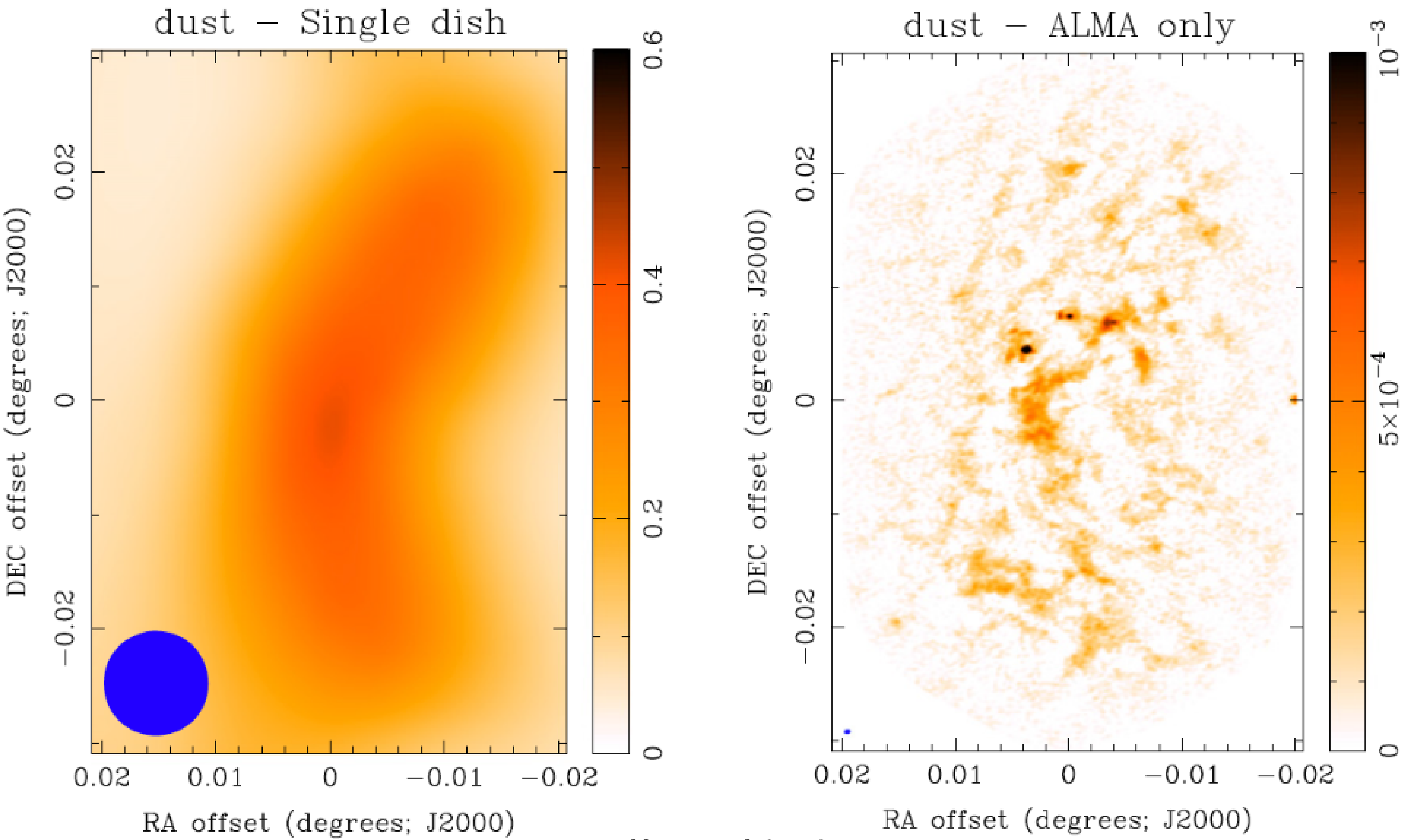
Combining Single Dish & Interferometer Data

--- Tapasi Ghosh, with many thanks to Natalie Butterfield and Snezana Stanimirovich

SDSS-2021

Imaging SD vs Interferometer Data

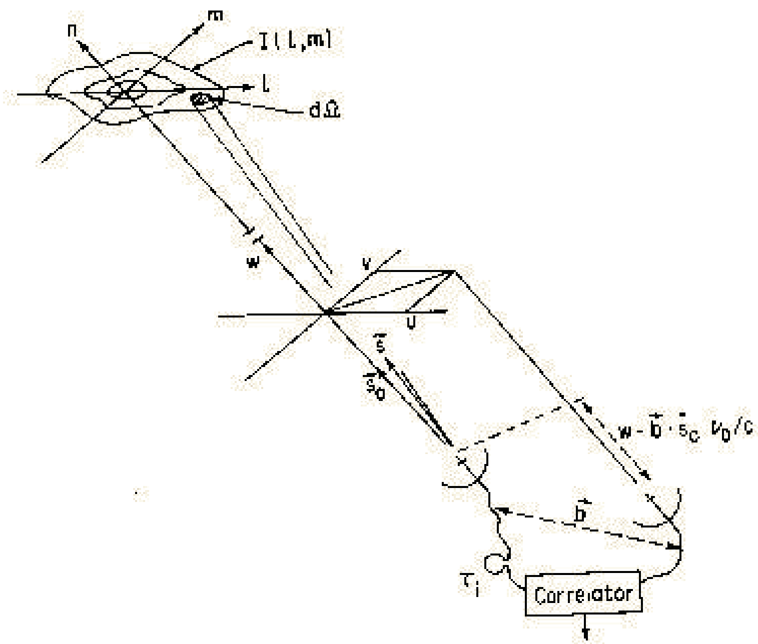


Rathborne et al. (2014)

“Single Dish data” <= extrapolated from to 3mm from 500-micron Herschel data (33” beam) using a “greybody” spectrum. Dust continuum and molecular line emission in a particular region of the CMZ

The fundamental idea behind Interferometry :

There exists a Fourier Transform relation between the sky brightness distribution, I and the response of a radio interferometer.

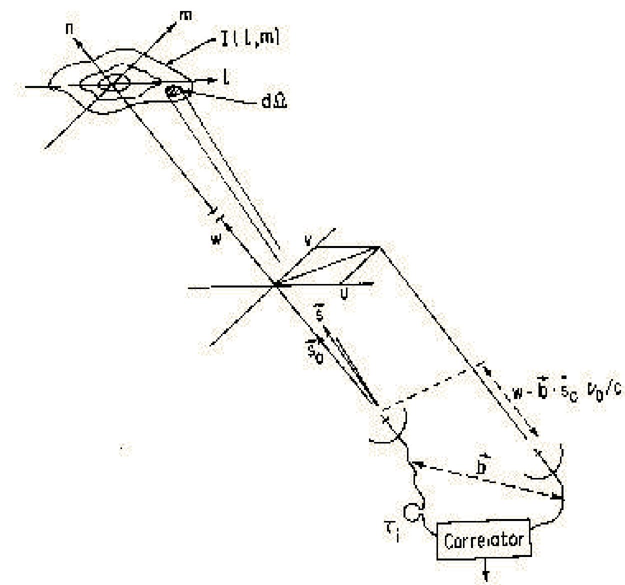


If the distance between two antennas (the baseline) is **d**, then the so-called visibility function, $V(\mathbf{d})$, is given by:

$$V(\mathbf{d}) = \int_{\text{source}} A(\boldsymbol{\sigma}) I(\boldsymbol{\sigma}) \exp\left[-2\pi i\, \mathbf{d} \cdot \boldsymbol{\sigma}/\lambda\right] d\Omega\ . \tag{1}$$

Here, $A(\boldsymbol{\sigma})$ is an antenna reception pattern, or **primary beam**, and $\boldsymbol{\sigma}$ is the vector difference between a given celestial position and the central position of the field of view. The **aperture synthesis technique** is a method of solving Equation 1 for $I(\boldsymbol{\sigma})$ by measuring V at suitable values of **d**.

