Methods for Calibrating GBT Data

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Introduction

Observing with a single-dish telescope and with the GBT in particular, has a flexibility and versatility that must be reflected in the techniques used in the calibration of the resulting data. The user of these telescopes will select calibration methods that are based partly on the type of observation, the observing strategy, observing frequency, performance of the telescope, weather conditions, and so forth. In addition, there's no one right way to calibrate single-dish data and observers will pick calibration techniques that match their own beliefs or a desired accuracy. The calibration also depends upon the science and nature of the observed source, making the total number of calibration schemes undeterminable.

The GBT, with its offset optics, wideband detectors, and mega-channel spectroscopes offers new challenges to the calibration process. We are just exploring such topics as DC offsets and non-linearities in the detection systems. Thus, we can expect a significant evolution of any 'recommended' calibration process for GBT data.

At best, an observatory can provide guidelines, recommendations, and 'typical' calibration schemes. Since it is impossible to provide all possible calibration schemes, it's important an observatory provides a suite of tools for spinning ones own calibration routines. Observers will need the tools whereby they can develop new calibration techniques as they develop observing techniques or use old techniques for new science.

This paper describes some of the 'standard' techniques or guidelines that can be used in the calibration of GBT data (and most data from any single-dish telescope). The paper does not cover the calibration of data other than standard continuum and spectral line data taken with backends like the GBT's DCR, Spectrometer, and Spectral Processor. Bolometer, pulsar, radar, VLBI … data are not covered.

I have broken the calibration process into two stages. The first is the conversion of the detected powers or A/D counts into either a flux density or antenna temperature. Sections 1 and 2 describe some recommended ways to achieve this for continuum and spectral line data. Next stage is the conversion of the data into intensity units that best match the user's science (section 3). Section 4 describes the myriad quantities that must be known or measured for the conversion of intensity units. Sections 5 and 6 contain suggested ways to astronomically measure the intensity of the noise diode, a quantity intrinsically necessary for many of the calibration techniques, and ways to derive a usable noise diode value from a table of values. Section 7 includes a list of requirements and priorities for any software system that is to help the user perform the calibration. This list is very subjective and open to debate.

1. Continuum Calibration

The details of the calibration of continuum data (Maddalena, 2000) depend upon the observing mode, the switching scheme, backend, and whether the observations are made with a single-beam or multi-beam receiver.

The general philosophy behind any observing method and switching scheme is to provide a way to calibrate the data into T_{SYS} and to subtract out the power from the instrument, ground, sky, and, in many cases, astronomical sources of little interest to the current observer. An example of the latter is the cosmic microwave background, which many radio astronomers gladly remove from their data. By far, the most difficult aspect of continuum calibration is defining how one does this subtraction.

1.1 Backends-Specific Issues

1.1.1 DCR

The design of the GBT's DCR backend, like many others, produces data that is not only proportional to the input power but also to the sample time. Doubling the sample time but keeping the input power levels alone doubles the raw counts. This tends not to be a problem for some observing modes but it's probably best to correct the data regardless of the observing mode.

Since the DCR can store data into multiple phases (e.g., noise diode on and off phases; signal or reference phases, and their combinations), and each phase can have its own duration, one needs to correct the raw counts for each phase.

For each phase, calculate the phase duration:

$$phase\_duration = cycle\_time \cdot (phase\_end - phase\_start) - phase\_blanking$$

where cycle_time is in seconds and is the time for a complete cycle of phases; phase_end and phase_start, given as a fraction of a cycle, define the start and end of a phase; and phase_blanking is the amount of blanking in seconds for that phase.

For each sample, i, in each phase, correct the raw counts using:

$$Counts(i) = Counts_{RAW}(i) / phase\_duration$$

Note: Many backends do not have their outputs counts dependent on their sampling time and the above corrections aren't needed.

Note: Even though all of the usual phase tables for the GBT have, for each phase in the table, the same phase_duration, the above corrections shouldn't be too computationally expensive and would help ensure that any future oddball phase table will produce correct results.

If the GBT were to ever support Doppler Tracking during a continuum observation, then the backends could receive occasional L.O. blanking signals. The phase_duration and correction factor would then need to be calculated for each integration since blanking would be changing from one integration to another.

1.1.2 Spectral Processor and Spectrometer

Both the Spectral processor and GBT Spectrometer can also be used for continuum observing. One would chose these devices over the DCR when in an RFI-rich environment or if the frequency flexibility and bandwidth options for the spectral-line backends have an advantage over that available for the DCR. Thus, we can expect observers to continue to uses these devices for continuum observing.

In general, for these backends, the calibration proceeds identical to that described under spectral line observations. After producing a $T_A(f)$ vector, the user would specify a mask of channels or frequencies that are to be ignored. A weighted average $T_A(f)$ of the unmasked channels would then be used as the equivalent of a continuum observation. That is:

$$\langle T_{SYS} \rangle = \frac{\sum_{\nu} w(f) \cdot T_A(f)}{\sum_{\nu} w(f)}$$

where w is a weight vector described in section XXX and ν is over all unmasked frequencies. Once $\langle T_{SYS} \rangle$ is formed, the calibration proceeds just as for DCR data.

1.2 Derivation of System Gain

The heart of the calibration of continuum observations is the accurate determination of the system gain. In general, for a linear system, for any time sample, i, the measured system temperature is derived from the measured counts:

$$T_{SYS}(i) = G_K \cdot Counts(i)$$
$$S_{SYS}(i) = G_S \cdot Counts(i)$$

where G_K is the gain of the total system in units of K/Counts and G_S is in units of Jy/Counts. S_{SYS} is T_{SYS} but in units of Jy instead of K. One can convert between G_K and G_S using:

$$G_S = \frac{2k \cdot G_K}{\eta_A A_P}$$

where k is Boltzman's constant, η_A is the telescope's aperture efficiency and A_p the physical, projected collecting area of the antenna

G must be measured with sufficient accuracy and must be measured often since the gain of the system will change with time. G will especially be prone to change whenever the system hardware configuration is changed. It must be determined for each backend sampler (e.g., each frequency band, each polarization, each beam, …)

G_K is typically derived from observations that use the receiver's noise diode when the diode's intensity is expressed in K (i.e., T_{CAL}). G_S is derived from either astronomical observations of flux calibrators or from the use of the receiver's noise diode when it is expressed in Jy (i.e., S_{CAL}).

Every signal and reference backend phase, every backend sampler, must have its own well-determined gain.

1.2.1 Derivation of System Gain using T_{CAL} and Noise Diodes

G_K is normally derived for the GBT by firing the noise diode. In the "… with Cal" switching schemes, the noise diode is fired on and off continuously during the observation and the value of G_K is derived from the same observations that are being calibrated. In the "… without Cal" schemes, there most often a separate "… with Cal" observation that is used to determine G_K that is then used to calibrate different "… without Cal" observations.

In these observations, the counts with the noise diode on and off are stored separately as distinct data phases. The antenna temperature with the diode on and off is:

$$T_{SYS}^{ON}(i) = G_K \cdot Counts^{ON}(i)$$
$$T_{SYS}^{OFF}(i) = G_K \cdot Counts^{OFF}(i)$$

and the difference between T^{ON} and T^{OFF} will be the intensity of the noise diode, T_{CAL} in K.

$$T_{CAL} = T_{SYS}^{ON}(i) - T_{SYS}^{OFF}(i) = G_K \cdot \left[Counts^{ON}(i) - Counts^{OFF}(i)\right]$$

The observer typically assumes T_{CAL} is a constant for many days or weeks and G_K is independent of input power levels and is a constant for a few seconds to many minutes. Solving for G_K:

$$G_K = \frac{T_{CAL}}{Counts^{ON}(i) - Counts^{OFF}(i)}$$

From the radiometer equation, the accuracy of G is:

$$\sigma_G(i) = \sqrt{\frac{T_{CAL}^2}{BW} \cdot \frac{\frac{\left(Counts^{ON}(i)\right)^2}{phase\_duration^{ON}} + \frac{\left(Counts^{OFF}(i)\right)^2}{phase\_duration^{OFF}}}{\left(Counts^{ON}(i) - Counts^{OFF}(i)\right)^4} + \frac{\sigma_{Tcal}^2}{\left(Counts^{ON}(i) - Counts^{OFF}(i)\right)^2}}$$

Unfortunately, the accuracy of G_K is such that one can almost never calculate it sample by sample. Instead, one usually has to average G_K over many samples. But, how many samples must one average over. Since:

$$\frac{\sigma_{TSYS}^2}{T_{SYS}^2} = \frac{\sigma_{\langle G \rangle}^2}{\langle G \rangle^2} + \frac{\sigma_{Counts}^2}{Counts^2}$$

we would like $\sigma_G/G << \sigma_{Counts}/Counts$ so that the inaccuracies of G don't contribute much to the resulting inaccuracies of T_{SYS}. If the accuracy of the measured counts is solely determined from the radiometer equation, and the two phase_durations are comparable, then, to first order, the number of samples to average over must satisfy the following:

$$\sum_{j=1}^{N} \left(Counts^{ON}(j)\right)^2 + \sum_{J=1}^{N} \left(Counts^{OFF}(j)\right)^2 << \left[\sum_{j=1}^{N} \left(Counts^{ON}(j) - Counts^{OFF}(j)\right)\right]^2$$

If the intensity of the noise diode is low and power levels aren't fluctuating too much, then one can simply use N >> Counts / ($Counts^{ON}$ – $Counts^{OFF}$) which is approximately N >> $2 \cdot (T_{SYS}/T_{CAL})^2$. For most GBT receivers, $T_{SYS} = 10\ T_{CAL}$ and N should then be at least 500 and, for the better accuracy, over 1000.

N is either specified by the user or chosen automatically by the analysis system so that the above holds true. Once a value for N is specified, the system can calculate an average G_K and use that value to turn counts into T_{SYS}. Namely:

$$G_K^{AVRG} = \frac{T_{CAL}}{N} \sum_{j=1}^{N} \frac{1}{Counts^{ON}(j) - Counts^{OFF}(j)}$$

$$\sigma_{Gavrg} = \sqrt{\frac{1}{\sum_{j=1}^{N} 1/\sigma_G^2(j)}}$$

1.2.2 Derivation of System Gain using Astronomical Sources

The derivation of G_S from astronomical sources follows the same principles as the determination of G_K except, here, the observer moves the telescope on and off a source that has a well-established flux, S_f. The source flux, then, is substituted for T_{CAL} in the above derivation.

$$G_S = \frac{S_f}{Counts^{ON}(i) - Counts^{OFF}(i)}$$

$$\sigma_G(i) = \sqrt{\frac{S_f^2}{BW} \cdot \frac{\frac{\left(Counts^{ON}(i)\right)^2}{phase\_duration^{ON}} + \frac{\left(Counts^{OFF}(i)\right)^2}{phase\_duration^{OFF}}}{\left(Counts^{ON}(i) - Counts^{OFF}(i)\right)^4} + \frac{\sigma_{Sf}^2}{\left(Counts^{ON}(i) - Counts^{OFF}(i)\right)^2}}$$

$$G_S^{AVRG} = \frac{S_f}{N} \sum_{j=1}^{N} \frac{1}{Counts^{ON}(j) - Counts^{OFF}(j)}$$

$$\sigma_{Gavrg} = \sqrt{\frac{1}{\sum_{j=1}^{N} 1/\sigma_G^2(j)}}$$

The suggested minimum value for N remains the same and, roughly, should be N >> 2k T_{SYS} / (η_A A_p S_f).

1.2.3 Derivation of System Gain using S_{CAL} and Noise Diodes

As I will show below, one can also determine the value of the noise diode in units of Jy from astronomical observations. The derivation of G_S from S_{CAL} follows the same principles as the determination of G_K except one substitutes S_{CAL} for T_{CAL}.

$$G_S = \frac{S_{CAL}}{Counts^{ON}(i) - Counts^{OFF}(i)}$$

$$\sigma_G(i) = \sqrt{\frac{S_{CAL}^2}{BW} \cdot \frac{\frac{\left(Counts^{ON}(i)\right)^2}{phase\_duration^{ON}} + \frac{\left(Counts^{OFF}(i)\right)^2}{phase\_duration^{OFF}}}{\left(Counts^{ON}(i) - Counts^{OFF}(i)\right)^4} + \frac{\sigma_{Scal}^2}{\left(Counts^{ON}(i) - Counts^{OFF}(i)\right)^2}}$$

$$G_S^{AVRG} = \frac{S_{CAL}}{N} \sum_{J=1}^{N} \frac{1}{Counts^{ON}(J) - Counts^{OFF}(J)}$$

$$\sigma_{Gavrg} = \sqrt{\frac{1}{\sum_{j=1}^{N} 1/\sigma_G^2(J)}}$$

The determination of N remains the same and, roughly, $N >> 2k\,T_{SYS} / (\eta_A\, A_p\, S_{CAL})$.

1.2.4 Practical Matters in the Determination of Gain and Noise Diode Values

Some practicalities in the determination of N and G are:

- If the scan is less than a few minutes, it's reasonable to assume N is the number of samples in the scan and, thereby, derive G for each scan.
- In many cases phase_durations can be assumed to be equal for all phases, which will greatly simplify the equations used for determining N
- If N is less than the number of samples in a scan, then the user should be able to specify that N should be determined from a sliding mean around the sample that is being calibrated. That is, to calibrate sample i, the limits of the sums in the above equations for G^{AVRG} go from i-N/2 to i+N/2. The first and last N samples in the scan should use a G that is the average over the first or last N samples.
- For back-to-back continuum scans, as in an OTF map, the sliding mean should be able to go across scan boundaries. The first and last N samples in the map should use a G that is the average over the first or last N samples in the map.
- In order to combine data at some later stages, the calibration software should calculate and store with the data an estimate of G_S or G_K and σ_G.

Some pitfalls to avoid in the determination of G are:

- If the counts are fluctuating because of changing atmospheric conditions, variable RFI, or the telescope is slewing across a strong source, then the difference between $Counts^{ON}$ and $Counts^{OFF}$ will be corrupted and so will the resulting G. In these cases, the user should be able to specify which samples to eliminate in the calculation of G. A system should also have the ability to automatically eliminate samples in cases like PEAK observations where it is known which samples or on and off the source.
- For short scans with narrow bandwidths, there may not be enough samples within a scan to derive a sufficiently accurate G.
- N should not be chosen larger than the time scale for the expected gain changes of the system. Yet, it must be large enough that inaccuracies in G don't contribute to the calibration.

