Millimeter Wave Calibration

Single Dish Summer School

August 2003

Phil Jewell

National Radio Astronomy Observatory

Green Bank

Millimeter-wave Calibration

Millimeter wave calibration practices are usually different from those at centimeter waves because of

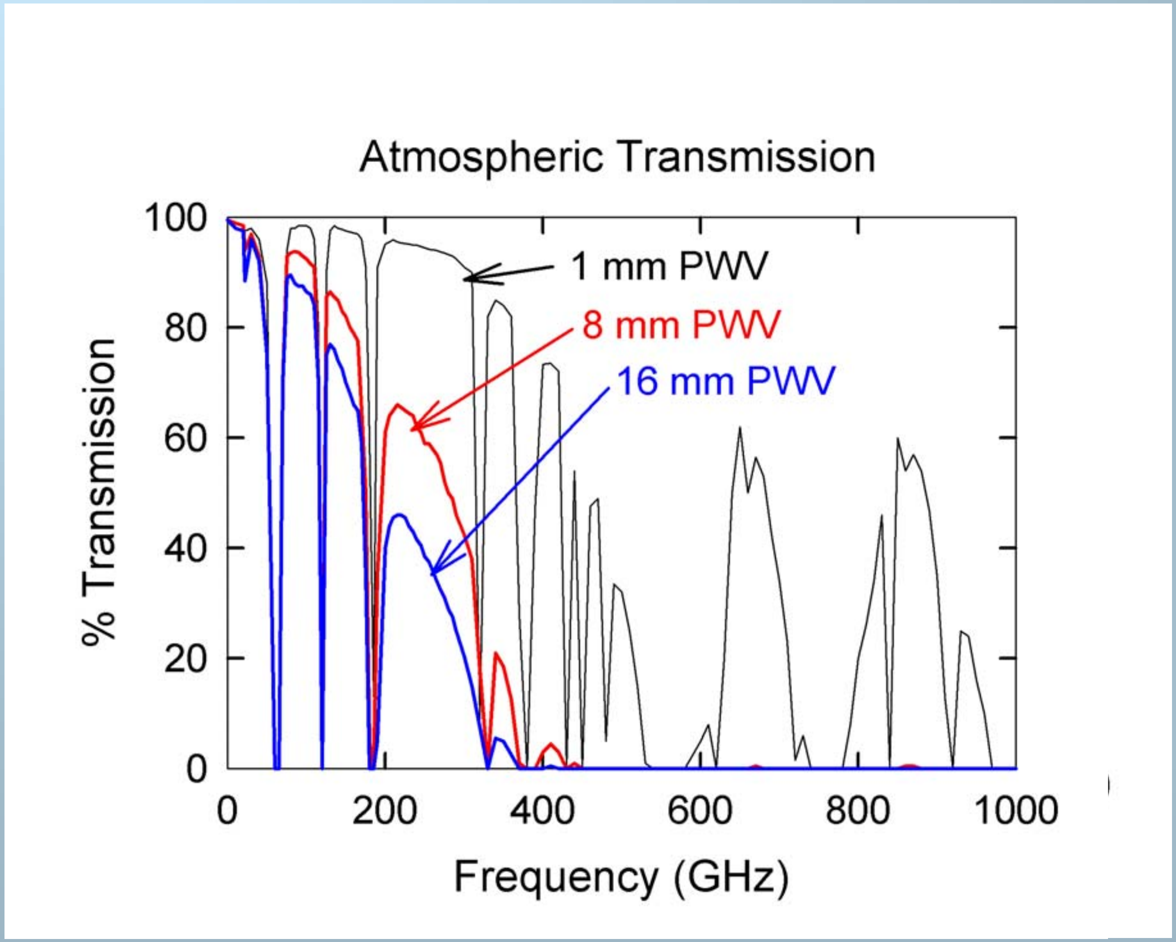
- The increased importance of the atmosphere
- Technology Differences
- A bit of history

Outline of Presentation

- The qualitative story
  - Differences between cm and mm-wave calibration
    - The atmosphere
    - Calibration technology
- Approximate mm-wave calibration formalism
  - Simplified “Chopper wheel” calibration
- More complete & rigorous calibration theory
  - Antenna Losses
  - Formal derivation of calibration equations

Why millimeter-wave calibration techniques and formalism came to differ from traditional centimeter wave formalism

Reason 1: The atmosphere is <u>very</u> important at mm-waves



Traditional atmospheric and efficiency correction

$$T_A' = \frac{T_A e^{\tau}}{\eta}$$

Where τ is atmospheric attenuation and η is antenna efficiency factor

τ is normally measured by telescope tipping scans.
This is completely valid in principle, but…

- Tipping scans take away considerable observing time
  - so they may not be done often enough
- They assume a homogeneous, plane-parallel atmosphere
  - The sky may be "lumpy" and time-variable
- Atmospheric correction is often done as a post-processing step

Total Power Antenna temperature response in the presence of the atmosphere

So we'd like an alternative, more accurate treatment:

$$T_A(sky) \approx T_{Rx} + T_{atm}(1 - e^{-\tau_{atm}})$$

T_{Rx} is the receiver noise temperature, T_{atm} is the mean temperature of the sky, a τ_{atm} is the atmospheric attenuation

But more accurately, not all power comes from the sky as the antenna also sees spillover pickup:

$$T_A(sky) = T_{Rx} + \eta_l T_{atm}(1 - e^{-\tau}) + (1 - \eta_l)T_{spill} + \eta_l T_{CBR} e^{-\tau}$$

Once one writes this equation down and accounts properly for all sources of telescope noise (i.e., atmosphere, spillover, cosmic background), it leads toward a different calibration formalism.

Calibration Technology

- For many years, cm-wave antenna temperature calibration has been done by injecting noise from a noise diode via a waveguide probe or cross-coupler.

- Such devices do not exist are are impractical for mm wavelengths (particularly short mm wavelengths where waveguides are small and noise diodes may not exist).

- This is second reason that mm-wave calibration formalism has taken a different path.

