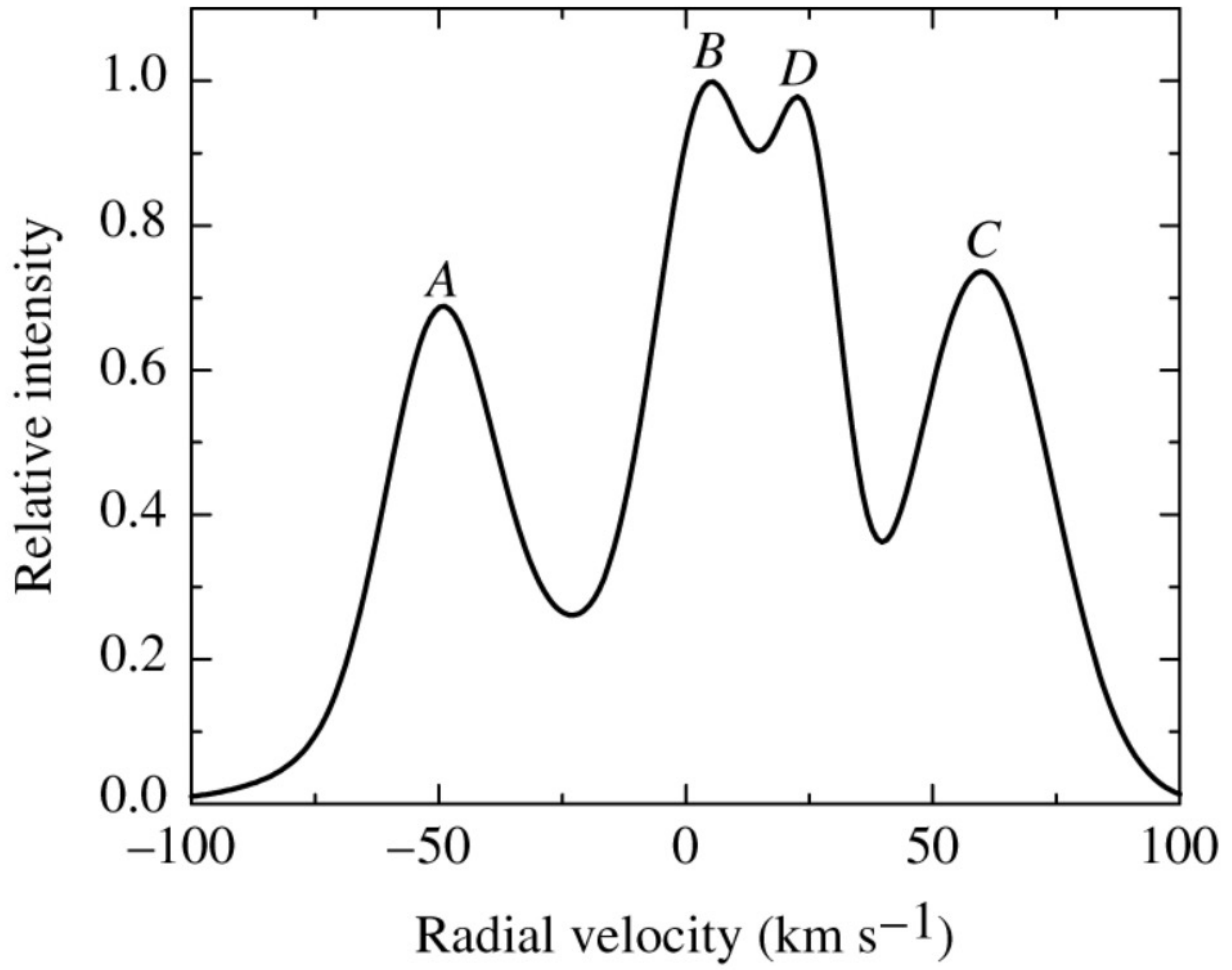

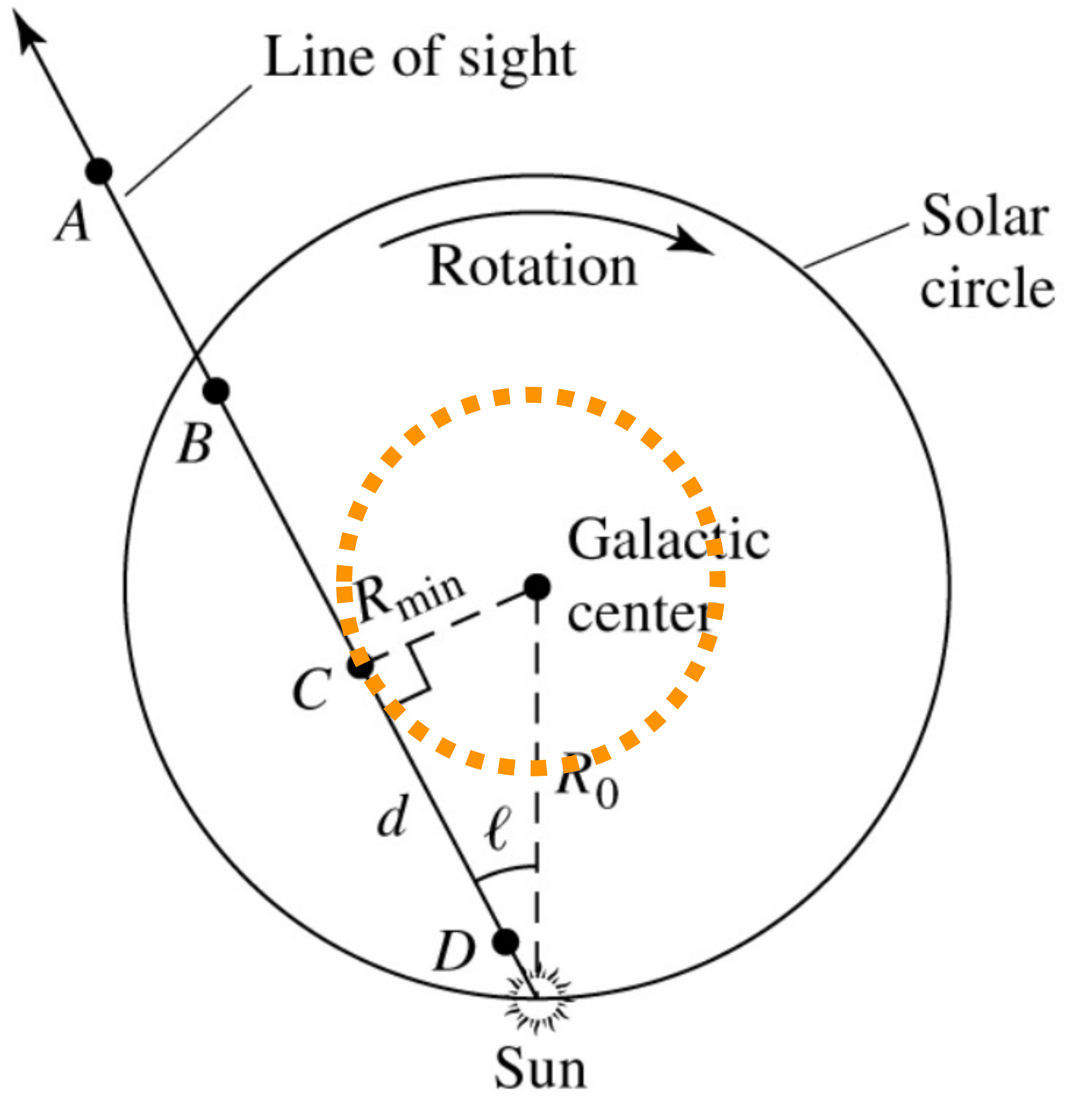

- Maximum velocity is reached when the cloud is closest to the GC along the LoS.

