

# The GBT Sensitivity Calculator User's Guide

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*Ron Maddalena*

*David Frayer*

*Dan Perera*

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## Brief History of the GBT Sensitivity Calculator

This calculator tool was originally designed and maintained by Ron Maddalena. It leverages his software tools for forecasting the weather and the algorithms used by the Dynamic Scheduling system to compute typical conditions for which an observation may be scheduled as a function of frequency. David Frayer took over the maintenance of the tool in 2020. New for 2020 is a restructuring of the ascii output, the inclusion of the  $T_{\text{a}}$  temperature scale, and the ability to support multiple instruments operating over the same frequency range with different receiver temperatures.

## Known Issues (as of July 2022)

- If selecting Topocentric Frame for spectral-line observations and choosing velocity, the tool will label the bandwidth in MHz instead of km/s. The calculations are correct; only the label is wrong.
- For pulsar modes, the sensitivity is based on the total effective bandwidth which is the number of windows times the effective bandwidth per window. There are no safeguards on the total bandwidth a user can select, i.e., a user can select more bandwidth than would be available for a receiver and obtained unrealistic results. It is up to the user to select the appropriate bandwidth for their observations. The calculator does not know about the bandwidth limits for each receiver.
- The calculations are based on the input frequency and the results are not based on integrating parameters across the full bandwidth. This can be important for UWB where parameters vary significantly across the band.
- For the  $1/f$  noise warnings, the calculator scales based on an observing time of 0.1 sec using the bandwidths in GBT Memo#302, where the values in the Memo were based on a time of 0.2 sec. The current warnings are more conservative (lower  $BW \cdot t$  limit).

# 1. Introduction to the GBT Sensitivity Calculator

The GBT Sensitivity Calculator has been designed to provide observers an easy way to determine the time needed to complete a proposed project or the expected sensitivity achieved by a project of a given length. The calculator should simplify the writing of the proposal technical justification since a user can save the input parameters and results to a file which can then be attached to the proposal to help minimize any misinterpretations for the technical reviewer.

The calculator has been designed to satisfy 95% or so of ‘traditional’ spectral line, pulsar and continuum observations and is not guaranteed to produce results better than 10% in sensitivity or 20% in time. It is up to the user on how best to derive from the calculator’s output observing time for their sessions and projects. Thus, we expect those writing proposals to continue to use the technical justification section of their proposals to describe how the results of the calculator were used to derive the time estimates for their projects. Users can also include in their technical justifications as many output logs from the calculator as they feel are needed.

This document has been compiled to guide users through the flow of questions asked by the calculator in order to produce a final sensitivity or time estimate which can then be included in the technical justification of the proposal. The structure and common features of the sensitivity calculator can be found in [Chapter 2](#). The following chapters are devoted to separate frames within the sensitivity calculator that require user input and should each be completed sequentially.

- 3. General Information
- 4. Hardware Information
- 5. Source Information
- 6. Data Reduction
- 7. Results

The Appendix provides a detailed discussion of the parameters and equations used by the calculator.

## 1.1 How Start the GBT Sensitivity Calculator

The Sensitivity Calculator is located at [https://dss.gb.nrao.edu/calculator-ui/war/Calculator\\_ui.html](https://dss.gb.nrao.edu/calculator-ui/war/Calculator_ui.html). You will need to supply your ‘My NRAO’ username and password.

## 2. Structure and Common Features of the Sensitivity Calculator

When you first start the GBT Sensitivity Calculator you will be presented with the screen shown in [Figure 2.1](#).

**Figure 2.1: The GBT Sensitivity Calculator**

Help Desk | Users Guide

Sensitivity Calculator

**General Information**

Derive: ☒ Observing Time from Desired Sensitivity  
☐ Sensitivity from Observing Time

Sensitivity Units: ☒ Flux Density (mJy)  
☐ Antenna Temp., Ta (mK)  
☐ Main Beam Temp., Tmb (mK)  
☐ Radiation Temp., Tr (mK)

Desired Sensitivity:

**Hardware Information**

Answer questions from top to bottom. If you change a question that was answered previously, check all answers that follow. Some answers will dictate the answer for other questions.

Backend:

Mode:

Receiver:

Beams:

Polarization:

BandWidth (MHz):

Number of Spectral Windows:

Switching Mode:

**Source Information**

Topocentric Frequency (MHz):

Source Diameter (arc minutes):

**Source Contribution Corrections**

Source Contribution to System Temperature: ☒ User Estimated Correction  
☐ Internal Galactic Model

**Controls**

**Results**




Results

Please fill out the questions to the left to begin.

### 2.1 Structure

The GBT Sensitivity calculator has been designed in a way that guides the user through various sequential steps. In general you should start by completing boxes from top to bottom, as the questions asked of you will depend on your answers to previous questions. For example, you will not be asked how many beams you wish to use if you have selected a single beam receiver. Any questions that are not applicable to your proposed observations or that cannot be asked at an early stage will be grayed out. As you select your answer to a question, the calculator will automatically update and if necessary, further questions may be asked of you. Questions asked of the user are organized by topics given in the frames on the left hand side of the screen ([General Information](#), [Hardware Information](#), [Source Information](#) and [Data Reduction](#)). At any time you may press the ‘[Update Results](#)’ button to view any parameters that the calculator can return to you based on the answers you have given. If the calculator is not able to return a derived sensitivity or total observing time then you still have some questions to answer.

## 2.2 Features

- [Help Desk](#) – You may select this link to send an email to the Dynamic Scheduling System (DSS) helpdesk using your default email application.
-  – Press the up arrow button to minimize its associated frame.
-  – Press the down arrow button to expand its associated frame.
-  – Red triangles denote questions that must be answered in order for the calculator to proceed or denote fields that have been altered, requiring you to press the ‘[Update Results](#)’ button on the right side of the calculator.

## 3. General Information Frame

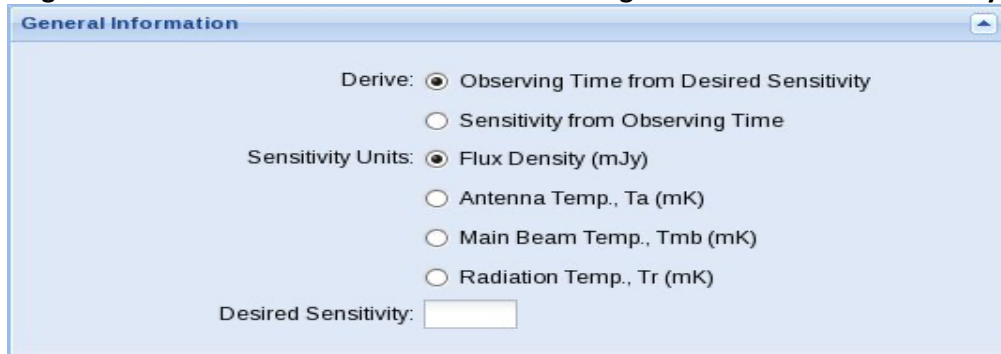
You must select whether you wish to derive the total observing time from a given sensitivity or vice versa. In either case, you must first need to choose your units for sensitivity. The allowed units are:

- **Flux Density (mJy)** - ( $10^{-29}$  Watts  $\text{m}^2\text{Hz}^{-1}$ ), and as if measured from above the Earth’s atmosphere (Default).
- **Antenna Temp., Ta (mK)** - as measured below the Earth’s atmosphere.
- **Radiation Temp., Tr (mK)** - as if measured from above the Earth’s atmosphere and derived for a function of source size. This is the  $T_R^*$  temperature scale corrected for source size.
- **Main Beam Temp., Tmb (mK)** – This scale represents the temperature measured in the main beam for an extended source, corrected for the atmosphere. It similar to Tr, but defined for sources whose diameter extends to the first nulls in the telescope’s beam.
- **Forward Beam Temp.,  $T_A^*$  (mK)** – This scale represents the power in the forward (on-sky) direction corrected for atmosphere. It is used by Argus and is the standard measured antenna temperature for mm-observations using the chopper/vane calibration technique.