Why you should worry about the atmosphere even at short cm wavelengths – an example:

Suppose you are observing near 22 GHz (1.3 cm) with the GBT. Under good weather conditions and at high elevation angles, the conventional T_{sys} of the K-band receiver is ~45 K (the receiver by itself is about 21 K)

The weather begins to deteriorate, and T_{sys} rises to about 300 K. This is bad, but you know from the radiometer equation that integration time goes as T_{sys}^2, so what took 1 minute to observe before is going to take $(300/45)^2 = 44$ minutes to observe now. You need the data badly, so you soldier on, knowing that you will get the needed signal to noise eventually. Right?

WRONG! Conventional T_{sys} measures the emission of the atmosphere, *but not its attenuation*.

$$T_{sys} = T_A(sky) \approx T_{Rx} + T_{atm}(1 - e^{-\tau_{atm}})$$

For the example quoted, T_{sys} = 300 K, T_{Rx} = 21 K, T_{atm} ~280 K,
$\Rightarrow \tau$ **~ 5.6**

ALMOST NO PHOTONS ARE MAKING IT THROUGH THE ATMOSPHERE!!

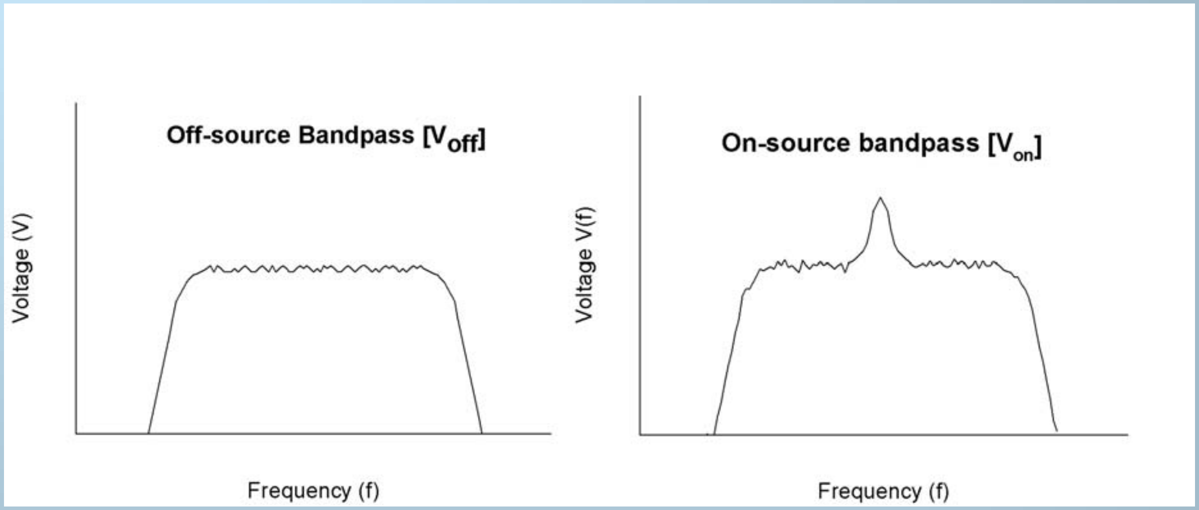
Using the millimeter-wave expression for effective system temperature referenced to above the atmosphere, T_{sys}^*

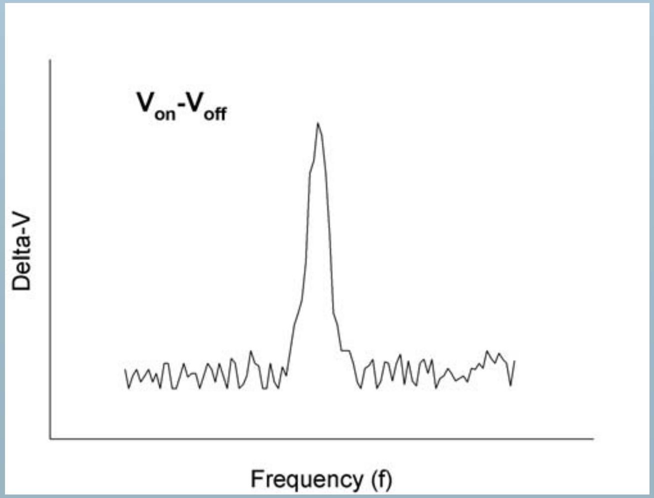
$$T_{sys}^* \approx \frac{T_{Rx} + T_{sky}}{\eta_\ell \exp(-\tau)}$$

T_{sys}^* = 101,000 K ! GIVE IT UP!

Simplified mm-wave “chopper wheel” calibration

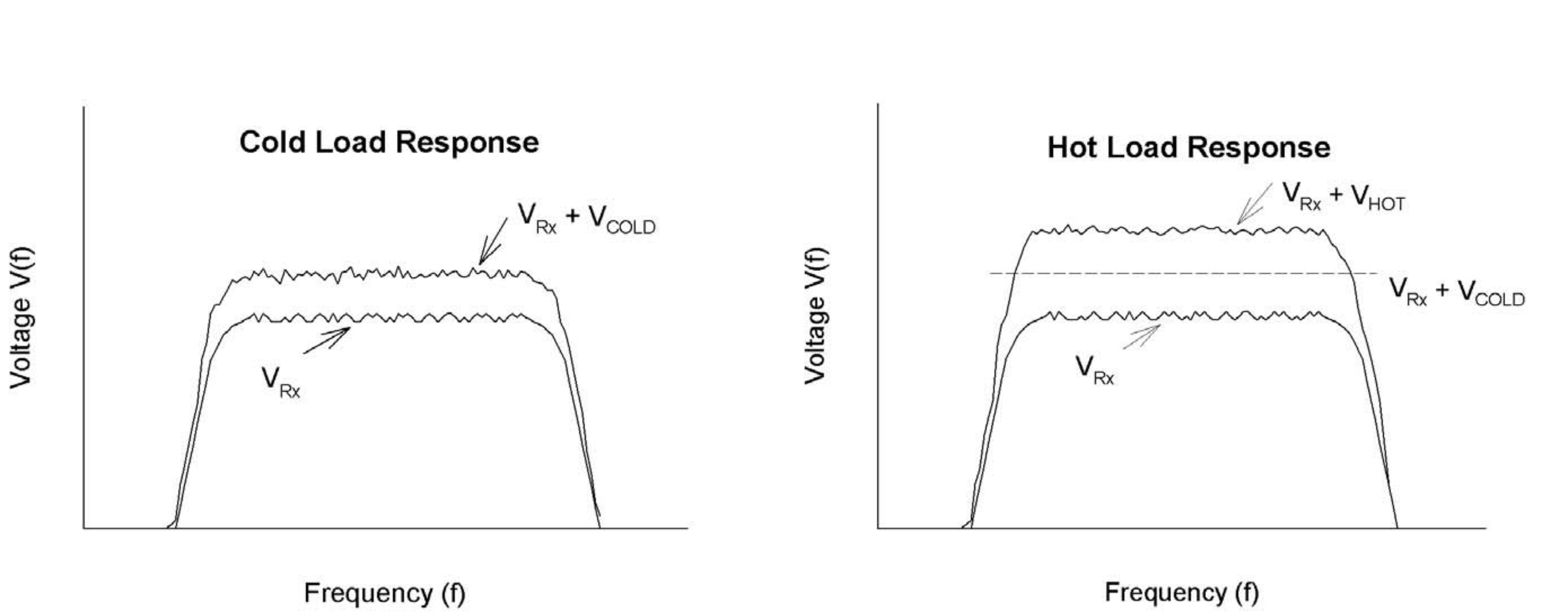
- Let’s develop millimeter-wave calibration by first considering basic calibration and signal processing:





Establish a temperature (or flux) calibration scale to our voltage bandpass.

Determine a "Kelvins per Volt conversion factor by doing a "Hot/Cold Load" calibration where T_{HOT} & T_{COLD} are the known temperature of blackbody loads and V_{HOT} & V_{COLD} are the corresponding system voltage responses.



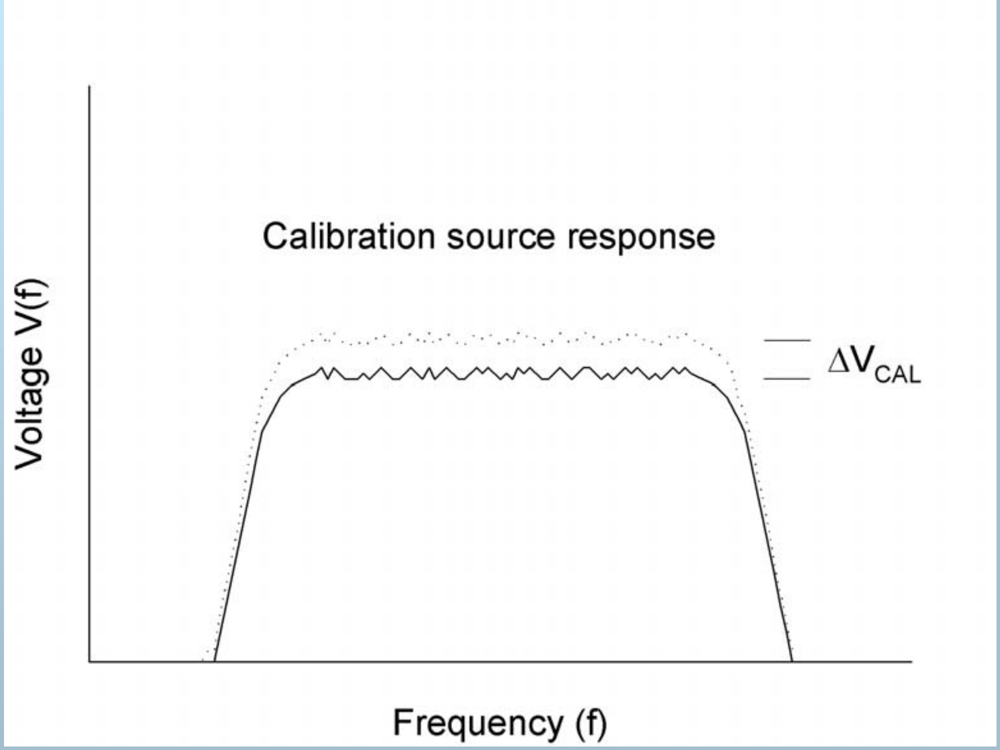
The temperature scale factor is then given by

$$g[Kelvins / Volt] = \frac{T_{HOT} - T_{COLD}}{[V_{RX} + V_{HOT}] - [V_{RX} + V_{COLD}]}$$

We can also determine receiver noise temperature $T_{Rx:}$

$$T_{RX} = \frac{T_{HOT} - YT_{COLD}}{Y - 1}$$ where $$Y = \frac{V_{RX} + V_{HOT}}{V_{RX} + V_{COLD}}$$

We can then use our temperature scale to calibrate a secondary noise source such as a waveguide coupled noise diode.



$$T_{CAL} = g[V_{CAL\_ON} - V_{CAL\_OFF}] = g\Delta V_{CAL}$$

System noise is defined as the voltage response from both the instrument & sky (which includes telescope effects):

$$V_{sys} = V_{Rx} + V_{sky}$$

$$\frac{V_{sys} + V_{cal}}{V_{sys}} = \frac{T_{sys} + T_{cal}}{T_{sys}} = \frac{V_{cal\_on}}{V_{cal\_off}}$$

$$V_{cal\_on} = V_{cal} + V_{Rx} + V_{sky}$$

$$V_{cal\_off} = V_{Rx} + V_{sky}$$

Which allows us to write T_{sys} as

$$T_{sys} = T_{cal} \cdot \frac{V_{sys}}{(V_{sys} + V_{cal}) - V_{sys}} = V_{sys} \frac{T_{cal}}{\Delta V_{cal}}$$

Normal signal processing is formulated as:

$$\Delta T_{on-off} \equiv T_{on} - T_{off} \equiv T_A$$

Or by the ratio

$$T_A = \frac{V_{on} - V_{off}}{V_{off}} T_{sys}$$

Recall that for noise diode calibration, the sky is present in both the cal on and cal off phases:

$$V_{cal\_on} = V_{cal} + V_{Rx} + V_{sky}$$

$$V_{cal\_off} = V_{Rx} + V_{sky}$$

Note that we could also write our signal processing difference as

$$\Delta T_{on-off} = (V_{on} - V_{off})\frac{T_{cal}}{\Delta V_{cal}} = (T_{on} - T_{off})\frac{T_{cal}}{\Delta T_{cal}}$$

At millimeter wavelengths, absorbing blackbody loads are used instead of noise diodes. In the calibration phase, the absorbing load blocks all emission from the sky. This is a difference from coupled noise diode calibration, for which the sky is present in both cal on and cal off phases.