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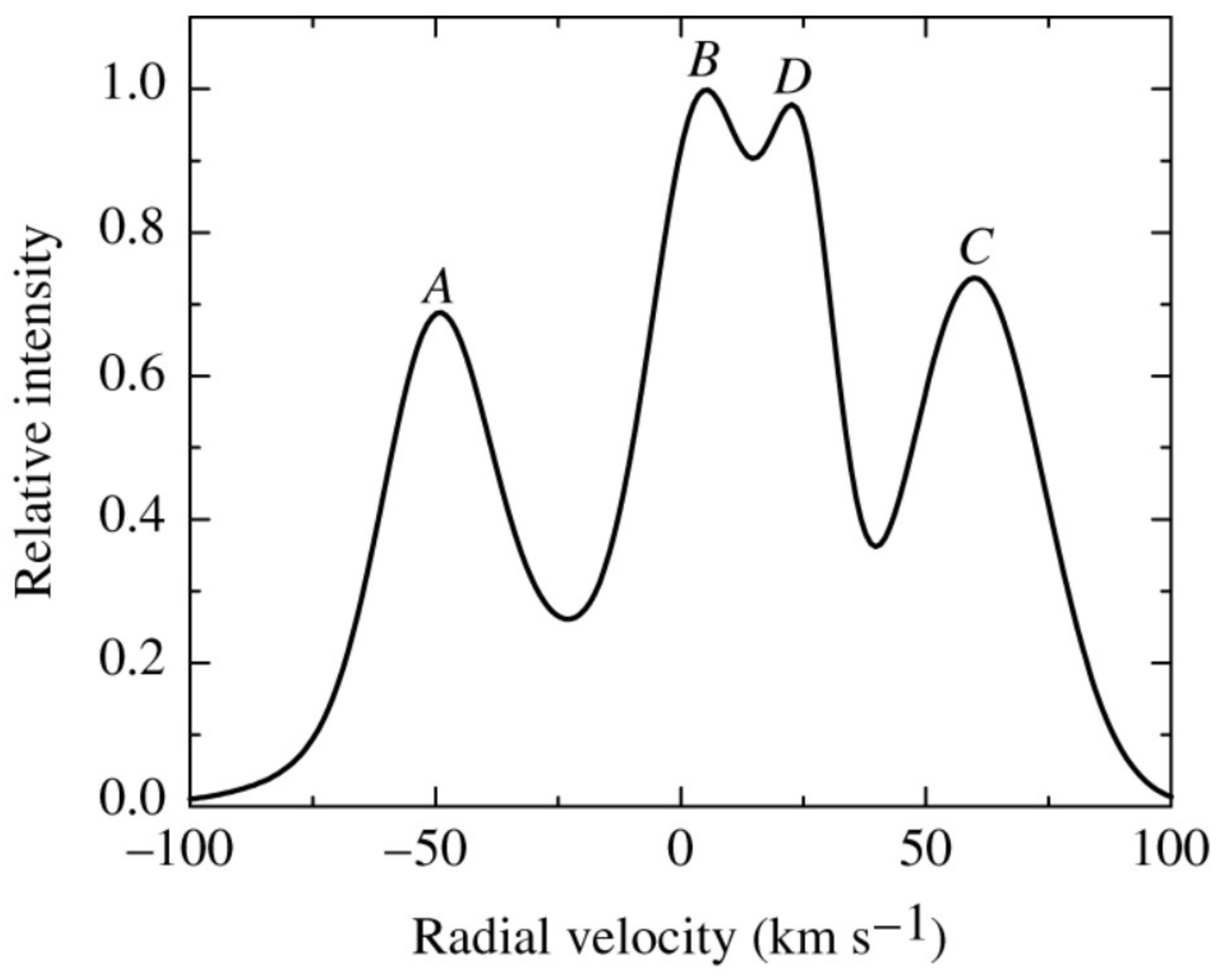
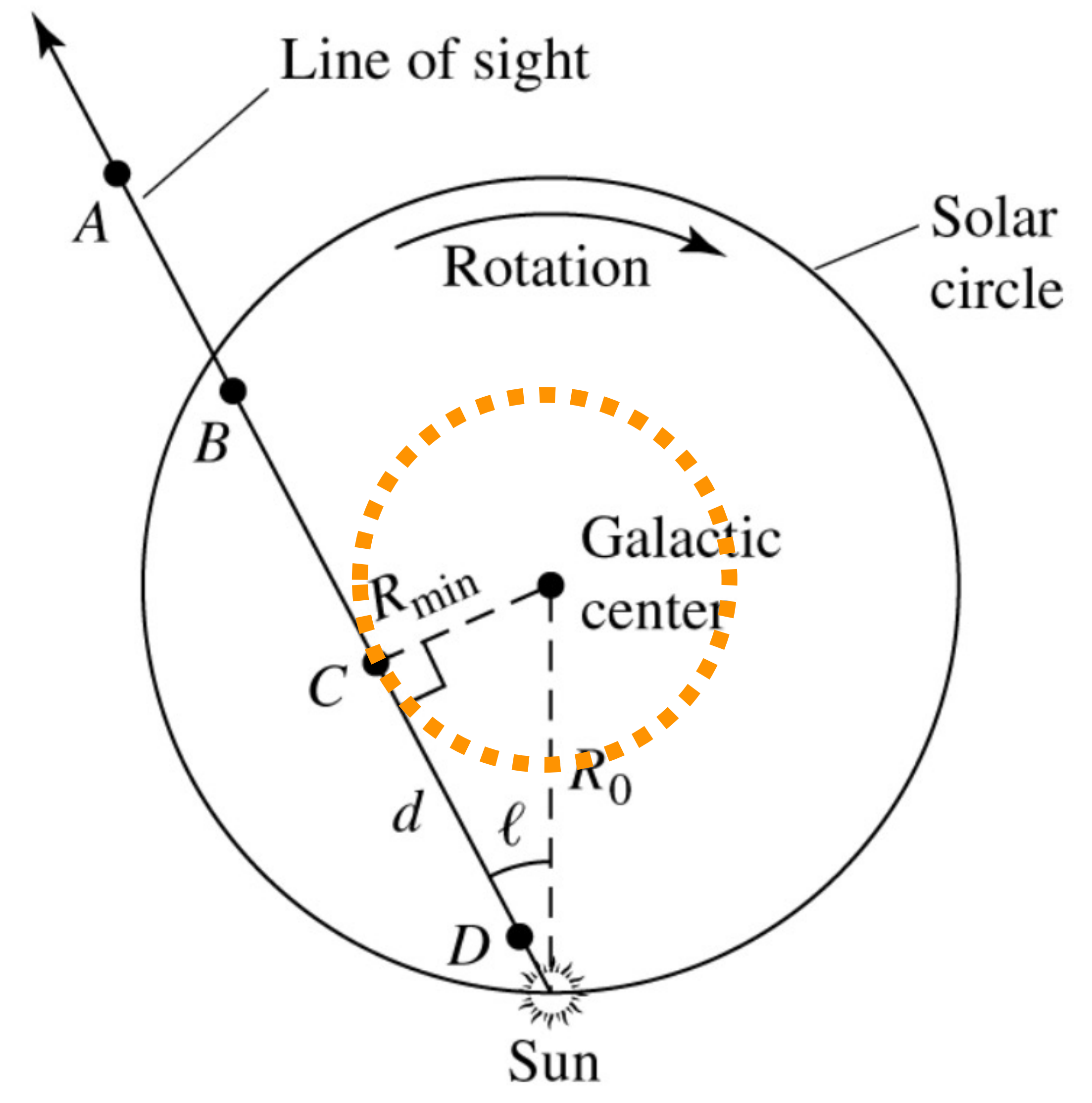
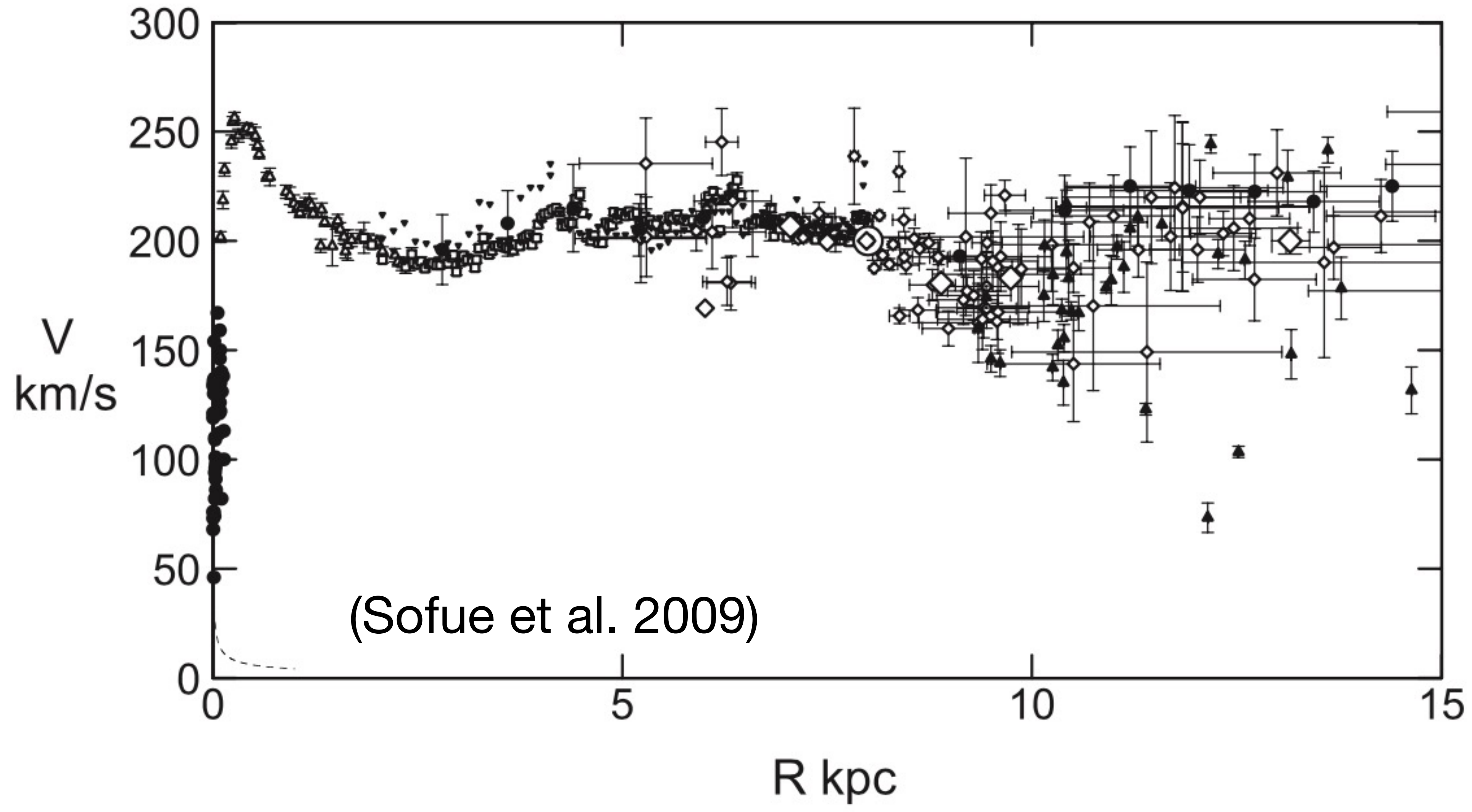