1.3 Continuum Observing Methods

Each of the current GBT observing procedures falls into one of two categories:

1.3.1 Various On-Off-like Procedures

Observing procedures like TRACK, ONOFF, OFFON, POINTMAP… exemplifies on-off observing. Here, the telescope observes multiple positions that are labeled as either signal and reference positions. On-off observing differs from OTF observing in that, for the GBT, in on-off observing the telescope is deemed to be observing a single position in a scan. The gist of the calibration and observing is to subtract the power of the reference positions (off or reference scans) from the signal positions (on or signal scans).

The current POINTMAP procedure also provides the user with an option to offset the telescope at a specified interval for a reference observation. In many cases, the user won't take this option and, instead, will define some subset of positions in the map body as reference observations.

Note: In some cases the observations are constructed so as to provide information that will be needed to calibrate the data. For example, performing an ONOFF observation of a calibrator. Or, making a POINTMAP that includes somewhere within the map (or as the optional reference position) a calibrator. Or TRACK observations, or their equivalent, made at various elevations could be used to determine T_{SYS}(el) and, thereby, an atmospheric opacity suitable for converting measured antenna temperature, T_A, to other more useful flavors of intensity.

1.3.2 Various OTF Mapping Procedures

Observing procedures like PEAK, CROSS, RALONGMAP, DECLATMAP use the on-the-fly observing technique. In OTF observing, the telescope is deemed to be moving while data are being collected into a scan. Mostly, the intention is for the telescope to be off of the source of interest at some point during a scan or group of scans. The observer defines some of the data samples as sufficiently away from the source of interest, and, thereby, suitable as reference samples.

The current RALONGMAP and DECLATMAP procedure also provides the user with an option to offset the telescope at a specified interval for a reference observation. In many cases, the user won't take this option and, instead, will define some subset of positions in the map body as reference observations.

In some cases the observations are constructed so as to provide further means to calibrate the data. For example, performing a PEAK observation of a calibrator. Or, making a RALONGMAP that includes somewhere within the map a calibrator. The TIPPING procedure is a special OTF Mapping procedure whose sole purpose is to derive the atmospheric opacity that goes into full data calibration.

1.4 Switching Schemes

In addition to specifying an observing method, the user picks the 'switching' mode. The nomenclature for the switching modes varies from telescope to telescope. For the GBT, those that are suitable for continuum observing are called: Total Power with Cal, Total Power without Cal, Switched Power with Cal, and Switched Power without Cal. The type of Switched Power observations depend mostly on the receiver design but typically the user may be beam switching or polarization switching. In the near future, we will have either secondary or tertiary switching.

The choice of switching mode provides in some cases a means to calibration or provides a ready reference observation

Since Total Power with Cal observations is by far the most common, I will use this method to provide the necessary background to discuss the other switching methods.

1.4.1 Total Power with Cal

Single-Beam Observations

For "Total Power with Cal" observing, one can determine $T_{SYS}(i)$ for each samplers in the backend for both the noise diode on and off phases:

$$T_{SYS}^{ON}(i) = G_K^{AVRG} \cdot Counts^{ON}(i)$$

$$T_{SYS}^{OFF}(i) = G_K^{AVRG} \cdot Counts^{OFF}(i)$$

T_{SYS} will have an accuracy of:

$$\sigma_{CAL\_ON} = T_{SYS}^{ON}(i)\sqrt{\left(\frac{\sigma_G}{G_K^{AVRG}}\right)^2 + \frac{1}{phase\_duration^{ON} \cdot BW}}$$

$$\sigma_{CAL\_OFF} = T_{SYS}^{OFF}(i)\sqrt{\left(\frac{\sigma_G}{G_K^{AVRG}}\right)^2 + \frac{1}{phase\_duration^{OFF} \cdot BW}}$$

The system should perform a weighted average of the on and off phase data. Since the diode will be on for part of the observation, the resulting average is adjusted accordingly.

$$T_{SYS}(i) = \left(\frac{T_{SYS}^{ON}/\sigma_{ON}^2 + T_{SYS}^{OFF}/\sigma_{OFF}^2}{1/\sigma_{ON}^2 + 1/\sigma_{OFF}^2}\right) - \left(\frac{T_{CAL} \cdot phase\_duration^{ON}}{phase\_duration^{ON} + phase\_duration^{OFF}}\right)$$

which has an accuracy of:

$$\sigma_{Tsys} = \sqrt{\frac{1}{1/\sigma_{ON}^2 + 1/\sigma_{OFF}^2} + \left(\frac{\sigma_{Tcal} \cdot phase\_duration^{ON}}{phase\_duration^{ON} + phase\_duration^{OFF}}\right)^2}$$

Displayed, Single-Value T_{SYS}

Astronomers often request quick feedback on system temperatures. For the most part, they are asking for a time-averaged estimate of T_{SYS}. A time averaged T_{SYS} can also be handy during further analysis when one wants to compare and combine data. One can derive such an average by performing a root-mean-square, weighted average of T_{SYS} that is either over a specified duration or, for most practical purposes, over the duration of a scan.

$$T_{SYS}^{AVRG} = \sqrt{\frac{\sum_{j=1}^{N} T_{SYS}^2(j) \big/ \sigma_{Tsys}^2}{\sum_{j=1}^{N} 1 \big/ \sigma_{Tsys}^2}}$$

$$\approx \sqrt{\frac{N}{\sum_{j=1}^{N} \frac{1}{T_{SYS}^2(j)}}}$$

$$\sigma_{TSYS}^{AVRG} = \sqrt{\frac{1}{\sum_{j=1}^{N} 1 \big/ \sigma_{Tsys}^2}}$$

Source Antenna Temperature

The antenna temperature of a source cannot be derived directly from a single-beam, total-power observation since T_{SYS} will have contributions from the receiver, atmosphere, ground pickup, scattering, and even large-angular-size astronomical sources the observer may not be interested in (e.g., the Cosmic Microwave Background). Instead, the observer differences two measures of T_{SYS} to derive T_A.

$$T_A(i) = T_{SYS}^{SIG}(i) - T_{SYS}^{REF}(j)$$

which will have an accuracy of:

$$\sigma_{T_A} = \sqrt{\frac{1}{1 \big/ \sigma_{TSYS\_SIG}^2 + 1 \big/ \sigma_{TSYS\_REF}^2}}$$

There are a myriad of ways an observer will want to pick the samples that are to be used as signal and reference samples. For example, for On-Off observing, one may want to subtract the weighted average of the 'reference/off' scans from the 'signal/on' scans. T_{SYS}^{REF} may also be generated as a model such as a one or two-dimensional polynomial. The current processing of PEAK within IARDS and GFM determines source strength by modeling the baseline. If it were a 2-dimensional OTF mapping experiment, one could fit a baseline to areas that were designated to be 'reference' samples and then use a 2-dimensional polynomial as a model of T_{SYS} for samples designated as 'signal'.

Other Units

So far, this section has described how to use G_K to derive T_{SYS}. If, instead, one wanted to use G_S then one would use the following equations, which are just simple modified versions of the ones already given.

$$S_{SYS}^{ON}(i) = G_S^{AVRG} \cdot Counts^{ON}(i)$$

$$S_{SYS}^{OFF}(i) = G_S^{AVRG} \cdot Counts^{OFF}(i)$$

$$\sigma_{CAL\_ON} = S_{SYS}^{ON}(i)\sqrt{\left(\frac{\sigma_G}{G_S^{AVRG}}\right)^2 + \frac{1}{phase\_duration^{ON} \cdot BW}}$$

$$\sigma_{CAL\_OFF} = S_{SYS}^{OFF}(i)\sqrt{\left(\frac{\sigma_G}{G_S^{AVRG}}\right)^2 + \frac{1}{phase\_duration^{OFF} \cdot BW}}$$

$$S_{SYS}(i) = \left(\frac{S_{SYS}^{CAL\_ON}/\sigma_{ON}^2 + S_{SYS}^{OFF}/\sigma_{OFF}^2}{1/\sigma_{ON}^2 + 1/\sigma_{OFF}^2}\right) \cdot \left(\frac{S_{CAL} \cdot phase\_duration^{ON}}{phase\_duration^{ON} + phase\_duration^{OFF}}\right)$$

$$\sigma_{Ssys} = \sqrt{\frac{1}{1/\sigma_{ON}^2 + 1/\sigma_{OFF}^2} + \left(\frac{\sigma_{Scal} \cdot phase\_duration^{ON}}{phase\_duration^{ON} + phase\_duration^{OFF}}\right)^2}$$

$$S_{SYS}^{AVRG} = \frac{\sum_{j=1}^{N} S_{SYS}(j)/\sigma_{Ssys}^2}{\sum_{j=1}^{N} 1/\sigma_{Ssys}^2}$$

$$\sigma_{Ssys}^{AVRG} = \sqrt{\frac{1}{\sum_{j=1}^{N} 1/\sigma_{Ssys}^2}}$$

$$S_A(i) = S_{SYS}^{SIG}(i) - S_{SYS}^{REF}(j)$$

$$\sigma_{S_A} = \sqrt{\frac{1}{1/\sigma_{Ssys\_SIG}^2 + 1/\sigma_{Ssys\_REF}^2}}$$

S_A is the antenna temperature of the source expressed in Jy instead of K and must be corrected for atmospheric attenuation and telescope efficiencies to derive true flux density.

Multi-Beam Observations

Multi-beam, total-power with Cal observations follow the same rules and equations as for a single beam observations. Here, T_{SYS} or S_{SYS} are calculated for each beam separately. Once the data are in units of T_{SYS} or S_{SYS}, it is common practice to take the data from a beam that has been designated as the 'reference' beam and subtract it from the other, 'signal' beams that have matching frequencies and polarizations. For example:

$$T_A(i) = T_{SYS}^{SIG\_Beam}(i) - T_{SYS}^{REF\_Beam}(i)$$
$$S_A(i) = S_{SYS}^{SIG\_Beam}(i) - S_{SYS}^{REF\_Beam}(i)$$

$$\sigma_{T_A} = \sqrt{\frac{1}{1/\sigma^2_{Tsys\_SIG\_Beam} + 1/\sigma^2_{Tsys\_REF\_Beam}}}$$
$$\sigma_{S_A} = \sqrt{\frac{1}{1/\sigma^2_{Ssys\_SIG\_Beam} + 1/\sigma^2_{Ssys\_REF\_Beam}}}$$

The resulting difference should have most of the atmospheric fluctuations removed as well as much of the common, systematic contributors to T_{SYS} or S_{SYS} and the large-angular-scale astronomical background.

1.4.2 Switched Power with Cal

The current practice of using "Switched Power" for high-frequency, multi-beam observing is flawed since the current beam switches are located after the first amplifiers in the receiver. Instead, for our current receiver design, one should observe using the "Total Power with Cal" mode and process the data using the "Multi-Beam" algorithms given above.

"Switched Power with Cal" should only be used for frequency-switched or polarization-switched observations. Or, for observations whenever we have a secondary or tertiary chopping system. (Frequency-switched continuum observations are very uncommon but are useful if one is interested in measuring the spectral index of a source.)

In these observations, the backends produce four-phase data – a phase for all combinations of noise diode on and off, signal and reference. The system calculates separate G's for the signal and reference phases. For example, if the user has decided to use T_{CAL}:

$$G_K^{SIG} = \frac{T_{CAL}^{SIG}}{Counts^{ON\_SIG}(i) - Counts^{OFF\_SIG}(i)}$$
$$G_K^{REF} = \frac{T_{CAL}^{REF}}{Counts^{ON\_REF}(i) - Counts^{OFF\_REF}(i)}$$

Note that T_{CAL} should be assumed to be different for the signal and reference phases, though for some types of observations, like tertiary chopping, they will be the same. As described above, one will usually need to calculate time-averaged gains, G^{AVRG_SIG} and G^{AVRG_REF}, in order to provide values with sufficient accuracy.

The system determines separate T_{SYS} for all four phases

$$
\begin{aligned}
T_{SYS}^{ON\_SIG}(i) &= G_K^{AVRG\_SIG} \cdot Counts^{ON\_SIG}(i) \\
T_{SYS}^{OFF\_SIG}(i) &= G_K^{AVRG\_SIG} \cdot Counts^{OFF\_SIG}(i) \\
T_{SYS}^{ON\_REF}(i) &= G_K^{AVRG\_REF} \cdot Counts^{ON\_REF}(i) \\
T_{SYS}^{OFF\_REF}(i) &= G_K^{AVRG\_REF} \cdot Counts^{OFF\_REF}(i)
\end{aligned},
$$

which will have accuracies of:

$$\sigma_{ON\_SIG} = T_{SYS}^{ON\_SIG}(i)\sqrt{\left(\frac{\sigma_G^{SIG}}{G_K^{AVRG}}\right)^2 + \frac{1}{phase\_duration^{ON\_SIG} \cdot BW_{SIGl}}}$$

$$\sigma_{OFF\_SIG} = T_{SYS}^{OFF\_SIG}(i)\sqrt{\left(\frac{\sigma_G^{SIG}}{G_K^{AVRG\_SIG}}\right)^2 + \frac{1}{phase\_duration^{OFF\_SIG} \cdot BW_{SIG}}}$$

$$\sigma_{ON\_REF} = T_{SYS}^{ON\_REF}(i)\sqrt{\left(\frac{\sigma_G^{REF}}{G_K^{AVRG}}\right)^2 + \frac{1}{phase\_duration^{ON\_REF} \cdot BW_{REF}}}$$

$$\sigma_{OFF\_REF} = T_{SYS}^{OFF\_REF}(i)\sqrt{\left(\frac{\sigma_G^{REF}}{G_K^{AVRG\_REF}}\right)^2 + \frac{1}{phase\_duration^{OFF\_REF} \cdot BW_{signal}}}$$