Simplified load calibration theory

$$\Delta T_{cal} = g\Delta V_{cal} = T_{load} - T_{off}$$

$$= [T_{Rx} + T_{hot}] - [T_{Rx} + \eta T_{atm}(1 - e^{-\tau}) + (1-\eta)T_{spill}]$$

Note that the load totally blocks the sky emission, which changes the calibration equations.

Let's simplify by assuming that

$$T_{hot} \cong T_{atm} \cong T_{spill} \cong T_{amb} \cong T_{cal}$$ i.e., all our loads are at ambient temp.

then

$$\Delta T_{cal} = \eta T_{amb} e^{-\tau}$$

$$\Delta T_{on-off} = (T_{on} - T_{off}) \frac{1}{\eta e^{-\tau}} = \frac{T_A}{\eta e^{-\tau}} \cong T_A{}^*$$

Thus, to first order, ambient absorber (chopper wheel) calibration corrects for atmospheric attenuation:

$$\Delta T_{ON-OFF} = \frac{T_A}{\eta e^{-\tau}} = T_A *$$

This is a very convenient property!

The signal processing equation shows qualitatively how this works:

$$T_A * = (T_{on} - T_{off}) \frac{T_{cal}}{T_{load} - T_{sky}}$$

As the sky gets warmer (I.e., more attenuating), the term in the denominator gets smaller, and the signal (and noise) are scaled up.

Conclusions of simplified derivation:

- Atmosphere is very important at mm-waves
- Calibration technology differs
- Ambient absorbing load (“chopper wheel”) provides the technology and has the very nice property that, to first order, it self-corrects for atmospheric attenuation.

- But… we’ve made a number of simplifying assumptions that need better treatment.

Antenna Losses and Efficiency Factors

Loss Type	Source
Ohmic Losses	Resistive heating in reflector Absorptive losses in paint
Blockage & Scattering	Obstructions in the optical path
Error pattern (or beam)	Phase errors from random imperfections in reflectors
Spillover (rear & forward)	Illumination pattern of feeds & optics
Diffraction sidelobes	From edge effects, blockage, panel gaps
Optical aberrations / sidelobes	Errors in optical figures or collimation
Source-beam coupling	Convolution of antenna pattern and source brightness distribution.

Ohmic Loss

Ohmic Loss (from resistive heating of the reflector or absorption of the paint on the reflector)

$$\eta_r \equiv \frac{G}{4\pi}\iint_{4\pi} P_n(\Omega)\,d\Omega$$

Terminates at ambient temperature
Generally very small (<<1%) for a single reflection, but can build up to significant values in complicated optical systems having many reflections (such as relay mirrors)

Blockage & Scattering

Derives from geometric blockage of the signal by support structures, including (in conventional, on-axis feeds)

- Feed support legs
- Subreflector & mount

Note that for an unblocked aperture such as the GBT, these losses are eliminated.

Error Pattern

Results from phase front distortions caused by deviations from an ideal reflecting surface (Ruze 1966).

<u>Random</u> deviations in the surface reduce the gain of the antenna:

$$G = \eta_o \left(\frac{\pi D^2}{\lambda} \right) \exp\left[-\left(\frac{4\pi\sigma}{\lambda} \right)^2 \right]$$

Aperture efficiency:

$$\eta_A = \eta_o \exp\left[-\left(\frac{4\pi\sigma}{\lambda} \right)^2 \right]$$

☞ is the rms surface accuracy of the reflector;

🗀$_o$ is the aperture efficiency in the long wavelength limit;

🗀$_A$ is the aperture efficiency at wavelength 🖰

From both theoretical analysis and actual measurements, Ruze found that if the dish errors followed Gaussian statistics, the error beam was typically a broad, Gaussian beam of FWHM

$$\theta_E = 2(\ln 2)^{1/2} \frac{\lambda}{\pi c_\sigma}$$

and relative amplitude

$$\frac{A_E}{A_M} = \frac{1}{\eta_o}\left[\frac{2c_\sigma}{D}\right]^2 \left\{\exp\left(\frac{4\pi\sigma}{\lambda}\right)^2 - 1\right\}$$

where the term c_σ is the correlation scale size of the errors, and A_M is the amplitude of the main beam. Surface errors often have a characteristic scale size, which may correspond to the size of the surface panels or some other structural element.

RUZE (1966)

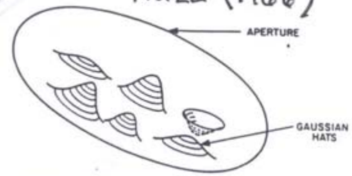


Fig. 4. Aperture subdivided into a number of hats.

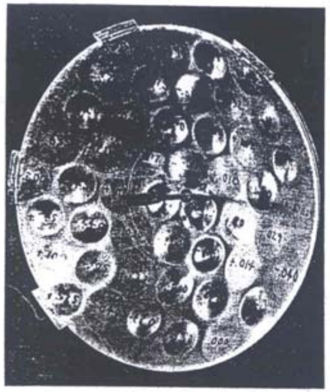


Fig. 5. Special model constructed to test theory.

the phase front distortions are assumed to be of Gaussian shape, the required integrations can again be performed [5] with the following result:

$$G(\theta, \phi) = G_0(\theta, \phi) e^{-\overline{\delta^2}} + \left(\frac{2\pi c}{\lambda}\right)^2 e^{-\overline{\delta^2}} \sum_{n=1}^{\infty} \frac{\overline{\delta^2}^n}{n \cdot n!} e^{-(\pi c u/\lambda)^2/n}. \tag{8}$$

Although (8) is more complex, the general effects are similar to those discussed previously.