- Maximum velocity is reached when the cloud is closest to the GC along the LoS.
- By sampling many lines-of-sight across the Galaxy, we can measure circular velocities as a function of R, and then generate the rotation curve.



Polarization

- The electric field of an electromagnetic wave traveling in the z-direction can be projected onto orthogonal x- and y-directions.

$$\overrightarrow{E} = [\hat{x}E_x \exp(i\phi_x) + \hat{y}E_y \exp(i\phi_y)] \;\; \exp[i(\overrightarrow{k} \cdot \vec{z} - \omega t)]$$

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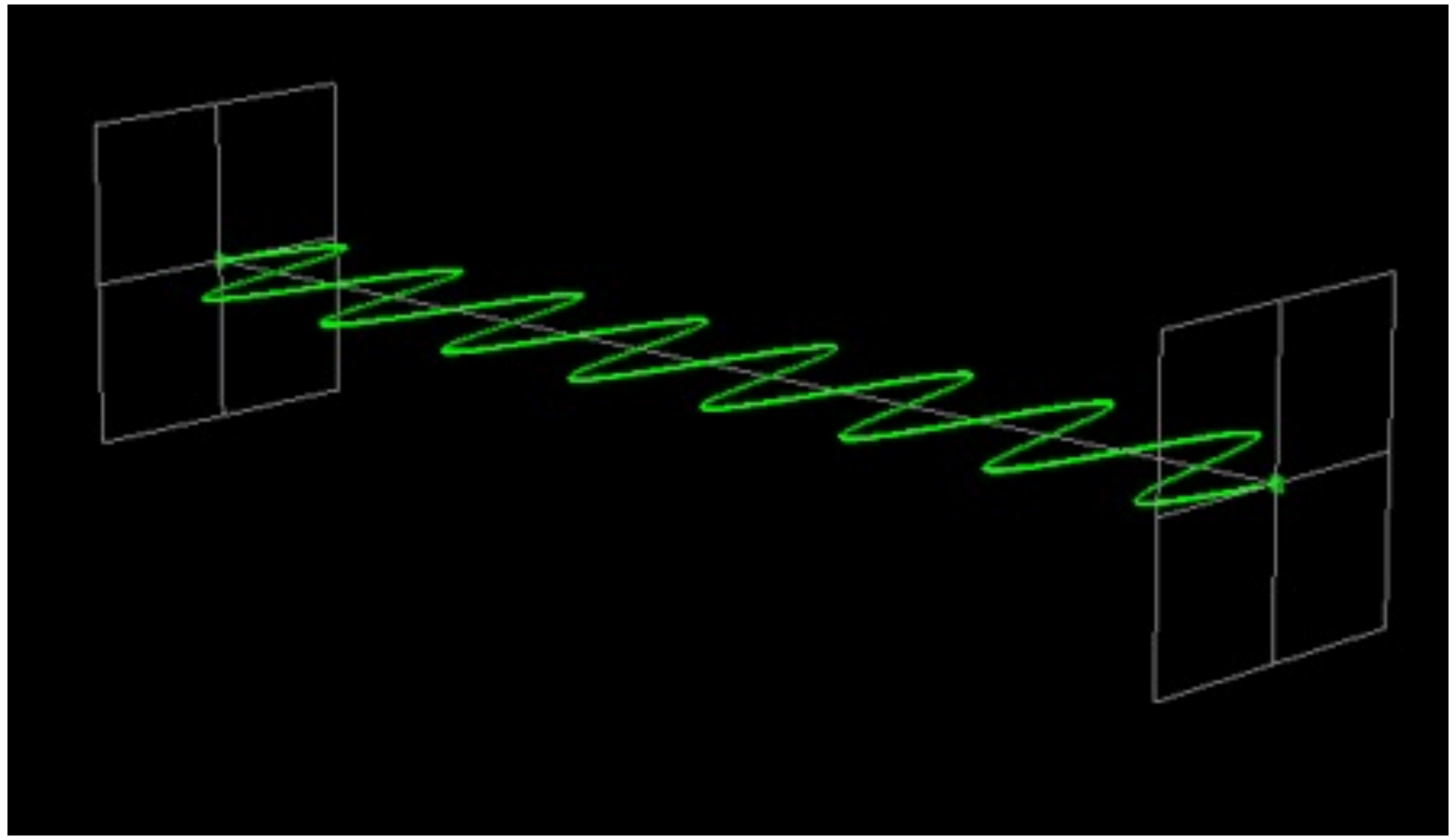
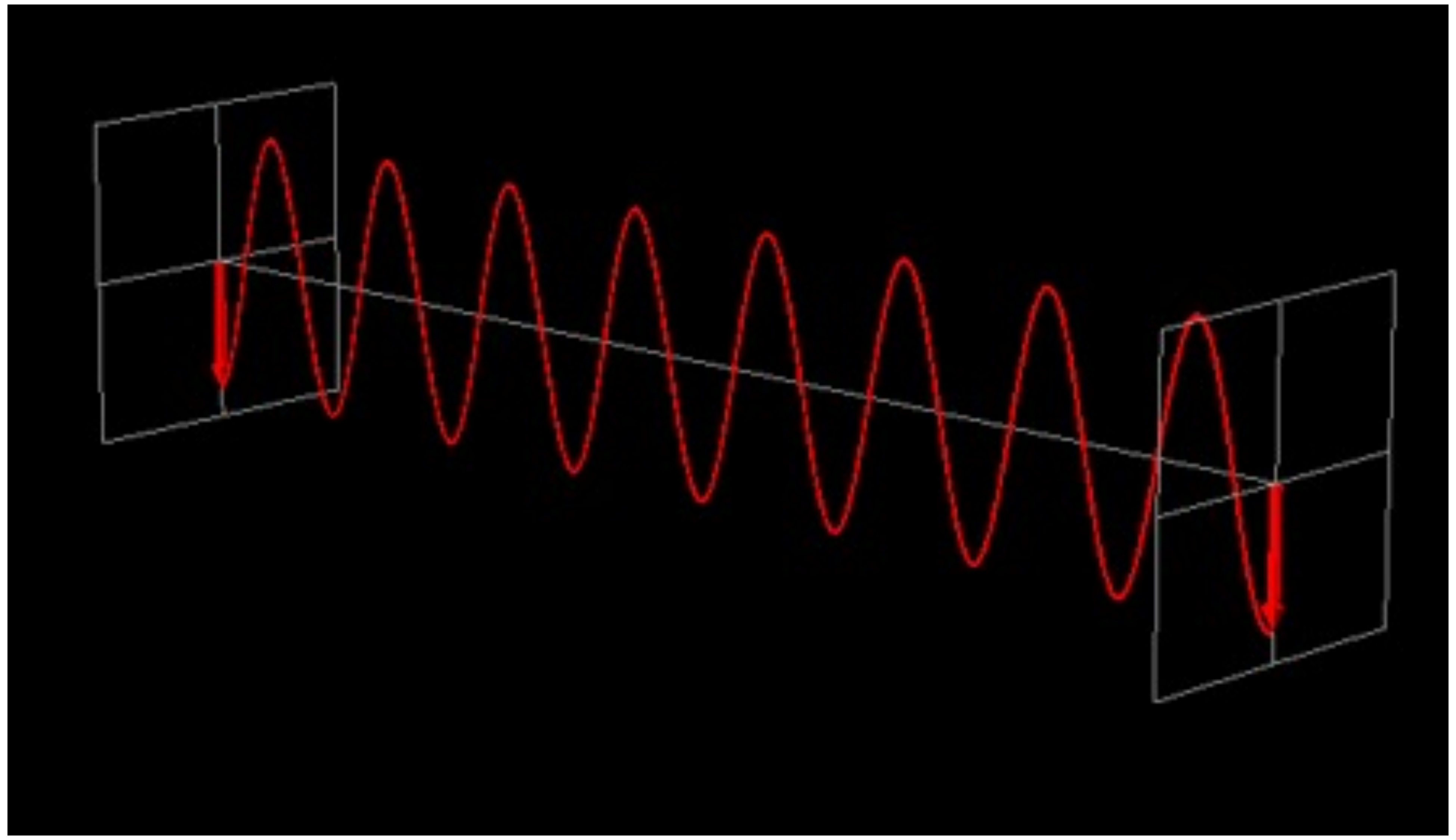
Horizontal linear polarization



