Radiation Fundamentals I

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GBO/AO SD workshop, Green Bank, September 13, 2021

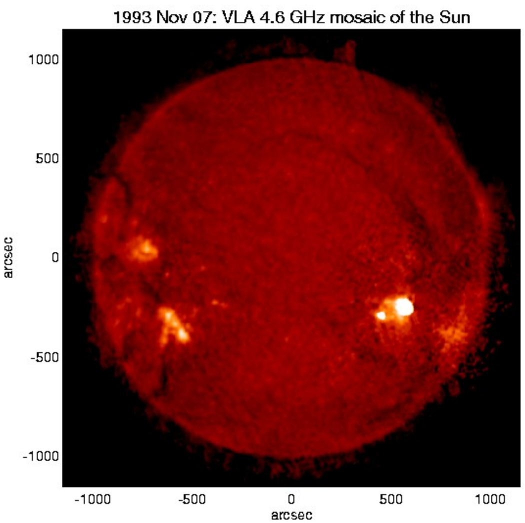
- Introduction
- Radiation fundamentals
- Radiative transfer (applicable to continuum and spectral lines)
- Black body radiation
- Larmor's equation
- Thermal free-free emission
- Synchrotron emission

Essential Radio Astronomy, Condon, J. & Ransom, S.

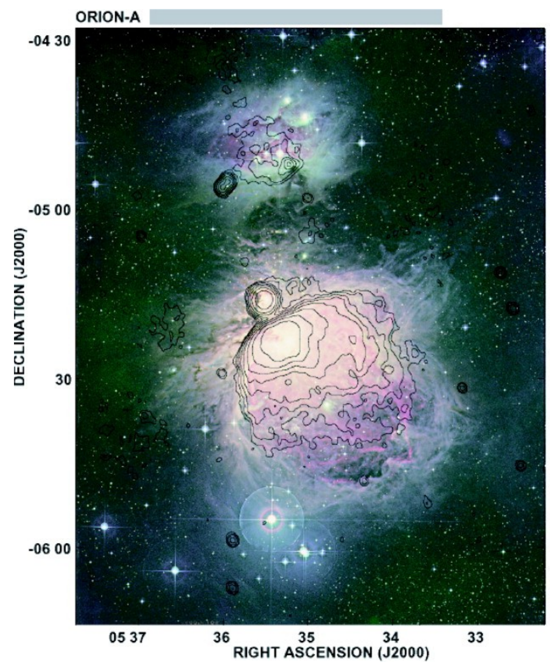
https://www.cv.nrao.edu/~sransom/web/xxx.html

Introduction: Radiation from astrophysical objects

Radio image of Sun

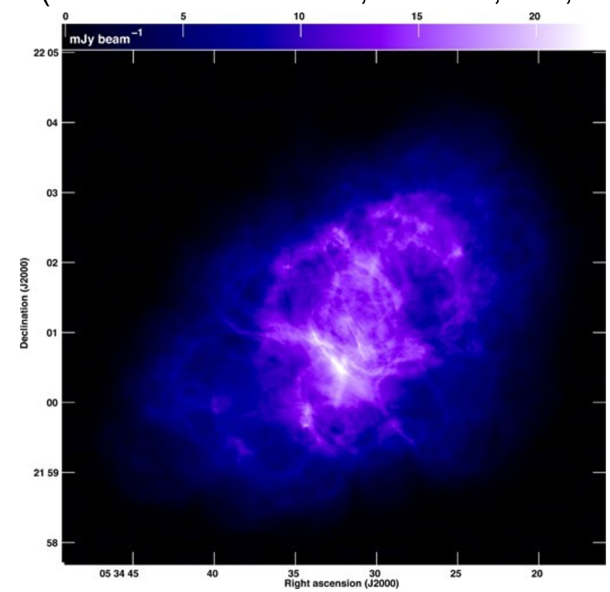


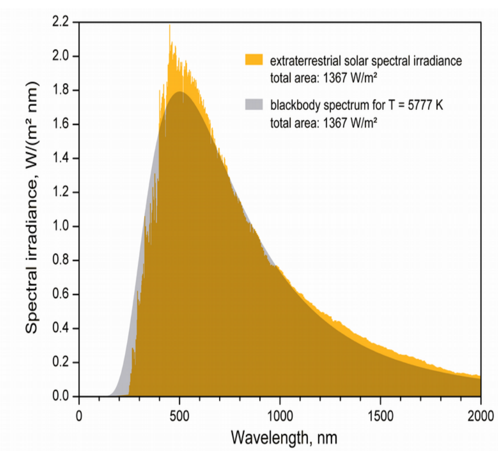
HII region

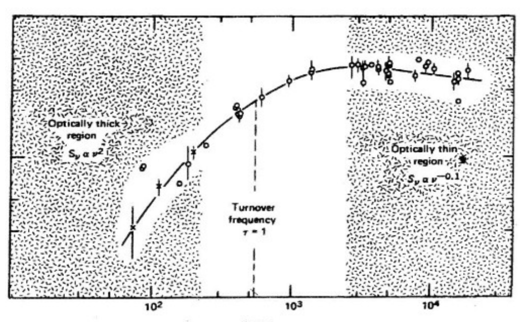


(Subrahmanyan et al. 2001, AJ, 121,399)

Crab Nebula @ 5.5 GHz
(Bietenholz et al. 2015, MNRAS, 446,205)







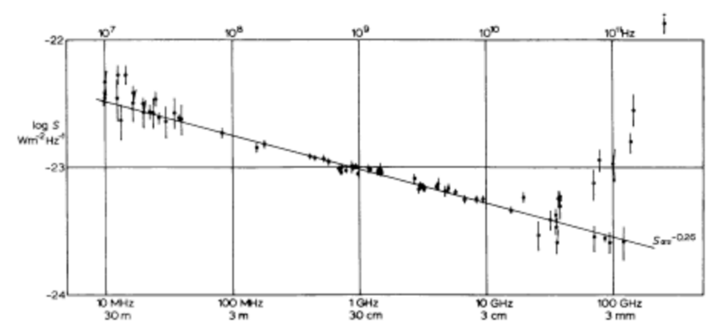
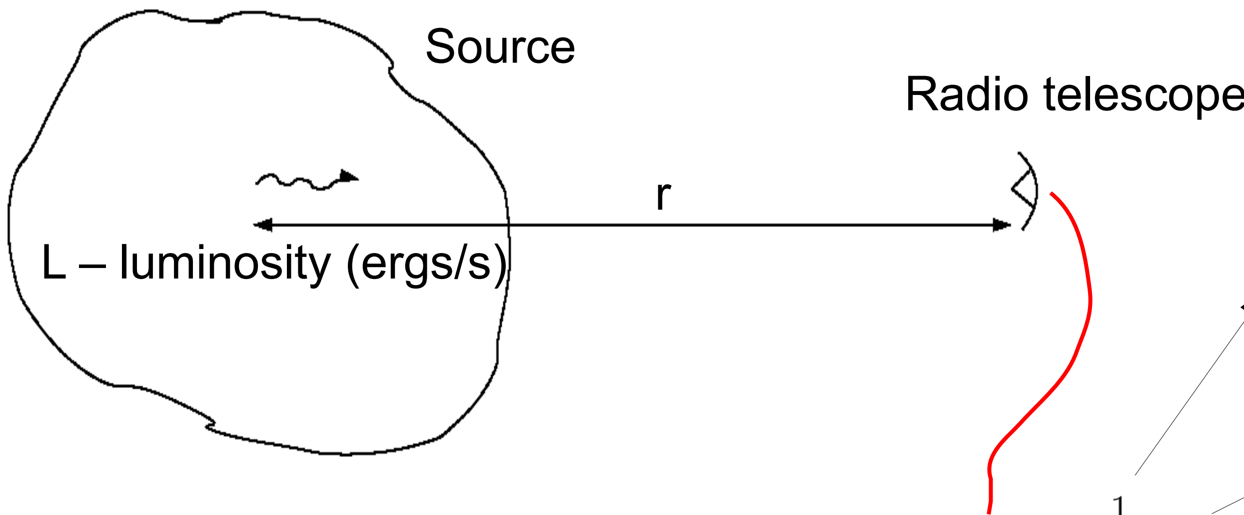


Fig. 1. Radio spectrum of total radiation from the Crab nebula.