Simplifying for a small, E-W array, with d=(u,v,w) and w axis pointing toward the celestial Pole, u –> N, V→E :



$$V(u,v) = \int\int A(l,m) I(l,m) \exp[-2\pi i(ul+vm)] \frac{dldm}{\sqrt{1-l^2-m^2}} \,. \qquad (2)$$

Therefore, the visibility function $V(u,v)$ can be expressed as the Fourier transform of a modified brightness distribution $A(l,m)I(l,m)$. Coordinates u and v $(w=0)$ are measured in units of wavelength and the $u-v$ plane is called **the spatial frequency domain**. These are effectively projections of a terrestrial baseline onto a plane perpendicular to the source direction. The $l-m$ plane is referred to as **the image domain**.

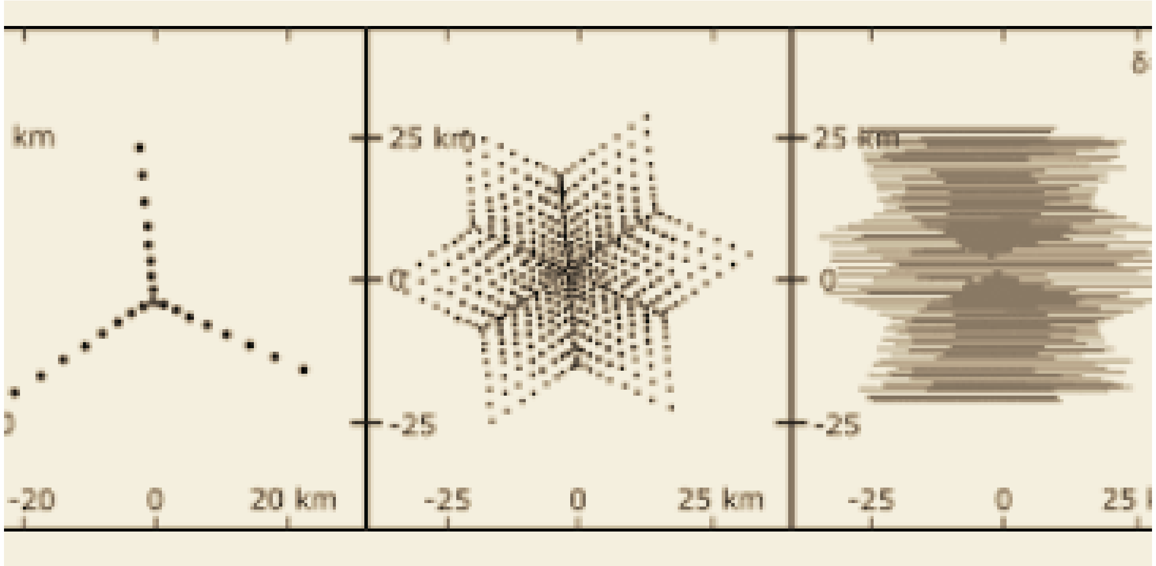
Hence, if $V(u,v)$ is a true (ideal) visibility function, the measured (observed) visibilities ($V'_{\rm int}$) can be expressed as:

$$V'_{\rm int}(u,v) = V(u,v)b_{\rm int}(u,v)\ . \quad (3)$$

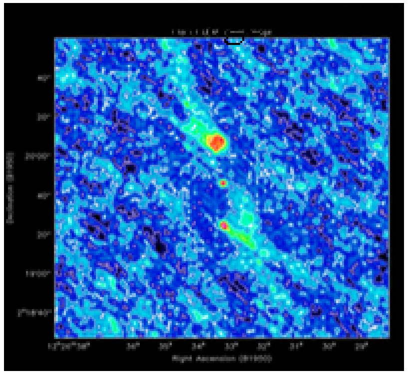
$b_{\rm int}$ is usually representable by a set of δ-functions, between the lowest and the highest spatial frequency sampled by the interferometer (corresponding to the shortest and the longest baselines, respectively). The Fourier transform of Equation 3 gives the observed sky brightness distribution $I^{\rm D}_{\rm int}$ (so called **'dirty' image**):

$$I^{\rm D}_{\rm int}(l,m) = I(l,m) * B_{\rm int}(l',m')\ , \quad (4)$$

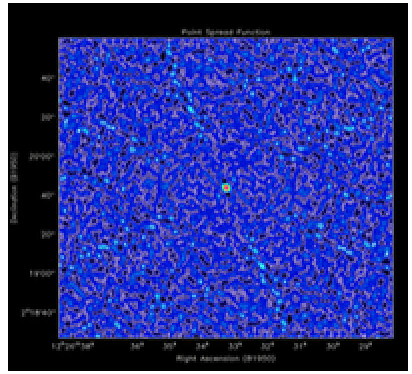
where $B_{\rm int}$ is **the synthesized or 'dirty' beam**, which is the point source response of the interferometer. As usually, asterisks ($*$) are used to denote convolution. When imaging, incomplete $u-v$ coverage leads to severe artifacts



Dirty Image

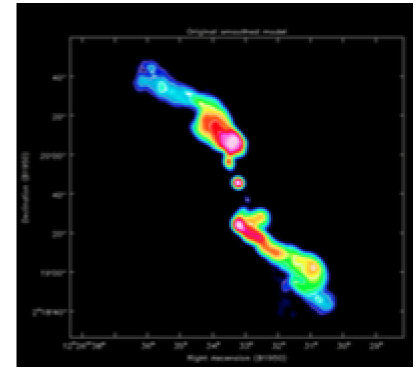


Dirty Beam

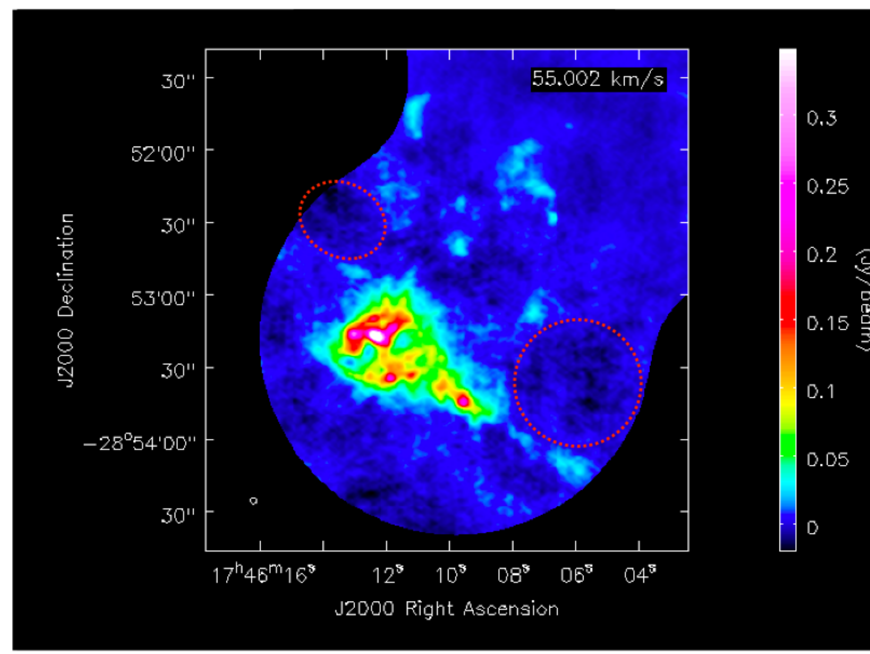


Deconvolution

“Cleaned” Image



For imaging larger objects, with angular sizes > λ / d_{min}, mosaic technique can help filling the u-v coverage, but the central (d_{min} – D/2) region can not be filled. This is often referred to as the “short-spacing problem”.