Vertical linear polarization



Credit: Applied Photophyics

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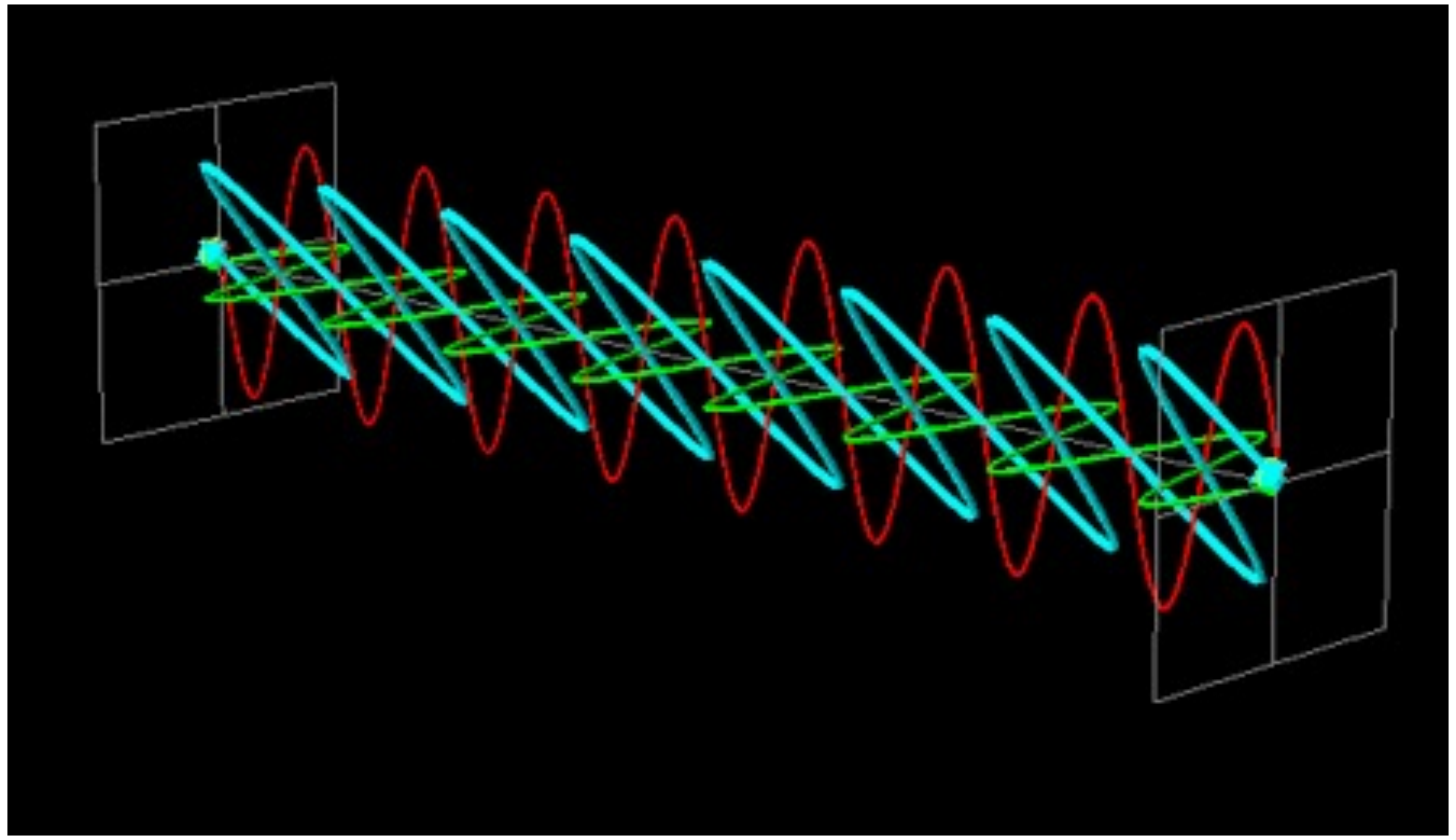
- Superposition of two orthogonally polarized components create different polarization states.

When $\delta = \phi_x - \phi_y = 0$, then we get linear polarization.

If $E_x = E_y$, then the linear polarization position angle ψ_{PPA} $= 45°$.



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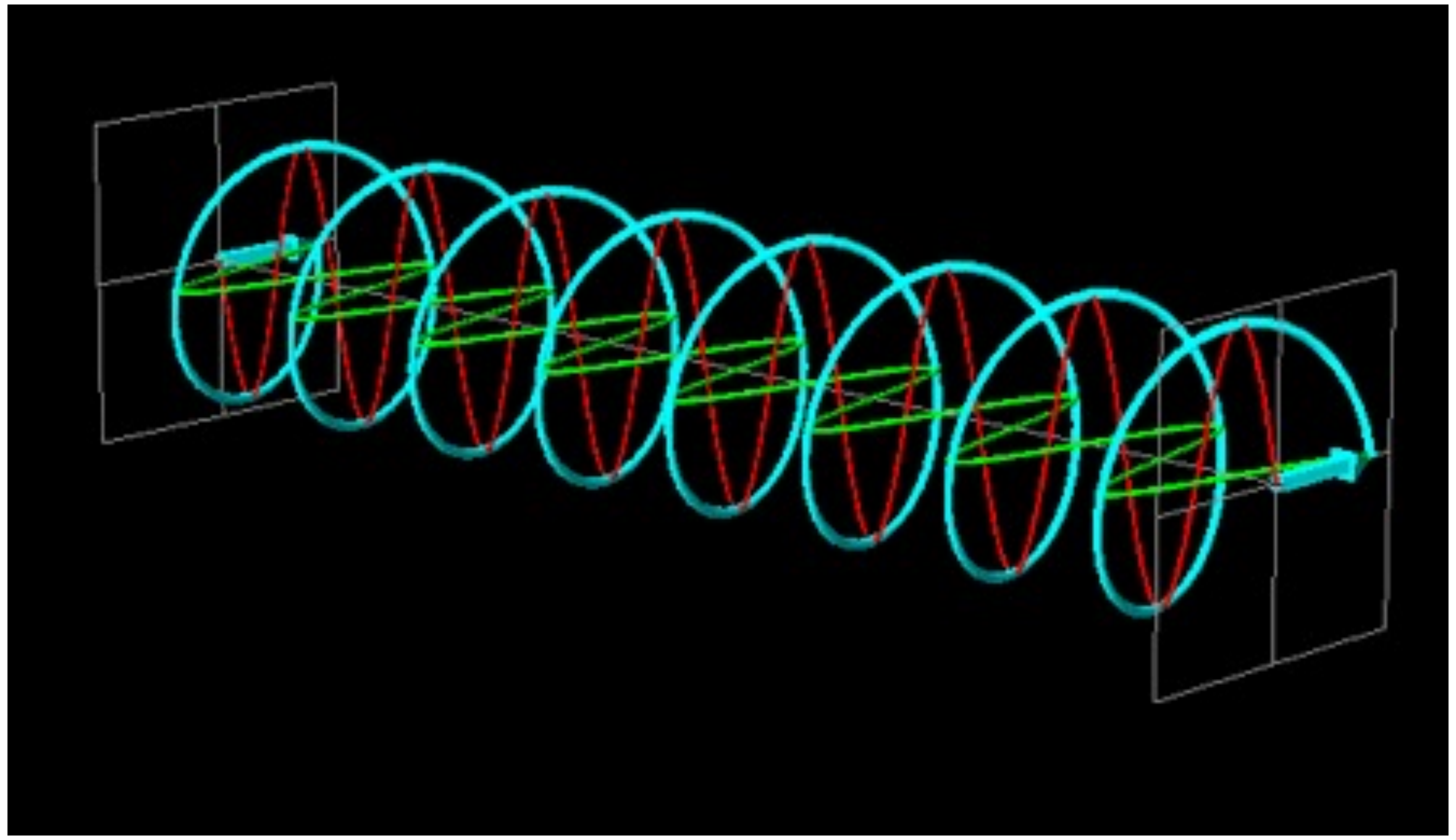
- If $E_x = E_y$, and $\delta = \phi_x - \phi_y = 90°$, then we get circular polarization.