(Baldwin, 1971, IAUS, 46, 22)

Bottom: Continuum spectra of sources – Flux density vs frequency or wavelength

Radiation Fundamentals: Flux density



One polarization

Effective collecting area

$$P_\nu = \frac{1}{2} S A_{eff} \text{ Spectral power (units: ergs/s/Hz)}$$

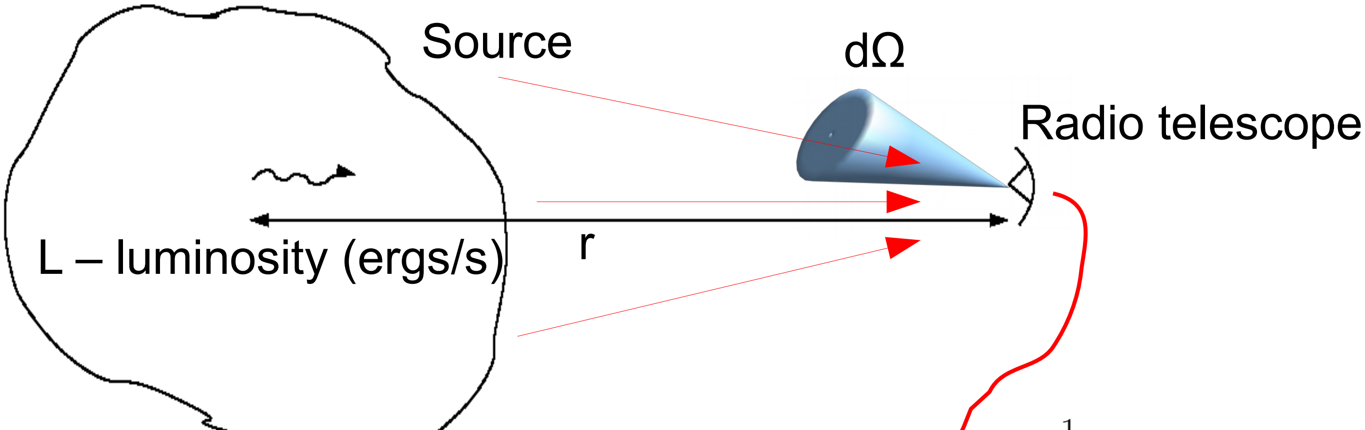
$$S \sim 10^{-26} \mathrm{J/s/m^2/Hz} \equiv 1 \text{ Jy}$$

$$L_\nu \qquad \text{Spectral luminosity (units: ergs/s/Hz)}$$

$$S = \frac{L_\nu}{4\pi r^2} \text{ Flux density (units: ergs/s/Hz/cm}^2\text{)}$$

Distance to source

Radiation Fundamentals: Specific Intensity



$$P_\nu = \frac{1}{2}\int_\Omega I_\nu A_e(\theta,\phi)\mathrm{d}\Omega \text{ Spectral power (units: ergs/s/Hz)}$$

Ω over the source solid angle or beam of the telescope

$$L_\nu \qquad \text{Spectral luminosity (units: ergs/s/Hz)}$$

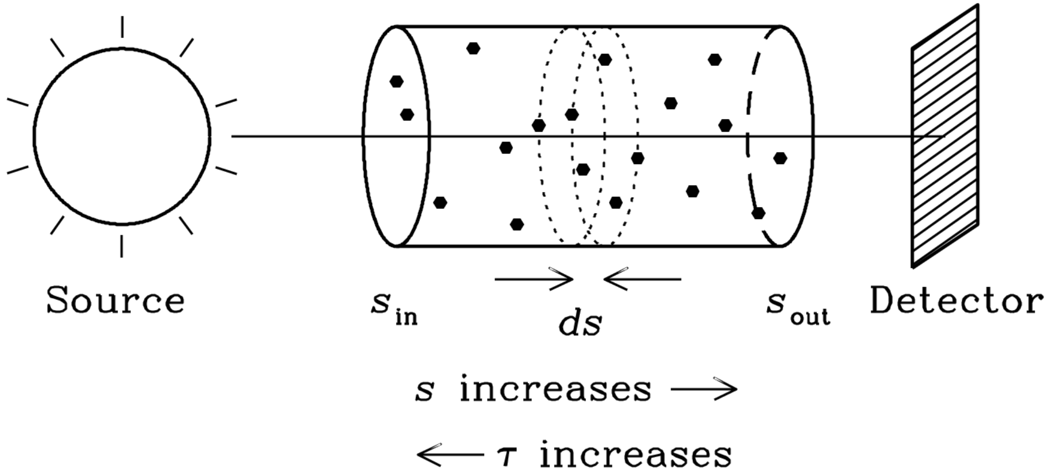
$$I_\nu = \frac{\mathrm{d}S}{\mathrm{d}\Omega} \text{ Intensity (units: ergs/s/Hz/cm}^2\text{/sr)}$$

$$S = \int_{\Omega_s} I_\nu \mathrm{d}\Omega \text{ Flux density (units: ergs/s/Hz/cm}^2\text{)}$$

Ω_s is the source solid angle

- Intensity is independent of the distance to the source.
- Intensity is conserved along a ray in free space.

Radiative Transfer



Change in intensity when the ray travels ds distance

Change due to absorption of radiation

Change due to emission of radiation

$$\frac{\mathrm{d}I_\nu}{\mathrm{d}s} = -\kappa_\nu I_\nu + j_\nu$$

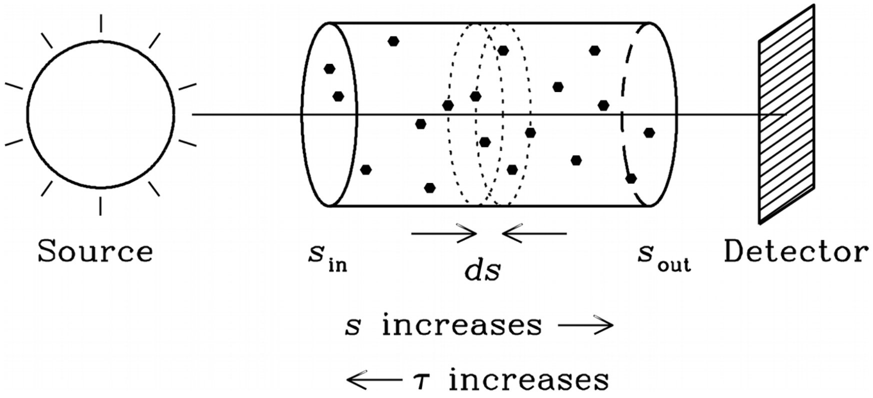
Absorption and stimulated emission

κ_ν absorption coefficient (units: cm^{-1})

j_ν emission coefficient (units: $\mathrm{ergs/s/cm^3/Hz/sr}$)

Radiative Transfer

$$\frac{dI_\nu}{ds} = -\kappa_\nu I_\nu + j_\nu$$



Case 1: Medium only absorbs radiation

$$\frac{dI_\nu}{I_\nu} = -\kappa_\nu \, ds$$

$$I_\nu(s_{out}) = I_\nu(s_{in}) \exp\left(-\int_{s_{in}}^{s_{out}} \kappa_\nu \, ds\right)$$

$$\tau_\nu \equiv -\int_{s_{out}}^{s_{in}} \kappa_\nu \, ds \quad \text{optical depth (unitless)}$$

$\tau_\nu > 0$ typically $\qquad \tau_\nu < 0$ Maser; exponential group

$$\tau_\nu >> 1 \quad \text{optically thick}$$

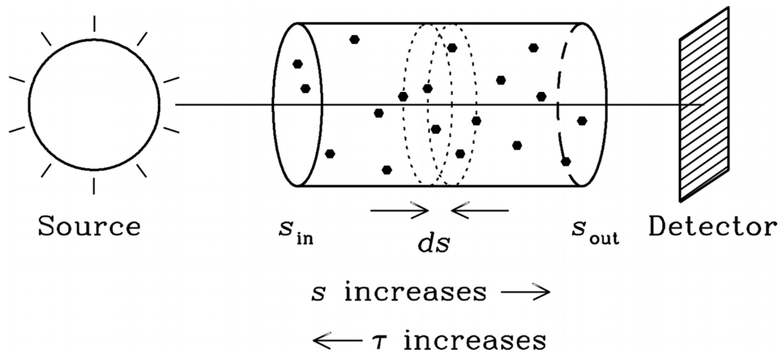
$$\tau_\nu << 1 \quad \text{optically thin}$$

Case 2: Medium only emits radiation

$$\frac{dI_\nu}{ds} = j_\nu$$

$$dI_\nu = j_\nu \, ds$$

Kirchhoff's Law



$$\frac{\mathrm{d}I_\nu}{\mathrm{d}s} = -\kappa_\nu I_\nu + j_\nu = 0$$

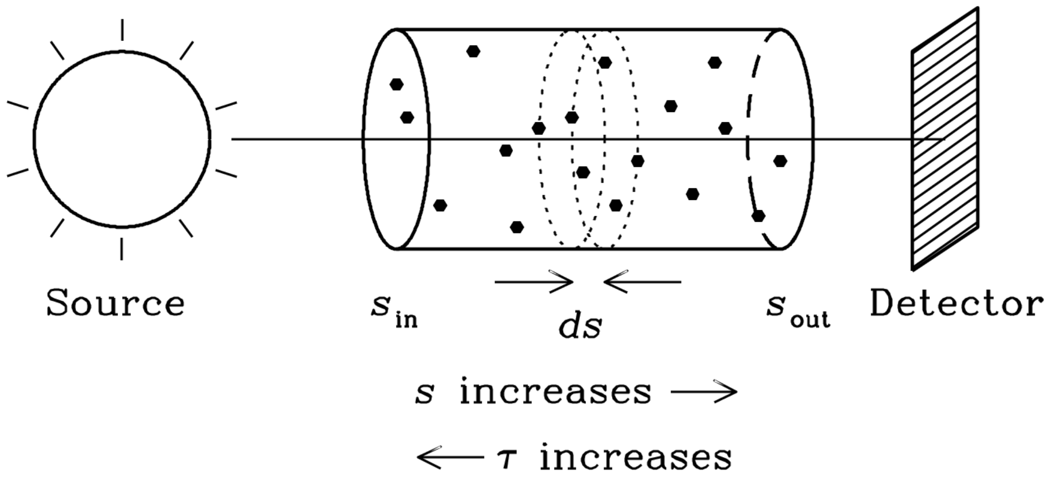
$$I_\nu = \frac{j_\nu}{\kappa_\nu}$$

$$I_\nu \equiv B_\nu(T) \ \text{Blackbody radiation}$$

$$\frac{j_\nu}{\kappa_\nu} = B_\nu(T) \longrightarrow$$

Kirchhoff's law; valid for TE and LTE;
In LTE, T will be kinetic temperature.
(In general this ratio is called the source function)