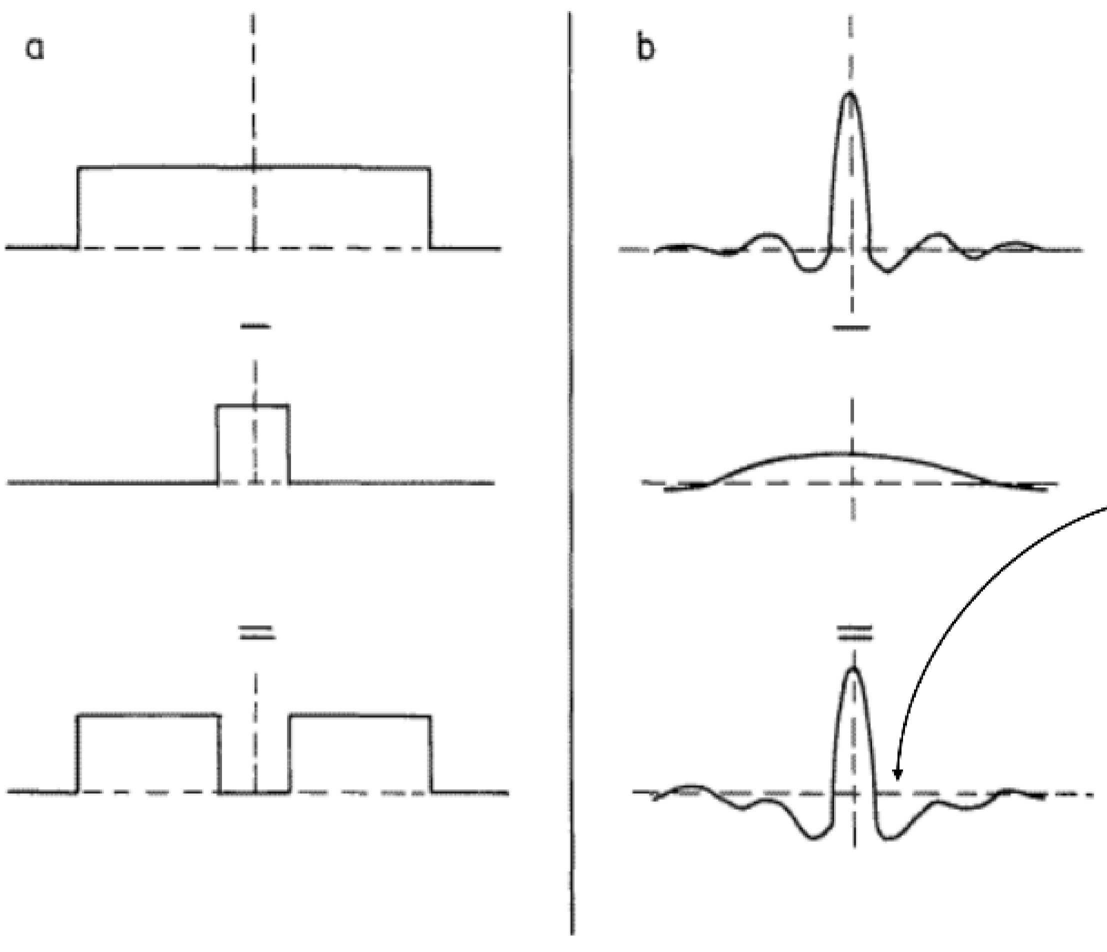
Science behind ‘Negative Bowls’

- UV plane
- PSF
- Braun & Walterbos (1985)

