THE NAIC-NRAO SCHOOL ON SINGLE-DISH RADIO ASTRONOMY TECHNIQUES & APPLICATIONS

THE MEASUREMENT PROCESS WITH FULLY-FILLED APERTURES

C.J. Salter, NAIC, Arecibo Observatory, HC3 Box 53995, Arecibo, PR 00612.

ACCURACY IN RADIO ASTRONOMY

- **Pulsar Periods: 1 part in 10^{12}**
- **HI Redshifts: For $z \approx 0.1$, about 1 part in 10^4**
- **Source Intensities: Typically, $\pm 10\%$**

BASIC DEFINITIONS

A: Intensity/Surface Brightness

$$\mathbf{I}(\theta,\ \phi,\ \nu,\ \mathbf{t}), \text{ where,}$$

θ and ϕ are the spherical coordinates of a given direction on the two-dimensional sphere,

ν is the frequency,

and, t is time, i.e. the emission may be time variable.

$$\mathbf{I}(\theta,\ \phi,\ \nu,\ \mathbf{t}) = \mathbf{B}(\theta,\ \phi,\ \nu,\ \mathbf{t}) = \lim_{\mathbf{dt},\,\mathbf{dA},\,\mathbf{d}\nu,\,\mathbf{d\Omega} \to \infty} \frac{\mathbf{dE}(\theta,\ \phi,\ \nu,\ \mathbf{t})}{\mathbf{dt\ dA\ d}\nu\ \mathbf{d\Omega}}$$

Standard units are $\mathbf{W\,m^{-2}Hz^{-1}ster^{-1}}$.

B: Brightness Temperature, T_B

T_B is a function of frequency, and related to the Intensity, $I(\theta, \phi, \nu, t)$, by the Rayleigh-Jeans approximation (unless the frequency is very high, and the temperature very low).

$$\mathbf{I}(\theta, \phi, \nu, \mathbf{t}) = \frac{2\,\mathbf{k}\,\mathbf{T_B}(\theta, \phi, \nu, \mathbf{t})}{\lambda^2} = \frac{2\,\mathbf{k}\,\nu^2\,\mathbf{T_B}(\theta, \phi, \nu, \mathbf{t})}{\mathbf{c}^2}$$

where, λ is the wavelength, k is Boltzmann's constant, and c is the velocity of light.

Flux Density

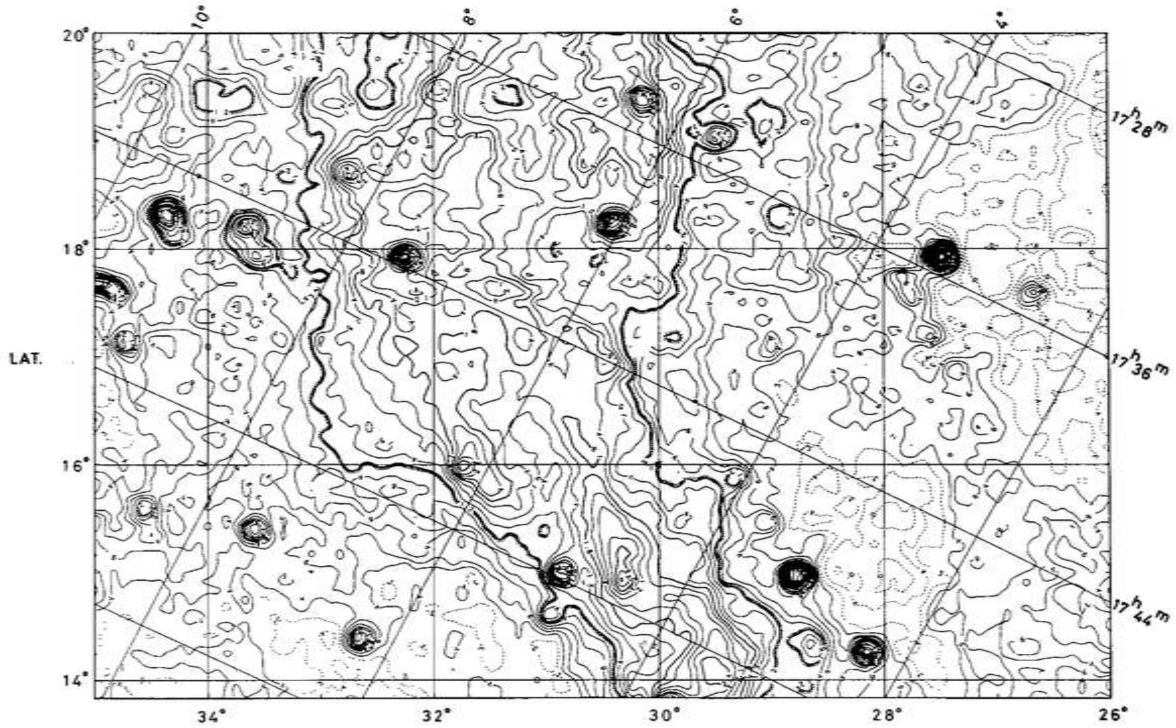

Figure 1: Contour maps of the intermediate latitude total-intensity emission from the North Polar Spur at 1.4 GHz made with the Effelsberg telescope (Sofue & Reich, 1979, A&AS, 38, 251). The image is in Galactic coordinates, and the HPBW$\sim$ 10 arcmin.