Solution to radiative transfer equation



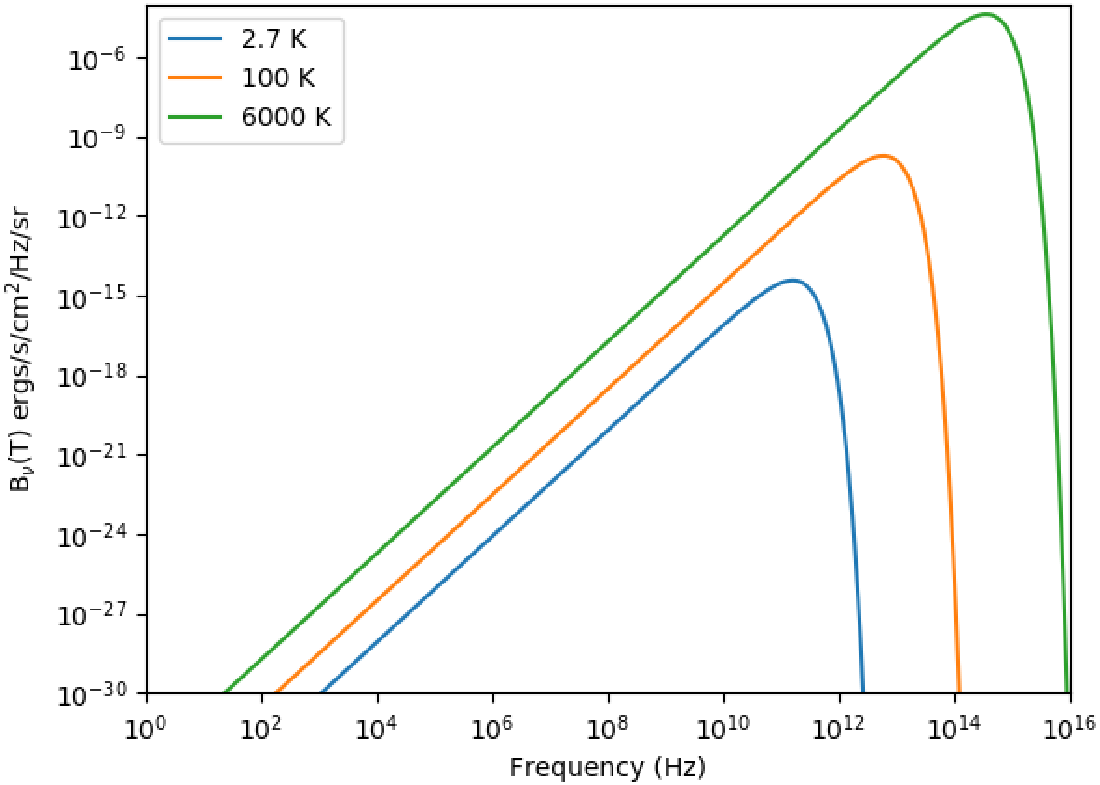
$$\frac{\mathrm{d}I_\nu}{\mathrm{d}s} = -\kappa_\nu I_\nu + j_\nu$$

$$I_\nu(s_{out}) = I_\nu(s_{in})\, e^{-\tau_\nu} + S_\nu(1 - e^{-\tau_\nu})$$

$$S_\nu = \frac{j_\nu}{\kappa_\nu} \text{ source function (units: ergs/s/cm}^2\text{/Hz/sr)}$$

$$S_\nu = B_\nu(T) \text{ For LTE emission}$$

Blackbody radiation



- Matter and radiation in equilibrium ==> radiation spectrum is blackbody
- Spectrum is characterized by T

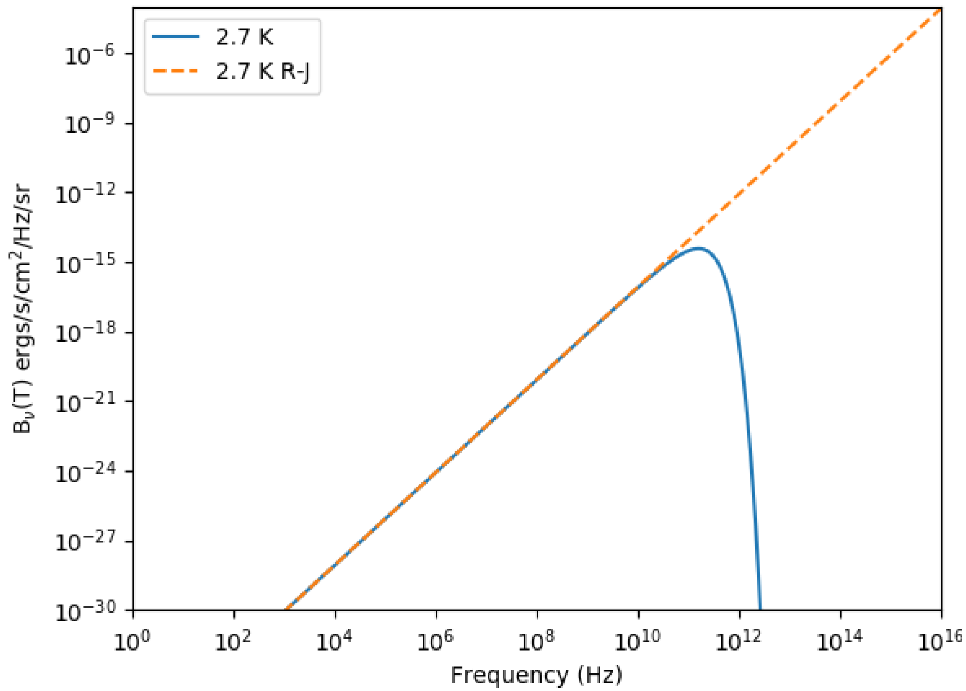
$$B_\nu(T) = \frac{2h\nu^3}{c^2}\frac{1}{e^{\frac{h\nu}{kT}} - 1}$$

h - Planck constant; c - speed of light

k - Boltzmann constant

$$\nu_{max} \approx 59\,T \;\; \text{GHz} \;\; \text{Freq of Peak}$$

BB radiation: Rayleigh-Jeans approx.



$$B_\nu(T) \;\approx\; \frac{2k\nu^2}{c^2}T = \frac{2kT}{\lambda^2}$$

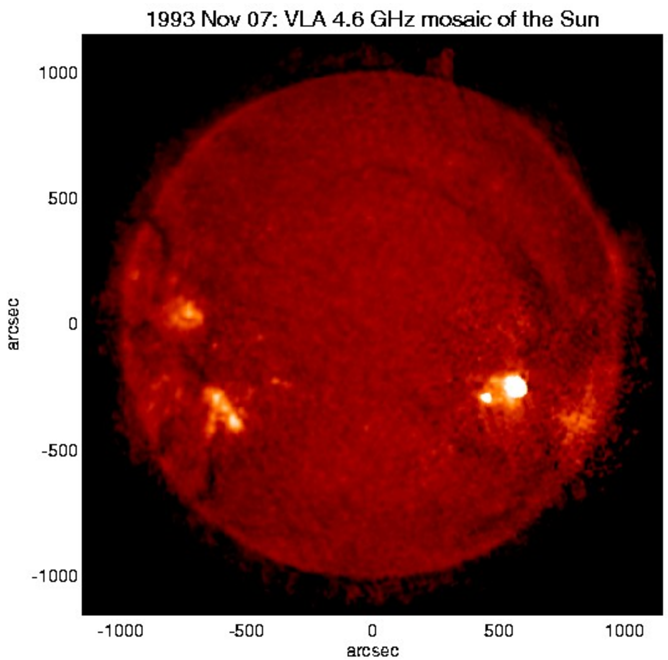
λ - wavelength; For frequencies below ν_{max}

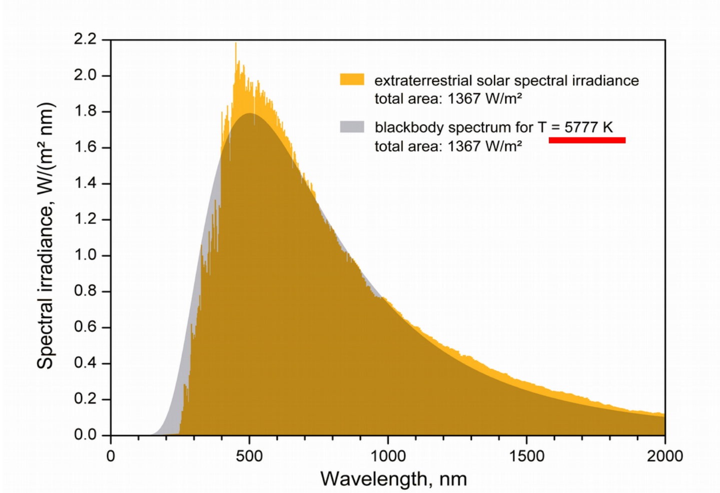
Brightness temperature definition:

$$I_\nu \;=\; \frac{2k\nu^2}{c^2}T_b \quad \text{brightness temperature - } T_b$$