ideal

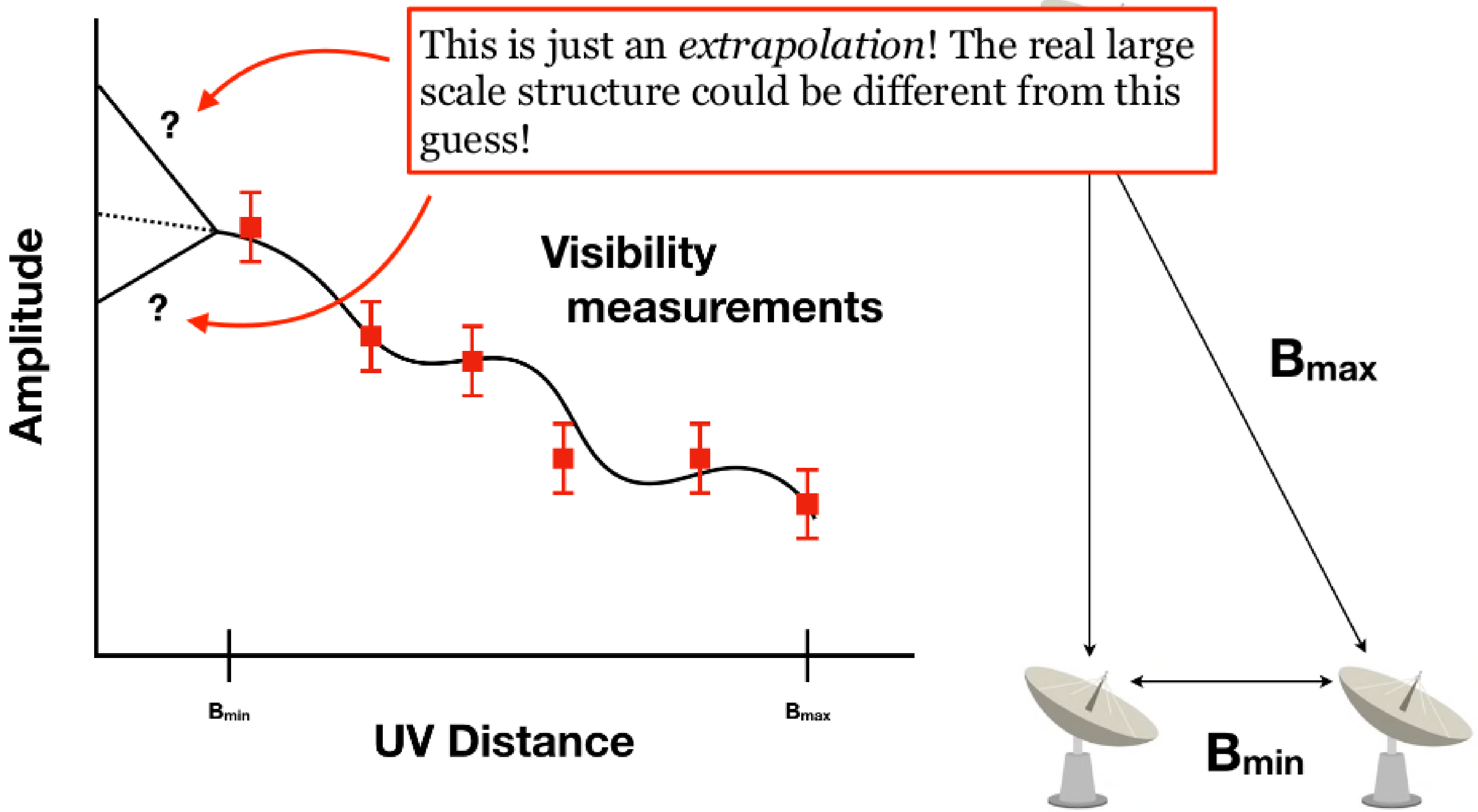
Central hole

Typical interferometer



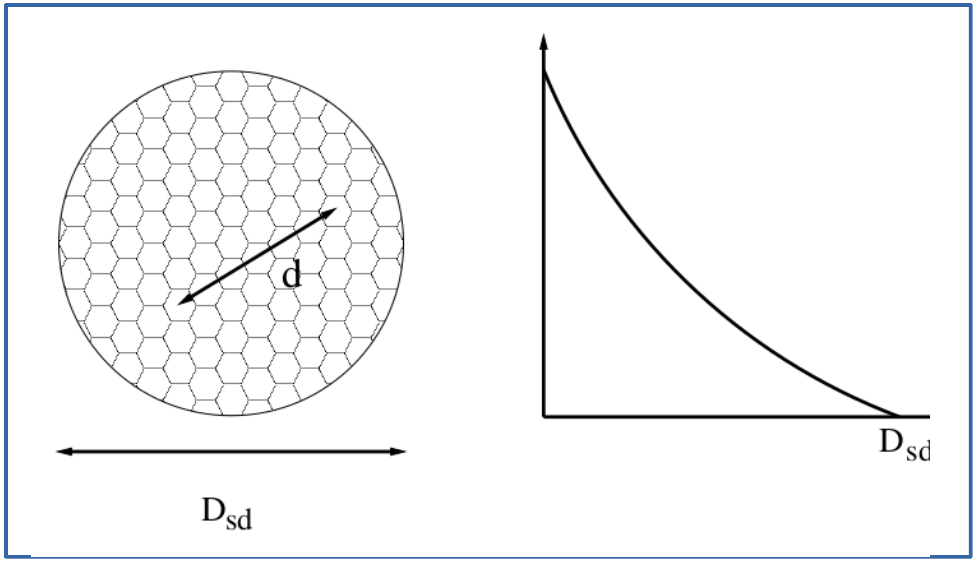
- Negative ‘bowl’

Measuring Visibilities



Single Dish as an Interferometer

A Single Dish can be thought of to be a collection of many small panels, acting as interferometer elements with their signals being combined at the focus.



The observed sky brightness distribution $I^{\mathrm{D}}_{\mathrm{sd}}$ in the case of single-dish observations is then given by:

$$I^{\mathrm{D}}_{\mathrm{sd}}(l,m) = I(l,m) * B_{\mathrm{sd}}(l',m'), \tag{5}$$

with B_{sd} being the **single-dish beam** pattern. The Fourier transform of Equation 5 gives the observed single-dish 'visibilities', V'_{sd}:

$$V'_{\mathrm{sd}}(u,v) = V(u,v) \times b_{\mathrm{sd}}(u,v) \tag{6}$$

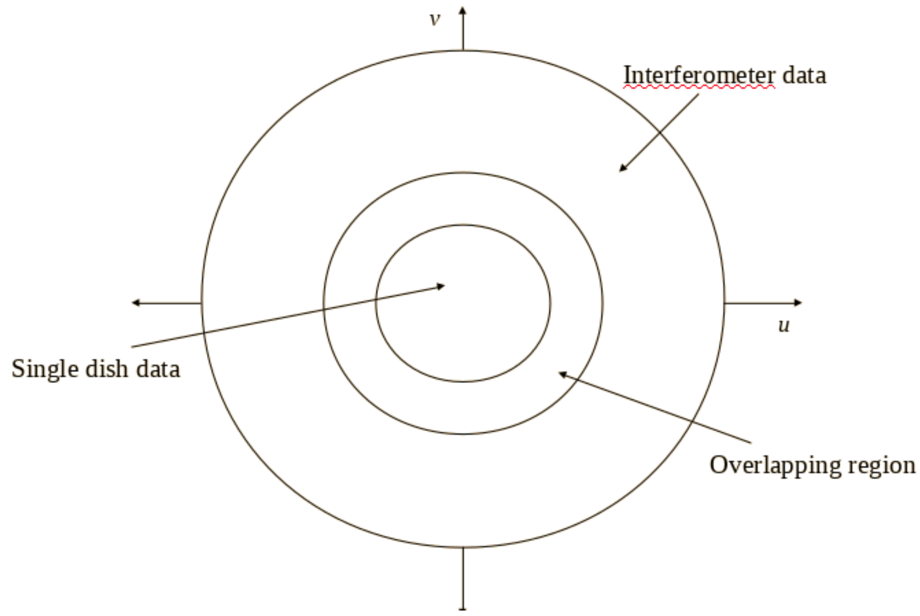
where b_{sd} is the Fourier transform of the single-dish beam pattern which, unlike b_{int}, is a continuous function between zero and the highest spatial frequency sampled by the single-dish. Determination of I from $I^{\mathrm{D}}_{\mathrm{sd}}$ requires deconvolution,

- Before adding single dish data in any manner ,one needs a relative calibration factor