We have considered a two-dimensional distribution of errors. It is of interest to present the one-dimensional case derived by Bramley [6] in our notation

$$G(\theta) = G_0(\theta) e^{-\overline{\delta^2}} + \frac{\sqrt{\pi} c}{\lambda} e^{-\overline{\delta^2}} \sum_{n=1}^{\infty} \frac{\overline{\delta^2}^n}{\sqrt{n} \cdot n!} e^{-(\pi c u/\lambda)^2/n}. \tag{9}$$

The gain reduction and pattern degradation predicted by (8) was checked in the original reference [5] by the construction of a special model, Fig. 5, which fulfilled the statistical assumptions necessary for the theoretical development.

II. Discussion

From (8), we can write the reduction of axial gain as

$$\frac{G}{G_0} = e^{-\overline{\delta^2}} + \frac{1}{\eta}\left(\frac{2c}{D}\right)^2 e^{-\overline{\delta^2}} \sum_{n=1}^{\infty} \frac{\overline{\delta^2}^n}{n \cdot n!}. \tag{10}$$

In the region of interest, i.e., reasonable tolerance losses, and for correlation regions that are small compared to the antenna diameter, the second term may be neglected and we have for the gain

$$G = G_0 e^{-\overline{\delta^2}} = \eta \left(\frac{\pi D}{\lambda}\right)^2 e^{-(4\pi\epsilon/\lambda)^2}, \tag{11}$$

where we define "ϵ" as the effective reflector tolerance in the same units as λ; i.e., that rms surface error on a shallow reflector (large f/D), which will produce the phase front variance $\overline{\delta^2}$. In Fig. 6 we plot the loss of gain (11) as a function of the rms error and the peak surface error. The ratio used, 3:1, is one found experimentally for large structures and results, in part, from the truncation used in the manufacturing process (i.e., large errors are corrected).

It should be noted that for small errors (11) is identical with (3), with the exception that the former is independent of the illumination function and the latter is not. For the statistical analysis, it was necessary to assume a uniform distribution of errors, for which case the illumination dependence factors out in (3) and becomes identical to (11).

For deep (nonshallow) reflectors, the surface tolerance is not exactly equal to the effective tolerance "ϵ." In addition, structural people at times measure the reflector deformations normal to the surface and at times in the axial direction. The relation between these quantities is

$$\epsilon = \frac{\Delta z}{1 + (r/2f)^2} \tag{12a}$$

$$\epsilon = \frac{\Delta n}{\sqrt{1 + (r/2f)^2}}. \tag{12b}$$

The result is that the tolerance gain loss in dB, as computed from the reflector axial or normal mean square error, is too high by a factor A. This factor is given in Fig. 7. For shallow reflectors, this correction factor approaches unity.

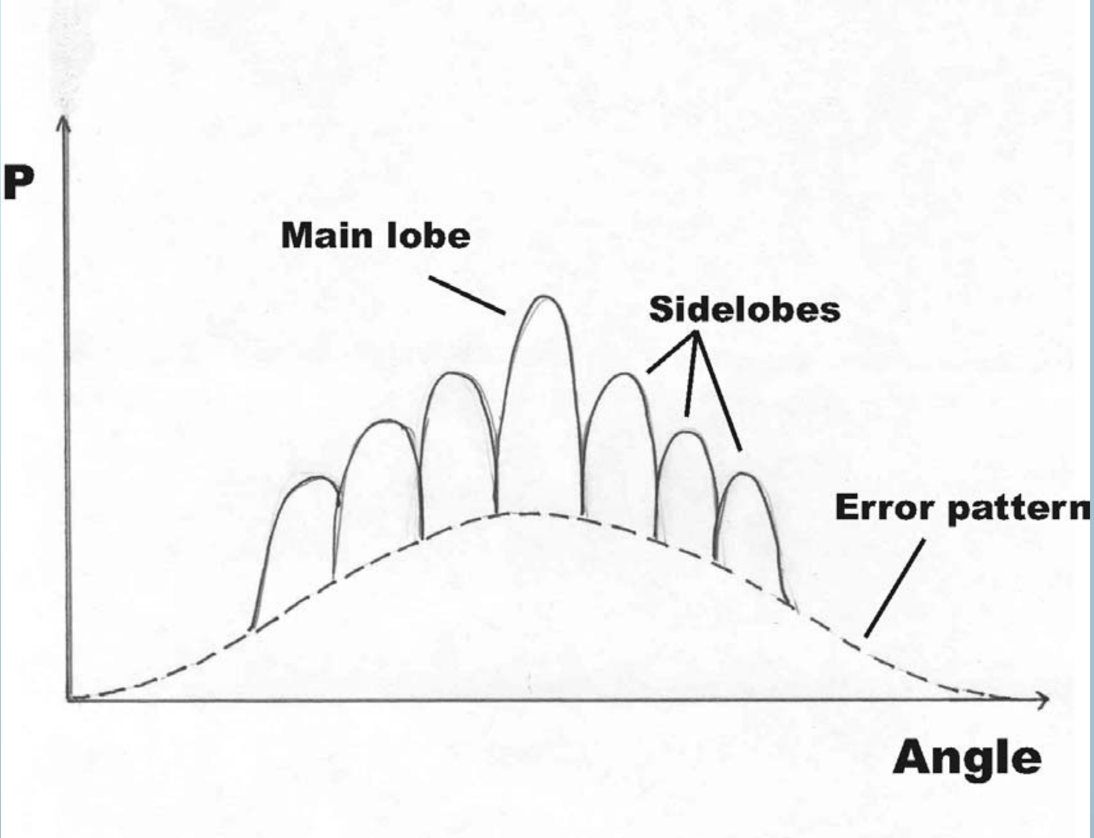
Equation (11) indicates that if a given reflector is operated at increasing frequency, the gain, at first, increases as the square of the frequency until the tolerance effect take over and then a rapid gain deterioration occurs. Maximum gain is realized at the wavelength of