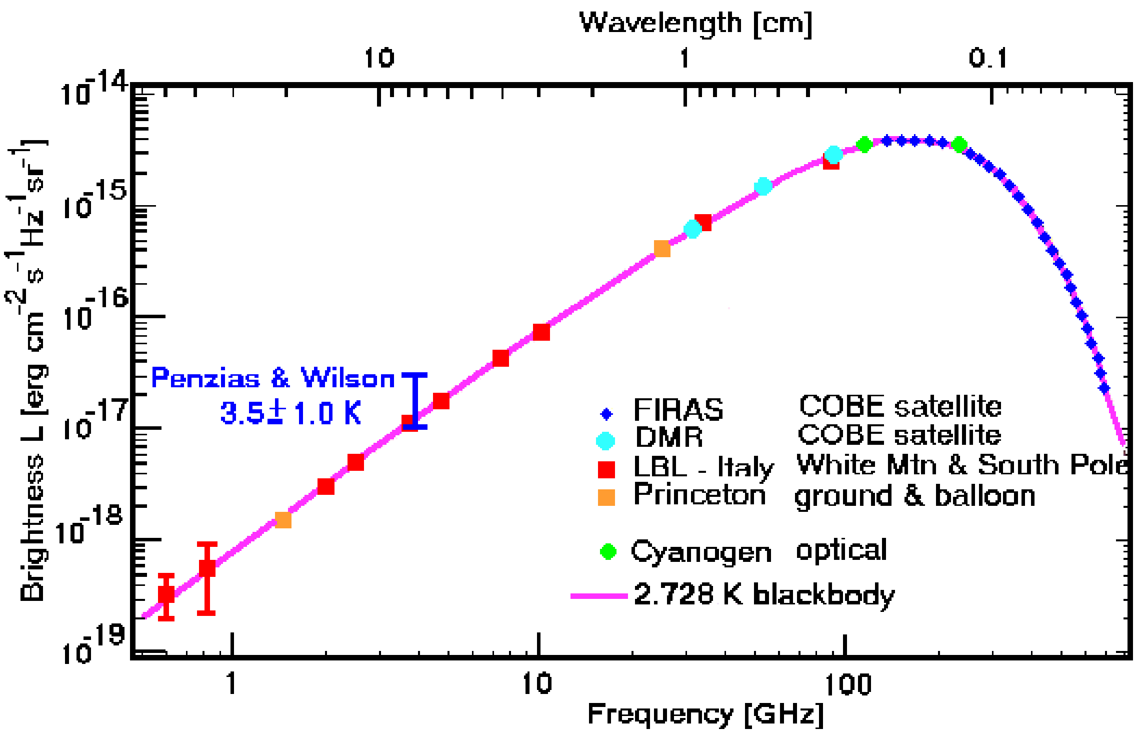
(for BB T_b=T for all freq in R-J approx)

Blackbody radiation in astrophysis



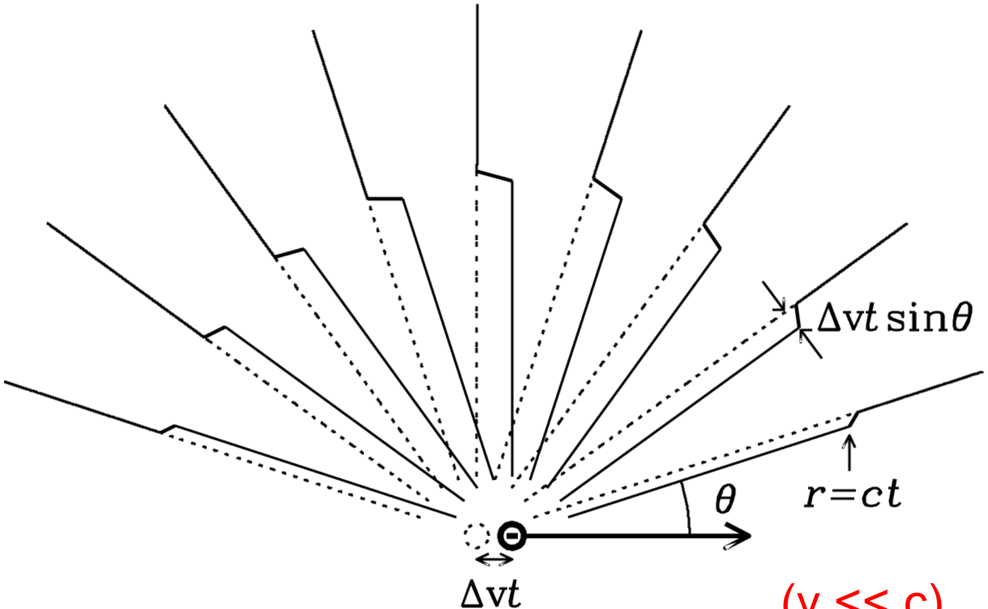


Spectrum of Cosmic microwave background radiation

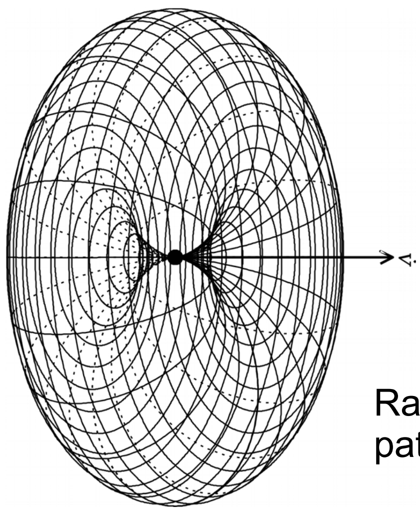


•Why are they blackbody radiations if the absorption and emission are balanced?

Larmor's Equation



(v << c)



Radiation pattern

$$E_\perp = \frac{q\dot{v}\sin(\theta)}{rc^2}$$

q - charge; $\dot{v}$ - acceleration; c - speed of light

r - distance from the charge

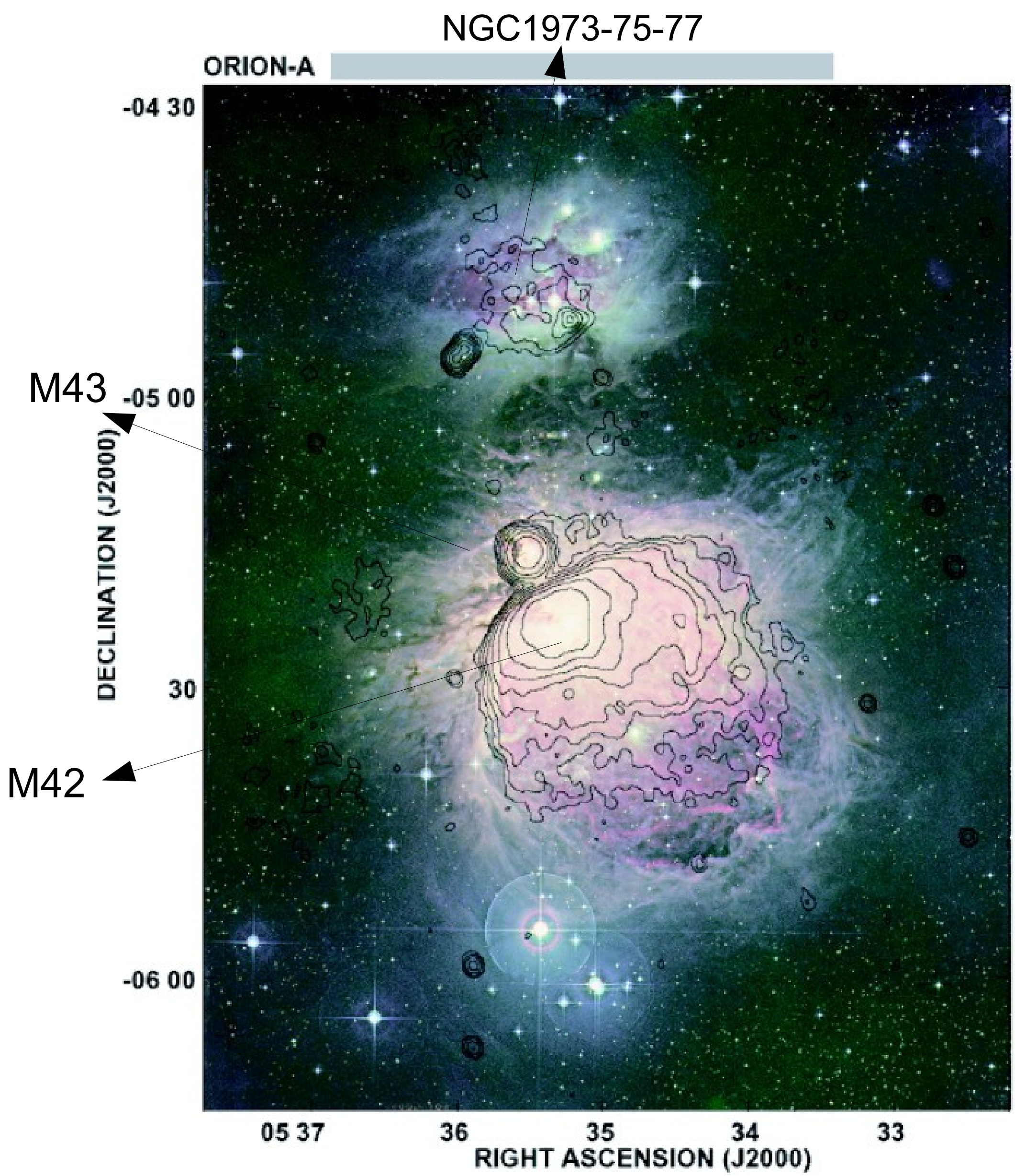
(Note the 1/r dependence compared to $1/r^2$ of Coulomb field; the time dependence of field amplitude comes from time dependence of acceleration.)