Now, the Flux Dennsity, S, is given by,

$$\mathbf{S}(\nu,\ \mathbf{t}) = \iint_{\mathbf{source}} \mathbf{I}(\theta,\ \phi,\ \nu,\ \mathbf{t})\ \mathbf{d\Omega}$$

Hence, if we consider our tiny cone of sky as a discrete source,

$$\begin{aligned}\mathbf{S}(\nu,\ \mathbf{t}) &= \mathbf{I}(\theta,\ \phi,\ \nu,\ \mathbf{t})\ \mathbf{d\Omega} \\ &= \frac{\mathbf{dE}(\theta,\ \phi,\ \nu,\ \mathbf{t})}{\mathbf{dt\ dA\ d}\nu\ \mathbf{d\Omega}} \times \mathbf{d\Omega} \\ &= \lim_{\mathbf{dt},\,\mathbf{dA},\,\mathbf{d}\nu \to \infty} \frac{\mathbf{dE}(\theta,\ \phi,\ \nu,\ \mathbf{t})}{\mathbf{dt\ dA\ d}\nu}\end{aligned}$$

The standard units of flux density are $\mathbf{W\,m^{-2}\,Hz^{-1}}$.

However, a practical unit, the JANSKY, has been adopted, where, $\mathbf{1\,Jy = 10^{-26}\,W\,m^{-2}Hz^{-1}}$.

With, $\mathbf{1\,mJy = 10^{-3}\,Jy}$ and $\mathbf{1\,\mu Jy = 10^{-6}\,Jy}$.

Distance Dependencies

If our small radio source is situated at a distance r, then by the inverse square law, the energy we receive from it is dE $\propto r^{-2}$. However, the solid angle that it subtends is $d\Omega \propto r^{-2}$. Hence, I = dE/(dt dA dν dΩ) is distance independent.

In contrast, the flux density, S = dE/(dt dA dν) $\propto r^{-2}$, and hence the flux density of a "standard candle" falls as the inverse square of its distance.

THE BASIC RADIO RECEIVER

Celestial radio emissions can be subdivided into two main spectral categories;

1. **Broadband emission or continuum radiation.**

   Typical examples: (i) planetary surfaces, (ii) HII regions, (iii) the Cosmic Microwave Background, and (iv) those Galactic and extragalactic sources that emit via the synchrotron process.

2. **Line emission due to low-energy transitions within atoms and molecules in space.**

The Super-Heterodyne Receiver

Mixer

Why frequency change rather than amplifying the signal at ν_{RF} right down the receiver chain?

1. **Long lengths of cable are less lossy for lower signal frequencies. (Nowadays, less importance due to use of optical fibers.**
2. **The fear of positive feedback causing the receiver to oscillate.**
3. **If ν_{LO} can be varied over a wide range, a single IF chain can service a wide range of receiver front-ends.**

Detector

The detector brings the band-center frequency down to DC (0 Hz). Usually, a square-law detector is employed, for which,

$$\begin{aligned}\mathbf{OutputVoltage} &\propto (\mathbf{InputVoltage})^2\\ &\propto \mathbf{InputPower}\end{aligned}$$

SIGNAL-TO-NOISE CONSIDERATIONS

System Noise

Cascaded Amplifiers **If our receiver chain consists of N amplifiers, each with gain, $\mathbf{g}_i$, and noise temperature, $\mathbf{T}_i$, the total system gain, g, is,**

$$\mathbf{g} = \mathbf{g_1} \times \mathbf{g_2} \times \mathbf{g_3} \times \ldots \times \mathbf{g_N}$$

and the effective receiver temperature is,

$$\mathrm{T_R} = \mathrm{T_1} + \frac{\mathrm{T_2}}{\mathrm{g_1}} + \frac{\mathrm{T_3}}{\mathrm{g_1 g_2}} + \frac{\mathrm{T_4}}{\mathrm{g_1 g_2 g_3}} + \ldots$$

(As each $\mathbf{g}_i \gg 1$, clearly, $\mathbf{T}_\mathrm{R} \approx \mathbf{T}_1$)

Lossy Components

$$\mathrm{T'_R} = (\frac{1}{\eta} - 1)\,\mathrm{T_\circ} + \frac{\mathrm{T_R}}{\eta}$$

(As $\eta < 1$, $\mathbf{T'_R} > \mathrm{T_R}$.)

The System Temperature

There are extra noise power contributions from such things as;

a) Emission from the neutral atmosphere.

b) Ground radiation in the far sidelobes.

c) Celestial noise contributions from,

i) the background radiation from the Milky Way,

ii) the 2.7 K radiation of the CMB, and,

iii) perhaps a contribution from the target source itself!