$$\lambda_m = 4\pi\epsilon, \tag{13}$$

where a tolerance loss of 4.3 dB is incurred. This maximum gain is

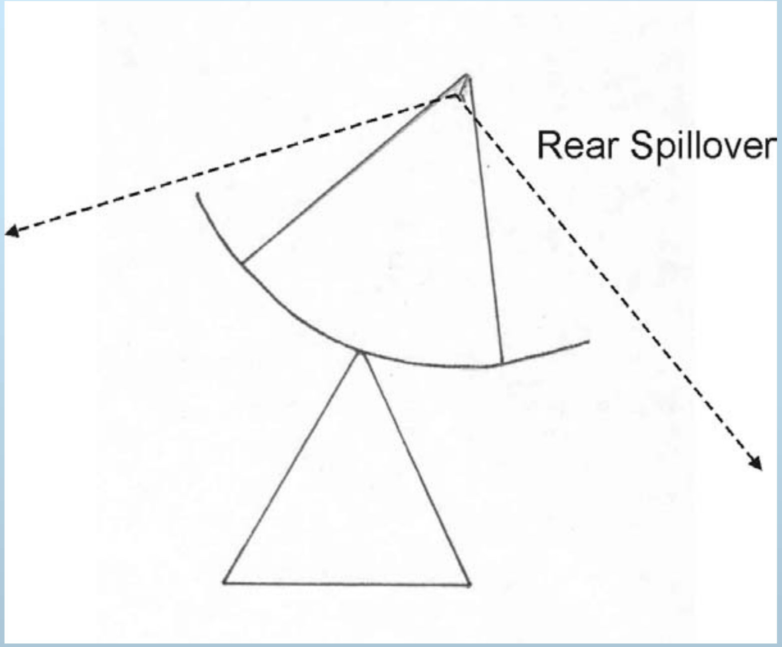
From Ruze (1966)

The total forward beam of the antenna may look like this:



Note: All antennas have error patterns. They become more pronounced as antennas are pushed to the upper end of (or beyond!) their design frequency limit.

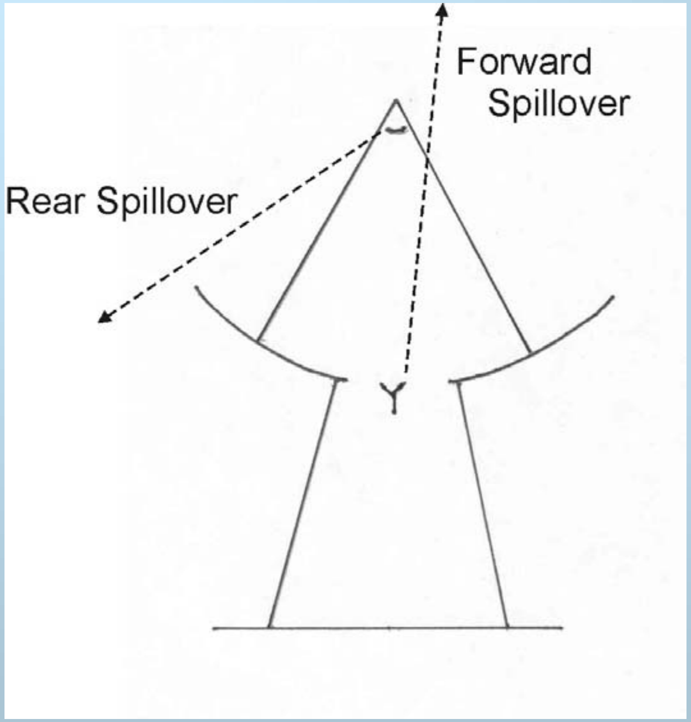
Rear Spillover



$$\eta_{rss} \equiv \frac{\displaystyle\iint_{2\pi(forward)} P_n(\Omega)\,d\Omega}{\displaystyle\iint_{4\pi} P_n(\Omega)\,d\Omega}$$

Defined as the fraction of power falling on the forward hemisphere.

Forward Spillover



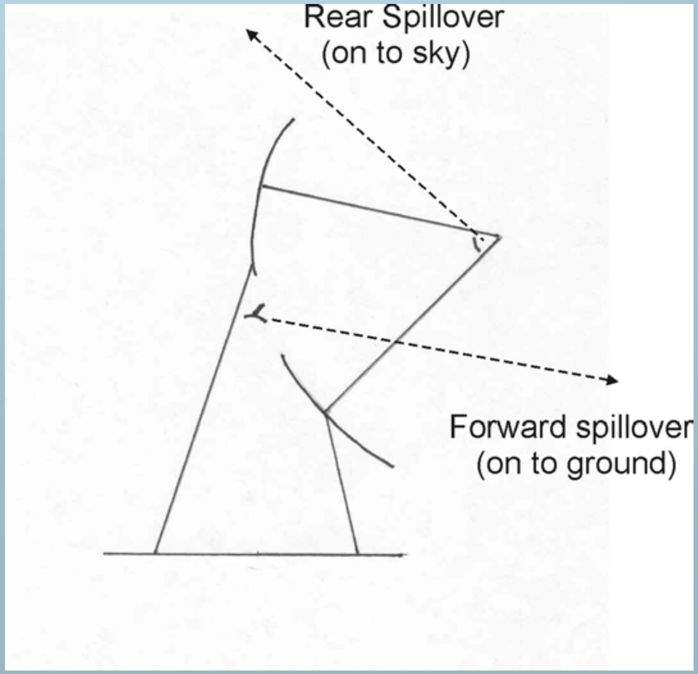
Antennas with secondary optics will have forward spillover.

$$\eta_{fss} \equiv \frac{\iint\limits_{\Omega_d} P_n(\Omega)d\Omega}{\iint\limits_{2\pi} P_n(\Omega)d\Omega}$$

Defined as the fraction of power falling within a diffraction zone Ω_d, usually defined to contain the near sidelobes and error pattern (both of which can significantly couple with source emission).

Rear spillover and scattering is usually terminated at ambient temperature == “warm spillover.”
Forward spillover and scattering typically terminates on the sky == “cold spillover.”