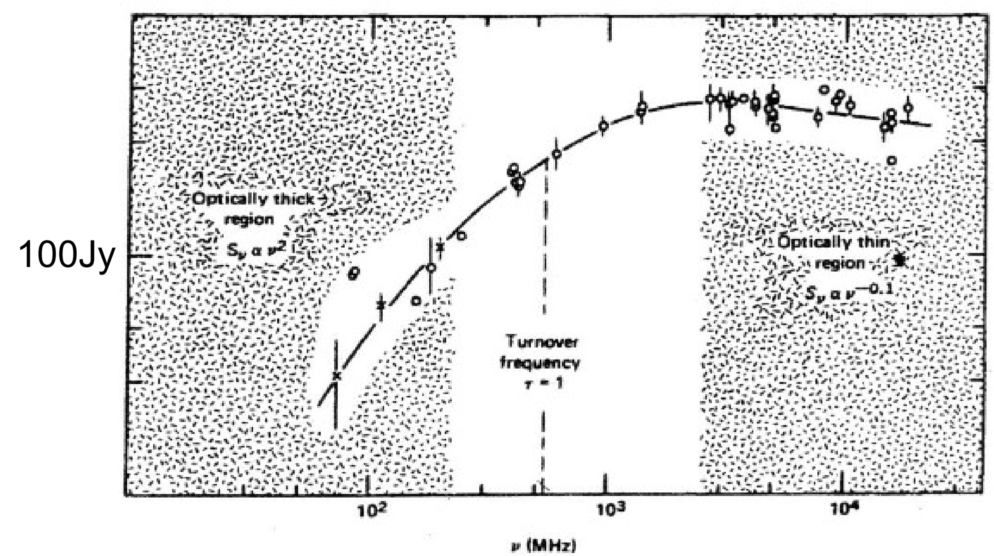
$$S \propto E_\perp^2 \propto \sin^2(\theta)$$

$$P = \frac{2}{3}\frac{q^2\dot{v}^2}{c^3}$$

Larmor's equation; total power radiated by an accelerating charge (depends on the square of the acceleration)

Continuum radiation from HII regions

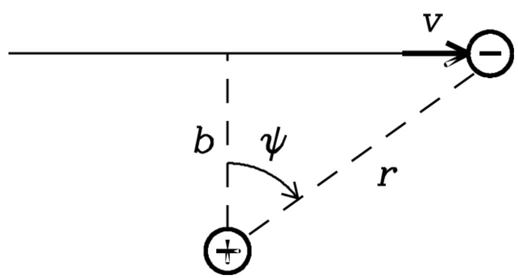


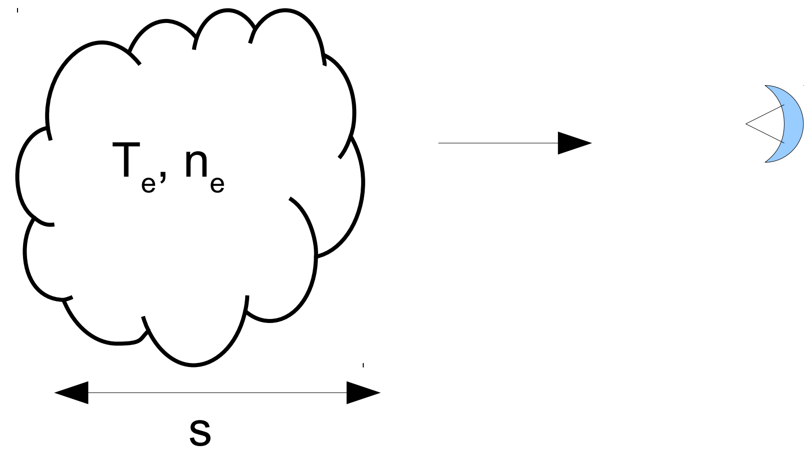


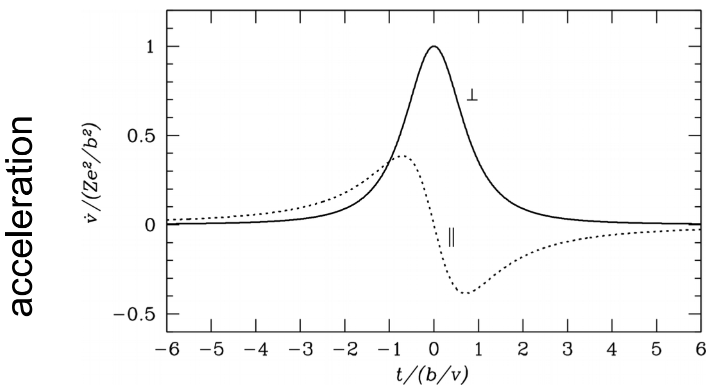
Free-free emission or
bremsstrahlung radiation

VLA 330 MHz radio contours on a UK Schmidt optical photograph of the Orion region (Subrahmanyan et al 2001)

Thermal free-free radiation







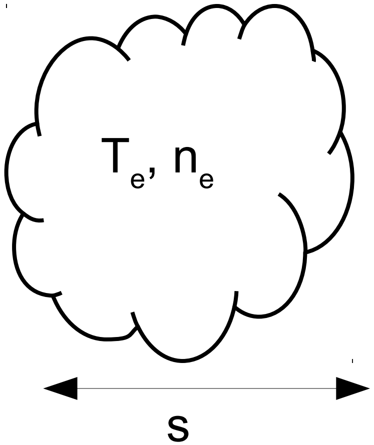
Normalized time

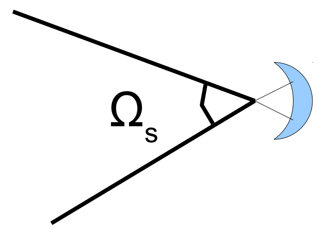
$$\tau \approx 3.28 \times 10^{-7} \left(\frac{T_e}{10^4\ \mathrm{K}}\right)^{-1.35} \left(\frac{\nu}{\mathrm{GHz}}\right)^{-2.1} \left(\frac{\mathrm{EM}}{\mathrm{pc\ cm^{-6}}}\right).$$

Emission measure $$\frac{\mathrm{EM}}{\mathrm{pc\ cm^{-6}}} \equiv \int_{\mathrm{los}} \left(\frac{n_e}{\mathrm{cm^{-3}}}\right)^2 d\left(\frac{s}{\mathrm{pc}}\right).$$

- Radiation from a thermal plasma
- Electrons accelerated by interaction with ions
- Larmor’s eq gives the power radiated by a single interaction
- Emissivity (j_ν) obtained by averaging over all possible interactions
- Absorption coefficient is then obtained using Kirchhoff’s law

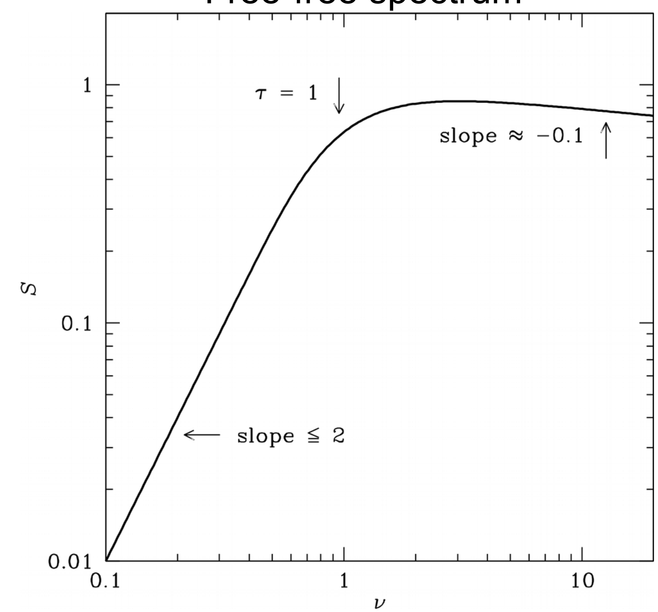
Thermal free-free radiation





$$\boxed{\tau \approx 3.28 \times 10^{-7} \left(\frac{T}{10^4\ \mathrm{K}}\right)^{-1.35} \left(\frac{\nu}{\mathrm{GHz}}\right)^{-2.1} \left(\frac{\mathrm{EM}}{\mathrm{pc\ cm^{-6}}}\right).}$$

Free-free spectrum



$$I_\nu(s_{out}) \; = \; \cancelto{0}{I_\nu(s_{in})} \, e^{-\tau_\nu} + S_\nu(1 - e^{-\tau_\nu})$$