### 3.1 Observing Time from Desired Sensitivity

Enter the sensitivity that you wish to achieve in the Desired Sensitivity box in your chosen units. Note that the units are in milli-Jansky or milli-Kelvin.

**Figure 3.1: General Information Frame - Observing Time from Desired Sensitivity**

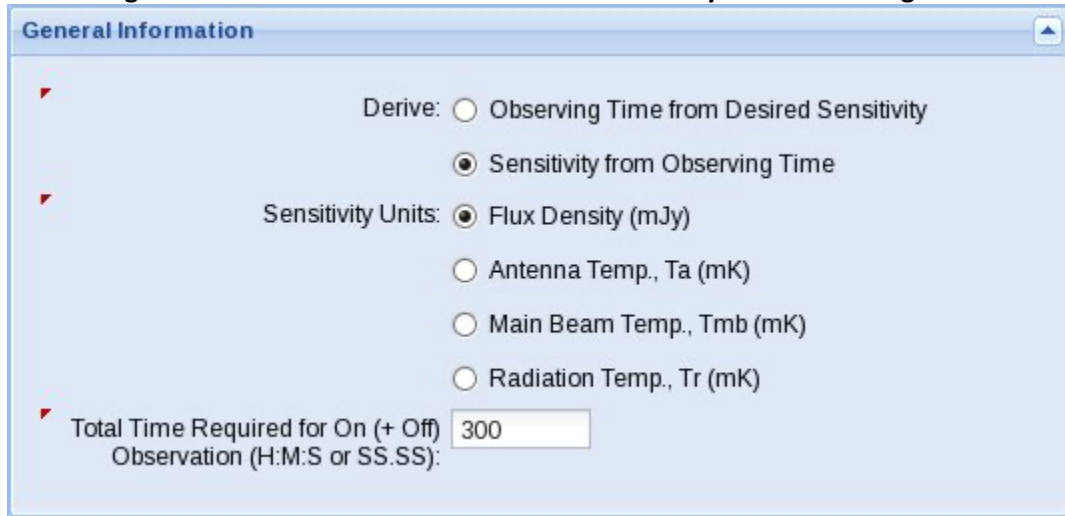


The screenshot shows a window titled "General Information" with a maximize button in the top right corner. Inside the window, there are two sections of radio buttons. The first section, labeled "Derive:", has two options: "Observing Time from Desired Sensitivity" (which is selected with a black dot) and "Sensitivity from Observing Time". The second section, labeled "Sensitivity Units:", has four options: "Flux Density (mJy)" (selected with a black dot), "Antenna Temp., Ta (mK)", "Main Beam Temp., Tmb (mK)", and "Radiation Temp., Tr (mK)". At the bottom of the window, there is a label "Desired Sensitivity:" followed by a text input box.

### 3.2 Sensitivity from Observing Time

Enter the total time required for your observations. Units of time can be entered in seconds or sexagesimal format.

Figure 3.2: General Information Frame - Sensitivity from Observing Time



The screenshot shows a window titled "General Information". Inside, there are two radio button options under the heading "Derive:". The first option is "Observing Time from Desired Sensitivity" (unselected). The second option is "Sensitivity from Observing Time" (selected). Below these, there are four radio button options under the heading "Sensitivity Units:". The first option is "Flux Density (mJy)" (selected). The other three options are "Antenna Temp., Ta (mK)", "Main Beam Temp., Tmb (mK)", and "Radiation Temp., Tr (mK)", all of which are unselected. At the bottom, there is a text label "Total Time Required for On (+ Off) Observation (H:M:S or SS.SS):" followed by a text input field containing the number "300".

### 4. Hardware Information Frame

You will be asked a series of questions concerning your choice of hardware and the calculator will check the answers to make sure that they can be accommodated by the hardware. Only those questions needed for the sensitivity calculator will be asked. For example, if you select the Caltech Continuum Backend (CCB) as your backend, then all subsequent choices will be filled out for you with the default CCB values, and you will not be required (or be able) to select other options.

Figure 4.1: Hardware Information Frame



The screenshot shows a window titled "Hardware Information". At the top, there is a text instruction: "Answer questions from top to bottom. If you change a question that was answered previously, check all answers that follow. Some answers will dictate the answer for other questions." Below this instruction are seven dropdown menus, each preceded by a red triangle icon. The labels and current values for the dropdowns are: "Backend:" (empty), "Mode:" (NOTHING), "Receiver:" (NOTHING), "Beams:" (NOTHING), "Polarization:" (NOTHING), "BandWidth (MHz):" (NOTHING), and "Number of Spectral Windows:" (NOTHING). The "Switching Mode:" dropdown is partially visible at the bottom and also shows "NOTHING".

The available fields within the Hardware Information Frame are:

- **Backend** – You may currently choose from the following backends for GBT observations:
  - Caltech Continuum Backend (CCB)

- GBT Digital Continuum Receiver (DCR)
  - VErstile GB Astronomical Spectrometer (VEGAS)
- **Mode** – The available modes currently available for each backend are:
    - Continuum – CCB and DCR
    - Spectral Line – VEGAS
    - Pulsar – VEGAS

The VEGAS backend can also be used for continuum observations, but the data collection is done in spectral-line mode.

- **Receiver** – If you have selected VEGAS or the DCR as your backend then you will need to select which GBT receiver you wish to use. The CCB only works with the Ka-band receiver. For 3mm continuum observations, the GBT uses Mustang, and the sensitivity estimates and mapping times are maintained by the Mustang instrument team (<https://www.gb.nrao.edu/mustang/>).
- **Beams** – If the receiver you have selected is capable of using more than one beam then you will be prompted to enter how many beams you wish to use here.
- **Polarization** - You will be given a choice between Single, Dual, and Full for VEGAS. The Ka-band receivers and Argus are single polarization instruments. When selecting Dual polarization, the default for data processing is to average both orthogonal polarizations, while for Full (Stokes) the default processing does not average the polarizations and returns a result consistent with a single polarization.
- **Bandwidth** – You will be given a choice of available bandwidths depending on which backend you have selected. The bandwidth represents the size of the frequency window that will be sampled by the backend. For the broad bandwidth VEGAS modes, the effective bandwidth is provided by the calculator (1250 MHz and 850 MHz) for the 1500 MHz and 1080 MHz bandwidth modes, respectively.
- **Spectral Windows** – You will be given a choice of how many spectral windows you wish to use for your observations. The number of windows available for VEGAS depend on the receiver, number of beams, and bandwidth selected. For Pulsar mode, the sensitivity will be based on the total effective bandwidth which is the number of windows multiplied by the effective bandwidth per window.
- **Switching Mode** – You may choose between ‘In-Band Frequency Switching’, ‘Out-of-Band’ Frequency Switching’ and ‘Position Switching’. Multi-beam receivers also allow for the options of ‘Nodding between beams’, and ‘Sub-reflector nodding between beams’.