Next, the system averages the data for the noise-diode On and Off phases and calculates the resulting accuracy of the average:

$$
\begin{aligned}
T_{SYS}^{SIG}(i) = &\left(\frac{T_{SYS}^{ON\_SIG}/\sigma_{ON\_SIG}^2 + T_{SYS}^{OFF\_SIG}/\sigma_{OFF\_SIG}^2}{1/\sigma_{ON\_SIG}^2 + 1/\sigma_{OFF\_SIG}^2}\right) \\
&- \left(\frac{T_{CAL}^{SIG} \cdot phase\_duration^{ON\_SIG}}{phase\_duration^{ON\_SIG} + phase\_duration^{OFF\_SIG}}\right)
\end{aligned}
$$

$$
\begin{aligned}
T_{SYS}^{REF}(i) = &\left(\frac{T_{SYS}^{ON\_REF}/\sigma_{ON\_REF}^2 + T_{SYS}^{OFF\_REF}/\sigma_{OFF\_REF}^2}{1/\sigma_{ON\_REF}^2 + 1/\sigma_{OFF\_REF}^2}\right) \\
&- \left(\frac{T_{CAL}^{REF} \cdot phase\_duration^{ON\_REF}}{phase\_duration^{ON\_REF} + phase\_duration^{OFF\_REF}}\right)
\end{aligned}
$$

$$\sigma_{Tsys}^{SIG} = \sqrt{\frac{1}{1/\sigma_{ON\_SIG}^2 + 1/\sigma_{OFF\_SIG}^2} + \left(\frac{\sigma_{Tcal} \cdot phase\_duration^{ON\_SIG}}{phase\_duration^{ON\_SIG} + phase\_duration^{OFF\_SIG}}\right)^2}$$

$$\sigma_{Tsys}^{REF} = \sqrt{\frac{1}{1/\sigma_{ON\_REF}^2 + 1/\sigma_{OFF\_REF}^2} + \left(\frac{\sigma_{Tcal} \cdot phase\_duration^{ON\_REF}}{phase\_duration^{ON\_REF} + phase\_duration^{OFF\_REF}}\right)^2}$$

The user should then be given the option of whether to subtract the reference from the signal data to create an array of source antenna temperatures:

$$T_A(i) = T_{SYS}^{SIG}(i) - T_{SYS}^{REF}(i)$$

$$\sigma_{TA} = \sqrt{\frac{1}{1/\sigma_{T_{SYS}\_SIG}^2 + 1/\sigma_{T_{SYS}\_REF}^2}}$$

1.4.3 Total Power and Switched Power without Cal

Once the framework in place for "… Power with Cal" observing, the calibration of "… without Cal" observations will follow rather easily. In "… without Cal" observing, G, the gain of the system, cannot be determined from the firing of the noise diode. Instead, the user has two options:

1. Specify a "… with Cal" observation from which either G_K or G_S can be determined.
2. Use a calibrator of known flux to determine G_S as explained in 1.2.

The measured G is then used for the "… without Cal" observations and all of the above calibration expressions are the same. Thus, the most difficult part of "… without Cal" observing is providing a way for the observer to specify the separate observation to use for determining G.

2. Spectral Line Calibration

The discussion of continuum calibration paves the way for spectral line calibration. Like continuum calibration, there are going to be backend-specific issues and differences depending upon the users choice of switching schemes, observing procedures. Likewise, the user should have a choice on how one determines the telescope gain. An added complexity is that observers now have multiple differencing methods they can pick from that depends upon the science or the object observed.

2.1 Backend-Specific Issues

TBD

2.1.1 Backend Sensitivity (K) Factor

For the purpose of proper averaging of data, it's important that one has an estimate of the weights for every spectrum. Usually, one will want weights for every channel. Since statistical weights are the inverse-square of the rms, to derive theoretical weights one must be able to derive proper estimates of the theoretical noise. The radiometer equation provides a means to a theoretical rms. But, we need to know the quantization and channel separation factor to use in the radiometer equation. This sensitivity factor (K) is backend dependent, and, sometimes, even mode dependent.

The tables in the GBT's manual (http://www.gb.nrao.edu/gbt/GBTMANUAL) provide estimates of K for our various backends. Currently, the following values should be adequate:

Backend	K
Spectral Processor	1.18
Spectrometer – 3 level	0.873
Spectrometer – 9 level	0.730

Values for the Spectrometer depend upon the chosen parameters for the windowing function used by the 'filler' software. The values in the table assume that the default is a Hanning function.

2.1.2 Averaging Integrations within a Scan

Scans are usually made out of multiple integrations that the observer will sometimes want to investigate separately. For example, those observing with OTF procedures or those observing in an RFI environment that changes rapidly with time. Some observers, however, won't be interested in the individual integrations and, instead, will want only the average of the integrations within a scan.

The observer should be given the option for the analysis system to perform a weighted average of the integrations within a scan. Each phase within a scan should be averaged separately. The user should be given a choice of how to estimate the weights. Possible choices are:

$$w^{i}_{phase}(f) = \frac{\Delta f \cdot t_{user}}{Counts^{i}_{user}(f)^{2}}$$

$$w^{i}_{phase}(f) = \frac{\Delta f \cdot (t^{i}_{phase} - t^{i}_{blanking})}{Counts^{i}_{phase}(f)^{2}}$$

$$w^{i}_{phase}(f) = \frac{\Delta f \cdot t_{user}}{Counts^{i}_{phase}(f)^{2}}$$

$$w^{i}_{phase}(f) = \frac{\Delta f \cdot t_{user}}{\left\langle Counts^{i}_{phase}(f) \right\rangle^{2}_{M}}, \text{ or}$$

$$w^{i}_{phase}(f) = \frac{\Delta f \cdot (t^{i}_{phase} - t^{i}_{blanking})}{\left\langle Counts^{i}_{phase}(f) \right\rangle^{2}_{M}}$$

where t_{user} and $Counts_{user}$ are user-supplied values for the phase time and frequency-dependent data values. The subscript i denotes the integration and < > indicates the analyis system should take a sliding mean, M channels wide, surrounding the frequency, f. The value of M should be under the user's control and should range from 1 to the number of channels in the spectrum.

Note how some of the denominators are proportional to $1/T_{SYS}^2$ and the numerators to the integration time, corrected for blanking time. Weighting in this way properly handles those cases where T_{SYS} changes from integration to integration or where either the phase duration or blanking times might change from integration to integration die to such things as L.O. blanking.

The average of the integration in a scan is then:

$$Counts^{Avrg}_{phase}(f) = \frac{\sum_{i=1}^{n} Counts^{i}_{phase}(f) \cdot w^{i}_{phase}(f)}{\sum_{i=1}^{n} w^{i}_{phase}(f)}$$

The integration time, weights, and rms that should be stored with the averaged data are then:

$$t^{Avrg}_{phase} = \sum_{i=1}^{n} \left(t^{i}_{phase} - t^{i}_{blanking} \right)$$

$$w^{Avrg}_{phase}(f) = \sqrt{\sum_{i=1}^{n} \left(w^{i}_{phase}(f) \right)^{2}}$$

$$\sigma^{Avrg}_{phase}(f) = \frac{K}{\sqrt{\Delta f \cdot w^{Avrg}_{phase}(f)}}$$

2.1.3 Averaging of Phases within an Integration or Scan for "…. With Cal" Observing.

In "… With Cal" observing, observers usually follow the practice of averaging together the noise diode on and off data. Since some noise diodes have strengths comparable to the system temperature, the average should be weighted appropriately.

$$Counts^{p}(f) = \frac{w_p^{ON}(f) \cdot Counts_p^{ON}(f) + w_p^{OFF}(f) \cdot Counts_p^{OFF}(f)}{w_p^{ON}(f) + w_p^{OFF}(f)}$$

$$t_p = t_p^{ON} + t_p^{OFF}$$

$$w_p(f) = w_p^{ON}(f) + w_p^{OFF}(f)$$

$$\sigma_p(f) = \frac{K}{\sqrt{\Delta f \cdot w_p(f)}}$$

The *p* represents either SIG or REF for the two phases in a Switched Power observation or the SIG and REF scans that are parts of a Total Power observation.

Although most observers will want to do the above averaging, sometimes it's better to ignore the data with the noise diode on. This is especially true if the noise diode has high-frequency structure or if the noise diode values change wildly across the observing band. By doing so, the observer is throwing away a good fraction of the data but this may be the only way to use such data. If the observer opts to ignore the noise-diode on data, then:

$$Counts^{p}(f) = Counts_p^{OFF}(f)$$

$$t_p = t_p^{OFF}$$

$$w_p(f) = w_p^{OFF}(f)$$

$$\sigma_p(f) = \frac{K}{\sqrt{\Delta f \cdot w_p(f)}}$$

2.2 Spectral-Line Differencing Methods

As in most of astronomy, spectral line observations involve differencing observations so as to remove instrumental and background affects. I will use the nomenclature REF and SIG to denote what the astronomer considers reference and signal data that are to be differenced.

There are a few differencing methods that observers tend to use:

2.2.1 Method 1

Most observers use a differencing equation that normalizes the spectrum so as to remove instrumental bandpasses that one believes scales with system gain:

$$T_{diff}(f) = \left\langle T_{SYS}^{REF}(f) \right\rangle_{N,M} \cdot \frac{Counts^{SIG}(f) - Counts^{REF}(f)}{Counts^{REF}(f)}$$

$$S_{diff}(f) = \left\langle S_{SYS}^{REF}(f) \right\rangle_{N,M} \cdot \frac{Counts^{SIG}(f) - Counts^{REF}(f)}{Counts^{REF}(f)}$$

The method requires a 'signal' observation with the telescope pointing on the source of interest at the frequency of interest and a separate 'reference' measurement that is either with the telescope off the source or at a slightly different frequency. The method may utilize a chopping mirror in the optics to perform the reference observations without having to move the larger telescope structure. Typically the chop is between, say, 0.1 and 10 Hz.

The method requires an accurate estimate of T_{SYS} at the reference position (the denominator of the right-hand quantity). The < > brackets illustrate that, in order to achieve enough accuracy in T_{SYS} one must usually average over N frequency channels or M integrations (or scans), or both. Section 2.5 deals with the determination of an accurate T_{SYS}.

The $Counts^{SIG}$ and $Counts^{REF}$ used in the differencing may actually be a combination of the different phases produced by the backend, as described in section 2.1. They can be from the same scan, as in Switched Power observing. Or, from different scans, as in Total Power observing, where the user should have some control as to what are the ISG and REF scans in an observation. For OTF mapping, the REF observations must be specified by the user to be either separate scans or integrations within a scan. For Pointed maps, the scans that are to be labeled SIG and REF should be specified by the user.

Some of the disadvantages to the method are: (1) the statistical noise is larger due to the differencing; (2) the extra time needed for the 'reference' observation; (3) the determination of a properly-accurate T_{SYS} is sometimes non trivial with wide bandpass spectrometers; and (4) the breakdown of the assumptions when there is a significant difference in power between the signal and reference observations due to, for example, either changing weather conditions, different observing elevations, or source continuum level.

The analysis system will probably require and store away for later use estimates of the T_{SYS}, theoretical measure of the noise, statistical weights, and integration time of the difference spectrum.

$$\left\langle T_{SYS}^{Diff}(f)\right\rangle_{N.M} = \sqrt{\frac{t^{REF}\left\langle T_{SYS}^{SIG}(f)\right\rangle_N^2 + t^{SIG}\left\langle T_{SYS}^{REF}(f)\right\rangle_N^2}{t^{REF}+t^{SIG}}}$$

$$\left\langle S_{SYS}^{Diff}(f)\right\rangle_{N,M} = \sqrt{\frac{t^{REF}\left\langle S_{SYS}^{SIG}(f)\right\rangle_N^2 + t^{SIG}\left\langle S_{SYS}^{REF}(f)\right\rangle_N^2}{t^{REF}+t^{SIG}}}$$

$$\sigma_{Tdiff}(f) = K\frac{\left\langle T_{SYS}^{Diff}(f)\right\rangle_N}{\sqrt{\Delta f\cdot\frac{t^{REF}t^{SIG}}{t^{REF}+t^{SIG}}}}$$

$$\sigma_{Sdiff}(f) = K\frac{\left\langle S_{SYS}^{Diff}(f)\right\rangle_N}{\sqrt{\Delta f\cdot\frac{t^{REF}t^{SIG}}{t^{REF}+t^{SIG}}}}$$

$$w_{Tdiff}(f) = \frac{1}{\sigma_{Tdiff}^2(f)}$$

$$w_{Sdiff}(f) = \frac{1}{\sigma_{Sdiff}^2(f)}$$

Note that the resulting rms is calculated exactly as if the observations had an effective integration time of:

$$t_{diff} = \frac{t^{REF}t^{SIG}}{t^{REF}+t^{SIG}}.$$

2.2.2 Method 2

An alternate method works well if one believes the bandpass shapes scale with incident power level, as might be the case when observing strong continuum sources. Here the observations consist of ‘signal’ and ‘reference’ measurements of the source of interest and another, ‘calibrator’ source whose continuum intensity is known and is usually about the same as the source of interest. Here:

$$T_{diff}(f) = T_{CALIB}(f)\cdot\frac{Counts_{SRC}^{SIG}(f) - Counts_{SRC}^{REF}(f)}{Counts(f)_{CALIB}^{SIG} - Counts_{CALIB}^{REF}(f)}$$

$$S_{diff}(f) = S_{CALIB}(f)\cdot\frac{Counts_{SRC}^{SIG}(f) - Counts_{SRC}^{REF}(f)}{Counts(f)_{CALIB}^{SIG} - Counts_{CALIB}^{REF}(f)}$$

Note how the results can be easily placed into the units of intensity of the 'calibration' source. An observer may opt to save some observing time by sharing reference observations between the source and the calibrator. For example:

$$T_{diff}(f) = T_{CALIB}(f) \cdot \frac{Counts_{SRC}^{SIG}(f) - Counts^{REF}(f)}{Counts(f)_{CALIB}^{SIG} - Counts^{REF}(f)}$$

$$S_{diff}(f) = S_{CALIB}(f) \cdot \frac{Counts_{SRC}^{SIG}(f) - Counts^{REF}(f)}{Counts(f)_{CALIB}^{SIG} - Counts^{REF}(f)}$$

In comparison to method 1, this method has its disadvantages: (1), the observations require more 'on-source' time to obtain the same signal-to-noise due to the statistical noise from the two differences; (2) the time on source is cut almost in half since a good fraction of the observing time must be spent observing the calibrator; (3) the accuracy with which one knows the frequency-dependent intensity of the 'calibrator' limits the accuracy of the calibration; and (4) finding an appropriate 'calibrator' source might require a large move of the telescope, increasing the observing overhead. The advantages to bandpass shapes often outweigh the disadvantages, especially when observing strong continuum sources.