$$f_{cal} = \frac{S_{\text{int}}}{S_{sd}}$$

by which the single dish data should be multiplied

The u-v plane



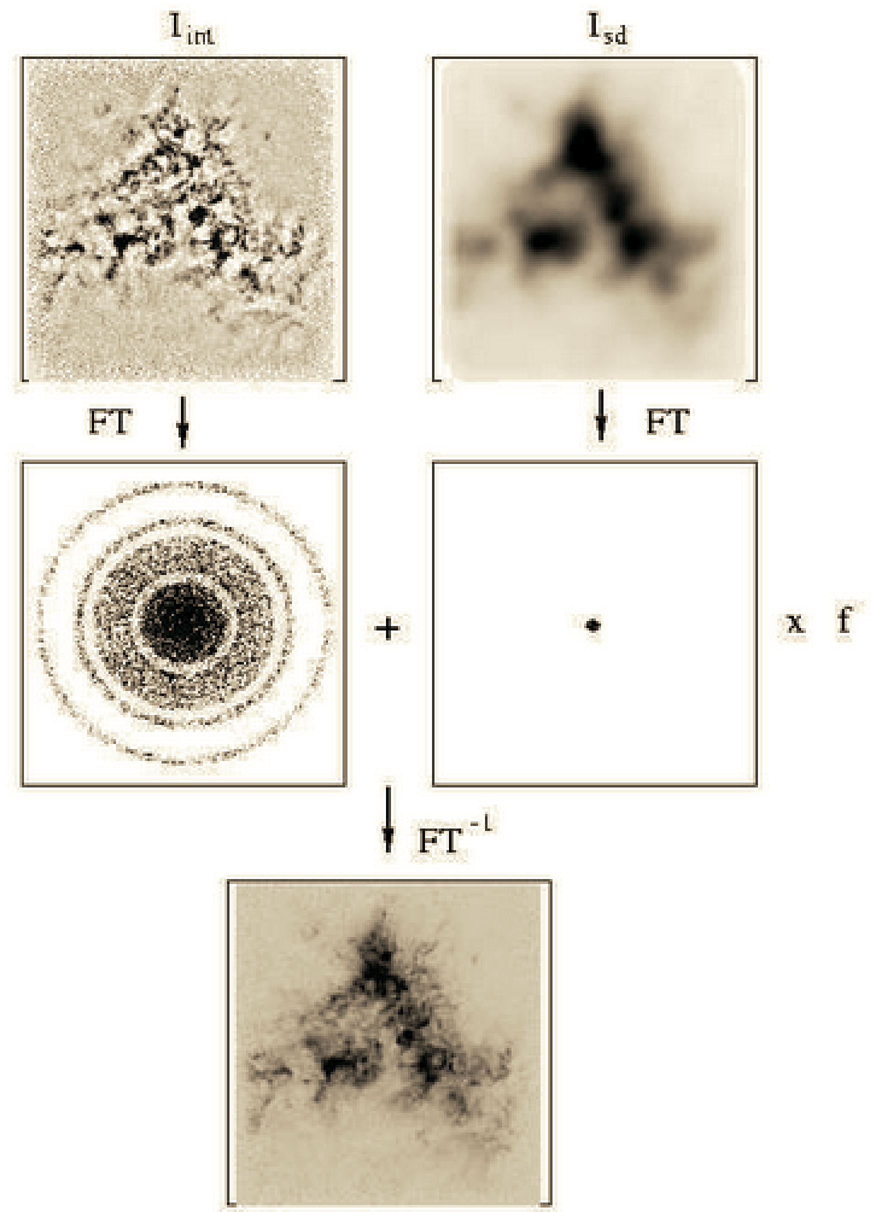
- If the calibration is perfect
- If $D_{sd} > b_{\min}$ one can compare the fluxes in the overlap region to determine

$$f_{cal} = \frac{I_{\text{int}}}{I_{sd}}$$

Method 1:

Adding the SD & Intf. Data in the UV plane.

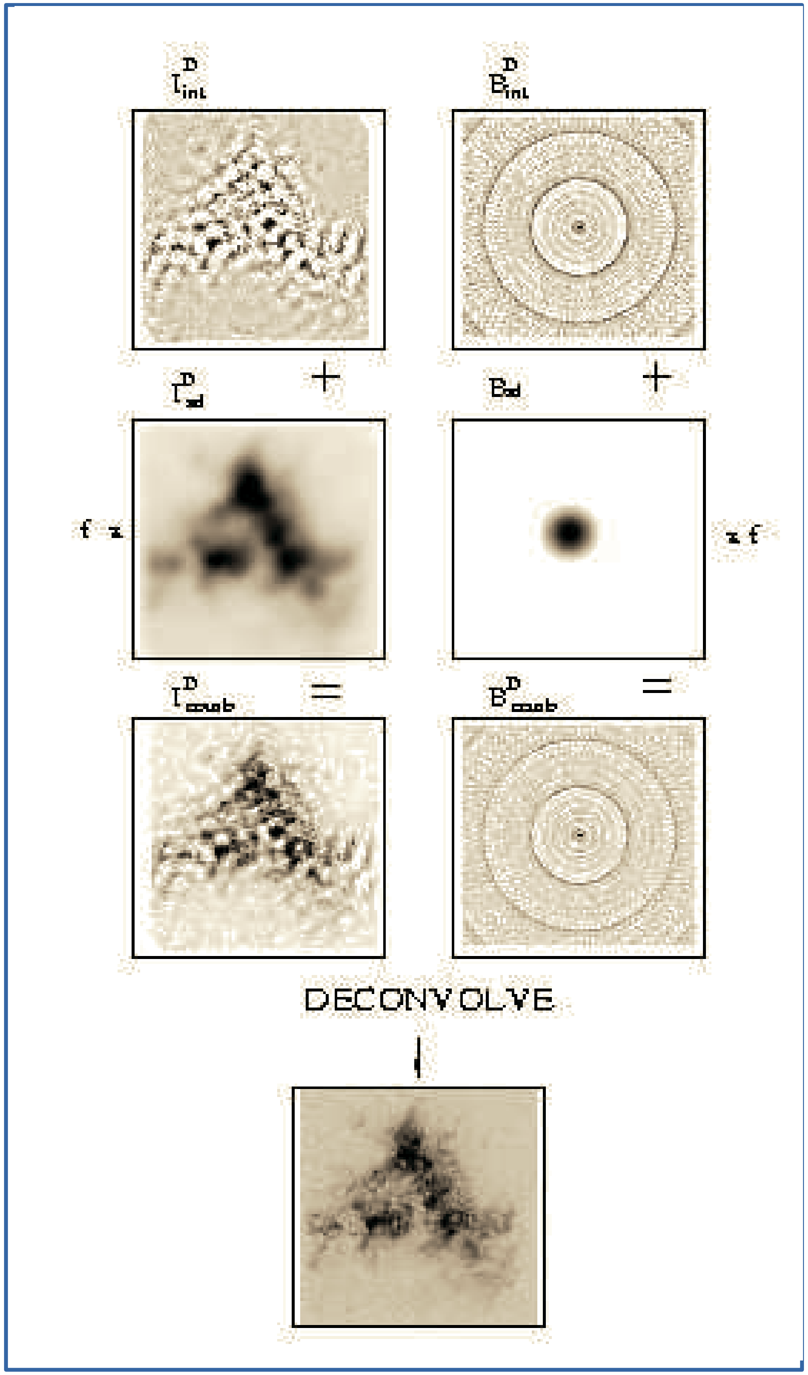
(a) FT each images
(b) Add –with Scaling Factor
(c) Inverse FT to the image plane



Method 2:

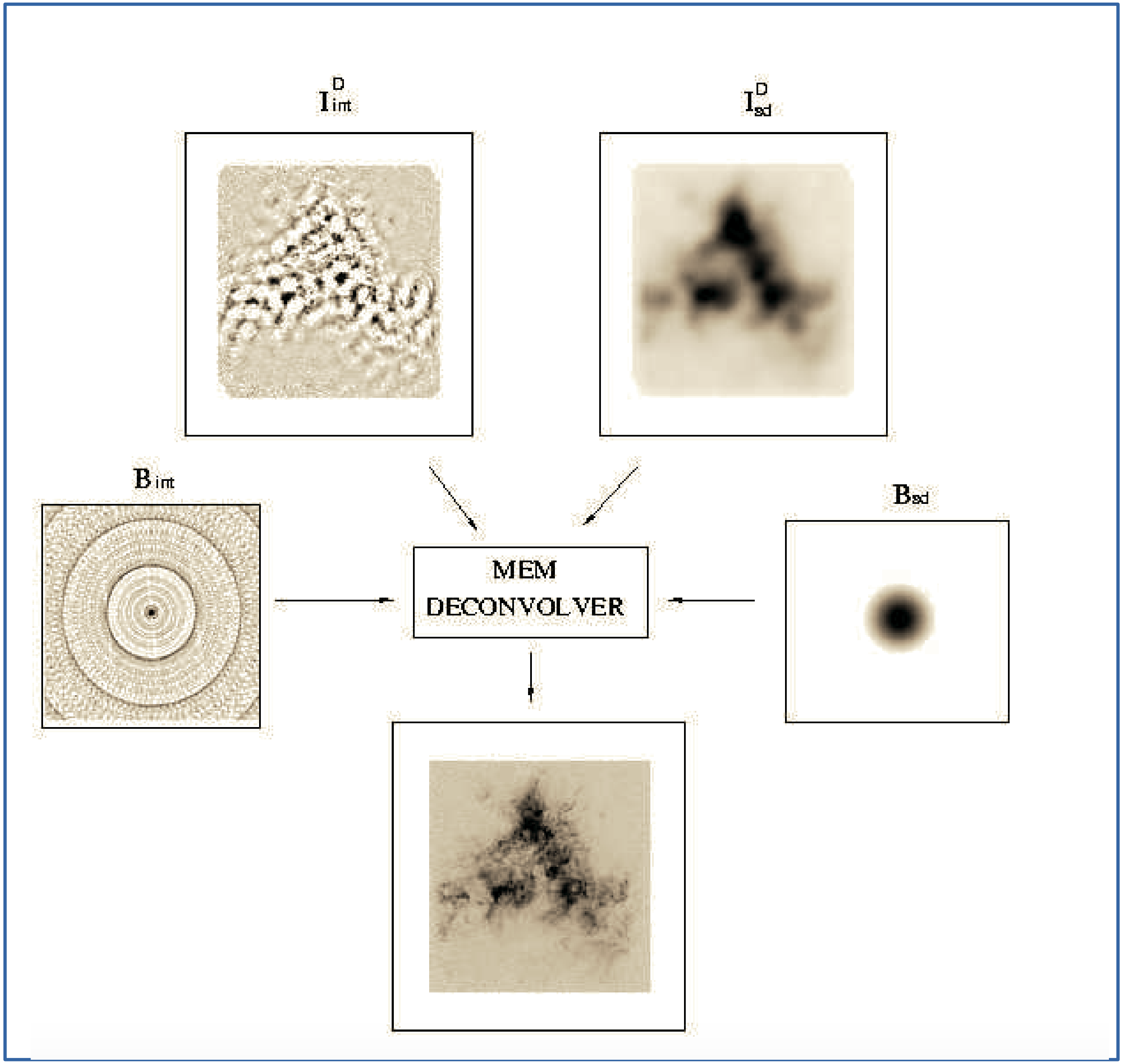
Adding the SD & Interferometer data in the Image plane.

(a) Add the images with Scaing Factor
(b) Add the beams
(c) Deconvolve combined dirty beam From the combined dirty image



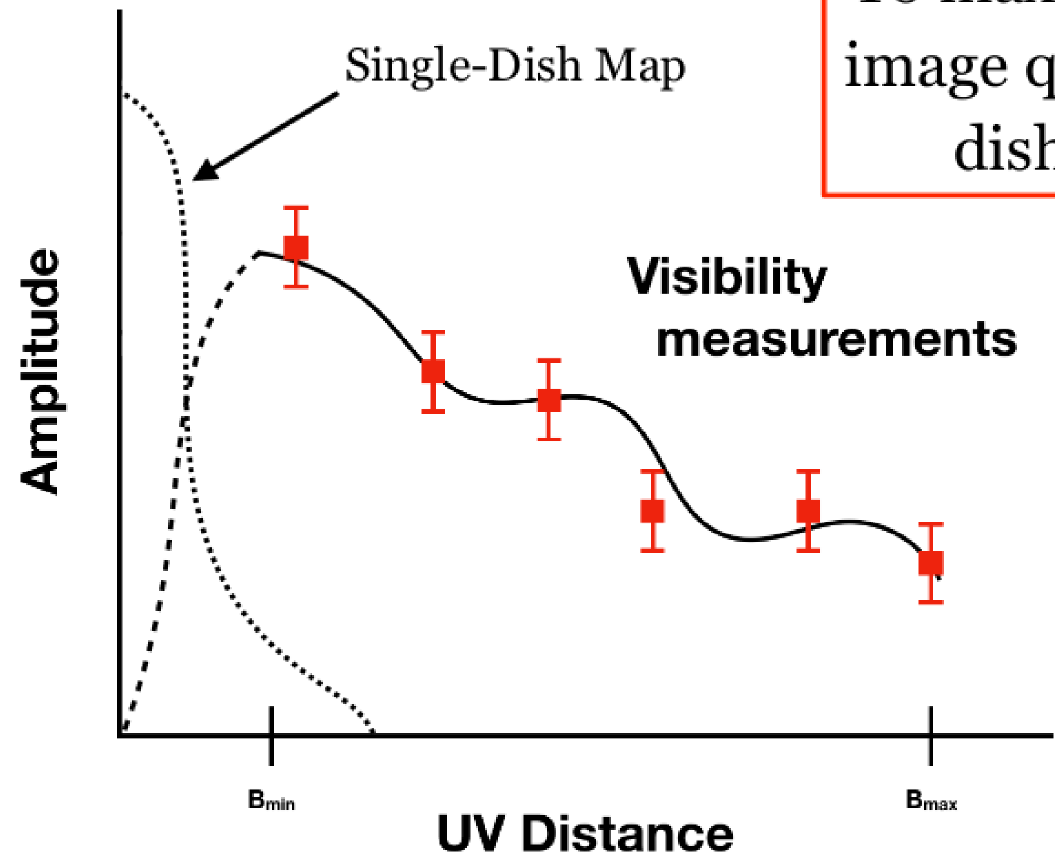
Method 3:

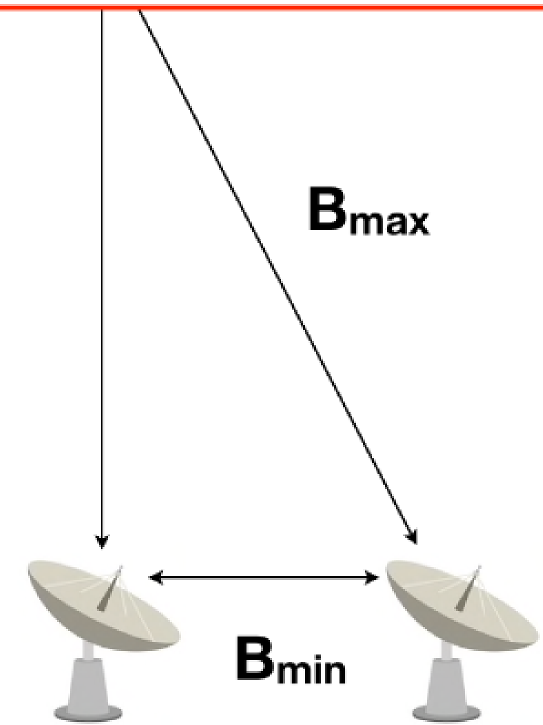
Non Linear deconvolution Technique using Maximum Entropy Method