## 5. Source Information Frame

Since projects usually observe multiple sources, you may want to run the sensitivity calculator for each, but it is also acceptable to run the calculator for a representative source, or a representative set of source parameters. If the latter approach is taken then you will need to infer how that calculation for a single or representative source translates into the time or sensitivity requirements for the body of your sources.



Figure 5.1: Source Information Frame

**Source Information**

Frequency Specified in the: ☐ Topocentric Frame  
☒ Rest Frame

Rest Frequency (MHz):

Doppler Correction:

Source Velocity (km/s):

Source Diameter (arc minutes):

**Source Contribution Corrections**

Source Contribution to System Temperature: ☒ User Estimated Correction  
☐ Internal Galactic Model

Contribution (K):

Source Declination (Deg):

Minimum Elevation (Deg):

User input is required for the following fields:

- **Topocentric Frame / Rest Frame** – You will be asked if the observing frequencies will be given in the line’s reset frame or in the topocentric frame (frame for the surface of the Earth). The default is ‘Rest Frame’
- **Topocentric / Rest Frequency (MHz)** – The default will be the middle of the selected receivers band.
- **Doppler Correction** – If you had previously selected ‘Rest Frame’ then you will be asked to select between an optical, radio and redshift Doppler correction. The default is ‘Optical’. For the standard optical definition ( $V = c * z$ ), the redshift and optical doppler correction is the same.
- **Source Velocity (km/s) / Redshift** - Depending on the selected Doppler correction you will be asked to supply a representative source velocity or redshift. The default in either case is zero.
- **Source Diameter (arc minutes)** – Use the slider to set a representative size for your source. The calculator assumes the source brightness distribution is that of a uniform disk. The slider is available for units of flux or Tr. If using flux density units, the available source size ranges from 0 (point source) to 2.6\*FWHM of the GBT beam (where the factor of 2.6 represents the size of the beam to the first nulls). For flux density units, a point-source will minimize the time needed. If using Tr units, the available source size ranges from 0.2 to 2.6\*FWHM of the GBT beam. For brightness temperature units, a source that fills the full beam<sup>\*\*\*</sup> will minimize the time required, and smaller sources would take longer due to beam dilution. For a source that fills the beam, the results for Tr and T<sub>mb</sub> are the same.
- **Source Contribution Corrections** – You will be asked whether to apply a correction for the representative source’s continuum level to the calculator’s system temperature calculations. The options are:

- **User Estimated Correction** – If you select this options then you will be asked to provide the background level in the units that you have selected in the ‘General Information’ frame.
- **Internal Galactic Model** – If selected then the calculator will ask for a representative J2000 Right Ascension of the source. The calculator will then augment the system temperature with the approximate continuum background from the Milky Way for the specified position using the 408 MHz survey of [Haslam et al \(1981, A&A, 100, 209\)](#).
- **Source Declination (Deg)** – Use the slider to set a representative source Declination in degrees (default = 0°).
- **Minimum Elevation (Deg)** – Use the slider to set the minimum allowable elevation for observations of the source (default = 5°). The calculator will provide a suggested minimum elevation, dependent on your source declination and observing frequency, if the value you entered is less than the recommended value.

## 6. Data Reduction Frame

Observers have a number of choices in how they collect and reduce their data that significantly affect the time they will need for an experiment and the corresponding sensitivity they will achieve. Only those that are most common have been included in the calculator.

**Figure 6.1: Data Reduction Frame**

Ratio of observing time spent on-source/on-frequency to that spent on a reference position/reference frequency.

1

In data reduction you have the option to average multiple reference observations in order to improve the noise. Enter number of reference observations that will be averaged together.

1

☒ Average Orthogonal Polarizations

☒ Difference Signal and Reference Observations

**Smoothing**

Smooth On-source Data to a ☒ Velocity Resolution in the Rest Frame  
Desired:

☐ Frequency Resolution in the Topocentric

☐ Frequency Resolution in the Rest Frame

Desired Resolution (km/s): 1

The available fields available for data reduction depend upon your answers in the [hardware frame](#). Possible questions include:

- **Ratio of observing time spent on-source/on-frequency to that spent on a reference position/reference frequency** – If you have selected any switching mode other than total power, then you will be asked:

- The ratio of time spent on your signal (on-source or on-frequency) observation to their reference observation. The default is set to 1.
  - The number of reference observations that will be averaged together. The default is set to 1. Some position-switching mode mapping observations will average several integrations at the beginning and end of each scan for the reference, and in these cases this parameter could be much larger than 1 (e.g., 10-50).
- **Average Orthogonal Polarizations** – You will be offered this choice based on your hardware configuration. If the receiver is dual polarization and dual polarization is selected, then the average polarization box will be checked by default. For the single polarization receivers (e.g., Ka-band and Argus) or the Full polarization option, the box will be left unchecked by default giving results for a single polarization. If you override the defaults for single polarization and select the average orthogonal polarization box, the tool will compute results assuming there are two polarizations to average.
  - **Difference Signal and Reference Observations** – You will be asked whether you plan on differencing signal and reference observations. By default this is set to on. For smooth baselines and narrow lines, improved single-to-noise may be achieved by removing a low order polynomial across the raw bandpass and not using a reference scan, but setting this option to off is not recommended in general. Turning this option off will decrease the total time by a factor of 4 (a factor of 2 is due to the lower noise when not subtracting an off and another factor of 2 since there are no reference scans and all time could be spent on the source). If this box is unselected then it is assumed all time is spent observing the on position when computing total time, and you will not be able to input a value for the ratio of time spent on-source to that spent on a reference position.
  - **Smoothing** – For spectral line observations you will be required to provide how you will smooth both the on-source and off-source data and whether you will be smoothing to a specified:
    - *Velocity Resolution* (km/s) in the source’s rest frame (default)
    - *Frequency Resolution in the Topocentric Frame* (MHz)
    - *Frequency Resolution in the Rest Frame* (MHz)
  - **Resolution** – Enter the desired resolution in the units selected above (Default = 1).

## 7. Results Frame


The results frame on the calculator’s right side is where all the output from the sensitivity calculator will be displayed. [Figure 7.1](#) shows the results frame prior to any entries by the user.

Figure 7.1: A Blank Results Frame



## 7.1 Controls

The only interface with the results frame is through the two buttons under 'Control'.