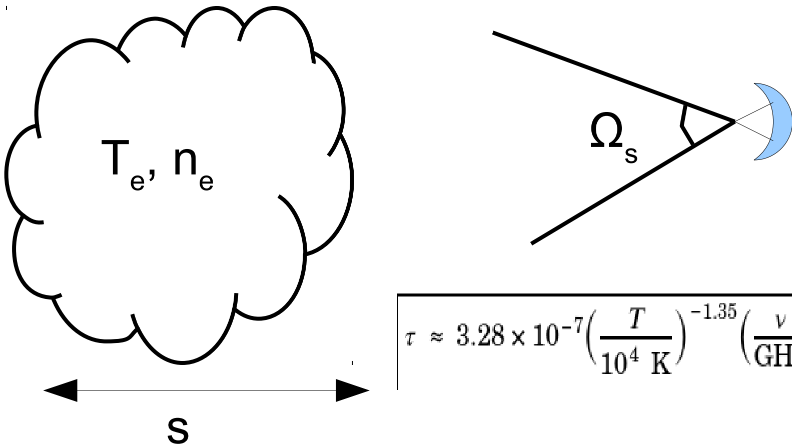
$$S_\nu \; = \; \frac{2k\nu^2}{c^2} T_e$$

$$S \; = \; \int_{\Omega_s} I_\nu(s_{out}) \, \mathrm{d}\Omega = I_\nu(s_{out})\Omega_s \quad \text{Flux density}$$

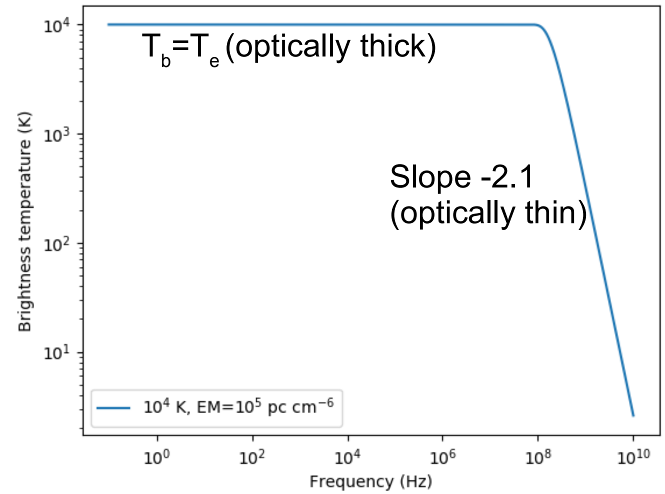
$$S \; \approx \; \frac{2k\nu^2}{c^2} T_e \quad \tau_\nu >> 1 \text{ optically thick}$$

$$S \; \propto \; \frac{2k\nu^2}{c^2} T_e \, \tau_\nu \quad \propto \quad T_e^{0.35} \, \nu^{-0.1} \, \mathrm{EM} \quad \tau_\nu << 1 \text{ optically thin}$$

Thermal free-free radiation



$$\tau \approx 3.28 \times 10^{-7} \left(\frac{T}{10^4\ \mathrm{K}}\right)^{-1.35} \left(\frac{\nu}{\mathrm{GHz}}\right)^{-2.1} \left(\frac{\mathrm{EM}}{\mathrm{pc\ cm^{-6}}}\right).$$



Solution of radiative transfer equation in terms of brightness temperature

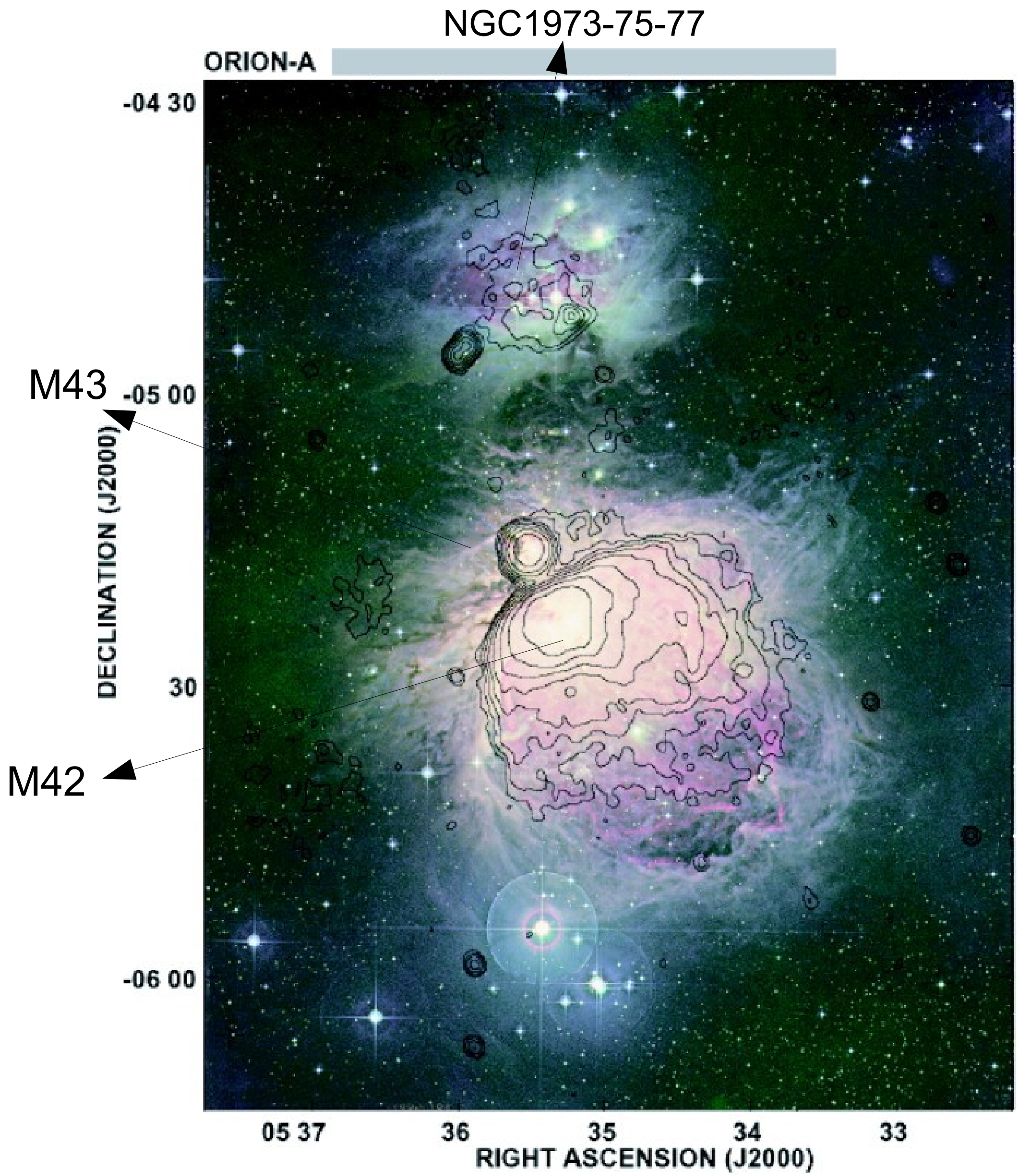
$$I_\nu(s_{out}) = I_\nu(s_{in})\, e^{-\tau_\nu} + S_\nu(1 - e^{-\tau_\nu})$$

$$T_b(s_{out}) = \cancelto{0}{T_{bg}(s_{in})}\, e^{-\tau_\nu} + T_e(1 - e^{-\tau_\nu})$$

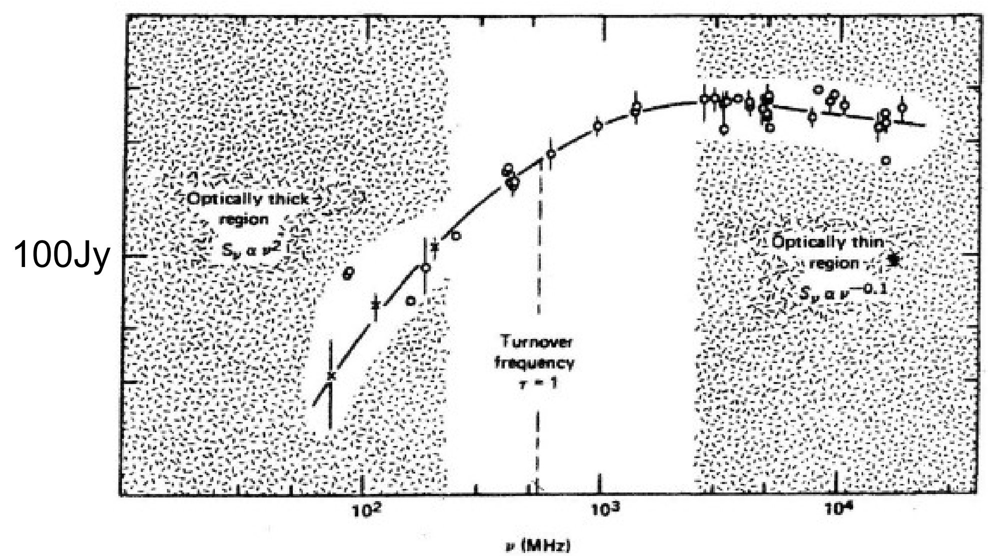
$$T_b(s_{out}) = T_e \quad \text{optically thick } \tau_\nu >> 1$$

$$T_b(s_{out}) = T_e\, \tau_\nu \;\propto\; T_e^{0.35}\, \nu^{-2.1}\, \mathrm{EM} \quad \text{optically think } \tau_\nu << 1$$