The appropriate statistical calculation for the first variant of method 2 is:

$$\sigma_{Tdiff}(f) = T_{diff}(f) \sqrt{\frac{\frac{\left(Counts_{SRC}^{SIG}\right)^2}{t_{SRC}^{SIG}} + \frac{\left(Counts_{SRC}^{REF}\right)^2}{t_{SRC}^{REF}}}{\left(Counts_{SRC}^{SIG} - Counts_{SRC}^{REF}\right)^2} + \frac{\frac{\left(Counts_{CALIB}^{SIG}\right)^2}{t_{CALIB}^{SIG}} + \frac{\left(Counts_{CALIB}^{REF}\right)^2}{t_{CALIB}^{REF}}}{\left(Counts_{CALIB}^{SIG} - Counts_{CALIB}^{REF}\right)^2}}$$

$$\sigma_{Sdiff}(f) = S_{diff}(f) \sqrt{\frac{\frac{\left(Counts_{SRC}^{SIG}\right)^2}{t_{SRC}^{SIG}} + \frac{\left(Counts_{SRC}^{REF}\right)^2}{t_{SRC}^{REF}}}{\left(Counts_{SRC}^{SIG} - Counts_{SRC}^{REF}\right)^2} + \frac{\frac{\left(Counts_{CALIB}^{SIG}\right)^2}{t_{CALIB}^{SIG}} + \frac{\left(Counts_{CALIB}^{REF}\right)^2}{t_{CALIB}^{REF}}}{\left(Counts_{CALIB}^{SIG} - Counts_{CALIB}^{REF}\right)^2}}$$

$$t_{diff} = \frac{t_{SRC}^{SIG} \cdot t_{SRC}^{REF} \cdot t_{CALIB}^{SIG} \cdot t_{CALIB}^{REF}}{t_{SRC}^{SIG} t_{SRC}^{REF} \cdot \left(t_{CALIB}^{SIG} + t_{CALIB}^{REF}\right) + t_{CALIB}^{SIG} t_{CALIB}^{REF} \cdot \left(t_{SRC}^{SIG} + t_{SRC}^{REF}\right)}$$

and, for the second variant:

$$\sigma_{Tdiff}(f) = T_{diff}(f)\sqrt{\frac{\dfrac{\left(Counts_{SRC}^{SIG}\right)^2}{t_{SRC}^{SIG}} + \dfrac{\left(Counts^{REF}\right)^2}{t^{REF}}}{\left(Counts_{SRC}^{SIG} - Counts^{REF}\right)^2} + \frac{\dfrac{\left(Counts_{CALIB}^{SIG}\right)^2}{t_{CALIB}^{SIG}} + \dfrac{\left(Counts^{REF}\right)^2}{t^{REF}}}{\left(Counts_{CALIB}^{SIG} - Counts^{REF}\right)^2}}$$

$$\sigma_{Sdiff}(f) = S_{diff}(f)\sqrt{\frac{\dfrac{\left(Counts_{SRC}^{SIG}\right)^2}{t_{SRC}^{SIG}} + \dfrac{\left(Counts^{REF}\right)^2}{t^{REF}}}{\left(Counts_{SRC}^{SIG} - Counts^{REF}\right)^2} + \frac{\dfrac{\left(Counts_{CALIB}^{SIG}\right)^2}{t_{CALIB}^{SIG}} + \dfrac{\left(Counts^{REF}\right)^2}{t^{REF}}}{\left(Counts_{CALIB}^{SIG} - Counts^{REF}\right)^2}}$$

$$t_{diff} = \frac{t_{SRC}^{SIG} \cdot t^{REF} \cdot t_{CALIB}^{SIG}}{t_{SRC}^{SIG} \cdot \left(t_{CALIB}^{SIG} + t^{REF}\right) + t_{CALIB}^{SIG} \cdot \left(t_{SRC}^{SIG} + t^{REF}\right)}$$

The remaining quantities that need to be calculated and stored with the data are:

$$w_{Tdiff}(f) = \frac{1}{\sigma_{Tdiff}^2(f)}$$

$$w_{Sdiff}(f) = \frac{1}{\sigma_{Sdiff}^2(f)}$$

$$T_{SYS}^{Diff}(f) = \frac{\sigma_{Tdiff}(f)}{K}\sqrt{\Delta f \cdot t_{diff}}$$

$$S_{SYS}^{Diff}(f) = \frac{\sigma_{Sdiff}(f)}{K}\sqrt{\Delta f \cdot t_{diff}}$$

2.2.3 Method 3

A variant of method 1 arises if it's necessary to have T_{SYS} determined for every channel. As I'll show below:

$$T_{SYS}^{REF} \approx \frac{T_{CAL}}{2} \cdot \frac{\left(Counts_{CAL\_ON}^{REF} + Counts_{CAL\_OFF}^{REF}\right)}{\left(Counts_{CAL\_ON}^{REF} + Counts_{CAL\_OFF}^{REF}\right)}$$

$$S_{SYS}^{REF} \approx \frac{S_{CAL}}{2} \cdot \frac{\left(Counts_{CAL\_ON}^{REF} + Counts_{CAL\_OFF}^{REF}\right)}{\left(Counts_{CAL\_ON}^{REF} + Counts_{CAL\_OFF}^{REF}\right)}$$

Since $T_{SYS}^{REF}(f) = G_K(f) \cdot Counts^{REF}(f)$ and $S_{SYS}^{REF}(f) = G_S(f) \cdot Counts^{REF}(f)$ the formulation of method 1 can be rewritten as:

$$T_{diff}(f) = \left\langle G_K^{SIG}(f) \right\rangle_{N,M} \cdot Counts^{SIG}(f) - \left\langle G_K^{REF}(f) \right\rangle_{N,M} \cdot Counts^{REF}(f)$$

$$S_{diff}(f) = \left\langle G_S^{SIG}(f) \right\rangle_{N,M} \cdot Counts^{SIG}(f) - \left\langle G_K^{REF}(f) \right\rangle_{N,M} \cdot Counts^{REF}(f)$$

or, if one believes the gain doesn't change between the signal and reference observations, the variants:

$$T_{diff}(f) = \left\langle G_K(f) \right\rangle_{N,M} \cdot \left(Counts^{SIG}(f) - Counts^{REF}(f) \right)$$

$$S_{diff}(f) = \left\langle G_S(f) \right\rangle_{N,M} \cdot \left(Counts^{SIG}(f) - Counts^{REF}(f) \right)$$

Note that for this method to be appropriate, G can only be averaged over time, not frequency. The disadvantages of this method are similar to those in method 1 except now one is required to determine the gain very accurately, and channel-by-channel.

The appropriate statistical calculation for the first variant of method 3 is:

$$\sigma_{Tdiff}(f) = \sqrt{ Counts^{SIG}(f)^2 \cdot \left(\frac{K^2 \cdot \left\langle G_K^{SIG}(f) \right\rangle_{N,M}^2}{\Delta f \cdot t^{SIG}} + \sigma_{G_K}^{SIG}(f)^2 \right) + Counts^{REF}(f)^2 \cdot \left(\frac{K^2 \cdot \left\langle G_K^{REF}(f) \right\rangle_{N,M}^2}{\Delta f \cdot t^{REG}} + \sigma_{G_K}^{REF}(f)^2 \right) }$$

$$\sigma_{Sdiff}(f) = \sqrt{ Counts^{SIG}(f)^2 \cdot \left(\frac{K^2 \cdot \left\langle G_S^{SIG}(f) \right\rangle_{N,M}^2}{\Delta f \cdot t^{SIG}} + \sigma_{G_S}^{SIG}(f)^2 \right) + Counts^{REF}(f)^2 \cdot \left(\frac{K^2 \cdot \left\langle G_S^{REF}(f) \right\rangle_{N,M}^2}{\Delta f \cdot t^{REG}} + \sigma_{G_S}^{REF}(f)^2 \right) }$$

And, for the second:

$$\sigma_{Tdiff}(f) = \sqrt{\begin{array}{l}\sigma_{G_K}(f)^2 \cdot \left(Counts^{SIG}(f) - Counts^{REF}(f)\right)^2 + \\ \frac{K^2}{\Delta f}\left\langle G_K(f)\right\rangle_{N,M}^2 \left(\frac{Counts^{SIG}(f)^2}{t^{SIG}} + \frac{Counts^{REF}(f)^2}{t^{REF}}\right)\end{array}}$$

$$\sigma_{Sdiff}(f) = \sqrt{\begin{array}{l}\sigma_{G_S}(f)^2 \cdot \left(Counts^{SIG}(f) - Counts^{REF}(f)\right)^2 + \\ \frac{K^2}{\Delta f}\left\langle G_S(f)\right\rangle_{N,M}^2 \left(\frac{Counts^{SIG}(f)^2}{t^{SIG}} + \frac{Counts^{REF}(f)^2}{t^{REF}}\right)\end{array}}$$

The remaining quantities that need to be calculated and stored with the data are:

$$w_{Tdiff}(f) = \frac{1}{\sigma_{Tdiff}^2(f)}$$

$$w_{Sdiff}(f) = \frac{1}{\sigma_{Sdiff}^2(f)}$$

$$t_{diff} = \frac{t^{REF} t^{SIG}}{t^{REF} + t^{SIG}}$$

$$T_{SYS}^{Diff}(f) = \frac{\sigma_{Tdiff}(f)}{K}\sqrt{\Delta f \cdot t_{diff}}$$

$$S_{SYS}^{Diff}(f) = \frac{\sigma_{Sdiff}(f)}{K}\sqrt{\Delta f \cdot t_{diff}}$$

2.2.4 Method 4

The last difference method I'll explore is similar to method 3 except the observer believes he or she can model the bandpass shape well enough that a reference measurement isn't necessary. Essentially,

$$T_{diff}(f) = \left\langle G_K^{SIG}(f)\right\rangle_{N,M} \cdot Counts^{SIG}(f) - BandPassModel(f)$$

$$S_{diff}(f) = \left\langle G_S^{SIG}(f)\right\rangle_{N,M} \cdot Counts^{SIG}(f) - BandPassModel(f)$$

Gain can be averaged over time or frequency, or both. The following section deals with the accurate determination of gain.

The method works best with very narrow spectral lines and very smooth, time-stable, bandpasses over a very limited range of frequencies. The significant limiting factor is in determining the bandpass model, which usually requires observations using one of the other methods on a source known not to produce the line of interest. Each observer will probably have their own way of determining the model, though the analysis software should facilitate the use of a user's model.

The major advantage is that, once the model is known, there's no need for a reference observation and the results will be less noisy since the method doesn't require differencing two noisy quantities. If we ignore the time needed to determine the model, most observations that use method 4 could achieve the same signal-to-noise in four times less time than if method 1 were used.

The statistical formulae are:

$$\sigma_{Tdiff}(f) = Counts^{SIG}(f)\sqrt{\left(\frac{K^2 \cdot \left\langle G_K^{SIG}(f)\right\rangle^2_{N,M}}{\Delta f \cdot t^{SIG}} + \sigma_{G_K}^{SIG}(f)^2\right)}$$

$$\sigma_{Sdiff}(f) = Counts^{SIG}(f)\sqrt{\left(\frac{K^2 \cdot \left\langle G_K^{SIG}(f)\right\rangle^2_{N,M}}{\Delta f \cdot t^{SIG}} + \sigma_{G_K}^{SIG}(f)^2\right)}$$

$$w_{Tdiff}(f) = \frac{1}{\sigma^2_{Tdiff}(f)}$$

$$w_{Sdiff}(f) = \frac{1}{\sigma^2_{Sdiff}(f)}$$

$$t_{diff} = t^{SIG}$$

$$T_{SYS}^{Diff}(f) = \frac{\sigma_{Tdiff}(f)}{K}\sqrt{\Delta f \cdot t_{diff}}$$

$$S_{SYS}^{Diff}(f) = \frac{\sigma_{Sdiff}(f)}{K}\sqrt{\Delta f \cdot t_{diff}}$$

2.3 Spectral-Line Observing Methods

Each of the current GBT observing procedures falls into the same two categories as continuum observing: On-Off-like and OTF Mapping procedures. The discussion of section 1.3 fully apply here.

2.4 Switching Schemes

2.4.1 Total Power with Cal

TBD

Single Beam Observations

Multi-Beam Observations

TBD

2.4.2 Switched Power with Cal

TBD

Single-Beam Observations

TBD

Frequency Switching

TBD

Single switch

TBD

Dual switch

TBD

Beam Switching

TBD

Single switch

TBD

Dual switch

TBD

Multi-Beam Observations

TBD

2.4.3 Total Power and Switched Power without Cal

TBD

2.5 Determination of Gain, T_{SYS}, and S_{SYS}

Three of the four differencing methods require either knowledge of the system gain or some measure of the system temperature. Ignoring all of the details for now,

$$T_{SYS} = G_K \cdot Counts$$
$$S_{SYS} = G_S \cdot Counts$$

Thus, one also needs the gain to estimate the system temperature.

The derivation of gain for spectral line observations follows closely that of continuum observing. Again, there are three ways in which gain can be determined, the choice depending upon what hardware and astronomical source an observer will want to use. Much of what follows is a repeat or an abbreviation of section 1.2.

G must be measured with sufficient accuracy and must be measured often since the gain of the system will change with time. G will especially be prone to change whenever the system hardware configuration is changed. It must be determined for each backend sampler (e.g., each frequency band, each polarization, each beam…)

G_K is typically derived from observations that use the receiver's noise diode when the diode's intensity is expressed in K (i.e., T_{CAL}). G_S is derived from either astronomical observations of flux calibrators or from the use of the receiver's noise diode when it is expressed in Jy (i.e., S_{CAL}).

2.5.1 Derivation of System Gain using T_{CAL} and Noise Diodes

As with continuum observations, G_K is normally derived for the GBT by firing the noise diode. In the "… with Cal" switching schemes, the noise diode is fired on and off continuously during the observation and the value of G_K is derived from the same observations that are being calibrated. In the "…. without Cal" schemes, there is a separate "… with Cal" observation that is used to determine G_K that is then used to calibrate different "… without Cal" observations.