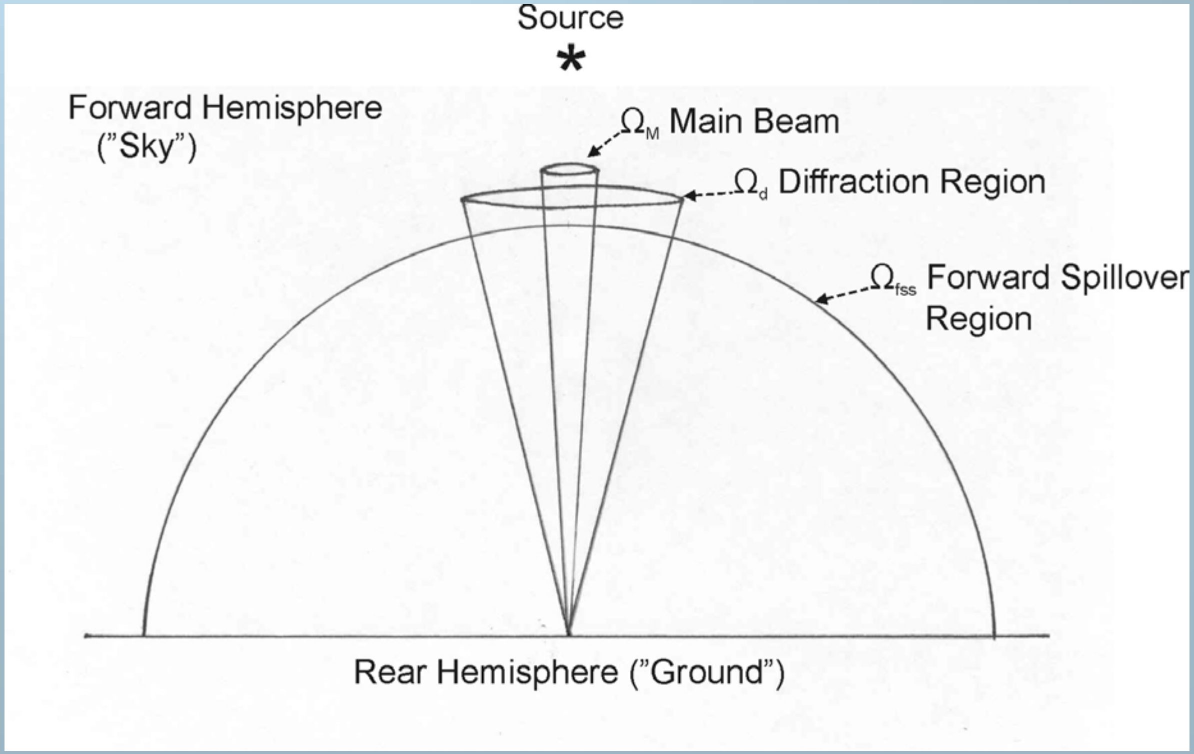
At low elevation angles, some rear spillover may actually terminate on the sky, and some forward spillover on the ground:



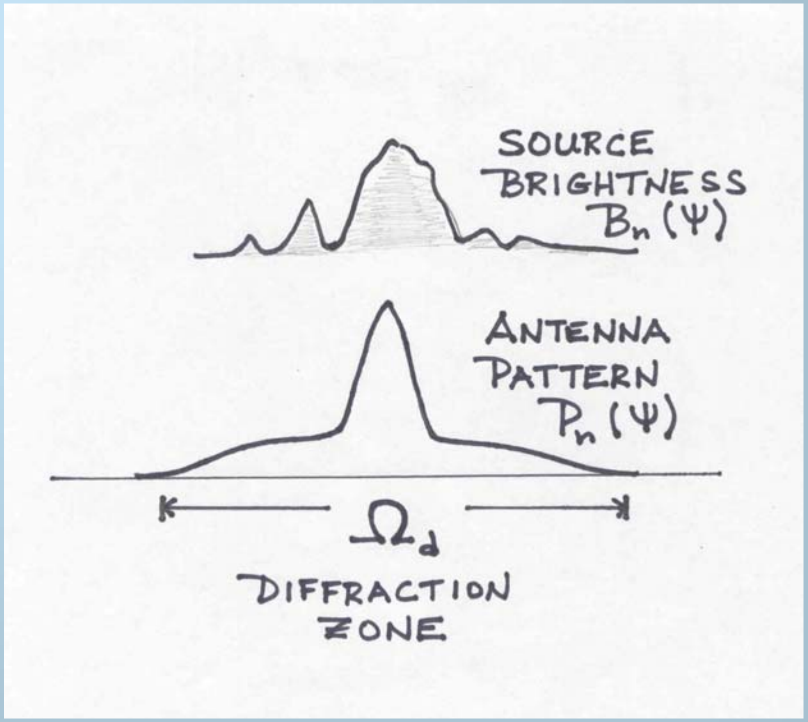
The two loss terms that terminate at ambient temperature, η_{rss} and η_r (ohmic loss) are usually grouped into a single efficiency, η_l, defined as

$$\eta_l = \eta_r \eta_{rss}$$

Graphical representation of illumination regions:



Source-Beam Coupling



Define a source-beam coupling efficiency:

$$\eta_c \equiv \frac{\iint\limits_{\Omega_{source}} P_n(\Psi - \Omega) B_n(\Psi) d\Psi}{\iint\limits_{\Omega_d} P_n(\Omega) d\Omega}$$

For a source brightness temperature T_B, define the corrected antenna temperature T_A*:

$$T_A^* \equiv \eta_{fss}\eta_c T_B$$

If the source angular extent is comparable to or smaller than the main beam, we can define a Main Beam Brightness Temperature as:

$$T_{MB} \equiv \frac{T_A^*}{\eta_{fss}\eta_M^*} = \frac{T_A}{\eta_B e^{-\tau}}$$

Where η_M* is the corrected main beam efficiency

$$\eta_M^* = \frac{\eta_B}{\eta_l \eta_{fss}}$$

Atmospheric effects on calibration

Attenuation of plane-parallel atmosphere:

$$Attenuation = e^{-\tau} = e^{-\tau_o \sec(z)} = e^{-\tau_o / \sin(el)} = e^{-\tau_o A}$$

Sky noise equation:

$$T_A(sky) = T_{Rx} + \eta_l T_{atm}(1 - e^{-\tau}) + (1 - \eta_l) T_{spill} + \eta_l T_{CBR} e^{-\tau}$$