Continuum radiation from HII regions



(Subrahmanyan et al 2001)



Multi-freq obs can be used to determine T_e & EM

$$S \approx \frac{2k\nu^2}{c^2} T_e \quad \tau_\nu >> 1 \text{ optically thick}$$

$$S \propto \frac{2k\nu^2}{c^2} T_e \, \tau_\nu \;\propto\; T_e^{0.35}\, \nu^{-0.1}\, \text{EM}$$

(optically thin)

$T_e \sim 10^4$ K; EM$\sim 10^4$ pc cm^{-6} (typical values)

Continuum radiation from Supernovae Remnants

Radio image of Crab Nebula at 5 GHz

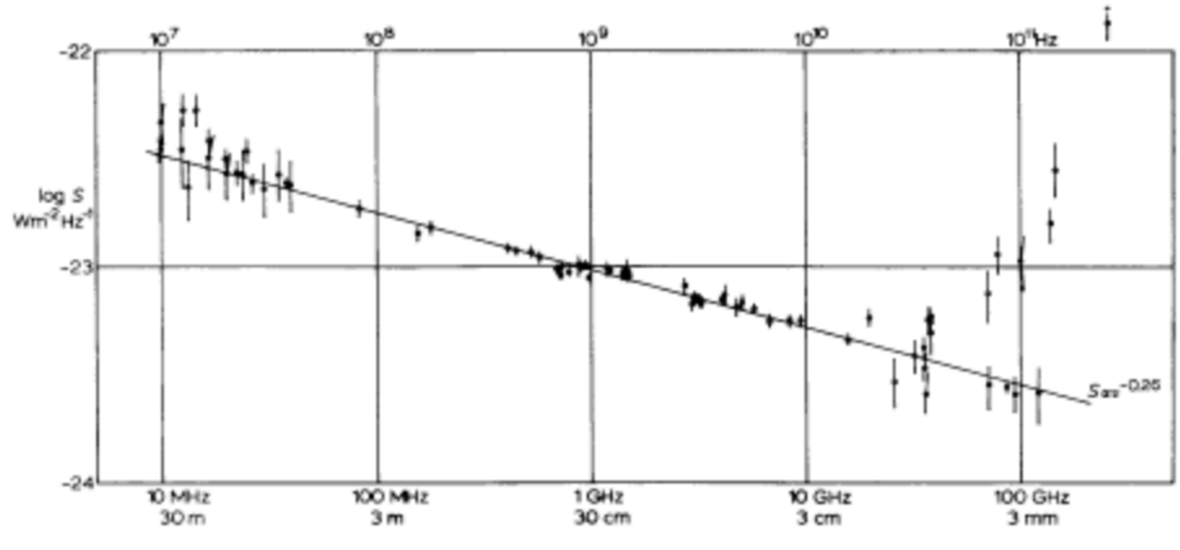
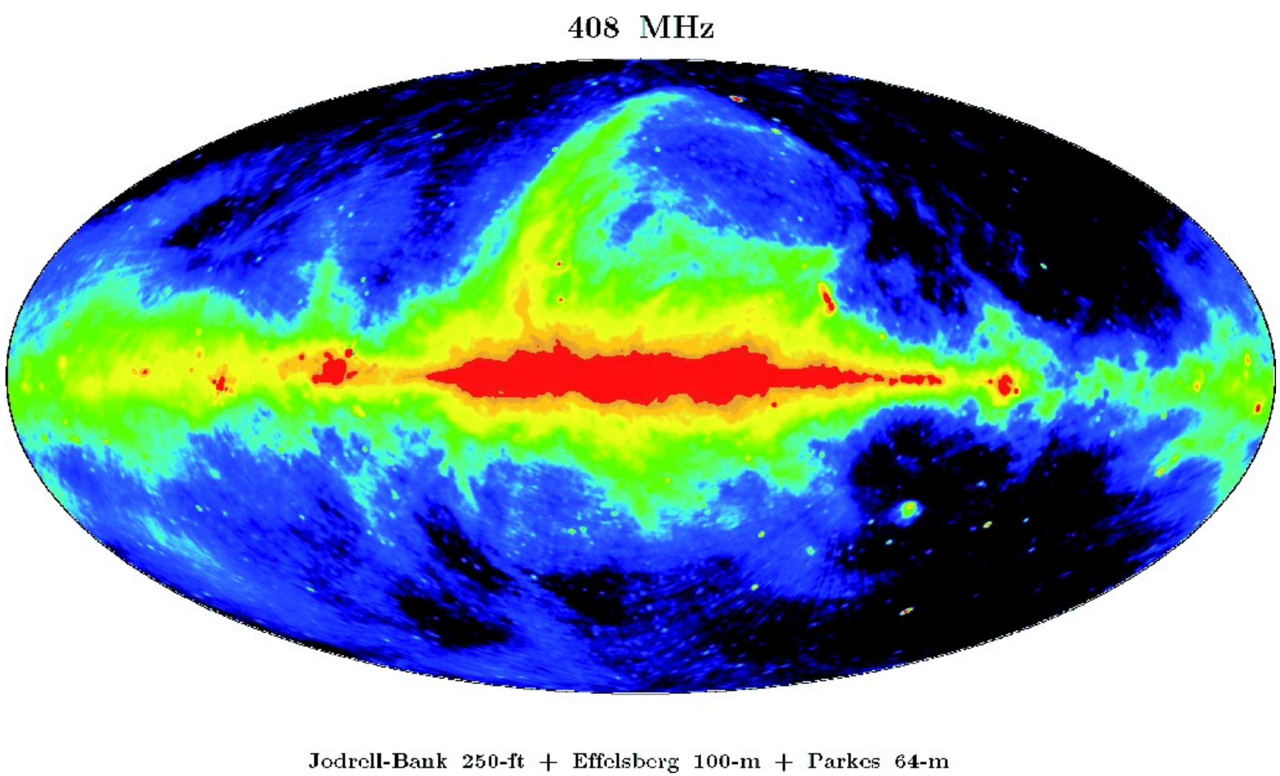


Fig. 1. Radio spectrum of total radiation from the Crab nebula.

Synchrotron radiation due to relativistic electrons gyrating in the nebular magnetic field

Radiation is linearly polarized (~ 8%)

Continuum radiation from the Galaxy



Haslam, Salter, et al. 1982, AA&S, 47, 1

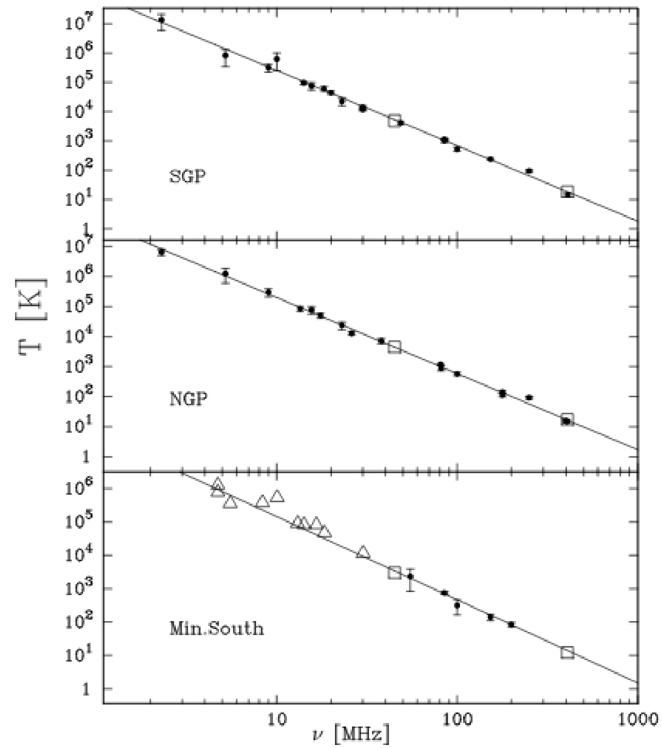
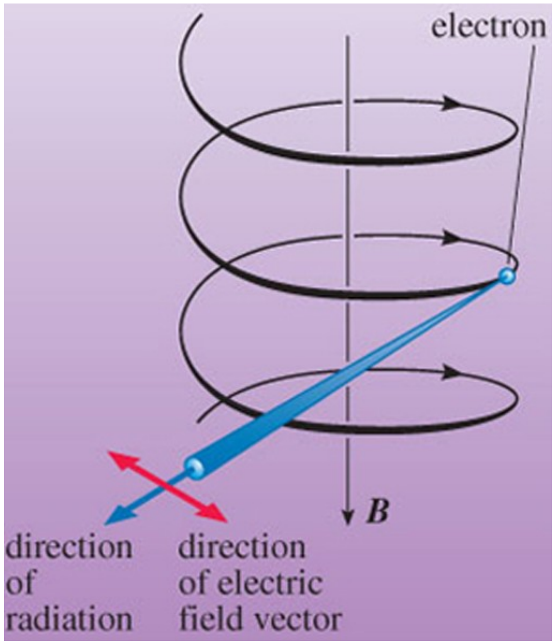