Thus, there always has to be a "… with Cal" observation. In these observations, the counts with the noise diode on and off are stored separately as distinct data phases. For spectral-line observations, gain may be different for every channel and can be derived from:

$$G_K(f) = \frac{T_{CAL}(f)}{Counts^{ON}(f) - Counts^{OFF}(f)}$$

$$G_S(f) = \frac{S_{CAL}(f)}{Counts^{ON}(f) - Counts^{OFF}(f)}$$

$$\sigma_{G_K}(f) = \sqrt{\frac{T_{CAL}(f)^2}{\Delta f} \cdot \frac{\frac{\left(Counts^{ON}(f)\right)^2}{phase\_duration^{ON}} + \frac{\left(Counts^{OFF}(f)\right)^2}{phase\_duration^{OFF}}}{\left(Counts^{ON}(f) - Counts^{OFF}(f)\right)^4} + \left(\frac{\sigma_{Tcal}(f)}{Counts^{ON}(f) - Counts^{OFF}(f)}\right)^2}$$

$$\sigma_{G_S}(f) = \sqrt{\frac{S_{CAL}(f)^2}{\Delta f} \cdot \frac{\frac{\left(Counts^{ON}(f)\right)^2}{phase\_duration^{ON}} + \frac{\left(Counts^{OFF}(f)\right)^2}{phase\_duration^{OFF}}}{\left(Counts^{ON}(f) - Counts^{OFF}(f)\right)^4} + \left(\frac{\sigma_{Scal}(f)}{Counts^{ON}(f) - Counts^{OFF}(f)}\right)^2}$$

Unfortunately, the accuracy of G_K is such that one can almost never calculate it channel by channel. Instead, one usually has to average G_K over many channels or many integrations.

The three "difference" methods have different requirements for determining the number of channels or integrations to average over.

Method 1

Method 1 requires an accurate $T_{SYS}(f)$ or $S_{SYS}(f)$. To achieve the required accuracy, one must usually perform a sliding average over frequency or time or both. I'll first cover the derivation of an adequate $T_{SYS.}$

The system temperature used in Method 1 must be something that represents the system temperature of whatever is in the denominator of T_{SYS}*(SIG-REF)/REF. For example, in some "Total Power with Cal" observing, it's useful to ignore the data with the noise diode on. If so, T_{SYS} should be calculated from:

$$T_{SYS}^{REF}(f) = G_K(f) \cdot Counts_{REF}^{OFF}(f)$$

$$\sigma_{Tsys} = T_{SYS}^{REF}(f) \cdot \sqrt{\left(\frac{\sigma_{G_K}}{G_K}\right)^2 + \frac{1}{\Delta f \cdot t_{REF}^{Off}}}$$

But, if the observer picks the usual way of not ignoring the noise-diode-on data, then:

$$T_{SYS}^{REF}(f) = G_K(f) \cdot \frac{\left(t_{REF}^{OFF} Counts_{REF}^{OFF}(f) + t_{REF}^{ON} Counts_{REF}^{ON}(f)\right)}{t_{REF}^{OFF} + t_{REF}^{ON}}$$

$$\sigma_{Tsys} = T_{SYS}^{REF}(f) \cdot \sqrt{\left(\frac{\sigma_G}{G_K}\right)^2 + \frac{1}{\Delta f \cdot t_{REF}^{Off}}}$$

The percentage uncertainty in the results of Method 1 is:

$$\left(\frac{\sigma_{Tdiff}(f)}{T_{diff}(f)}\right)^2 = \left(\frac{\sigma_{\langle T_{SYS}^{REF}\rangle}(f)}{\langle T_{SYS}^{REF}(f)\rangle_{N,M}}\right)^2 + \left(\frac{\sigma_{Counts^{SIG}}(f)}{Counts^{SIG}}\right)^2 + \left(\frac{\sigma_{Counts^{REF}}(f)}{Counts^{REF}}\right)^2$$

N and M are the number of frequency channels and integrations or scans that are to be averaged over to derive a suitably accurate measure of T_{SYS}. One would like to have the percentage uncertainty introduced by the uncertainty in T_{SYS} to be much lower than the other two sources of statistical uncertainty. That is:

$$\left(\frac{\langle \sigma_{T_{SYS}^{REF}}(f)\rangle_{N,M}}{\langle T_{SYS}^{REF}(f)\rangle_{N,M}}\right)^2 << \left(\frac{\sigma_{Counts^{SIG}}(f)}{Counts^{SIG}}\right)^2 + \left(\frac{\sigma_{Counts^{REF}}(f)}{Counts^{REF}}\right)^2$$

Since:

$$T_{SYS}^{REF}(f) = \langle Counts^{REF}(f)\rangle_{N,M} \cdot \langle G_K^{REF}(f)\rangle_{N,M}$$

then:

$$\left(\frac{\left\langle \sigma_{T_{SYS}}(f)\right\rangle_{N,M}}{\left\langle T_{SYS}^{REF}(f)\right\rangle_{N,M}}\right)^2 \approx \left(\frac{\left\langle \sigma_{Counts}(f)\right\rangle_{N,M}}{\left\langle Counts^{REF}(f)\right\rangle_{N,M}}\right)^2 + \left(\frac{\left\langle \sigma_{G_K}(f)\right\rangle_{N,M}}{\left\langle G_K^{REF}(f)\right\rangle_{N,M}}\right)^2$$

The result is only approximately correct since, for some calibration techniques, the determination of G might involve using the $Counts^{REF}$ data in which case the statistical errors of G and $Counts^{REF}$ are not independent. Nevertheless, we're only trying to determine approximate, minimum values for M and N.

The assumption is that N and M will be chosen to be shorter than the frequency scale and temporal scale for fluctuations in G and T_{SYS}. Then, one can use the assumption that the ratio of sums of near constant values is nearly that of the sum of the ratios. N and M should then be chosen such that:

$$\frac{1}{(N \cdot \Delta f)\cdot(M \cdot t^{REF})} + \left(\frac{\left\langle \sigma_{G_K}(f)\right\rangle_{N,M}}{\left\langle G_K^{REF}(f)\right\rangle_{N,M}}\right)^2 << \left(\frac{\sigma_{Counts^{SIG}}(f)}{Counts^{SIG}}\right)^2 + \left(\frac{\sigma_{Counts^{REF}}(f)}{Counts^{REF}}\right)^2$$

For back-of-the-envelope estimates, and assuming the time spent on reference observations is the same as for signal, and the time with the noise diode on equals that with the diode off, then:

$$N \cdot M \approx \frac{1}{p}\left(\frac{\sigma_{T_{CAL}}^2 + 2T_{SYS}^2}{T_{CAL}^2}\right)$$

where p is selected by the observers and represents the percentage error they are willing to tolerate in the calibration from the determination of T_{SYS}. For most observing, p should be, at worse, 0.1 and should be more like 0.01. For many GBT receivers, T_{SYS} ~ 10 T_{CAL}, and N·M should be at least 1000 and more likely 10,000.

It is very important to realize that for some observing frequencies, it will be impossible to pick N·M with such large values. Either the frequency structure of T_{CAL}, gain, or T_{SYS} will be too high, precluding a large value for N; or gain or T_{SYS} will be changing with time precluding a large value for M. In these cases, the observer must realize the compromises that need to be made between calibration accuracy and, say, bandpass shapes.

For those who will be using S_{SYS}, the above formulae become:

$$\frac{1}{(N \cdot \Delta f)\cdot(M \cdot t^{REF})} + \left(\frac{\left\langle \sigma_{G_S}(f)\right\rangle_{N,M}}{\left\langle G_S^{REF}(f)\right\rangle_{N,M}}\right)^2 << \left(\frac{\sigma_{Counts^{SIG}}(f)}{Counts^{SIG}}\right)^2 + \left(\frac{\sigma_{Counts^{REF}}(f)}{Counts^{REF}}\right)^2$$

$$N \cdot M \approx \frac{1}{p}\left(\frac{\sigma_{S_{CAL}}^2 + 2S_{SYS}^2}{S_{CAL}^2}\right)$$

Method 2

Method 2 does not require an estimate of gain of system temperature.

Method 3

TBD

Method 4

TBD

2.5.2 Single-Values for T_{SYS}, Statistical Weights, and Theoretical RMS

As in continuum observations, during observers or as a summary of the observations, astronomers would like a single-valued system temperature that is representative of an observation. For spectral-line observations, this is usually a frequency-averaged estimate of T_{SYS} (or S_{SYS}) for each sampler they are using. Additionally, some analysis systems cannot handle a vector of T_{SYS} values and require a single-valued T_{SYS}. One should use such analysis systems with some cautions since they will not be able to properly calibrate multiple spectral lines within a wide bandwidth data if T_{SYS} varies significantly across the band. This will be especially true for the extra wide bandwidths of some of the GBT systems. Nevertheless, because there will be users of these limited systems it is necessary the analysis systems provide a scalar estimate of system temperature.

One can derive an average T_{SYS} and S_{SYS} by performing a root-mean-squared, weighted average over the frequency range of a spectrum:

$$T_{SYS}^{AVRG} = \sqrt{\frac{\sum_{f_i=f_1}^{f_2} T_{SYS}^2(f_i) \big/ \sigma_{T_{sys}}^2(f_i)}{\sum_{f_i=f_1}^{f_2} 1 \big/ \sigma_{T_{sys}}^2(f_i)}}$$

$$\approx \sqrt{\frac{(f_2 - f_1) \big/ \Delta f}{\sum_{f_i=f_1}^{f_2} \frac{1}{T_{SYS}^2(f_i)}}}$$

$$\sigma_{T_{SYS}}^{AVRG} = \sqrt{\frac{1}{\sum_{f_i=f_1}^{f_2} 1 \big/ \sigma_{T_{sys}}^2(f_i)}}$$

$$S_{SYS}^{AVRG} = \sqrt{\frac{\sum_{f_i=f_1}^{f_2} S_{SYS}^2(f_i) \big/ \sigma_{S_{sys}}^2(f_i)}{\sum_{f_i=f_1}^{f_2} 1 \big/ \sigma_{S_{sys}}^2(f_i)}}$$

$$\approx \sqrt{\frac{(f_2 - f_1) \big/ \Delta f}{\sum_{f_i=f_1}^{f_2} \frac{1}{S_{SYS}^2(f_i)}}}$$

$$\sigma_{S_{SYS}}^{AVRG} = \sqrt{\frac{1}{\sum_{f_i=f_1}^{f_2} 1 \big/ \sigma_{S_{sys}}^2(f_i)}}$$

For some backends, the summation should avoid the corrupted few channels at the start and end of the spectrum.

Some analysis systems probably don't allow for channel-by-channel weight vectors as well. In which case a scalar value for the weights, and theoretical rms, can be determined via:

$$\sigma_{T_i}^{AVRG} = \frac{K \cdot T_{SYS}^i}{\sqrt{(f_2 - f_1) \cdot t_i}}$$

$$\sigma_{S_i}^{AVRG} = \frac{K \cdot S_{SYS}^i}{\sqrt{(f_2 - f_1) \cdot t_i}}$$

$$w_i^{AVRG} = \left(1 \big/ \sigma_i^{AVRG}\right)^2$$

The super/subscript i indicates the ‘type’ of data for which the weights are being estimated and could represent, for example, a phase (signal, reference, …) for data that have yet to be combined, or the difference spectrum, or a frequency-switched folded spectrum.

2.6 Averaging Observations

TBD

$$T_{avrg}(f) = \frac{\sum_i w^i(f) \cdot T(f)}{\sum_i w^i(f)}$$

$$\left\langle w_{avrg}(f) \right\rangle_N = \sum_i \left\langle w^i(f) \right\rangle_N$$

$$\left\langle \sigma_{avrg}(f) \right\rangle_N = \sqrt{\frac{1}{\left\langle w_{avrg}(f) \right\rangle_N}}$$

$$t_{avrg} = \sum_i t^i$$

$$\left\langle T_{SYS}^{Avrg}(f) \right\rangle_N = \frac{\left\langle \sigma_{avrg}(f) \right\rangle_N}{K_{avrg}} \sqrt{\Delta f \cdot t_{avrg}}.$$

2.7 Folding Frequency-Switched Observations

TBD

$$T_{fold}(f) = \frac{1}{\left\langle w_{diff}(f) \right\rangle_N + \left\langle w_{diff}(f+\Delta) \right\rangle_N} \left(\begin{array}{l} \left\langle w_{diff}(f) \right\rangle_N \left\langle T_{SYS}^{REF}(f) \right\rangle_N \left(\dfrac{Counts^{SIG}(f) - Counts^{REF}(f)}{Counts^{REF}(f)} \right) \\ + \left\langle w_{diff}(f+\Delta) \right\rangle_N \left\langle T_{SYS}^{SIG}(f+\Delta) \right\rangle_N \left(\dfrac{Counts^{REF}(f+\Delta) - Counts^{SIG}(f+\Delta)}{Counts^{SIG}(f+\Delta)} \right) \end{array} \right)$$

$$T_{fold}(f) = \left(T_{diff}(f) - T_{diff}(f+\Delta) \right) / 2$$

$$\langle w_{rap}(f)\rangle_N = \langle w_{diff}(f)\rangle_N + \langle w_{diff}(f+\Delta)\rangle_N$$

$$\langle \sigma_{rap}(f)\rangle_{N,M} = \sqrt{\frac{1}{\langle w_{rap}(f)\rangle_{N,M}}}$$

$$t_{fold} = 2t_{diff}$$

$$\langle T_{SYS}^{fold}(f)\rangle_{N,M} = \frac{\langle \sigma_{fold}(f)\rangle_{N,M}}{K}\sqrt{\Delta f \cdot t_{fold}}$$

2.8 RFI Excision

TBD

3. Units of Intensity

The process of calibration takes the raw counts, powers, or squared voltages produced by a backend and convert them into some more meaningful unit of intensity. The units of intensity that the data are to be converted into depend upon the science or object observed. In the history of radio astronomy, many units of intensity have been defined (See Baars, 1973; Dent 1972; Ulich et al 1980; Kutner and Ulich, 1981).

The factors used to accomplish the conversions are sometimes elevation dependent, frequency dependent, and sometimes change as the weather changes

The most common units of intensity are:

3.1 T_A – Antenna Temperature

Antenna temperature (T_A) in units of Kelvin is usually considered the most basic unit of intensity. Essentially, it is from T_A that all other units of intensity are derived.