- If $\delta = \phi_x - \phi_y = 90°$, but $E_x \neq E_y$, then we get elliptical polarization.

- If $E_x = E_y$, but the phases are completely uncorrelated, then the light is unpolarized.

Right-handed circular polarization



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Polarization

$$\overrightarrow{E} = [\hat{x}E_x \exp(i\phi_x) + \hat{y}E_y \exp(i\phi_y)] \;\; \exp[i(\overrightarrow{k} \cdot \vec{z} - \omega t)]$$

- Polarization of a source can be characterized by Stokes parameters (for a “linear feed” with a cross dipole)

$$I = < E_x^2 + E_y^2 > /R_0$$
$$Q = < E_x^2 - E_y^2 > /R_0$$
$$U = < 2E_xE_y \cos(\delta) > /R_0$$
$$V = < 2E_xE_y \sin(\delta) > /R_0$$

$R_0 = 4.19 \times 10^{-10}\ s\ cm^{-1}$ is the radiation resistance of free space.

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- Polarized flux density is $I_p = \sqrt{Q^2 + U^2 + V^2}$
- Degree of polarization is $p = \frac{I_p}{I}$
- If the light is unpolarized, then $Q = U = V = 0; I_p = 0; p = 0$).

Polarization

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- Linear polarization $L = \sqrt{Q^2 + U^2}$
- Linear polarization position angle $\psi = \frac{1}{2}\tan^{-1}\left(\frac{U}{Q}\right)$
- Circular polarized flux is $|V|$

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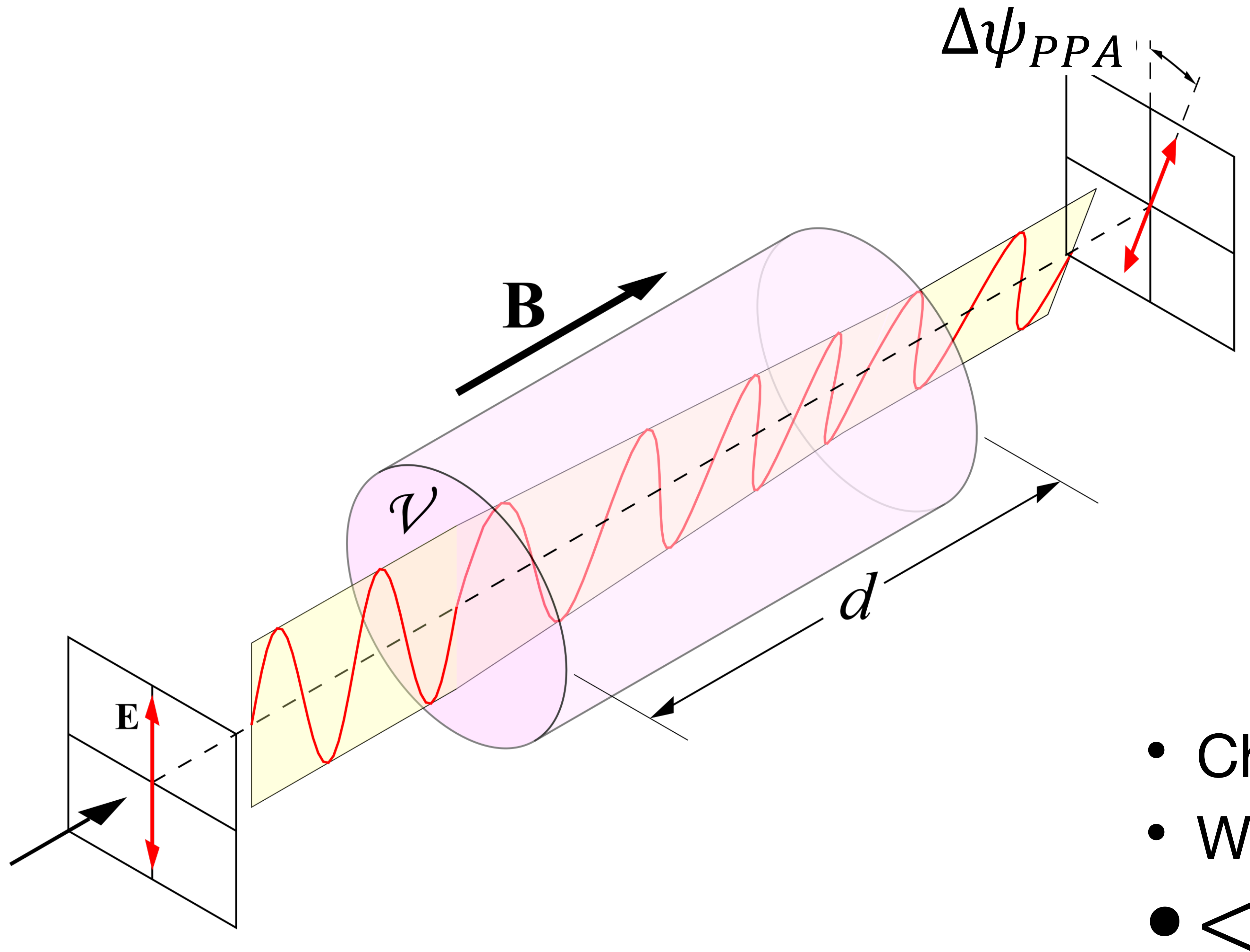
Polarization

- Faraday rotation: When the linearly polarized light propagates through ISM, the polarization position angle can be rotated.



Credit: Wikimedia Commons

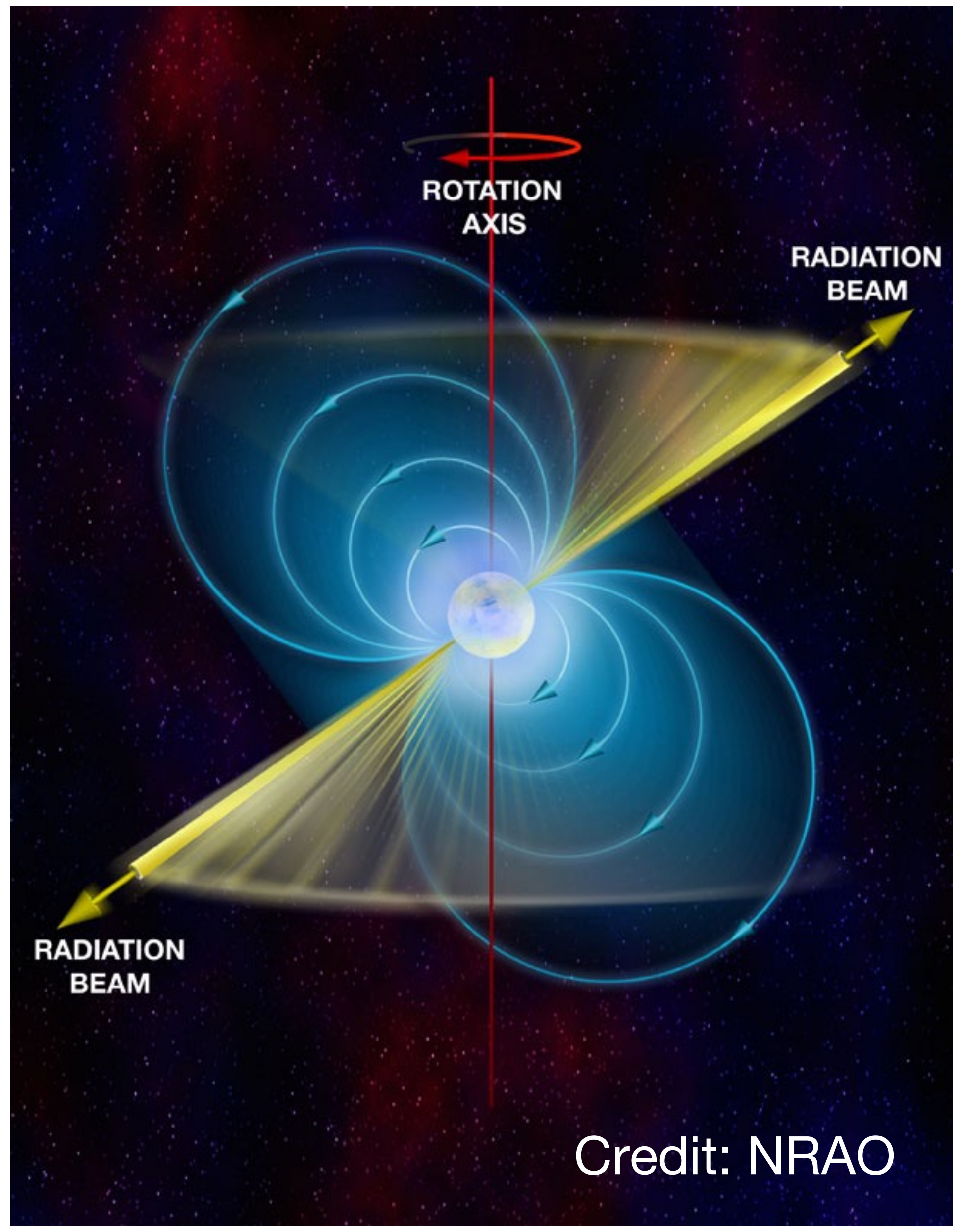
$$\Delta\psi_{PPA} = \lambda^2 \times RM$$

Rotation measure

$$RM \approx 8.1\times10^5 \int_{los} \left(\frac{n_e}{cm^{-3}}\right)\left(\frac{B_\parallel}{gauss}\right)\left(\frac{dl}{pc}\right) rad/m^2$$

- Change in ψ_{PPA} can be measured with multi-frequency observations.
- We can constrain RM.
- $< B_\parallel >$ can be estimate.