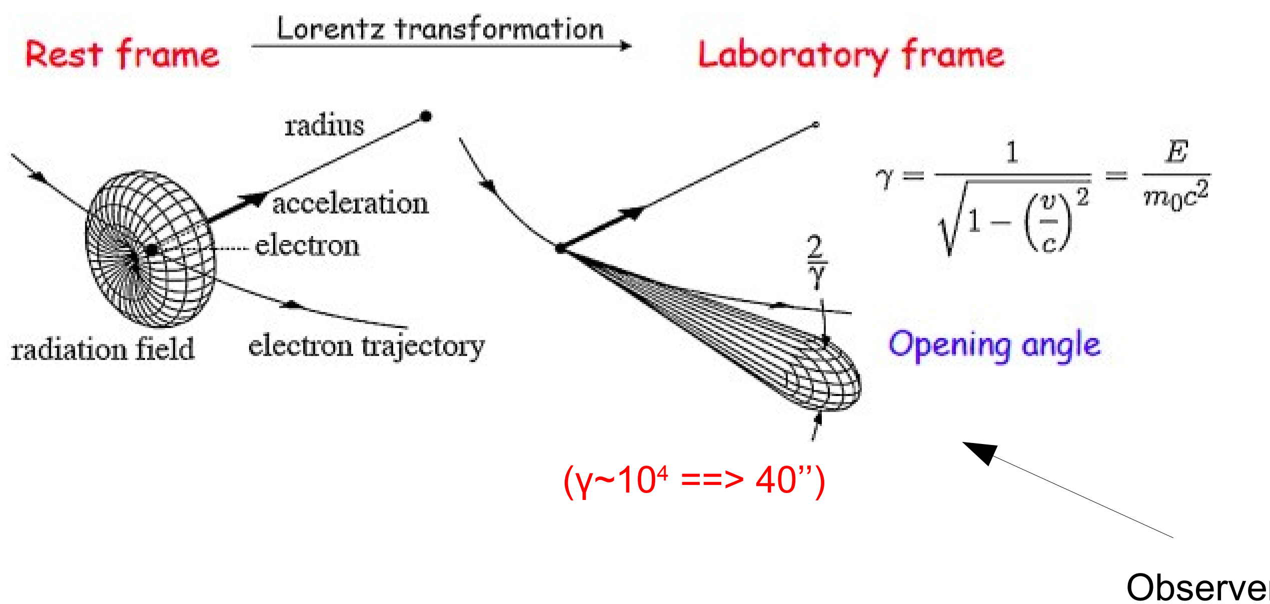
Fig. 3. Multi-frequency spectra of the Galactic component for (*from top to bottom*): south Galactic Pole, North Galactic Pole, and minimum-south zones. The triangles are not considered in the least squares fitting of the line. The squares represent the 45 and 408 MHz data.

Guzman et al 2011, A&A, 525, A138

Synchrotron radiation due to cosmic ray electrons gyrating in the galactic magnetic field. (~ 10% polarized ...)

Synchrotron radiation





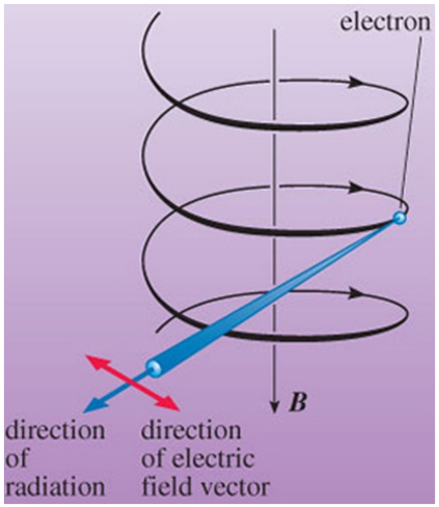
•Radiation produced when ultra-relativistic (E >> $m_e c^2$) electrons are accelerated by magnetic field.

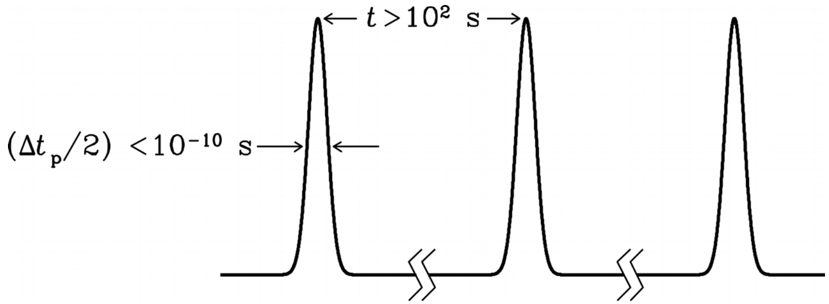
•The radiated power in the rest frame of the electron can be calculated using Larmor's equation.

•The $\sin^2(\theta)$ pattern is transformed to a narrow beam with width ~$2/\gamma$ in the lab frame (i.e. all the radiation power will be in the narrow beam)

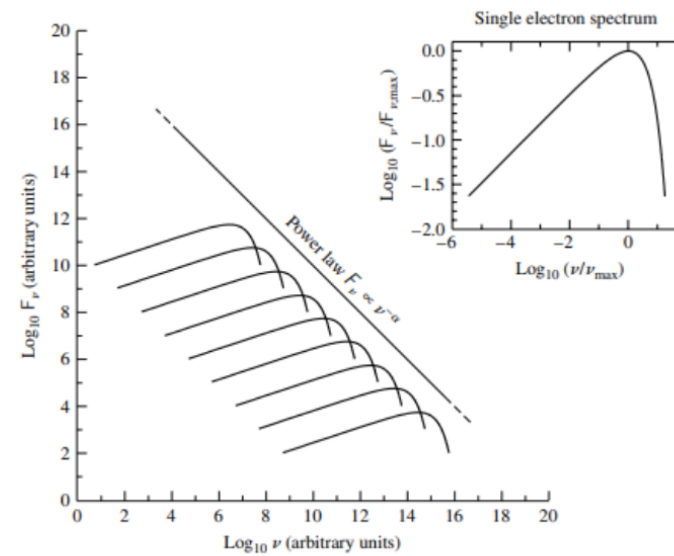
•For a single electron case, the observer sees a train of pulses

Synchrotron radiation





B ~ 10μG; γ ~ 10^4



•Power in each pulse is computed using Larmor's equation.

•Fourier transform of the train of pulses gives the spectrum of radiation by a single electron

$$\nu_{max} \propto E^2 B$$

•Typically sources have electrons with power law distribution

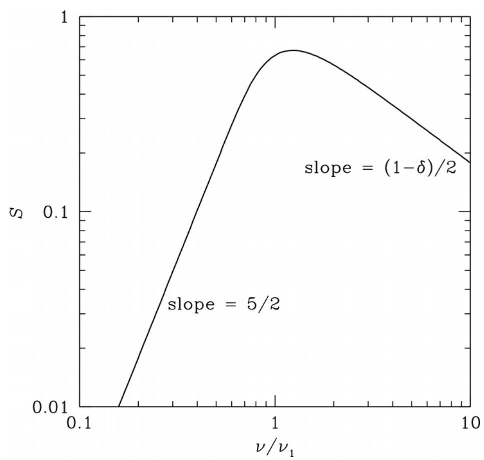
$$n(E)dE \propto E^\delta dE$$

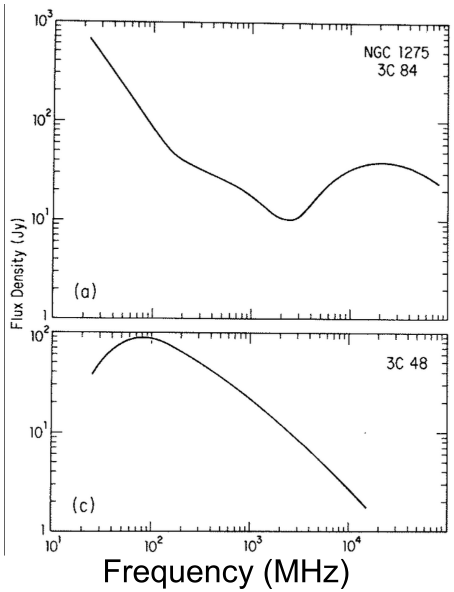
•For optically thin case, the net spectrum from the source is the sum of emission from all electrons

$$\boxed{j_\nu \propto B^{(1+\delta)/2}\nu^{(1-\delta)/2}}$$

Slope of the spectrum determined by δ

Synchrotron self-absorption





Synchrotron spectrum of astronomical sources

Effective temperature $$E = 3kT_{eff} \quad \text{not } \frac{3}{2}kT_{eff} \text{ since ultra-relativistic}$$

Electrons of energy E emits at $\nu_{max} \propto E^2 B$

$$T_{eff} = 1.18 \times 10^6 \, \nu^{1/2} B^{-1/2} \quad \nu \text{ in Hz; } B \text{ in G}$$

$T_{eff} \sim 10^{12}$ K (100 MHz, B= 10μG)

Brightness temperature cannot exceed effective temperature

$$I_\nu = \frac{2k\nu^2}{c^2} T_b \propto \nu^{5/2} B^{-1/2} \quad \text{setting } T_b = T_{eff}$$ (optically thick)

$$S \propto \nu^{5/2} B^{-1/2}$$

Thank You