3.1.1 S_A – Antenna Temperature in units of Jy

It is often convenient to use Jy for the units of antenna temperatures, especially when one is using noise diodes in units of Jy (S_{CAL}) or astronomical sources as calibrators. The relationship between S_A and T_A is:

$$S_A = \frac{2k \cdot T_A}{A_P \cdot \eta_A}$$

where η_A is the aperture efficiency, A_P is the physical area of the antenna, k is Boltzman's constant, and S is in units of Jy (= 10^{-19} ergs/m^2). In these units, $2k/A_P$ = 2.84 Jy/K for the GBT.

3.2 T_A'– Antenna Temperature Prime

T_A' is defined as the antenna temperature in units of Kelvin, corrected for atmospheric attenuation. That is:

$$T_A' = T_A \cdot e^{\tau_0 A}$$

where T_A is the conventional antenna temperature, τ_o is the total atmospheric opacity at the zenith, and A is the number of air masses relative to the zenith.

3.3 T_A^* – Corrected Antenna Temperature

T_A^* is defined as the antenna temperature in units of Kelvin, corrected for atmospheric attenuation and for rear spillover, ohmic loss, and blockage efficiency:

$$T_A^* = T_A \cdot e^{\tau_0 A} / \eta_l \ .$$

Here, η_l is an efficiency that corrects for rear spillover, ohmic loss, and blockage.

3.4 T_{MB} – Main-Beam Brightness Temperature

The main beam brightness temperature scale is defined as the convolution of the brightness temperature distribution of the source and the main diffraction beam of the antenna. This scale is appropriate for observation of sources whose angular sizes are comparable to the main beam. It is defined as:

$$T_{MB} = T_A \cdot e^{\tau_0 A} / \eta_{MB}$$

where η_{MB} is the main-beam efficiency.

3.5 T_R^*– Corrected Radiation Temperature

T_R^* is related to T_A by:

$$T_R^* = T_A \cdot e^{\tau_0 A} / (\eta_{fss} \cdot \eta_l)$$

where η_{fss} is the forward spillover efficiency.

Though not always the case, and in a very loss way, T_R^* is most appropriate for observations that used a receiver at the secondary focus and T_A^* for prime focus observations.

3.6 S – Flux Density

Unlike most of the previous intensity units, flux density is an absolute, physically significant unit of intensity since all telescopes will reproduce the same flux density. It can be derived from T_A units via:

$$S_f = \frac{2k \cdot T_A \cdot e^{\tau_0 A}}{A_P \cdot \eta_A}$$

where η_A is the aperture efficiency, A_P is the physical area of the antenna, k is Boltzman's constant, and S is in units of Jy (= 10^{-19} ergs/m^2). In these units, $2k/A_P$ = 2.84 Jy/K for the GBT. Also note that if the flux density of a source is known, one can drive the above equation backwards to provide useful calibration information, as illustrated below.

3.7 Rayleigh-Jeans Law and Effective Radiation Temperature

All of the above temperature intensity units (i.e., all except S) use the assumption of the Rayleigh-Jean law that h·f « kT or f « 2 x 10^9 T. This assumption breaks down at the highest frequencies of the GBT. Let T represents any of the above units of intensity and J the effective radiation temperature. Then:

$$J = \frac{hf/k}{e^{hf/kT} - 1},$$

and the inverse of this equation is:

$$T = \frac{hf/k}{\ln\left(1 + hf/kJ\right)}.$$

Because of the wide bandwidths and high frequency capabilities of the GBT, if the user has opted to convert to Rayleigh-Jeans equivalent temperature, then the above calculation should be performed on every channel in a spectral line observation.

For similar reasons, for continuum observations one shouldn't use the frequency at the center of the band in the above calculation. Rather, one must use a weighted average of the above equations. Namely:

$$\langle J \rangle_f = \frac{\int_{f_1}^{f_2} \frac{hf/k}{e^{hf/kT}-1} \cdot df}{f_2 - f_1} ,$$

and, for the inverse translation:

$$\langle T \rangle_f = \frac{\int_{\nu_1}^{\nu_2} \frac{hf/k}{\ln(1+hf/kJ)} \cdot df}{f_2 - f_1} .$$

Here, f_1 and f_2 are the approximate lower and upper frequency limits of the continuum observation.

4. Calibration Factors

Many of the calibration factors presented above will also need to be determined by the observer or NRAO staff. The following sections describes various methods to determine each of these factors.

4.1 Atmospheric opacity at the zenith

Over the high frequency range of the GBT, the atmosphere should be considered multiple layers. In many cases, one can assume a two-layer model that consists of: (1) a thick oxygen layer whose opacity depends solely on the dry-air pressure and temperature profiles over the observatory; (2) and a thin water layer whose opacity depends upon the water partial pressure and temperature profiles over the observatory, and the existence of hydrosols. At frequencies below a few GHz, water vapor and variations in the oxygen opacity are not very important and the observer can assume that zenith opacity is weather invariant.

No attempt will be made here to discuss the affects of the ionosphere at the lowest frequencies.

We have the capability to provide a number of methods that the observers can choose from for determining opacity.

4.1.1 Vertical Weather Data: One of only two absolute ways to determine opacity is from knowing the vertical profiles of dry-air and water vapor pressures, hydrosols, and temperatures above the observatory. Liebe (1985) provides a very comprehensive model that would use vertical weather profiles to estimate opacities. Lehto (1989, pp 165-173) has beautifully simplified Liebe's work to a very manageable series of equations, tables, and just the essential parameters from vertical profiles. Between 2 and 60 GHz, Lehto estimates his simplifications differ by at most 10% from Liebe's much more complicated model. Liebe's and Lehto's discussions are too long to recreate here. The WWW page http://www.gb.nrao.edu/~rmaddale/Weather illustrates the derivation of opacities from vertical profiles.

4.1.2 Ground Weather Data: Lehto also provides a comprehensive discussion on how one can estimate opacity from just ground-based weather data. Here, typical, 'model' vertical profiles are substituted for the measured profiles. The estimated accuracy after this further simplification of Liebe's model is 20%.

4.1.3 Values in the Literature or local database: At low frequencies, the atmospheric opacity is weather independent. One can create a table of suggested opacity values, either from a literature search, from running models at various frequencies, or by keeping a database of opacities that staff has measured at various low frequencies.

4.1.4 T_{SYS} vs. Elevation (Tippings or standard observations): Due to radiative processes, the atmosphere not only absorbs but also emits radiation. One can measure T_{SYS} at various elevations and use a model of the atmospheric emission to derive opacity. These measurements can be either from observations that used a procedure like GO's **TIPPING** or from the T_{SYS} estimates that are a by-product of most types of continuum or spectral line observations.

The literature gives either one or two-layer models suitable for measuring opacities. A one-layer model, good for frequencies below about 5 GHz, is:

$$T_{sys} = T_{rx} + T_{ATM} \cdot (1 - e^{\tau_0 A}).$$

For very low opacities or high elevations, the first-order expansion of the model is:

$$T_{sys} = T_{rx}^{'} + T_{ATM} \tau_0 A.$$

T_{ATM} is some 'effective' temperature of the atmospheric layer that is producing the opacity and is best determined from vertical weather data but, most often, is taken to be 20 to 40 K colder than the ground air temperature.

One then does either a non-linear, least-squares fit (first model) or a linear, least-squares fit (second model) of measured T_{SYS} at a wide range of elevations to derive T_{rx}' and τ_0. T_{rx}' is the classic receiver temperature (T_{rx}) that engineers usually measure plus contributions from the 3K cosmic microwave background and ground pickup/spillover.

At frequencies above 15 GHz, a more suitable atmospheric model for determining opacity is the two-layer model proposed by Kutner (1978), Ulich (1980), and Kutner and Ulich (1981):

$$
\begin{aligned}
T_{sys} &= T_{rx} \\
&+ \eta_{ms} \left\{ T_{CMB} \exp\left(\left(-\tau_W + \tau_{O_2}\right)A\right) + T_W - \exp\left(-\tau_W A\right) \cdot \left[T_W - T_{O_2}\left(1 - \exp\left(-\tau_{O_2} A\right)\right)\right]\right\} \\
&+ \left(1 - \eta_{ms}\right) T_{GRND}
\end{aligned}
$$

T_{rx} is the classic value for the receiver temperature (and only loosely related to T_{rx}' from above). η_{MS} is either $\eta_{fss}\eta_l$ for Gregorian observations or η_l for prime focus observations. T_{CMB}=2.8K, T_W and T_{O2} are the effective temperatures of the water and oxygen layers that are producing the opacity, and τ_W and τ_{O2} are the opacities from water and oxygen. T_{GRND} is a representative ground temperature. Resenkranz (1975) provides estimates of τ_{O2} as a function of frequency and height above sea level for the observatory. T_W and T_{O2} are usually estimated from ground air temperatures, as suggested by Kutner (1981). One then does a non-linear, least-squares fit for T_{rx} and τ_W with T_{SYS} as the dependent (measured) quantity and A the independent quantity. Finally, $\tau_0 = \tau_W + \tau_{O2}$.

Using TSYS in these ways to determine opacity is fraught with complications:

- A large fraction of the uncertainty with these models stems from the usual practice of taking ground temperature values to guess an effective atmospheric temperature. To first order, the percentage error in the resulting opacity is the same as the percentage error in the estimate of T_{ATM}.
- On days with variable cloud cover, the **TIPPING** observing procedure will produce data that has bumps and dips as the antenna moves on and off the clouds. Tippings on days for which opacity is most needed, are often useless.
- At low frequencies, **TIPPING** will often stumble across the galactic plane whose contribution to T_{SYS} must be removed over the affected elevation range.
- The GBT, even at its fastest slew rates, is confusion limited below about 2 GHz and the **TIPPING** data will be contaminated by point sources that have to be low-pass filtered in the data analysis.

4.1.5 T_{SYS} - T_{rx}': If one has an accurate determination of T_{rx}', either estimated from the engineers values for T_{rx} or one obtained from a previous tipping, then T_{SYS} - T_{rx}' is a estimate of the atmospheric opacity. Rearranging the above one-layer model gives:

$$
\tau_0 = \frac{\ln\left(1 - \frac{T_{sys} - T_{rx}^{'}}{T_{ATM}}\right)}{A}
$$

This method of determining opacity uses T_{SYS}, a very common by-product of most types of observations. Except for a one-time determination of T_{rx}', there's no need for extra observations. As clouds go by or the elevation of the source changes, the measured increase in T_{SYS} is exactly related to the increase in opacity one should use. That is, the observer can determine opacities minute by minute throughout an observing run. The significant problems with this method is, again, the reliance on an estimate of T_{ATM} and the assumption that T_{rx}' is stable and accurately determined. The method is slightly less accurate at frequencies where the two-layer model is required.

4.1.6 T_A vs. Elevation: If one were to measure the T_A for a source as it rises or sets, one can rearrange equations 3 or 5 to determine directly opacity:

$$\ln(T_{MB}\eta_{MB}) - \ln(T_A) = \tau_0 A$$
$$\ln\left(\frac{A_P \cdot \eta_A \cdot S_\nu}{2k}\right) - \ln(T_A) = \tau_0 A.$$

The first equation is most suitable for an extended source and the second for a point source. If T_{MB} or S_f are also known (i.e., one has used a calibrator), as well as the efficiencies at the elevation of the observation, then a single observation can be used to derive opacity.

If the source intensities are not known, or if one wants a more reliable estimate of opacity, one can measure opacity at various elevations and fit for τ_0. If the efficiency is independent of elevation, this is a linear least-squares fit where τ_0 is the fitted slope, A is the independent variable, and $\ln(T_A)$ is the dependent (measured) quantity. The Y-intercept of the fit is the first term in the above equation. If the source intensity and efficiencies are known one can use a linear least squares fit that is constrained to go through the already-known intercept.. If the source intensity and efficiencies are not known, then the Y-intercept will contain information on the product of the source intensity and efficiency.

If the efficiency is dependent on the elevation and is known, then this becomes a non-linear, least-squares problem. If the efficiency is dependent on the elevation but the dependence isn't known, then one cannot use this method to determine opacity.

This method has the great advantage that one need not take up significant extra observing time to derive opacities (e.g., one can use pointing measurements of a nearby source) and, like using vertical profiles, is the only direct determination of opacity. It has the disadvantage that one needs some knowledge of the elevation-dependent efficiencies. The method works best if the data were taken over a wide range in elevations.

4.1.7 Extrapolation from Tipping Radiometer: The 86 GHz radiometer measures opacity every few minutes. The 86-GHz opacity can be extrapolated to other frequencies using models like that in Liebe (1985). These models are too detailed to discuss here. The radiometer suffers from all the problems with variable cloud cover that antenna tippings have. Opacities are sometime unphysical when the weather is even slightly cloudy. Extrapolating under these conditions to other frequencies is a serious source of concern that must weigh into a users decision as to what method to use to determine opacity.

4.2 Number of air masses relative to the zenith (A).

The "Number of air masses" refers to the path length the signal takes though the atmosphere. If the vertical profile of the atmosphere is known, then one can derive a value for A that doesn't assume a plane-parallel atmosphere.

$$A = \frac{\displaystyle\int_0^\infty \frac{\rho(h)\cdot dh}{\sqrt{1-\left(\frac{R}{R+h}\,\frac{n_0}{n(h)}\right)^2 \sin^2(el)}}}{\displaystyle\int_0^\infty \rho(h)\cdot dh}$$

(Rohlfs and Wilson 1996) Here, R is the radius of the Earth at the observatory (~6370.6 km for the GBT), el is the actual, **refraction-corrected** elevation of the observations, h is the height above the observatory, ρ(h) is the density of the atmosphere at height h, n(h) is the index of refraction at height h, and n_0 is the index of refraction at h = 0.

From the temperature (T(h) in C), total pressure (P(h) in mmHg), and water partial pressure (P_w(h) in mmHg), an adequate estimate of the index of refraction at height h is (Froome and Essen 1969, Maddalena 1994):

$$n(h) = 1 + 10^{-6}\left[\frac{102.49P(h)}{273.15+T(h)} - \frac{91.14P_w(h)}{273.15+T(h)} + \frac{4.958\cdot 10^5 P_w(h)}{(273.15+T(h))^2}\right]$$

For elevations above 5 degrees, an approximation for A that is said to be good to about 1% (Rohlfs and Wilson, 1996) is:

$$A = -0.0045 + \frac{1.00672}{\sin(el)} - \frac{0.002234}{\sin^2(el)} - \frac{0.0006247}{\sin^3(el)}$$

If one assumes a plane-parallel model of the atmosphere, then:

$$A = 1/\sin(el)$$

This simplification produces a 1% and 11% error in A at elevations of 16 and 5 degrees respectively.