Pulsar emission

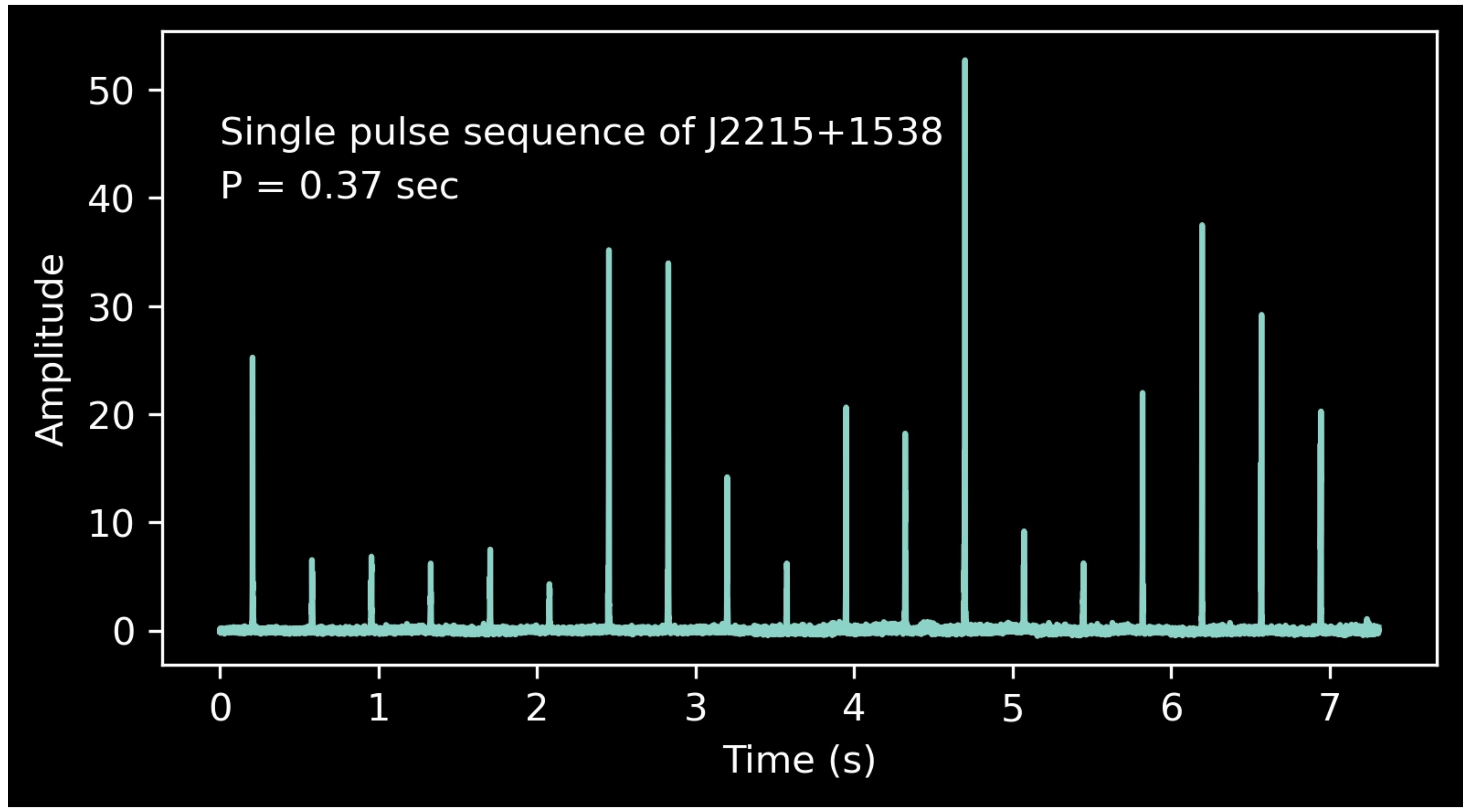


- Pulsars are excellent rotators.
- Pulsar are powered by loss of rotational kinetic energy.
- Energy is dissipated as dipole radiation……….

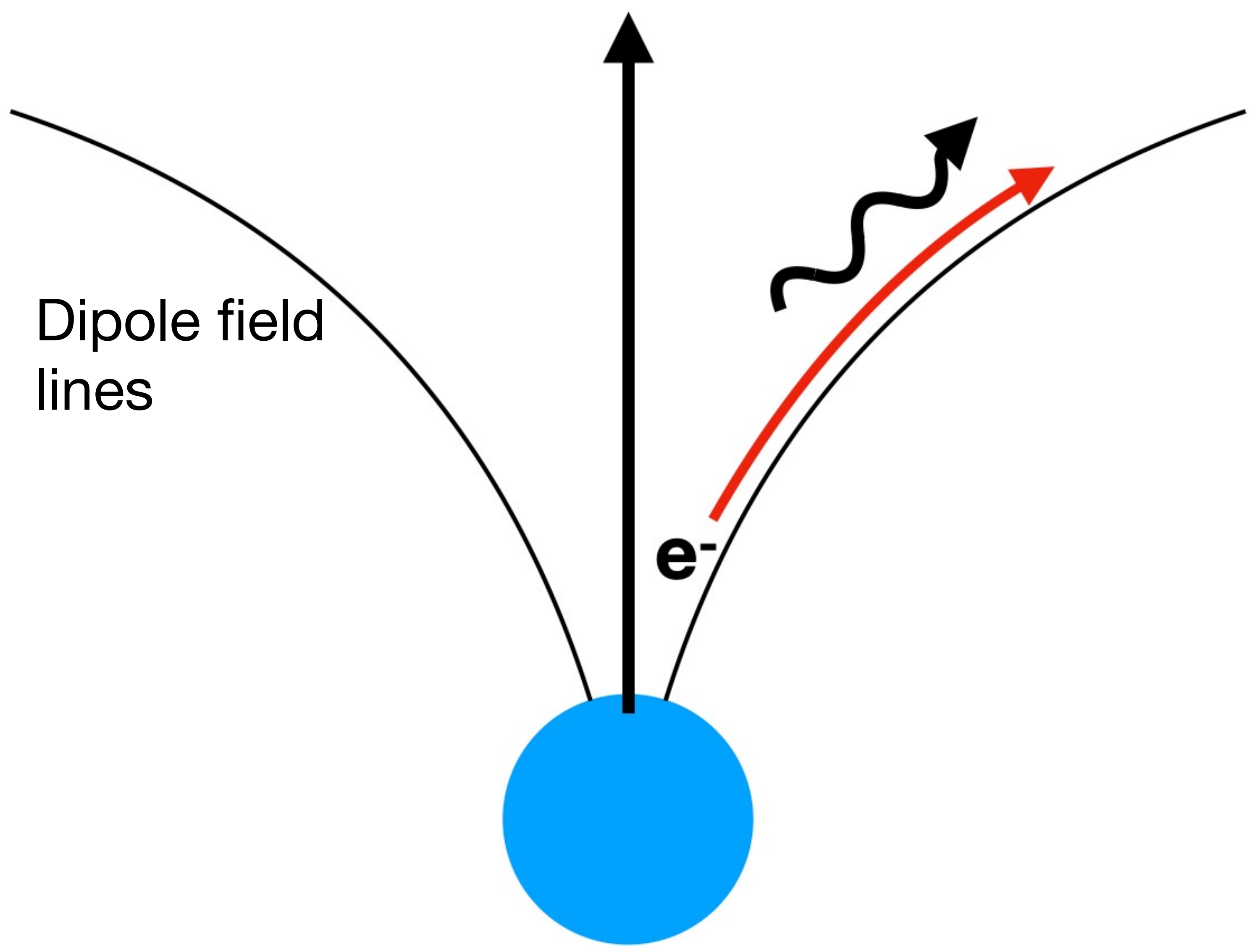
$$\dot{E} = -\frac{dE_{rot}}{dt} \approx 3.95\times10^{31}\left(\frac{\dot{P}}{10^{-15}}\right)\left(\frac{P}{s}\right)\ erg/s$$

Pulsar period increases over time and spin-down gradually!