- **Update Results** - Press the 'Update Results' button to display all results that the sensitivity calculator can currently return based on the information you have given. The calculator will also prompt you to press this button if you have changed any of the entries in the user input frames on the left of the screen by displaying a red arrow  next to any field that has been altered.
- **Save to File** - If you press the 'Save to File' button then you will be given the option of saving the information displayed in the results frame as a text file. The information in this file can be cut and paste into time estimate technical box of the proposal.

## 7.1 The Results Window

The results grid is for debugging as it presents all of the internal variables and values used by the calculator's underlying code. [Figure 7.2](#) gives an example of the 'Results' tab layout after all information has been provided by the user.

Figure 7.2: Results Frame – ‘Results’ Tab

Results	
Result Grids	
Results	
Derived Sensitivity:	7.045358 mJy
Time at Signal Position or Frequency:	00:02:30.0 s
Time at Reference Position or Frequency:	00:02:30.0 s
Effective Integration Time:	00:01:15.0 s
Obs. Mode Time Mult. Factor:	4
FWHM Beamwidth:	8.56 '
Aperture Efficiency:	0.70
Extended Source Efficiency:	0.70
Confusion Limit:	58.57 S (mJy)
# Hrs Above Min Elevation:	8.55 hours
Topocentric Frequency:	1440.000 MHz
Min. Topocentric Channel Width:	0.095 kHz
Desired Freq. or Vel. Resolution:	1.000000 MHz or km/s
Typical Air Mass:	1.8
Typical Atmospheric Attenuation:	1.013
Typical System Temperature:	16.1 K
Backend Sampling Efficiency (K1):	1.0320
Backend Channel Weighting (K2):	1.2100
Other Results	
Maximum Elevation:	51.6 d
Pulsar Factor (bw / eff_bw * dc / (100.0 - dc)):	1.0000
eta_surf:	1.00

Outputs from the sensitivity calculator in this mode are given under the following headings:

- **Results** – This section contains information that will be of use to you when writing your technical justification. If you are unable to derive a final sensitivity or total observing time, then you will need to include additional information in the user input frames to the left of the screen (and also remember to press the ‘Update Results’ button). You may also find it helpful to look under the ‘User Input’ heading (Also in the results frame) and scan it for any fields which appear to be blank, in which case you can enter that missing information in the relevant frame.
- **Messages** – Information that the calculator may deem important will be displayed under this section. These messages include cases where the minimum selected elevation is lower than recommended, cases when the noise is affected by confusion or 1/f noise in cases of continuum observations. **Do not ignore the effects of confusion or 1/f noise in your time request when the calculator tool provides a warning.** Please contact observatory staff if you are unsure of the importance of the message(s) for your program.
- **Other Results** – The values of constants used for the sensitivity calculators calculations. Details of these constants and algorithms can be found in the Appendix.

- **User Input** – All of the information entered into the [General Information](#), [Hardware Information](#), [Source Information](#) and [Data Reduction](#) frames will be displayed under this heading.

### 7.1.1 The Grid Results Tab

Figure 7.3: Results Frame – ‘Result Grids’ Tab

Term	Value
FWHM Beamwidth	8.556098298902917
Obs. Mode Time Mult. Factor	4
Typical System Temperature	16.10573683827094
Typical Air Mass	1.8290468944511824
Time at Signal Position or Frequency	150

If you select the ‘Result Grids’ tab, then the output will be displayed in table format (Figure 7.3). Unlike the ‘Results’ tab, there are only two headings:

- **Results** – Contains all of the parameters that are given under the headings ‘Results’ and ‘Other Results’ in the [Result Tab](#).
- **User Input** – All of the information entered into the [General Information](#), [Hardware Information](#), [Source Information](#) and [Data Reduction](#) frames will be displayed under this heading.

## Appendix

### A. Definition and Terms

Term	Definition
<b>A</b>	Representative atmospheric attenuation, $e^{\tau \cdot AirMass}$ , for the typical weather condition and elevation of an observation at the user’s observing frequency
<b>AirMass</b>	Representative air mass through which the observations will be made
<b>BW</b>	Bandwidth in MHz, either native to the backend or the bandwidth to which the user smooths
<b>BW<sub>Ref</sub></b>	Bandwidth in MHz that the reference (off) observation will be smoothed to
<b>c</b>	Speed of light in $m\ s^{-1}$
<b>DishRadius</b>	Illumination radius for the selected receiver in m
<b><math>\Delta f_{REST}</math></b>	Frequency resolution in the rest frame in MHz
<b><math>\Delta V_{REST}</math></b>	Velocity resolution in the rest frame in $m\ s^{-1}$
<b>El<sub>Min</sub>, El<sub>Max</sub></b>	Range in observing elevation in °
<b>EST</b>	Effective system temperature in K ( $T_{sys} \cdot e^{\tau \cdot AirMass}$ )
<b>EST<sub>0</sub></b>	Effective system temperature under the best possible weather conditions

<b>EST<sub>TS</sub></b>	Effective system temperature but augmented by the expected loss in efficiency due to tracking and surface errors
<b>FeedTaper</b>	Feed taper illumination of the reflector in dB.
<b>Frequency</b>	Topocentric frequency in MHz
<b>FWHM</b>	Full-width, half-maximum beam width in '
<b><math>\eta_{MB}</math>, <math>\eta_I</math>, <math>\eta_{fss}</math>, and <math>\eta_S</math></b>	Efficiency which takes into consideration surface errors as a function of frequency and source size for various intensity conventions.
<b><math>\eta_0</math></b>	Aperture efficiency at long wavelengths and receiver and frequency dependent
<b><math>\eta_A</math></b>	Aperture efficiency which takes into consideration surface errors as a function of frequency
<b><math>\eta_{Dss}</math></b>	Normalized observing efficiency, in units of time, as suggested by DSS simulations and the product $\eta_{Track} \eta_{Surf} \eta_{Atm}$
<b><math>\eta_{Track}</math>, <math>\eta_{Surf}</math>, <math>\eta_{Atm}</math></b>	The normalized observing efficiency, in units of time, as suggested by DSS simulations due to tracking errors (e.g., wind induced), thermal-induced surface errors, and atmospheric conditions.
<b><math>\theta_{Source}</math></b>	Source size in '
<b>k</b>	Boltzmann's constant
<b><math>K_1</math></b>	Sampling sensitivity of a backend and, thus, is hardware dependent
<b><math>K_2</math></b>	Autocorrelation channel weighting factor for spectral line backends or a measure of the independence of samples for continuum observing; hence hardware dependent
<b><math>N_{RefAvrg}</math></b>	Number of reference observations that will be averaged together in the data reduction and used for every signal (on) observation
<b><math>R_{RefSmthAvrg}</math></b>	Amount by which a reference observation is smoothed or the number of reference observations that are averaged together beyond that of the signal observations
<b><math>N_{Samp}^{Uncorr}</math></b>	Degree to which backend inputs measure uncorrelated signals. (For example, = 2 when averaging orthogonal polarizations or when using nodding or in-band frequency switching )
<b><math>R_{SigRef}</math></b>	Ratio of time spent on the signal observation to the time on the reference observation
<b><math>\sigma_S</math></b>	Sensitivity in units of Jy (above atmosphere)
<b><math>\sigma_{T_R^*}</math></b>	Sensitivity in K in the $T_R^*$ (above atmosphere) temperature scale
<b><math>\sigma_{T_A}</math></b>	Sensitivity in K in the $T_A$ (below atmosphere) temperature scale
<b><math>\tau</math></b>	Representative zenith atmospheric opacity in units of Nepers
<b><math>\tau_0</math></b>	Best possible zenith atmospheric opacity in units of Nepers
<b><math>t_{eff}</math></b>	Effective integration time in s - essentially the time that satisfies the radiometer equation and is related to the actual observing time in ways that depend upon the observing tactics
<b><math>t_{Sig}</math></b>	Time spent in s on a source or signal position or frequency
<b><math>t_{Ref}</math></b>	Time spent in s on a reference position or frequency
<b><math>t_{Total}</math></b>	Total time in s needed to complete an observation
<b><math>T_{Atm}</math></b>	Temperature of the atmosphere in K to use in an estimate of $T_{sys}$ -- approximate temperature of the atmospheric layer that is contributing most to the opacity
<b><math>T_{CMB}</math></b>	The cosmic microwave background = 2.7 K
<b><math>T_{BG}</math></b>	Either the continuum temperature in K of the user's source or the temperature in K of the galactic background in the direction of the observation
<b><math>T_{Rcvrl}</math></b>	Contribution to $T_{sys}$ in K from the receiver
<b><math>T_{sys}</math></b>	Expected approximate system temperature in K at a representative elevation
<b>V</b>	Source velocity in m/s
<b>z</b>	Redshift (V/c)