4.3 η_A -- Aperture efficiency

For the GBT, aperture efficiency, η_A, is dependent upon frequency, elevation when the observations are above a frequency of about 5 GHz, and feed for multi-feed receivers. Below 5 GHz, one can assume η_A=0.70 at all elevations. We are currently attempting to measure η_A at high frequencies. A by-product of this work will be a parameterization of η_A as a function of elevation, frequency, and feed.

These attempts will take some time and, as we improve the surface of the GBT, η_A will also change. Observers will also want a fast way of estimating their own η_A. Thus, we need to provide users a practical way to determine η_A from their own astronomical observations. This can be done by inverting equation 5:

$$\eta_A = \frac{2k \cdot T_A \cdot e^{\tau_0 A}}{A_P \cdot S_f}$$

The user measures T_A toward a true point source of known flux density, S_f using either a continuum or spectral line backend. For example, one of the traditional results of a PEAK is the T_A of the source. The user must also determine an appropriate opacity and value for A or use the system-supplied default values.

4.4 η_R -- Ohmic loss efficiency

The ohmic losses for the GBT are probably extremely small and we can safely assume η_R=1.0. The software should allow the user the power to specify a different value..

4.5 η_l -- Rear spillover, ohmic loss, and blockage efficiency

The GBT has no blockage and should not have any appreciable ohmic losses. Its rear spillover should be typical of most radio telescopes with a 10-15 dB illumination taper. One can assume η_l ~1.0 over all elevations, though no one has attempted to confirm this. η_l probably is a constant over all of the GBT's anticipated range of frequency.

4.6 η_{MB} -- Main-beam efficiency

The main-beam efficiency for the GBT is dependent upon frequency, elevation when the observations are above a frequency of about 5 GHz, and feed for multi-feed receivers. η_{MB} is defined as:

$$\eta_{MB} = \frac{\eta_A A_P \Omega_{MB}}{\eta_R \lambda^2}.$$

Default values for η_A and η_R are described above. A_p is the physical, projected area of the dish, λ the observing wavelength, and Ω_{MB} is the solid angle of the main beam.

A very good way to determine Ω_{MB}, and simultaneously η_A, is by doing a **PEAK** observation on a true point source of known flux density. η_A would be calculated using the method described above. If we assume the GBT's beam is a Gaussian at the observing frequency and elevation, then an approximation that is good to about 5% (Rohlfs and Wilson, 1996) is

$$\Omega_{MB} = 1.133 \cdot \theta_{X@0.5\,power} \cdot \theta_{Y@0.5\,power}$$

where θ_X and θ_Y are the full-width, half-maximum beam widths in the two orthogonal directions, a by-product of the data reduction for a **PEAK** observation. According to Rohlfs and Wilson (1996) a better estimate that is good to 1% can be derived from the full-widths at the 10% power points in the profile:

$$\Omega_{MB} = 0.3411 \cdot \theta_{X@0.1\,power} \cdot \theta_{Y@0.1\,power}$$

The determination of these quantities should be added to the applications that analyze **PEAK** data.

4.7 η_{fss} -- Forward spillover and scattering efficiency

The GBT has no blockage and its forward spillover should be typical of most radio telescopes with a 10-15 dB illumination taper. η_{fss} is defined as η_{MB}/η_l and probably has a value close to 1, though no one has attempted to confirm this. η_{fss} probably is a constant over all of the GBT's anticipated range of frequency.

5. Astronomical Measurement of S_{CAL} and T_{CAL}

For the accurate calibration of data, some observers will opt to determine their own values of T_{CAL} instead of relying on values provided by the receiver engineer. This might be the case if there's some reason to believe the receiver's noise diode has changed or whether the 5-10% accuracy of the engineer's values isn't sufficient, or if the engineer's values don't have enough frequency resolution.

One can perform these measurements using either a spectral line or continuum backends.

5.1 Spectral Line

The measurements usually entail moving on and off a source of known intensity while firing the noise diode (Johnson and Maddalena, 2002). I will assume the observer will use a point source since all of the uses of astronomically-determined S_{CAL} and T_{CAL} presented in this paper are based on this assumption. It'll be the work of the observers to determine how to modify this paper for those times an extended source is used.

One determines S_{CAL} by comparing the power with the diode on and off to the difference in power between the signal and reference position. If the signal and reference observations are made back-to-back, as would mostly likely be the case when using the **OFFON** or **ONOFF**, "Total Power with Cal"**,** observing procedure, then:

$$S_{CAL}(f) = S(f) \cdot \frac{\eta_A}{e^{\tau_0 A}} \cdot \left\langle \frac{Counts_{REF}^{cal\_on}(f) - Counts_{REF}^{cal\_off}(f)}{Counts_{SIG}^{cal\_off}(f) - Counts_{REF}^{cal\_off}(f)} \right\rangle_M$$

$$\sigma_{Scal}(f) = \frac{S_{CAL}(f)}{\sqrt{\Delta f}} \left\langle \sqrt{ \frac{ \frac{\left(Counts_{REF}^{cal\_on}(f)\right)^2}{t_{REF}^{cal\_on}} + \frac{\left(Counts_{REF}^{cal\_off}(f)\right)^2}{t_{REF}^{cal\_off}} }{ \left(Counts_{REF}^{cal\_on}(f) - Counts_{REF}^{cal\_off}(f)\right)^2 } + \frac{ \frac{\left(Counts_{SIG}^{cal\_off}(f)\right)^2}{t_{SIG}^{cal\_off}} + \frac{\left(Counts_{REF}^{cal\_off}(f)\right)^2}{t_{REF}^{cal\_off}} }{ \left(Counts_{SIG}^{cal\_off}(f) - Counts_{REF}^{cal\_off}(f)\right)^2 } } \right\rangle_M$$

S(f) is the frequency-dependent flux of the source and Δf is the frequency resolution of the backend. One has to be careful to use a frequency-dependent source flux for wide bandwidths or sources with non-flat spectra. The < > indicates a sliding (boxcar) average that is M channels wide around frequency f. M should be chosen by the observer to be large enough so as to minimize the uncertainty in S_{CAL} but must be smaller than the anticipated frequency structure of the noise diode.

Note that it's not necessary that an observer use the procedures **OFFON** or **ONOFF** or the "Total Power with Cal" switching mode. Instead, the parts of the above equations could come from, say, a **TRACK** observation of a reference observation using "Total Power with Cal" and a **TRACK** observation of the signal position using either "Total Power with Cal" or "Total Power without Cal". Or, maybe an OTF observation that slews across a known source. Thus, the user must be given the ability to choose the various observations, integrations, etc. that are needed for the above equation.

In order for S_{CAL} to be independent of elevation, one must know and compensate for the elevation dependence of telescope efficiency, η_A and atmospheric opacity. Atmospheric opacity is a quantity that, if measured astronomically with a tipping, requires knowing T_{SYS} which, in turn, requires knowing T_{CAL}. Luckily, the dependence on opacity is usually slight and one can iterate down to a solution. One merely determines opacity using any estimate of T_{CAL}, determines a revised estimate of S_{CAL} and then T_{CAL} using the above equations re-reduces the tipping data using the new T_{CAL}, re-reduces the on-off observations, and so on until the results converge. For extremely wide bandwidths near the atmospheric H_2O or O_2, one may even want to specify a frequency-dependent opacity. At the highest frequencies, maybe a frequency-dependent efficiency.

For back-of-the-envelope purposes, σ_{Scal} is very roughly:

$$\sigma_{Scal} \approx \frac{2kT_{SYS}}{A_P}\sqrt{\frac{2\cdot\left(1+\frac{S_{CAL}^2}{S^2}\right)}{M\cdot\Delta f\cdot t}}$$

Usually, the uncertainty in the flux of the source is much larger than the radiometer uncertainties of T_{CAL} and S_{CAL}. To avoid the systematic errors introduced by using a single source, observers will typically repeat these observations with multiple sources and average the T_{CAL} and S_{CAL} values.

Then, T_{CAL} is derived from S_{CAL} using:

$$T_{CAL}(f) = \frac{A_P\cdot S_{CAL}(f)}{2k}$$

$$\sigma_{Tcal}(f) = \sigma_{Scal}(f)\cdot\frac{A_P}{2k}$$

5.2 Continuum

The determination of T_{CAL} and S_{CAL} from continuum data works along the same lines. Again, it's up to the observer whether to use data from an OTF observing procedure, two **TRACK** observations, or an **OFFON**-like procedure. The continuum formulae are:

$$S_{CAL}(f) = S_f\cdot\frac{\eta_A}{e^{\tau_0 A}}\cdot\frac{1}{N}\sum_{i=1}^{N}\frac{Counts_{REF}^{cal\_on}(i) - Counts_{REF}^{cal\_off}(i)}{Counts_{SIG}^{cal\_off}(i) - Counts_{REF}^{cal\_off}(i)}$$

$$\sigma_{Scal}(f) = \frac{S_{CAL}(f)}{N\sqrt{BW}} \sqrt{\sum_{i=1}^{N} \left[\frac{\frac{\left(Counts_{REF}^{cal\_on}(i)\right)^2}{t_{REF}^{cal\_on}} + \frac{\left(Counts_{REF}^{cal\_off}(i)\right)^2}{t_{REF}^{cal\_off}}}{\left(Counts_{REF}^{cal\_on}(i) - Counts_{REF}^{cal\_off}(i)\right)^2} + \frac{\frac{\left(Counts_{SIG}^{cal\_off}(i)\right)^2}{t_{SIG}^{cal\_off}} + \frac{\left(Counts_{REF}^{cal\_off}(i)\right)^2}{t_{REF}^{cal\_off}}}{\left(Counts_{SIG}^{cal\_off}(i) - Counts_{REF}^{cal\_off}(i)\right)^2} \right]}$$

$$T_{CAL}(f) = \frac{A_p \cdot S_{CAL}(f)}{2k}$$

$$\sigma_{Tcal}(f) = \sigma_{Scal}(f) \cdot \frac{A_P}{2k}$$

Now, f is the frequency at the center of the observing band. Instead of a sliding (boxcar) mean over frequency, here it's essentially a simple time average over N samples. The back-of-the-envelope value for the uncertainty is:

$$\sigma_{Scal} \approx \frac{2kT_{sys}}{A_P} \sqrt{\frac{2 \cdot \left(1 + \frac{S_{CAL}^2}{S^2}\right)}{N \cdot BW \cdot t}}$$

The source flux, S_f, used in this method is that which the observer believes represents the flux across the observing bandwidth. For sources with extreme spectral indices and observations with wide-enough bandwidths, the observer shouldn't specify an S(f) for the center of the band. Rather, it should be the mean of S(f) across the band:

$$S_f = \frac{\int_{f=f_1}^{f_2} S(f) \cdot df}{f_2 - f_1}$$

Here, f_1 and f_2 are the frequency limits of the continuum observation. For those sources whose fluxes in the literature are estimated by a power law ($S_f = 10^{c_1 + c_2 \cdot \log_{10}(f) + c_3 \cdot \log_{10}^2(f)}$), the integral is explicitly:

$$S_f = \frac{\int_{X=\log_{10}(f_1)}^{\log_{10}(f_2)} 10^{c_1 + c_2 \cdot X + c_3 \cdot X^2} \cdot dX}{f_2 - f_1}$$

6. Interpolating and Averaging T_{CAL} and S_{CAL} from Noise Diode Values

In most of the calibration methods described, noise diode values, either represented as T_{CAL} or S_{CAL}, must be either known or determined for the calibration to proceed. In many cases, the accuracy of the determined T_{CAL} limits the final accuracy of the calibration.

We should expect that the T_{CAL} and S_{CAL} values used in the calibration process will be different for every signal and reference backend phase and every backend sampler. This is because diode values depend upon the receiver, feed, polarization, and frequency of the observations. In fact, with the high spectral resolution and wide bandwidths of the GBT backends, noise diode values may be needed for every channel in a spectral-line observation.

The engineers usually provide a table of frequency-dependent diode values that depend upon the feed and polarization of the observation. Observers will create their own table of diode values. But, most observations will not have the same bandwidth or spectral-resolution of the tables. So, tabular diode values will need to be either interpolated or averaged to the resolution of the observations.

6.1 Continuum

For continuum calibration, one requires a T_{CAL} value that is representative of the bandpass over which the observation is made. Since continuum observations usually have wide bandwidths, the required T_{CAL} value will probably have lower spectral resolution than the engineer's table of values. Thus, one usually must determine an average T_{CAL} value from a table of values.

Because the gain of the system changes with frequency within the wide continuum bandwidths, a straight average of T_{CAL} values isn't appropriate. Rather, one must perform a gain-weighted average across the band:

$$T_{CAL}^{AVRG} = \frac{\int_0^\infty G_K(f) \cdot T_{CAL}(f) \cdot df}{\int_0^\infty G_K(f) \cdot df}$$

$$S_{CAL}^{AVRG} = \frac{\int_0^\infty G_S(f) \cdot S_{CAL}(f) \cdot df}{\int_0^\infty G_S(f) \cdot df}$$

Although the formal limits are zero and infinity, gain is essentially zero outside the bandwidth of the observations. Since the noise diode values are usually presented in a table with equally spaced frequencies, the integral turns into the summation:

$$T_{CAL}^{AVRG} = \frac{\sum_{f_i=f_1}^{f_2} G_K(f_i) \cdot T_{CAL}(f_i)}{\sum_{f=f_1}^{f_2} G_K(f_i)}$$

$$S_{CAL}^{AVRG} = \frac{\sum_{f_i=f_1}^{f_2} G_S(f_i) \cdot S_{CAL}(f_i)}{\sum_{f=f_1}^{f_2} G_S(f_i)}$$

$$\sigma_{Tcal}^{AVRG} \approx \frac{\Delta f \cdot T_{CAL}}{f_2 - f_1} \sqrt{\sum_{f_i=f_1}^{f_2} 2 \cdot \frac{\sigma_G^2(f_i)}{G_K^2(f_i)} + \frac{\sigma_{Tcal}^2(f_i)}{T_{CAL}^2(f_i)}}$$

$$\sigma_{Scal}^{AVRG} \approx \frac{\Delta f \cdot S_{CAL}}{f_2 - f_1} \sqrt{\sum_{f_i=f_1}^{f_2} 2 \cdot \frac{\sigma_G^2(f_i)}{G_S^2(f_i)} + \frac{\sigma_{Scal}^2(f_i)}{S_{CAL}^2(f_i)}}$$

Here, f_1 and f_2 are the frequency range of the observation and Δf is the frequency granularity of the noise diode table. The statistical errors are only approximations since errors in the determination of G are usually correlated with those in the determination of diode values..