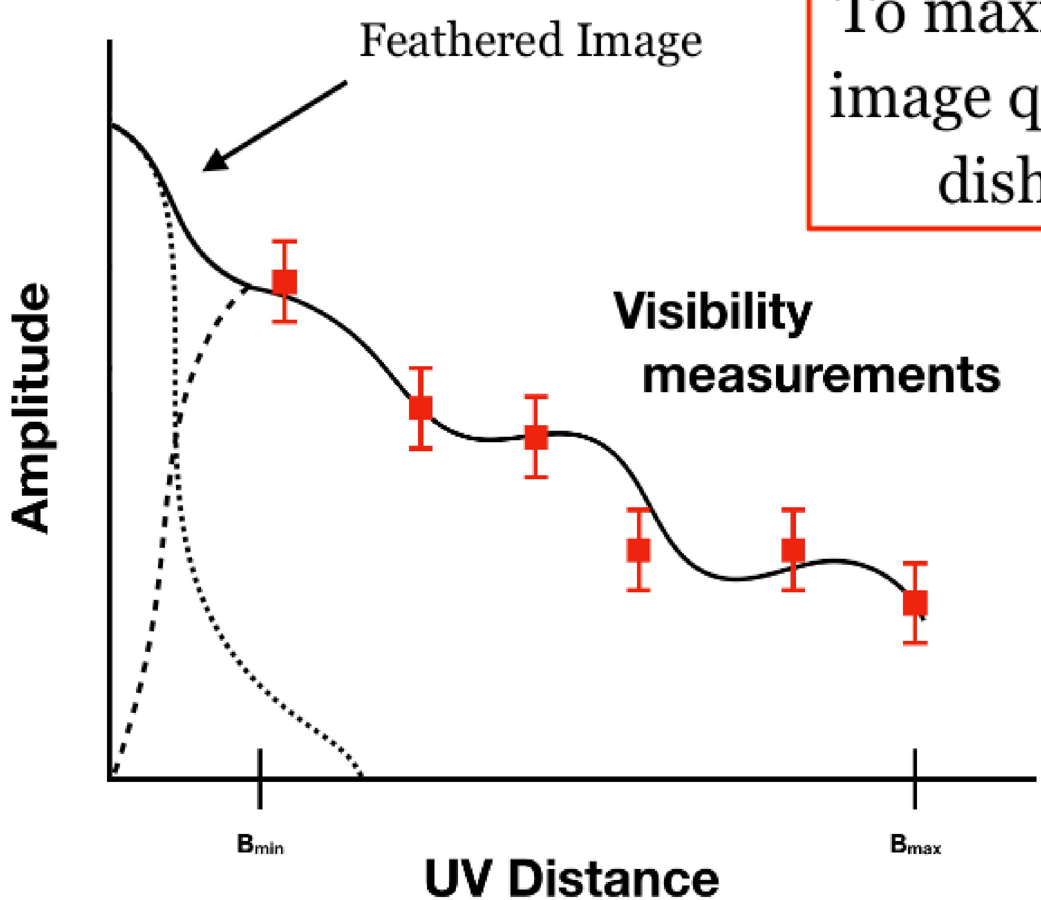
Measuring Visibilities

To maximize flux recovery and image quality you want a single dish size of $D>1.5x\ B_{min}$

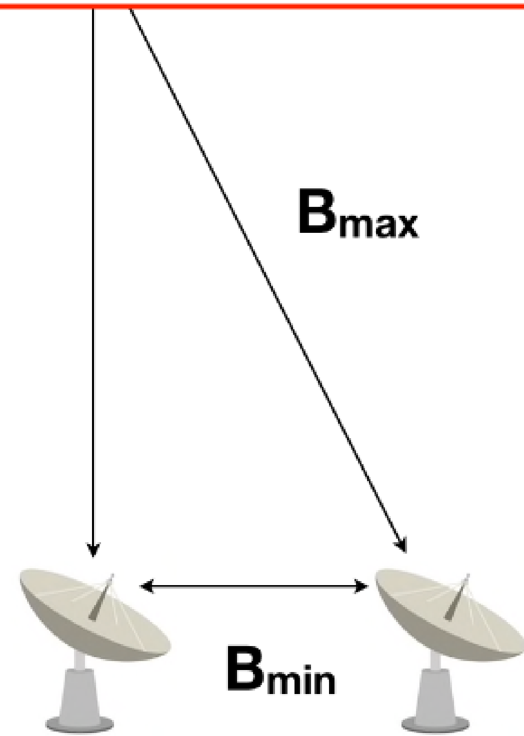




Measuring Visibilities

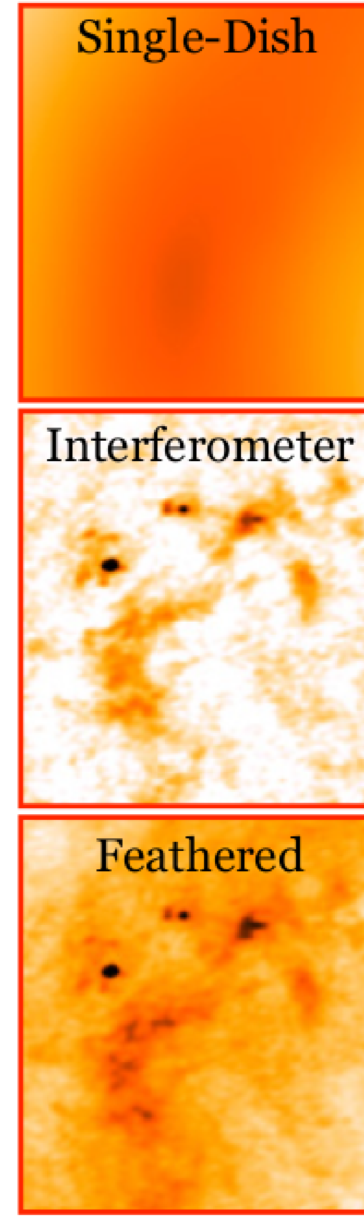


To maximize flux recovery and image quality you want a single dish size of $D > 1.5x\ B_{min}$



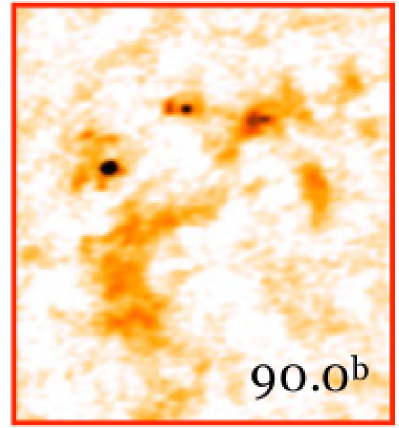
Mathod 1 : Feathering in CASA

- Combination of SD+IF data is ‘feathering’
    - Fill in short UV-spacing information
- To maximize flux recovery and image quality you want a single dish size of D>1.5x B_{min}
    - For the GBT: VLA arrays D & C; ALMA arrays C43-1 —C43-7
    - For Arecibo: VLA array D, C, & B
- CASA task: ‘Feather’
- Valid Flux Measurements
    - Need single-dish data to get valid flux measurements



Valid Flux Measurements

Missing flux from large scale structures can effect measurements!