Millimeter-wave Calibration Formalism

Corrections we must make:

1. At millimeter wavelengths, we are no longer in the R-J part of the Planck curve, so define a Rayleigh-Jeans equivalent radiation temperature of a Planck blackbody at temperature T.

$$J(\nu, T) = \frac{h\nu / k}{\exp(h\nu / kT) - 1}$$

2. Let all temperatures be different:

$$T_{atm} \neq T_{spill} \neq T_{chop} \neq T_{amb}$$

3. Most millimeter wave receivers using SIS mixers have some response to the image sideband (even if they are nominally "single sideband"). HEMT amplifiers probably have negligible response to the image sideband.
   - The atmosphere often has different response in the signal & image sidebands
   - Receiver gain must be known in the signal sideband

$$G_i + G_s = 1$$

G_s = signal sideband gain, G_i image sideband gain.

Defining relations:

$$\Delta T_{source} = T_{source} - T_{sky} = T_{on} - T_{off}$$

$$T_A^* \equiv \frac{\Delta T_{source}}{G_s \eta_l \exp(-\tau_s A)}$$

Definition of corrected antenna temperature

In terms of signal processing of measured quantities,

$$T_A^* = \Delta T_{source} \frac{T_c^*}{\Delta T_{cal}}$$

Where T_c^* is a calibration scale factor and ΔT_{cal} is the Load-Sky cal difference.

$$T_c{}^* = \frac{\Delta T_{cal}}{G_s \eta_l \exp(-\tau_s A)}$$

Definition

$$\Delta T_{cal} = T_{load} - T_{sky}$$

Load and sky noise has contributions from both sidebands:

$$T_{load} = G_s J(\nu_s, T_{chop}) + G_i J(\nu_i, T_{chop}).$$

$$T_{sky} = G_s \{\eta_l J(\nu_s, T_{atm})(1 - e^{-\tau_s A}) + (1-\eta_l) J(\nu_s, T_{spill}) + \eta_l J(\nu_s, T_{CBR}) e^{-\tau_s A}\}$$
$$+ G_i \{\eta_l J(\nu_i, T_{atm})(1 - e^{-\tau_i A}) + (1-\eta_l) J(\nu_i, T_{spill}) + \eta_l J(\nu_i, T_{CBR}) e^{-\tau_i A}\}.$$

$$T_c{*} = \frac{\Delta T_{cal}}{G_s \eta_l \exp(-\tau_s A)}$$

$$T_c{*} = \left(1 + \frac{G_i}{G_s}\right)\left[J(\nu_s, T_{atm}) - J(\nu_s, T_{CBR})\right]$$

$$+ \left(1 + \frac{G_i}{G_s}\right)\left[J(\nu_s, T_{spill}) - J(\nu_s, T_{atm})\right] e^{\tau_s A}$$

$$+ \frac{G_i}{G_s}\left[J(\nu_s, T_{atm}) - J(\nu_s, T_{CBR})\right]\left\{\exp[\tau_s - \tau_i) A] - 1\right\}$$

$$+ \frac{1}{\eta_l}\left(1 + \frac{G_i}{G_s}\right)\left[J(\nu_s, T_{chop}) - J(\nu_s, T_{spill})] e^{\tau_s A}\right]$$

where we have assumed only that

$$J(\nu_s, T) = J(\nu_i, T)$$

Commonly used T_R* scale definition (recommended by Kutner and Ulich:

$$T_R^* \equiv \frac{T_A^*}{\eta_{fss}}$$

The advantage of T_R* is that it includes all telescope losses except direct source coupling of the forward beam in Ωd. The disadvantage is that η_{fss} is not a natural part of chopper wheel cal and must be added as an extra factor. T_A* is quoted most often. Either convention is OK, but know which one the observatory is using.

Finally, under these definitions, the effective system temperature, T_{sys}* is

$$T_{sys}^{*} = \frac{\left[1+\frac{G_i}{G_s}\right](T_{Rx}+T_{sky})}{\eta_\ell \exp(-\tau)}$$

(using the T_A* definition)

Possible sources of error in chopper wheel cal

- Gain compression of receiver when ambient load is switched in
  - Particularly bad when T_{Rx} and T_{sky} are low
- Uncertainties in input parameters to T_c^* equation
  - Mean atmospheric temperature T_{atm} is difficult to to determine. Can improve with atmospheric models and/or radiosonde data.

- There are alternatives to simple one-load ambient chopper wheels
  - 2 load chopper
  - 2 loads in center of subreflector
  - Semi-transparent loads

Absolute calibration of efficiencies and flux density conversion factors

- Efficiency and conversion factors needed:

η_l – Rear spillover and ohmic loss efficiency
η_{fss} – Forward spillover efficiency
$\eta_M{}^*$ or η_c -- Forward coupling efficiency

η_l can be determined from fits to atmospheric tipping scans

η_{fss} can measure from observations of the Moon, if $\Omega_{moon} = \Omega_d$

$$\eta_{fss} = \frac{T_A{}^*}{T_B(MOON)}$$

$\eta_M{}^*$ -- corrected main beam efficiency – can measure from observations of planets

$$\eta_M{}^* = \frac{T_A{}^*[Planet]}{\eta_{fss}\eta_c(disk)T_B[Planet]}$$

$$\eta_c(disk) = 1 - \exp\left\{-4\ln 2\left(\frac{r}{\theta_M}\right)^2\right\}$$

Absolute Calibration Measurements

Planet	T_B (90 GHz) (K)	T_B (227 GHz) (K)	Planet Unit Semi-Diameter (arcsec)
Venus	367±10	317±30	8.34
Mars	207±6	207±7	4.68
Jupiter	179±5	165±18	95.20
Saturn	149±4	140±14	78.15
Uranus	134±4	101±11	35.02
Neptune	127±4	104±11	33.50

Flux conversion factors (Jy/K)

$$S_v = \frac{2kT_A}{\eta_A A_p}$$

Conventional

$$S_v = \frac{\eta_l T_A *}{\eta_A} \frac{2k}{A_p}$$

T_A* definition