The sum of all these noise contributions is the SYSTEM TEMPERATURE, $\mathbf{T}_{\rm sys}$, expressed as,

$$\mathrm{T}_{\rm sys} = \mathrm{T}_{\rm R} + \underbrace{\mathrm{T}_{\rm atm} + \mathrm{T}_{\rm ground} + \mathrm{T}_{\rm gal} + \mathrm{T}_{\rm CMB} + \mathrm{T}_{\rm source}}_{\mathrm{T}_{\rm A}}$$

where sum of all contributions from outside the observing system is the ANTENNA TEMPERATURE, $\mathrm{T}_{\rm A}$.

The contribution from the Milky Way galaxy plus the CMB changes greatly with frequency, i.e.

- **at 38 MHz, $\mathbf{T}_{\rm gal} \geq 10^4 - 10^5\,\mathbf{K}$**
- **at 327 MHz, $\mathbf{T}_{\rm gal} \sim 20 - 2,000\,\mathbf{K}$**
- **at 5 GHz and above, $\mathbf{T}_{\rm gal}$ is negligible, and is dwarfed by $\mathrm{T}_{\rm CMB} = 2.7\,\mathbf{K}$**

The Radiometer Equation

How weak a source we can detect with a given Rx?

The RADIOMETER EQUATION for our total-power receiver gives the rms noise ripple against which we are trying to detect our target source. In units of temperature,

$$\mathrm{T_{rms}} = \frac{\mathrm{T_{sys}}}{\sqrt{\Delta\nu\ \tau}}$$

where, $\Delta\nu$ is the bandwidth in Hz, and τ is the integration time in sec.

Moving our telescope beam ON and OFF a source which gives an antenna temperature rise of $\Delta \mathrm{T_A}$, with a dwell time $\tau/2$ in each phase, signal-to-noise ratio will be,

$$\frac{\Delta\mathrm{T_A}}{\mathrm{T_{rms}}} = \frac{\Delta\mathrm{T_A}\sqrt{\Delta\nu\ \tau}}{2 \times \mathrm{T_{sys}}}$$

Usually a signal-to-noise ratio of $\sim$5 is considered necessary for a significant detection.

When considering a point source's flux density, S, rather than its $\mathrm{T_A}$, we know that the power received on a collecting area, $\mathrm{A_{eff}}$, from the source in a single polarization is $1/2\,\mathrm{S}\,\mathrm{A_{eff}}\,\Delta\nu$. The same power would be received were the feed to be replaced by a matched resistor of temperature, $\Delta\mathrm{T_A}$, so,

$$\frac{\mathbf{1}}{\mathbf{2}}\,\mathrm{S}\,\mathrm{A_{eff}}\Delta\nu = \mathrm{k}\Delta\mathrm{T_A}\Delta\nu$$

or,

$$\boldsymbol{\Delta}\mathrm{T_A} = \frac{\mathrm{SA_{eff}}}{\mathbf{2k}}$$

It often simpler when considering sensitivity in terms of point-source flux density to express $\mathrm{T_{sys}}$ in terms of the flux density that would exactly double the system temperature. This is called the SYSTEM EQUIVALENT FLUX DENSITY (SEFD), and is given by

$$\mathrm{SEFD} = \frac{2\,\mathrm{k}\,\mathrm{T_{sys}}}{\mathrm{A_{eff}}}$$

Then, the radiometer equation becomes,

$$\mathrm{S_{rms}} = \frac{\mathrm{SEFD}}{\sqrt{\boldsymbol{\Delta}\nu\ \tau}}$$

Confusion

SINGLE-DISH IMAGING & SAMPLING

How far apart should we space our scans (and how often should we sample along a scan) such as to lose no information which our telescope is capable of passing? Consider the one dimensional case;

If the normalized beam pattern of the telescope is $\mathbf{P}(\theta)$, and the true distribution of brightness temperature is $\mathbf{T}_{\mathrm{B}}(\theta)$, the antenna temperature recorded is,

$$\mathrm{T}_{\mathrm{A}}(\theta') = \frac{\mathbf{1}}{\mathbf{\Omega}_{\mathrm{A}}} \int \mathrm{T}_{\mathrm{B}}(\theta)\, \mathbf{P}(\theta' - \theta)\, \mathbf{d}\theta$$

This is a convolution, and the Fourier Transform of a convolution is equal to the multiplication of the Fourier Transforms of the two functions being convolved.

Sampling at discrete points is multiplying the recorded $\mathrm{T_A}(\theta')$ by a set of delta functions separated by the sampling interval, s rads. This is equivalent to convolving the Fourier transform of $\mathrm{T_A}(\theta')$, (zero outside $\pm$ $\mathbf{D}/\lambda$ cycles/rad), by a series of delta functions spaced by $\mathbf{1}/s$ cycles/rad.

The series of "islands" do not overlap, i.e. we retain full information, only if,

$$\frac{\mathbf{1}}{\mathbf{s}} > \frac{\mathbf{2}\,\mathrm{D}}{\lambda}$$

Or in other words,

$$\mathbf{s} < \frac{\lambda}{\mathbf{2}\mathrm{D}}\ \textbf{rad}$$