## B. Basic Equations

The algorithms used by the sensitivity calculator are a modified version of the classic radiometer equation. The modified equation describes the theoretical noise one would obtain if observing above the atmosphere with a given effective system temperature ( $EST_{TS}$ ; see §E.1 for the relationship between  $EST_{TS}$  and  $T_{SYS}$ ) and bandwidth (BW in MHz) for a given effective duration of an observation ( $t_{eff}$  in sec). If we ignore the contribution from 1/F noise:

$$\sigma_{T_R^*} = \left( \frac{K_1 \cdot EST_{TS}}{\eta_l \eta_{fss}} \right) \frac{1}{\sqrt{N_{Samp}^{Uncorr} \cdot 10^6 \cdot BW \cdot t_{eff}}}$$

$N_{Samp}^{Uncorr}$  depends upon the details of the observing tactics (§E.5).  $K_1$  = the sampling sensitivity of a backend (§E.3).

The calculator uses the definition of the  $T_R^*$  temperature scale of Kutner and Ulich (1981). Accordingly,  $T_R^*$  is the brightness temperature of an equivalent black-body source with the assumption that the source is a circular disc of uniform brightness.  $\eta_l \eta_{fss}$  corrects for both the efficiency of the telescope's optics as well as the convolution of the source brightness distribution with the telescope's beam (§E.4).

Due to the rough accuracy needed, the calculator makes some simplifying assumptions:

- The Rayleigh-Jeans approximation holds at all frequencies,
- The ohmic-loss efficiency factors,  $\eta_r$  in Kutner and Ulich is taken to be equal to 1.

One should not use the results of the calculator for the accurate calibration of astronomical measurements.

Observers will be specifying (or requesting) sensitivities in the following units:

- Flux density (S in Jy =  $10^{-26}$  Watts  $m^{-2} s^{-1}$ ), as if observed from above the Earth's atmosphere
- Temperature as defined by the  $T_A$  temperature scale, as observed from the surface of the Earth
- Temperature as defined by the  $T_R^*$  ( $T_r$ ) temperature scale, as if observed from above the Earth's atmosphere
- Temperature as defined by the  $T_{MB}$  ( $T_m$ ) temperature scale, which is identical to the definition of  $T_R^*$  but with the assumption that the source size is the same as the telescope's beam to its first null. Here,  $\eta_l \eta_{fss}$  is equal to the standard definition of beam efficiency,  $\eta_{MB}$ .
- Temperature as defined by  $T_A^*$  ( $T_{star}$ ) temperature scale, which is the antenna brightness temperature corrected for atmosphere in the forward (on-sky) direction.

With the above simplifying assumptions, and the standard conversions between these units of intensity, the relationships of the theoretical noise for the various intensity scales are:

$$\sigma_{T_A} = \frac{\eta_s \cdot \pi \cdot DishRadius^2}{2k \cdot e^{\tau \cdot AirMass}} \cdot \sigma_S$$

$$\sigma_{T_A} = \frac{\eta_l \eta_{fss}}{e^{\tau \cdot AirMass}} \cdot \sigma_{T_R^*}$$

$$\sigma_{T_A} = \frac{\eta_{MB}}{e^{\tau \cdot AirMass}} \cdot \sigma_{T_{MB}}$$



$$\sigma_{T_A} = \frac{\eta_l}{e^{\tau \cdot AirMass}} \cdot \sigma_{T_A^*}$$

$e^{\tau \cdot AirMass}$ , hereafter called AtmosAtten, is the representative atmospheric attenuation for the elevation range of the observation;  $k$  = Boltzman's constant. Section E.4 describes how  $\eta_s$ ,  $\eta_{MB}$  and  $\eta_l$   $\eta_{fss}$  depend upon the efficiency of the telescope's optics and source size.

Most observing tactics require differencing observations of a signal/source position (or signal frequency) with a reference position (or reference frequency). Since both the signal and reference observations have noise, the resulting difference will be noisier than either the signal or reference observation. Since  $t_{eff}$  is related to the noise in the resulting difference,  $t_{eff}$  is always less than the total duration of the signal plus reference observation.

Often one may spend a different amount of time on each phase of an observations. For example, to save overhead it's common practice to use one reference observation for multiple signal observations. The calculator defines  $R_{SigRef}$ , which has a default value of 1, as the ratio of the time spent on each phase of an observation.

$$R_{SigRef} = t_{Sig}/t_{Ref}.$$

To help reduce the extra noise from differencing observations, it's a common practice during data analysis to smooth the reference observation or to average multiple reference observations. The calculator defines  $R_{RefSmthAvg}$  as the amount by which a reference observation is smoothed or averaged beyond any smoothing done to the signal observation. Early versions of the calculator allowed for different values for smoothing of the signal and reference scans, but this option has been removed. However, the same effect can be obtained by increasing the number of reference observations that will be averaged together. From the theory of the propagation of errors, the noise in the difference is:

$$\sigma^2 = \sigma_{Sig}^2 + \sigma_{Ref}^2$$

Assuming  $EST_{TS}$  is the same for the signal and reference observations, the above equation implies that:

$$\frac{1}{t_{eff}} = \frac{1}{t_{Sig}} + \frac{1}{R_{RefSmthAvg} \cdot t_{Ref}}$$