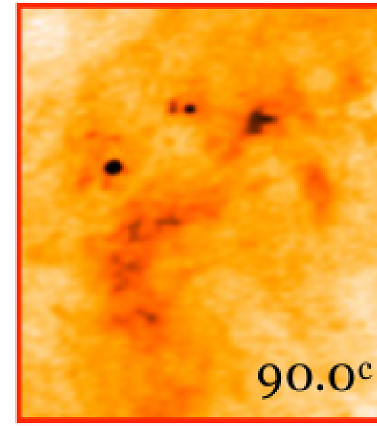


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Table 3
Continuum Regions

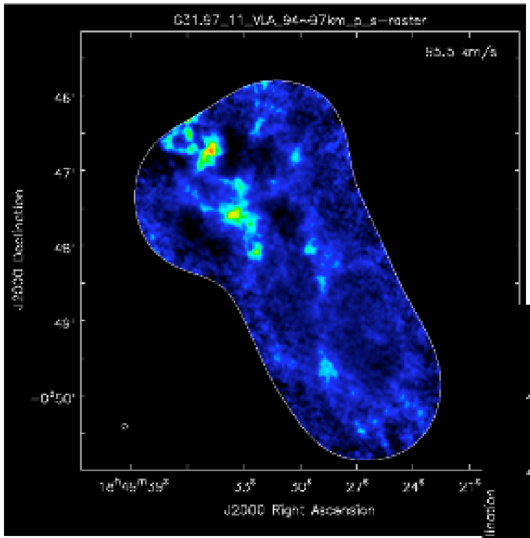
	Area	Cont.	Measured Flux (mJy)[a]						Spectral Index		
	Area	Cont.	24.1	25.4	27.5	36.4	90.0[b]	90.0[c]	(24–90 GHz)		log N_{Lyc}
	(sq″)	Level	(GHz)	(GHz)	(GHz)	(GHz)	(GHz)	(GHz)	Uncorrected[b]	Corrected[c]	(phot s^{-1})
C1	35.7	10σ	4.6 ± 0.1	4.6 ± 0.2	4.2 ± 0.2	2.6 ± 0.1	3.1 ± 0.1	6.6 ± 0.3	−0.29 ± 0.01	0.27 ± 0.03	46.5
C2	279.1	6σ	18.8 ± 0.1	18.7 ± 0.1	15.6 ± 0.1	8.2 ± 0.2	10.5 ± 0.1	27.3 ± 0.3	−0.43 ± 0.01	0.28 ± 0.03	47.2
C3	27.6	10σ	2.3 ± 0.1	2.4 ± 0.1	2.1 ± 0.1	2.6 ± 0.2	5.9 ± 0.1	11.3 ± 0.6	0.68 ± 0.01	1.17 ± 0.05	45.9
C4	14.8	6σ	1.0 ± 0.1	0.9 ± 0.1	0.9 ± 0.1	0.5 ± 0.1	1.9 ± 0.1	4.2 ± 0.5	0.52 ± 0.09	1.1 ± 0.1	45.9
C5	16.1	10σ	6.3 ± 0.1	5.6 ± 0.2	4.0 ± 0.1	NA	1.0 ± 0.2	1.9 ± 0.4	−1.31 ± 0.03	−0.86 ± 0.05	…
C6	81.6	6σ	4.5 ± 0.1	4.2 ± 0.1	5.5 ± 0.1	6.1 ± 0.1	11.8 ± 0.1	26.6 ± 0.5	0.74 ± 0.07	1.34 ± 0.09	46.5
C7	161.9	6σ	10.7 ± 0.1	8.4 ± 0.2	10.1 ± 0.2	5.9 ± 0.2	8.3 ± 0.1	28.4 ± 0.4	−0.1 ± 0.15	0.8 ± 0.18	46.9
C8	164.2	6σ	8.4 ± 0.1	5.7 ± 0.1	6.3 ± 0.1	2.8 ± 0.1	6.4 ± 0.1	32.3 ± 0.5	−0.1 ± 0.17	1.1 ± 0.14	46.8
C9	521.8	6σ	43.4 ± 0.1	34.1 ± 0.2	37.7 ± 0.2	35.8 ± 0.2	24.4 ± 0.3	80.9 ± 0.3	−0.3 ± 0.25	0.6 ± 0.3	47.5
C10	7.7	10σ	1.6 ± 0.1	1.6 ± 0.2	NA	NA	0.6 ± 0.1	1.0 ± 0.1	−0.73 ± 0.03	−0.35 ± 0.01	…

[a] "NA" indicates this region was outside or near the edge of the field of view.
[b] Values from 3 mm ALMA-only image of Rathborne et al. (2014b).
[c] Values from single-dish-corrected ALMA image of Rathborne et al. (2014b).

Mills et al. (2015)

Example of Feathering

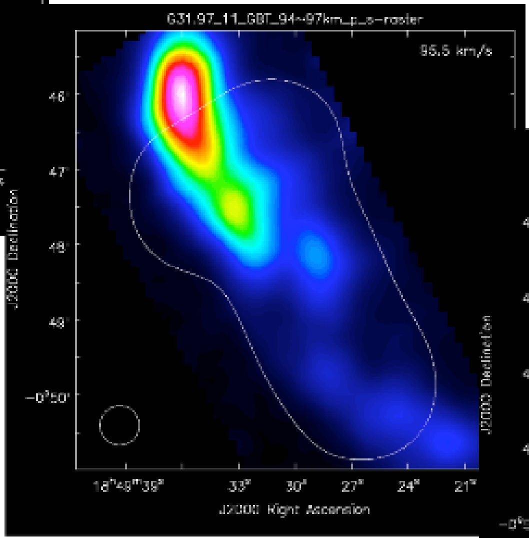
EVLA NH_3 (multi-scale CLEANed)



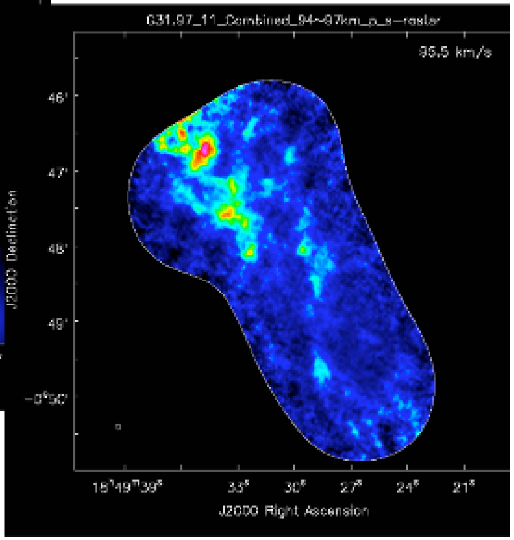
DiRienzo et al. (2015)

To maximize flux recovery and image quality you want a single dish size of $D > 1.5x\ B_{min}$

GBT NH_3



Feathered



Slide Credit: Brian mason

Summary

- Interferometers can produce higher resolution images than Single Dish telescopes, but are not sensitive to emission on size scales greater than the largest angular scale defined by its shortest baseline.
- The characteristic signs of missing extended flux-density in an image are negative bowls surrounding the main emission region.
- Single dish and interferometer data can be combined to get high resolution images that are also sensitive to diffuse emission.
- Several techniques to combine data are possible :
  - Image domain
  - Fourier domain ("Feathering")
  - During deconvolution
  - GBT can be used with JVLA-C and -D configurations & with ALMA
- Feathering is the default image combination method in CASA

“Give Me Back My Short Spacings!”