In most cases G(f) isn't known and must be measured since it is time variable and changes as attenuators, etc. are changed. Unfortunately, one requires $T_{CAL}(f)$ values in order to astronomically measure gain, a circular problem.

One can try to astronomically measure noise diode with a spectral-line backend with a frequency resolution that is smaller than the anticipated changes in G and make enough frequency-overlapped observations at various center frequencies to cover the bandwidth of the continuum observations. Above are various methods for astronomically measuring diode values in this way. Then, use those diode values to determine G(f), using the spectral-line methods described above, and, finally, perform the above summations. The hazard with this approach is that continuum and spectral-line backends usually do not share the same signal paths and the spectral-line estimate of G(f) may not be appropriate for continuum observations. The errors introduced are usually small enough to ignore.

Alternatively, observers may not want to go through this complexity and are willing to risk a systematic error in calibration by just using a straight average across the bandwidth of the continuum observation.

$$T_{CAL}^{AVRG} = \frac{\Delta f \sum_{f_i = f_1}^{f_2} T_{CAL}(f_i)}{f_2 - f_1}$$

$$S_{CAL}^{AVRG} = \frac{\Delta f \sum_{f_i = f_1}^{f_2} S_{CAL}(f_i)}{f_2 - f_1}$$

$$\sigma_{Tcal}^{AVRG} \approx \frac{\Delta f}{f_2 - f_1} \sqrt{\sum_{f_i = f_1}^{f_2} \sigma_{Tcal}^{2}(f_i)}$$

$$\sigma_{Scal}^{AVRG} \approx \frac{\Delta f}{f_2 - f_1} \sqrt{\sum_{f_i = f_1}^{f_2} \sigma_{Scal}^{2}(f_i)}$$

6.2 Spectral Line

TBD

7. Analysis System Requirements

There are a number of requirements that must be placed on the analysis system so as to provide the necessary flexibility and range of options to the observer. The following list of requirements is subjective and is by no means complete.

7.1 Continuum Calibration

- The software should allow the grouping of scans into an observation that is than handled as a single object.
- In some cases, the observing procedure (e.g. **OFFON, PEAK, and DECLATMAP**) can define a default grouping.
- The user should be able to define their own grouping of scans or modify the suggested default groupings. The user should be able to use a combination of ranges in positions, source names, UT and LST times, scans, and sample numbers when defining an observation grouping.
- In some cases, the observing procedure can define a set of default reference samples (e.g., the first and last few samples in a PEAK observation) or scans (e.g., the ‘Off’ scan in an **OFFON** observation).
- The user should be able to define their own sets of reference samples or reference scans, or modify the default set. The user should be able to use a combination of ranges in positions, UT and LST times, scans, source name, and sample numbers when defining reference samples and scans. All samples/scans not designated as reference samples/scans should be considered ‘signal’ samples/scans.

- Once reference observations are defined, the software should provide at least the following options:
  - Perform a weighted average of the reference samples/scans and subtract the average from the signal samples/scans. Note that this is essentially a removal of a DC offset since the same data value is subtracted from each signal sample/scan.
  - For each signal sample, create a unique, sliding weighted average of the reference samples/scans that surround in time or position the signal sample/scan. This 'sliding' average is then subtracted from the signal sample/scan. Note that each signal sample/scan is altered by a different amount.
  - Filter the reference samples/scans that surround in time or position a signal sample/scan. A median filter is commonly the filter of choice. This 'sliding' filtered version of the reference data is then subtracted from the signal sample/scan. Note that each signal sample/scan is altered by a different amount.
  - Fit a one, two, or three-dimensional function to the reference samples. The three possible dimensions can be based on some combination of telescope positions, either LST or UT, or scan/sample number. The fitted function is then removed from the signal samples/scans. Note that each signal sample/scan is altered by a different amount.

7.2 Spectral-Line Calibration

TBD

7.3 Intensity Units

- Data analysis systems should allow the user the ability to convert between most of the more commonly used units of intensities.
- T_A should be the default unit of intensity.
- The software should not assume the user wants a particular intensity scale. For example, it should not assume whether T_R^* or T_A^* is the better unit of intensity.
- Note that the conversions are mostly simple algebra and, as such, the user should be able to switch intensity units without having to go through a full calibration process. That is, unit conversions shouldn't involve more than a smidgen of CPU, typically a single division or multiplication. Note that converting to T_A from other intensity units and from T_A to other units is always possible.
- The software should keep track of the current intensity units. This is for both the user and software's later use.
- The software should supply default values for all quantities used in the conversions that are time independent and that have been properly determined by staff.

- The software should warn the user if they are converting to an intensity unit that requires time-dependent quantities (e.g., a unit that requires opacities) or factors that have not yet been fully determined by staff.
- The software should provide a concise summary of the factors that will go into any unit conversion.
- Once the data is converted, the software should provide an easy way to query the values of the factors that went into the conversion.
- The user must be given complete and easy control over the values used in the conversion of intensity units. That is, the defaults should be very easy to override.
- For those users who want to spin their own conversion routines, the software should allow one to display and manipulate data in the raw units of the backend without first converting to T_A.
- For those users who want to spin their own conversion routines, the software should treat the user's converted data in the same way it would treat data that was converted using the NRAO-supplied routines.
- Even after converting to a standard unit of intensity, the user should have the ability to manipulate (e.g., scale) the data.
- The user should have the ability to label the intensity units whatever they desire. This will be useful for those that spin their own conversion routines or have manipulated the data into an intensity unit that is not one of the supported standards.
- The analysis system should keep two sets of flags, one holds the units of intensity, the other whether the intensity is in terms of the effective radiation temperature or assumes the Rayleigh-Jean law.

7.4 Calibration Factors

7.4.1 Opacity

The user must be allowed to pick the method they want to use for estimating opacity. Each way in which an observer will measure opacity has its own sets of requirements.

All Methods

- For multi-frequency observing, each frequency band should have its own opacity since opacities can vary greatly across a receiver's bandpass.
- For wide-bandwidth spectral-line observations, it's possible for the opacity to vary greatly within a single band. The system should provide a flag that will calculate and correct for the opacity on a channel-by-channel basis.

Vertical Weather Data

- Eventually, the software should allow the user the choice between models of the atmosphere.

- The software must accept vertical profile data for the time of the observation.

Ground Weather Data

- The software should use for defaults the weather data at the time of the observation.
- Users should have the ability to override the defaults with their own values.

Values in the Literature or local database

- The software and documentation should make it clear how and when the tabular value of opacity was derived.
- Users should have the ability to override the default value with their own.

T_{SYS} vs. Elevation (Tippings or standard observations)

- The user must be able to specify which **TIPPING** observation they want to use for determining opacity.
- Like pointing, the results of each **TIPPING** observation should be maintained in an observatory database for future, statistical use.
- For multi-frequency observations, or very-wide-band spectral-line observations, opacities may be very different for the different observing frequencies. Because of limitations in the current I.F. system, a user may elect to use spectral-line observing for TIPPINGS than continuum observing. Thus, we need a spectral-line tipping analysis procedure.
- As an alternative to **TIPPING**, the user must be able to specify a set of continuum or spectral line observations. The measured T_{SYS} from these observations are to be used for modeling the atmosphere.
- The user must be given a choice between the one-layer model, its first order approximation, and a two layer model.
- The user should be able to use their own 'multi-layer' model.
- The software should present default values for T_{ATM}, T_{O2}, and T_W… but the user must be able to override these defaults.
- For **TIPPING**s, the user must be able to specify whether the data is to be low-passed filtered.
- The user must be able to specify ranges of elevation that are to be ignored in the least-square fits.

T_{SYS} - T_{rx}'

- The system should provide default values for T_{rx}', determined from the engineer's values and augmented by estimates of the ground pickup and cosmic microwave background.
- The software should use for defaults a suitable value for T_{ATM} at the time of the observation.

- Users should have the ability to override the defaults with their own values.

T_A vs. Elevation

- Automating the processing of data for this method will be difficult. Instead, we can ask that the user supply a table of measured T_A and number of air masses for a single source.
- The software should provide default values for efficiencies and, if possible, source intensities from its calibration database. The user must have the ability to override the default intensities.
- The user must select which form of equation 12 they will want to use.
- If only one entry is in the provided data table, the software should insist on having values for source intensity and efficiency.
- If more than one entry in the table, and the user or software has estimates of efficiency and source intensity, the user should be given the option of whether or not to constrain the intercept of the least-square fit.

7.4.2 Rayleigh-Jeans Law and Effective Radiation Temperature

- The analysis system should provide a toggle that would convert any flavor of intensity into a Rayleigh-Jeans equivalent temperature.
- Likewise, the system should allow one to convert from the effective source radiation temperature back to its corresponding flavor of temperature.
- The software should maintain a flag describing whether or not the Rayleigh-Jeans law is assumed or whether the data have been corrected for this assumption. The flag is in addition to the information on whether the units are T_A, T_R^*, etc.
- For continuum data, the software should provide default frequency limits but also must allow for the user to easily change them.

7.4.3 A, η_A, η_{MB}…

- The user should be able to specify whether to use equation 9 or 10 to determine A, with equation 9 being the default. The user should have the ability to override the default and specify A.
- Providing a way to enter an atmospheric profile to utilize equation 8 would be a nice future enhancement.
- To assist the user, the software should have a built-in database of useful calibrators and parameterization of their fluxes from which default flux densities could be derived. The usual way fluxes are parameterized in astronomical catalogs is:

$$S_f = 10^{c_1 + c_2 \cdot \log_{10}(f) + c_3 \cdot \log_{10}^2(f)}$$

where the constants are from various catalogs like that of Ott et al (Astron. Astrophys., Vol. 284, pp 331-339, 1994). Once the software has determined the observing frequency and object observed, it can then determine the source flux using the A, B, and C coefficients in the database and equation 9. The software should warn the user if they are trying to determine a flux that is an extrapolation for that source's parameterization. For spectral-line observations, S_f should be calculated for every frequency in the spectra. The users should be able to substitute for the default values their own source names, parameterization coefficients, and flux densities.

- We require a routine that will take user-specified values for T_A and opacity and values for either:
  - Source Name and observing frequency so that a flux density can be automatically calculated from a catalog
  - Source flux density in Jy.

  The routine then uses equation 8 to derive η_A.
- Furthermore, we need routines that take **ONOFF, OFFON, OFFONOFF, PEAK**, ... continuum and spectral-line observations, derive T_A from the data, use the header information for source name and observing frequencies to calculate source flux densities and opacities. The user should be able to override these default values and specify their own values. Then, the routine can calculate η_A.
- We require a routine that will take a **PEAK** observation, either continuum or spectral line, and derive η_{MB}. This will utilize the above-mentioned routine to derive η_A and equations 9 and 10. The user should have full control in overriding any of the default values (e.g., η_R) used by the routines.
- **PEAK** should be modified and return Ω_{MB} for both the 0.5 and 0.1 power levels. The user must be given a choice of which power level to use for their determination of η_{MB}.
- For multi-frequency observing, each frequency band should have its own efficiencies since efficiencies can vary greatly across a receiver's bandpass.

7.5 Measurement of S_{CAL} and T_{CAL}

TBD

7.6 Interpolating and Averaging T_{CAL} and S_{CAL}

TBD

Variables and Constants

A – Number of atmospheres through which observations are made.
A_P – Physical area of the telescope (= 7.854 x 10^7 cm^2 for the GBT)
Az – Azimuth of the observation.
BW – Bandwidth in Hz.
c – Speed of light = 299792.458 cm/sec.
Counts – Detected signal in the basic units of the backend
el – actual, refraction-corrected elevation of the observation.
f – Observing frequency in Hz
Δf – Frequency resolution (not the channel spacing) in Hz
f_1 and f_2 – Lower and upper frequency limits in Hz
G_S, G_K – Gain in units of Jy/Count or K/Count.
K – Backend sensitivity factor.
η_A – Aperture efficiency
η_{fss} – Forward spillover and scattering efficiency
η_l – Rear spillover, ohmic loss, and blockage efficiency
η_{MB} – Main-beam efficiency
η_R – Ohmic loss efficiency
h – Planck's constant = 6.6260 x 10^{-27} ergs sec.
J – Effective source radiation temperature in units of K
k – Boltzman's constant = 1.3807 x 10^{-16} ergs/K.
λ – Observing wavelength in cm (= c/f)
n(h), n_0 – Index of refraction of the atmosphere at height h above the observatory or at ground level.
$\rho(h)$, P(h), T(h), $P_w(h)$ – Density, pressure, temperature, water partial pressure of the atmosphere at height h above the observatory.
R – Distance of the observatory from the center of the Earth (=6370600 cm for the GBT).
S – Flux Density in units of Jy
S_A – Antenna temperature in units of Jy.
S_{CAL} – Noise diode value in units of Jy.
S_{SYS} – System temperature in units of Jy.
σ –Statistical standard deviation or theoretical noise (1 sigma)
t – Integration time in seconds.
T_A – Antenna temperature in units of K
T_A'– Antenna temperature prime in units of K
T_A^* – Corrected antenna temperature in units of K
T_{ATM} – Representative temperature of the atmosphere in K.
T_{CAL} – Noise diode value in K
T_{MB} – Main-beam brightness temperature in units of K
T_{O2} – Representative temperature of the O_2 layer in the atmosphere in K.
T_R^* – Corrected radiation temperature in units of K
T_{rx} – Receiver temperature in units of K
T_{rx}'– Receiver temperature, spillover, etc. in units of K; essentially, T_{SYS} minus any contribution from the atmosphere.

T_{GRND} – Ground temperature in units of K
T_{SRC} – Antenna temperature of a source in units of K
T_{SYS} – System temperature in units of K.
T_W – Representative temperature of the H_2O layer in the atmosphere in units of K.
θ – Beam width in radians
τ_W, τ_{O2} – Atmospheric opacities at the zenith for H_2O, O_2 in nepers
τ_0 – Total atmospheric opacity at the zenith in nepers.
w – Statistical weights.
Ω_{MB} – Beam solid angle in stereradians.

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