And, therefore,

$$t_{eff} = \frac{t_{Sig} \cdot (R_{RefSmthAvg} \cdot t_{Ref})}{t_{Sig} + (R_{RefSmthAvg} \cdot t_{Ref})}$$

The typical user isn't interested in the value of  $t_{eff}$ ,  $t_{Sig}$  or  $t_{Ref}$ . Rather, users are mostly interested in the total duration needed for an observation ( $t_{Total}$ ). If there is no overhead involved in the observing, then  $t_{Total} = t_{Sig} + t_{Ref}$ , and after a bit of algebra, one derives the following relationships between  $t_{eff}$ ,  $t_{Total}$ ,  $t_{Sig}$  and  $t_{Ref}$ :

$$\begin{aligned} t_{Total} &= t_{Sig} + t_{Ref} = t_{eff} \cdot \frac{(R_{SigRef} + R_{RefSmthAvg})(R_{SigRef} + 1)}{R_{SigRef} \cdot R_{RefSmthAvg}} \\ t_{eff} &= t_{Total} \cdot \frac{R_{SigRef} \cdot R_{RefSmthAvg}}{(R_{SigRef} + R_{RefSmthAvg})(R_{SigRef} + 1)} \\ t_{Sig} &= t_{Total} \cdot R_{SigRef} / (R_{SigRef} + 1) \\ t_{Ref} &= t_{Total} / (R_{SigRef} + 1). \end{aligned}$$

The above radiometer equations for the various intensity scales, in terms of  $t_{\text{Total}}$ , are then:

$$\begin{aligned}\sigma_S &= \left( \frac{2k \cdot K_1 \cdot EST_{TS}}{\eta_s \cdot \pi \cdot DishRadius^2} \right) \sqrt{\frac{(R_{\text{SigRef}} + R_{\text{RefSmthAvg}})(R_{\text{SigRef}} + 1)}{10^6 \cdot BW \cdot N_{\text{Samp}}^{\text{Uncorr}} \cdot R_{\text{SigRef}} \cdot R_{\text{RefSmthAvg}} \cdot t_{\text{Total}}}} \\ \sigma_{T_R^*} &= \left( \frac{K_1 \cdot EST_{TS}}{\eta_l \eta_{fss}} \right) \sqrt{\frac{(R_{\text{SigRef}} + R_{\text{RefSmthAvg}})(R_{\text{SigRef}} + 1)}{10^6 \cdot BW \cdot N_{\text{Samp}}^{\text{Uncorr}} \cdot R_{\text{SigRef}} \cdot R_{\text{RefSmthAvg}} \cdot t_{\text{Total}}}} \\ \sigma_{T_{MB}} &= \left( \frac{K_1 \cdot EST_{TS}}{\eta_{MB}} \right) \sqrt{\frac{(R_{\text{SigRef}} + R_{\text{RefSmthAvg}})(R_{\text{SigRef}} + 1)}{10^6 \cdot BW \cdot N_{\text{Samp}}^{\text{Uncorr}} \cdot R_{\text{SigRef}} \cdot R_{\text{RefSmthAvg}} \cdot t_{\text{Total}}}} \\ \sigma_{T_A} &= \left( \frac{K_1 \cdot EST_{TS}}{AtmosAtten} \right) \sqrt{\frac{(R_{\text{SigRef}} + R_{\text{RefSmthAvg}})(R_{\text{SigRef}} + 1)}{10^6 \cdot BW \cdot N_{\text{Samp}}^{\text{Uncorr}} \cdot R_{\text{SigRef}} \cdot R_{\text{RefSmthAvg}} \cdot t_{\text{Total}}}} \\ \sigma_{T_A^*} &= \left( \frac{K_1 \cdot EST_{TS}}{\eta_l} \right) \sqrt{\frac{(R_{\text{SigRef}} + R_{\text{RefSmthAvg}})(R_{\text{SigRef}} + 1)}{10^6 \cdot BW \cdot N_{\text{Samp}}^{\text{Uncorr}} \cdot R_{\text{SigRef}} \cdot R_{\text{RefSmthAvg}} \cdot t_{\text{Total}}}}\end{aligned}$$

Some observing tactics will not require taking a reference observation, which implies that  $t_{\text{Total}} = t_{\text{eff}}$ . In these cases:

$$\begin{aligned}\sigma_S &= \left( \frac{2k \cdot K_1 \cdot EST_{TS}}{\eta_s \cdot \pi \cdot DishRadius^2} \right) \sqrt{\frac{1}{10^6 \cdot BW \cdot N_{\text{Samp}}^{\text{Uncorr}} \cdot t_{\text{Total}}}} \\ \sigma_{T_R^*} &= \left( \frac{K_1 \cdot EST_{TS}}{\eta_l \eta_{fss}} \right) \sqrt{\frac{1}{10^6 \cdot BW \cdot N_{\text{Samp}}^{\text{Uncorr}} \cdot t_{\text{Total}}}} \\ \sigma_{T_R^*} &= \left( \frac{K_1 \cdot EST_{TS}}{\eta_{MB}} \right) \sqrt{\frac{1}{10^6 \cdot BW \cdot N_{\text{Samp}}^{\text{Uncorr}} \cdot t_{\text{Total}}}} \\ \sigma_{T_A} &= \left( \frac{K_1 \cdot EST_{TS}}{AtmosAtten} \right) \sqrt{\frac{1}{10^6 \cdot BW \cdot N_{\text{Samp}}^{\text{Uncorr}} \cdot t_{\text{Total}}}} \\ \sigma_{T_A^*} &= \left( \frac{K_1 \cdot EST_{TS}}{\eta_l} \right) \sqrt{\frac{1}{10^6 \cdot BW \cdot N_{\text{Samp}}^{\text{Uncorr}} \cdot t_{\text{Total}}}}\end{aligned}$$

Simple inversions of these equations allow one to derive a total time from a user-supplied sensitivity:

$$\begin{aligned}
t_{\text{Total}} &= \left( \frac{K_1 \cdot EST_{TS}}{\eta_l \eta_{fss} \cdot \sigma_{T_R^*}} \right)^2 \left( \frac{(R_{\text{SigRef}} + R_{\text{RefSmthAvg}})(R_{\text{SigRef}} + 1)}{BW \cdot N_{\text{Samp}}^{\text{Uncorr}} \cdot R_{\text{SigRef}} \cdot R_{\text{RefSmthAvg}}} \right) \\
t_{\text{Total}} &= \left( \frac{K_1 \cdot EST_{TS}}{\eta_{MB} \cdot \sigma_{T_{MB}}} \right)^2 \left( \frac{(R_{\text{SigRef}} + R_{\text{RefSmthAvg}})(R_{\text{SigRef}} + 1)}{10^6 \cdot BW \cdot N_{\text{Samp}}^{\text{Uncorr}} \cdot R_{\text{SigRef}} \cdot R_{\text{RefSmthAvg}}} \right) \\
t_{\text{Total}} &= \left( \frac{K_1 \cdot EST_{TS}}{\sigma_{T_A} \cdot \text{AtmosAtten}} \right)^2 \left( \frac{(R_{\text{SigRef}} + R_{\text{RefSmthAvg}})(R_{\text{SigRef}} + 1)}{10^6 \cdot BW \cdot N_{\text{Samp}}^{\text{Uncorr}} \cdot R_{\text{SigRef}} \cdot R_{\text{RefSmthAvg}}} \right) \\
t_{\text{Total}} &= \left( \frac{K_1 \cdot EST_{TS}}{\eta_l \cdot \sigma_{T_A^*}} \right)^2 \left( \frac{(R_{\text{SigRef}} + R_{\text{RefSmthAvg}})(R_{\text{SigRef}} + 1)}{BW \cdot N_{\text{Samp}}^{\text{Uncorr}} \cdot R_{\text{SigRef}} \cdot R_{\text{RefSmthAvg}}} \right)
\end{aligned}$$