Spin-down rate of J2215+1538 is $\dot{P} = (2.3672 \pm 2)\times10^{-15}\ s/s.$

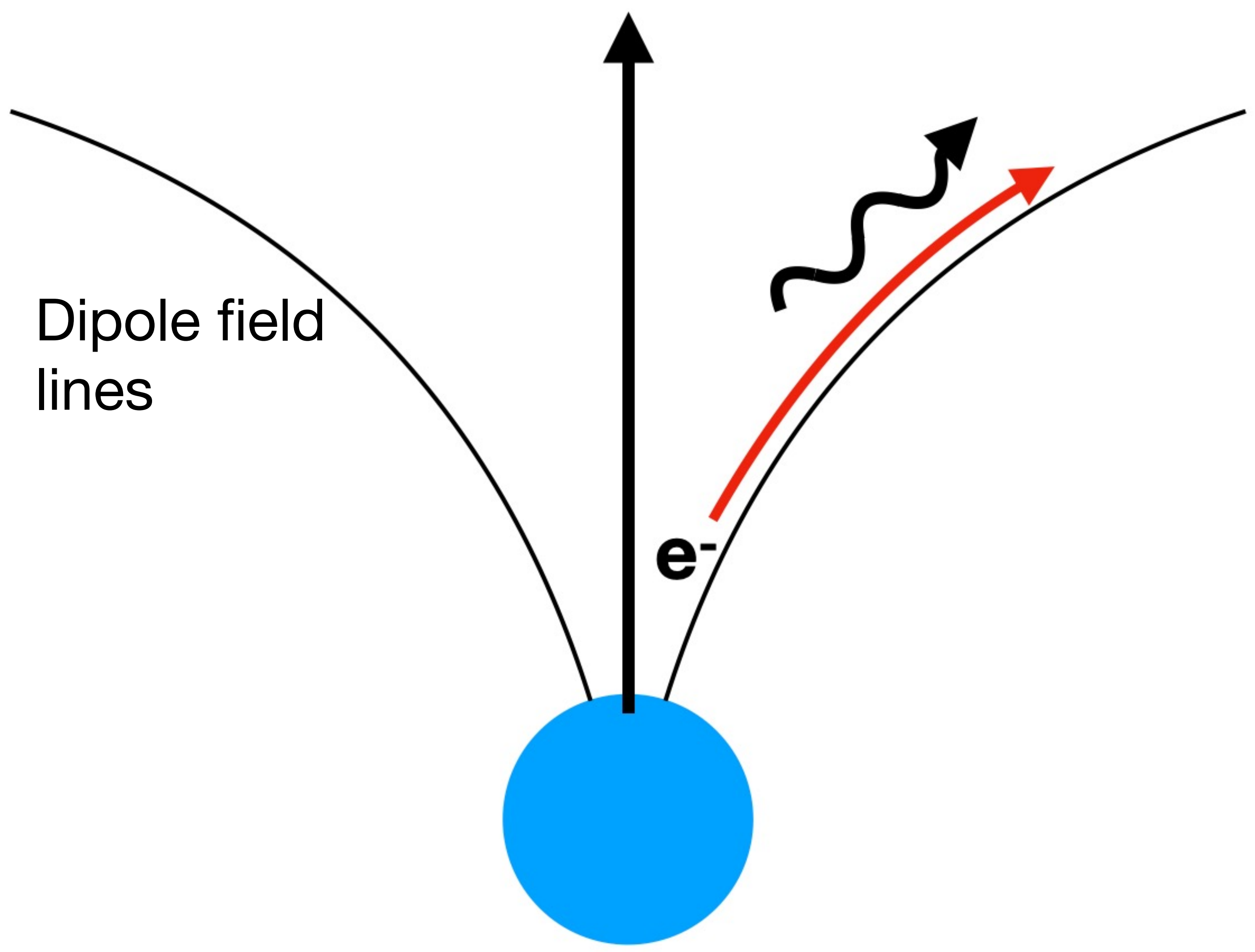


Pulsar polarization



- Pulsars are highly magnetized dipoles ($\approx 10^8 - 10^{12} G$).
- Particle accelerate along magnetic field lines (curvature radiation) —> photons emit long the plan of the magnetic field.
- Pulsar emission is highly linearly polarized.