Without reference observations:

$$\begin{aligned}
t_{\text{Total}} &= \left( \frac{2k \cdot K_1 \cdot EST_{TS}}{\eta_s \cdot \pi \cdot \text{DishRadius}^2 \cdot \sigma_s} \right)^2 \left( \frac{1}{10^6 \cdot BW \cdot N_{\text{Samp}}^{\text{Uncorr}}} \right) \\
t_{\text{Total}} &= \left( \frac{K_1 \cdot EST_{TS}}{\eta_l \eta_{fss} \cdot \sigma_{T_R^*}} \right)^2 \left( \frac{1}{10^6 \cdot BW \cdot N_{\text{Samp}}^{\text{Uncorr}}} \right) \\
t_{\text{Total}} &= \left( \frac{K_1 \cdot EST_{TS}}{\eta_{MB} \cdot \sigma_{T_{MB}}} \right)^2 \left( \frac{1}{10^6 \cdot BW \cdot N_{\text{Samp}}^{\text{Uncorr}}} \right) \\
t_{\text{Total}} &= \left( \frac{K_1 \cdot EST_{TS}}{\sigma_{T_A} \cdot \text{AtmosAtten}} \right)^2 \left( \frac{1}{10^6 \cdot BW \cdot N_{\text{Samp}}^{\text{Uncorr}}} \right) \\
t_{\text{Total}} &= \left( \frac{K_1 \cdot EST_{TS}}{\eta_l \cdot \sigma_{T_A^*}} \right)^2 \left( \frac{1}{10^6 \cdot BW \cdot N_{\text{Samp}}^{\text{Uncorr}}} \right)
\end{aligned}$$

### C. 1/F Gain Instabilities

All receivers have a sensitivity limit from 1/F gain instabilities. A few systems that were explicitly designed for continuum observations (e.g., Mustang and the Ka-Band receiver used with the CCB backend) have much less of an issue with 1/F instabilities. These gain instabilities essentially place an upper limit on  $t_{\text{Total}} \cdot BW$  – beyond a certain point, increasing the bandwidth or the integration time does not improve one's sensitivity.

*There are a number of tactics that lets one improve upon the 1/F limit (e.g., averaging multiple short difference observations; making multiple maps with fast slewing). Since the calculator interface would need to be made much more complicated to capture all necessary details about the observing tactics, it cannot decide whether or not ones observations will actually exceed the limit. Therefore, to be on the safe side, the calculator warns a user whenever it is possible the observations will exceed the limit. The calculator returns a warning if the If the user's planned observation exceeds the 1/F limit, the user should consider conferring with the local support staff, and maybe justify the tactics they will use to overcome the 1/F limit in their proposal.*

The calculator uses the following estimates:

**Table 1**

System	Upper Limit for $t_{\text{Total}} \cdot \text{BW}$
Mustang	$10^5 \text{ MHz} \cdot \text{s}$
Ka-Band with CCB	$3.5 \times 10^5 \text{ MHz} \cdot \text{s}$
All Other Systems	See Mason (2013); GBT Memo#282

GBT Memo #282 (Mason 2013) provides the maximum effective bandwidth for each receiver  $[\text{BW}_{\text{eff}}(\text{max})]$  based on the  $1/f$  noise performance. For mapping extended regions, the effective bandwidth  $[\text{BW}_{\text{eff}}]$  is smaller than the maximum bandwidth given in the GBT Memo based on the time scale between the ON and OFF/reference observations. Depending on the time scale ( $t$ ) between the ON and OFF/reference observations (or time to map over a region of interest), the effective bandwidth  $\text{BW}_{\text{eff}} = \text{BW}_{\text{eff}}(\text{max}) (0.2 \text{ sec}/t)$ , where the time ( $t$ ) is in seconds. The continuum sensitivity can be estimated using the sensitivity calculator by selecting a bandwidth (e.g., in spectral-line mode) that matches the  $1/f$  noise effective bandwidth. This is the minimum noise level that can be reached by a single observation. Improvements can be made by making multiple observations (the noise integrates down as  $(1/N)^{0.5}$  where  $N$  is the number of independent observations). Thus, for continuum observations it is important to take many short observations and to carry out fast mapping scans for imaging.

#### D. Confusion Limit

Desired sensitivities in some observing modes may not be reachable due to confusion within the beam from multiple background sources. The confusion limit depends upon the observing frequency (in MHz), the FWHM beam width of the telescope (in arc minutes) at that frequency, and the user's chosen units for sensitivity. Standard wisdom recommends that one stay under five times the confusion limit for a reliable detection. The calculator approximates the confusion limit using the following equations (Condon 2002):

**Table 2**

Units	5x Confusion Limit
S (Jy)	$\frac{0.13 \cdot \text{FWHM}^2}{\text{Frequency}^{0.7}}$
$T_{\text{R}}^*(\text{K})$ $T_{\text{MB}}(\text{K})$ $T_{\text{A}}^*(\text{K})$	$\frac{0.13 \cdot \pi \cdot (\text{DishRadius} \cdot \text{FWHM})^2}{2k \cdot \text{Frequency}^{0.7}}$
$T_{\text{A}}(\text{K})$	$\frac{0.13 \cdot \eta_{\text{A}} \cdot \pi \cdot (\text{DishRadius} \cdot \text{FWHM})^2}{2k \cdot \text{Frequency}^{0.7} \cdot \text{AtmosAtten}}$

If the user is entering a sensitivity to derive a  $t_{\text{Total}}$ , then the calculator warns the user whenever the sensitivity is smaller than the corresponding value from the above table. If the user enters a time in order to derive sensitivity, then the calculator warns the user whenever the calculated sensitivity is smaller than the corresponding value from the above table. In both cases, the warning presents the value of the confusion limit.

*As with 1/F limit, the calculator cannot distinguish whether or not the specified observing tactics are or are not limited by confusion. Instead, the calculator assumes the user is best qualified to make that judgment.*

### E. Determining Values for Various Quantities

Many of the above equations depend upon values for  $EST_{TS}$  which in turn depends upon  $\tau$ , air mass (i.e., elevation), receiver temperature,  $T_{Atm}$ , background source temperature, spillover and the cosmic microwave background as well as the yet-to-be defined quantities  $\eta_{Track}$  and  $\eta_{Surf}$ . The calculator, of course, doesn't know the details of the weather conditions or elevation ranges over which the observations will happen. Instead, it makes the assumption that the observations will happen under typical opacity and wind conditions, as determined by DSS simulations (Condon & Balser, 2011), for the chosen semester, topocentric frequency, and receiver. It also assumes observations will be taken symmetrically around the meridian to a frequency-dependent minimum elevation.