Pulsar polarization



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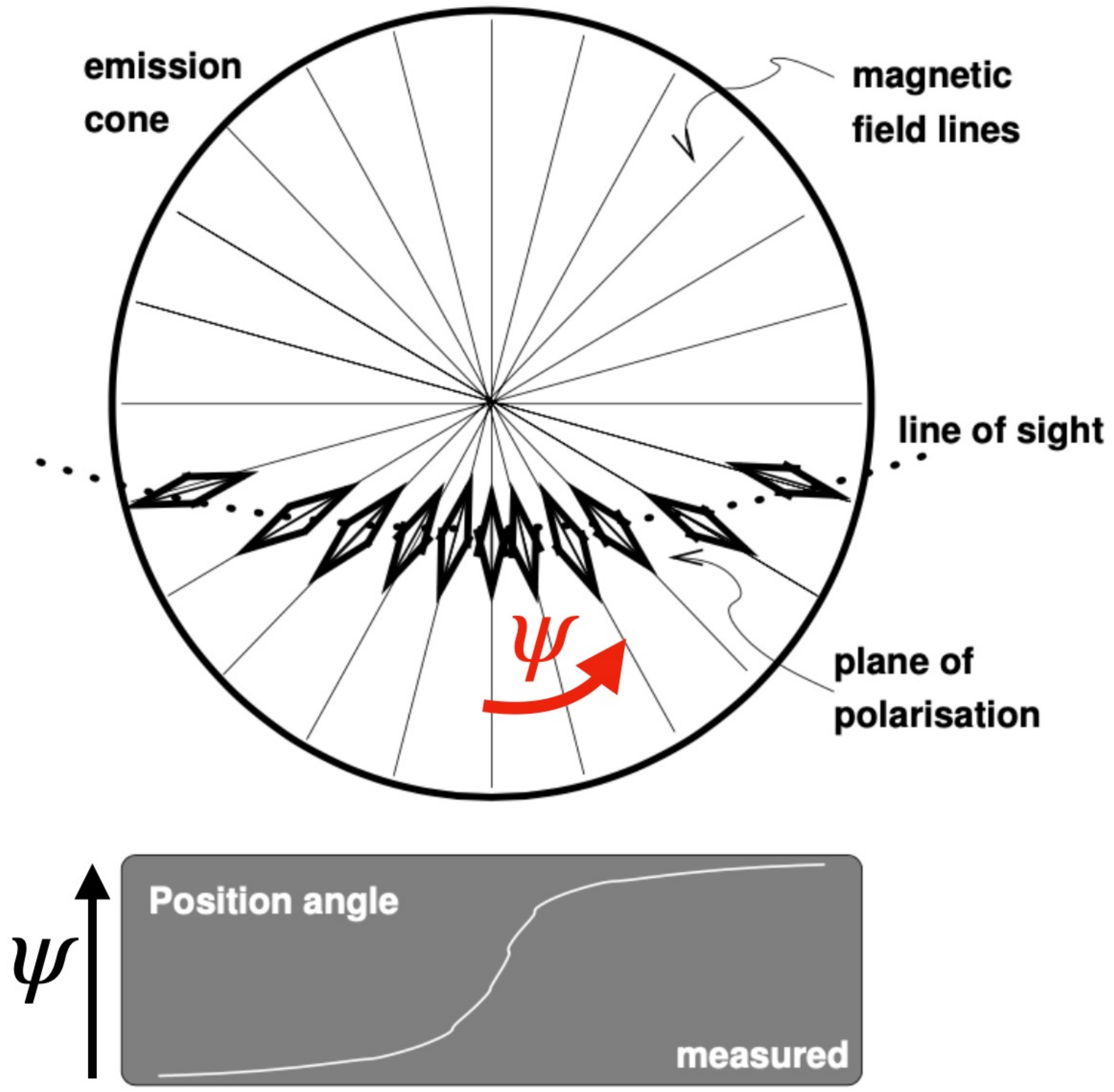
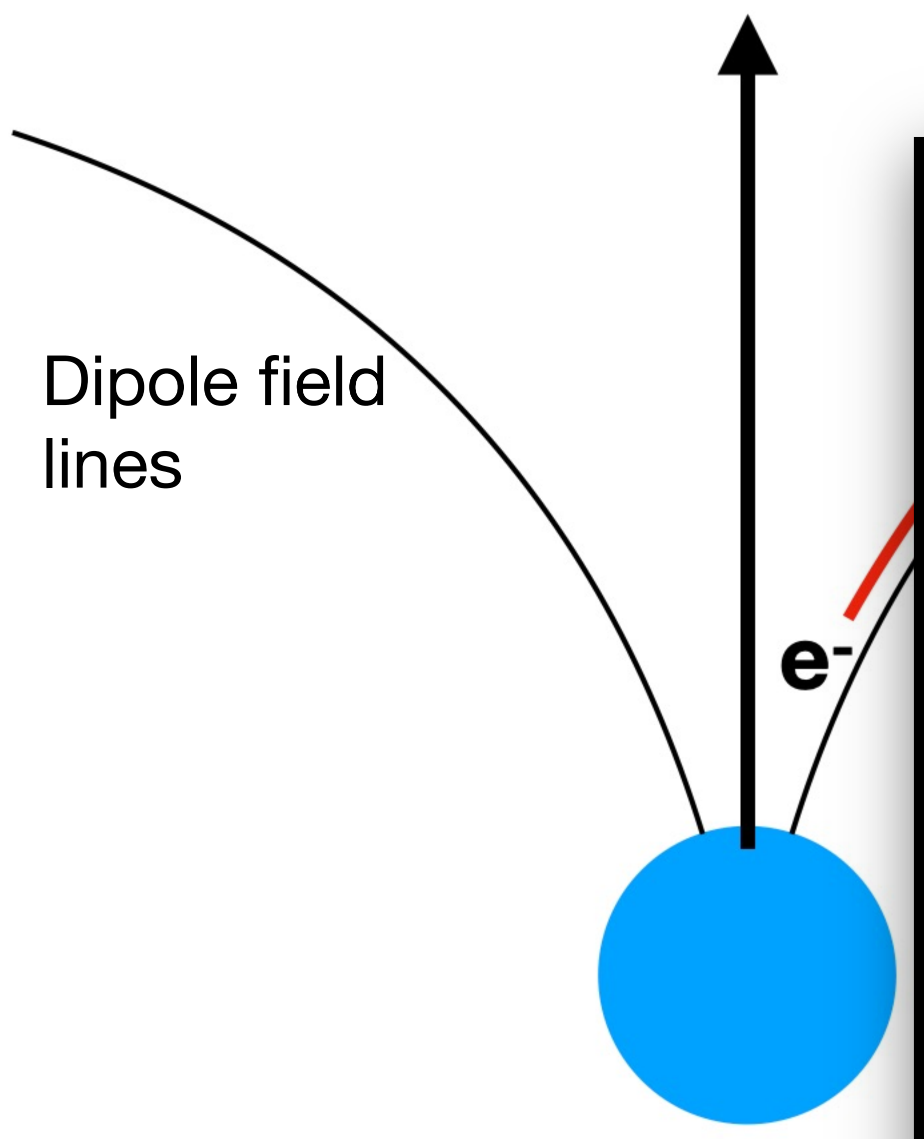
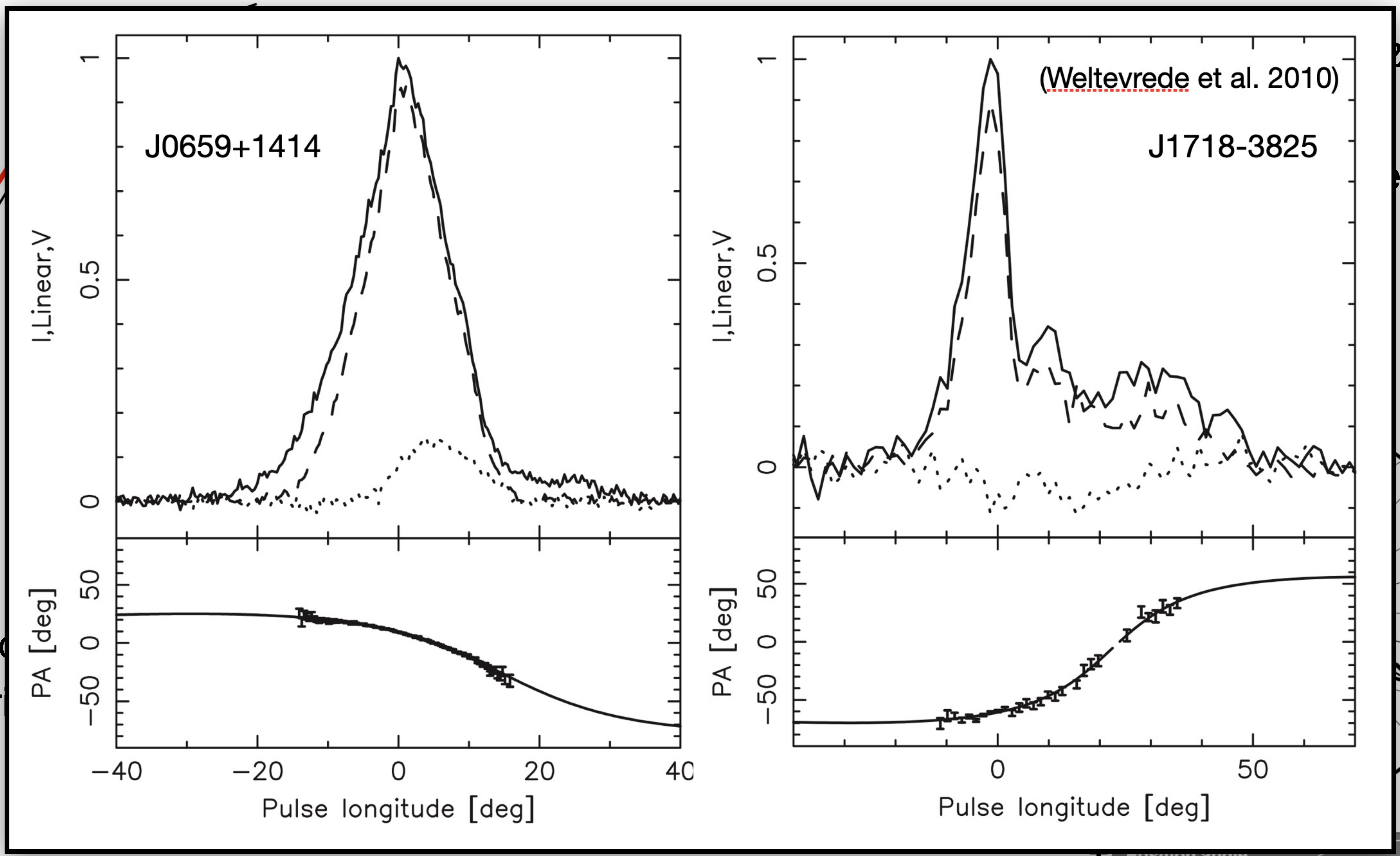
- Emission beam across our line of sight results in a S-shaped curve in the polarization position angle sweep ψ_{PPA}.



(Lorimer & Kramer 2005)

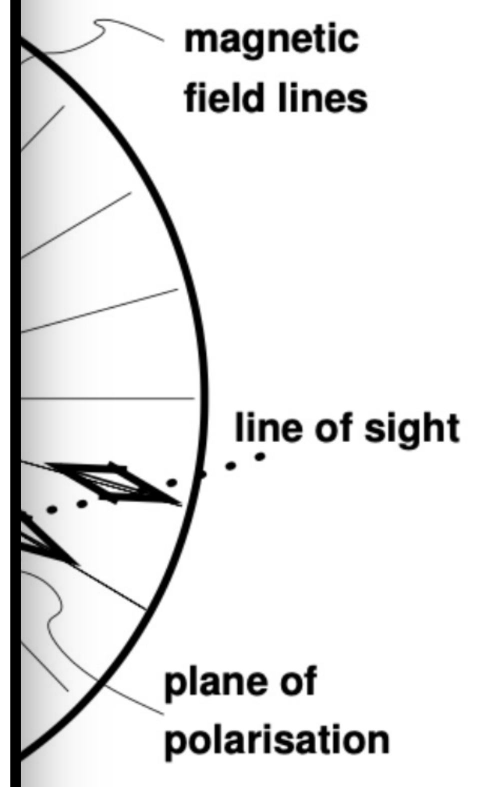
Pulsar polarization

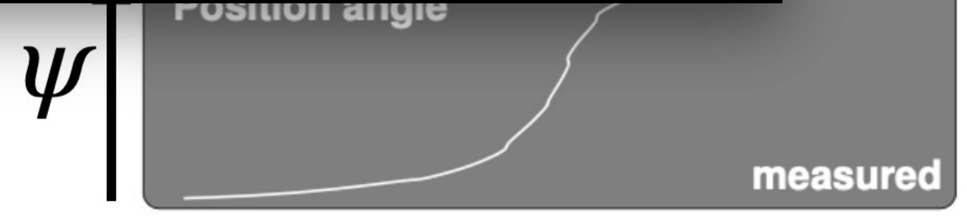




G).

radiation) —>





- Emission beam ac
  curve in the polar

Thank you!

The derivations of the equation and more details can be found in the textbook “Essential Radio Astronomy”

https://science.nrao.edu/opportunities/courses/era