#### E.1 $EST_{TS}$

In various DSS memos, we define the Effective System temperature as  $EST = T_{sys} \cdot AtmosAtten$ . This is the system temperature one would use in the radiometer equation to get the same sensitivity as if the observations were taken above the atmosphere.

In addition to atmospheric losses, the calculator also uses estimates for the relative loss of efficiencies due to tracking or surface errors ( $\eta_{Track}$ ,  $\eta_{Surf}$ , as defined in DSS memos) from winds or daytime observing. The quantity  $EST_{TS}$  used by the calculator is the EST with the extra losses due to tracking and surface errors.

$$EST_{TS} = \frac{EST}{\sqrt{\eta_{Track}\eta_{Surf}}} = \frac{T_{sys} \cdot AtmosAtten}{\sqrt{\eta_{Track}\eta_{Surf}}}$$

The DSS simulators provide the calculator with enough information that one can directly estimate a typical value for  $EST_{TS}$  for most observing setups. The DSS quantity  $\eta_{DSS}$  is an observing efficiency, with respect to time, that is normalized to the best conditions that are possible for the observing frequency, receiver, and source elevation. That is, a value of  $\eta_{DSS} = 0.5$  suggests one will need twice as much observing time to achieve the same sensitivity as one would under the best opacity, wind, surface, ... conditions.  $\eta_{DSS}$  is the product of the observing efficiency for atmospheric conditions ( $\eta_{Atm}$ ), tracking errors due to the telescope's pointing accuracy under various wind conditions ( $\eta_{Track}$ ) and surface errors ( $\eta_{Surf}$ ).

By the DSS definition,  $\eta_{Atm} = (EST_0/EST)^2$ , where  $EST_0$  is the effective system temperature for the best possible weather conditions (see §E.2). Thus:

$$\eta_{DSS} = \eta_{Track}\eta_{Surf} \left( \frac{EST_0}{EST} \right)^2 = \left( \frac{EST_0}{EST_{TC}} \right)^2$$

In most cases, one would use  $EST_{TS} = EST_0 / \sqrt{\eta_{DSS}}$ . However, this will not be the case if one were observing toward a strong background source. To compensate for a background source with an effective black-body temperature of  $T_{BG}$ , one uses instead:

$$EST_{TS} = \frac{T_{BG}}{\sqrt{\eta_{Track}\eta_{Surf}}} + \frac{EST_0}{\sqrt{\eta_{DSS}}}$$

The DSS simulations have supplied values for  $\eta_{DSS}$  and the product  $\eta_{Track} \eta_{Surf}$ , all of which can be assumed to be frequency and receiver dependent. At all but the highest frequencies  $\eta_{Track} \eta_{Surf} \approx 1$ . The user either supplies  $T_{BG}$  or asks the calculator to provide an estimate of the galactic background, derived from the 408 MHz observations of Haslam et al (1981), for a specified Right Ascension, Declination and observing frequency.

## E.2 $EST_0$

$EST_0$ , the effective system temperature under the best weather conditions, is obtained from:

$$EST_0 = (T_{Rcvr} + T_{Spill} + T_{Atm}) \cdot e^{\tau_0 \cdot \text{AirMass}} - (T_{Atm} - T_{CMB})$$

where  $\tau_0$  is the best possible opacity at the observing frequency and  $\text{AirMass}$  is the **typical** air mass for the range of desired elevations.  $T_{Atm}$  is the equivalent black-body temperature of the atmosphere at the observing frequency for the best of weather conditions. The calculator assumes that both  $T_{Spill}$  (= 3 K) and  $T_{CMB}$  (=2.7 K) are receiver and frequency independent and that  $T_{Spill}$  is independent of elevation.  $T_{Rcvr}$  is receiver and frequency dependent and take on values provided by the receiver engineers. Since  $\tau_0$  and  $T_{Atm}$  are frequency and weather dependent, the calculator uses values derived from historical weather data averaged over five years year. (Due to the NRAO's semester scheduling system that begin and end in mid winter/summer, using yearly averages provides sufficiently accurate values.)

## E.3 Atmospheric Attenuation (*AtmosAtten*)

Note that a number of equations require *AtmosAtten*, that is the expected, typical (not best) atmospheric attenuation under which one can expect to be scheduled. With a bit of algebra:

$$\text{AtmosAtten} = e^{\tau \cdot \text{Airmass}} = \frac{EST_0 \sqrt{\frac{\eta_{Track} \eta_{Surf}}{\eta_{DSS}}} + (T_{Atm} - T_{CMB})}{T_{Rcvr} + T_{Spill} + T_{Atm}}$$

## E.4 $\eta_l$ , $\eta_{fss}$ , $\eta_{MB}$ and $\eta_S$

The aperture efficiency of the GBT ( $\eta_A$ ) is based on a typical rms surface error of 235  $\mu\text{m}$  (Frayer et al. 2019).

The calculator uses the Ruze equation to determine the frequency dependence of  $\eta_A$  :

$$\eta_A = \eta_0 \cdot e^{-(4.19 \times 10^{-8} \cdot \text{rms}_{Surface} \cdot \text{Frequency})^2}$$

The calculator assumes  $\eta_0 = 0.71$  for the long-wavelength aperture efficiency. Earlier versions of the calculator assumed that the aperture efficiency varied significantly with elevation, but with improvements with the telescope Zernike-Gravity model this is no longer the case (Frayer et al. 2018) and no correction is made to the aperture efficiency as a function of elevation.

For extended sources, the calculator assumes a source with a uniform brightness distribution of a circular shape of diameter  $\theta_{Source}$ . From models of the telescope's beam with a typical feed illumination pattern (taper) of -13 dB, we derive the following approximations:

$$\eta_S = \eta_A / (1 - 0.03740x + 0.2842x^2 - 0.1282x^3)$$

$$\eta_l \eta_{fss} = \eta_A / [-0.1192 + 0.9722 / (1 - e^{-0.08568 \cdot x^2})]$$

$$\eta_{MB} = 1.16\eta_A$$

where  $x = \theta_{Source} / FWHM$ . In practice, at the highest frequencies the coefficient measured is closer to 1.3, associated with a Gaussian beam approximation (see Maddalena 2010 for background and Frayer et al. 2019 for GBT measurements) instead of 1.16, but this is not accounted for in the calculator currently.

Notes:

- The value of  $\eta_{MB}$  is independent of source size since, by definition, the source has a size that extends just to the first nulls in the antenna beam pattern ( $\theta_{Source} \sim 2.6$  FWHM).
- As the source size decreases  $\eta_S$  goes to  $\eta_A$  while  $\eta_{fss}$  goes to zero.

- The calculator’s upper limit for the source diameter that is to first null ( $\sim 2.6$  FWHM). Given the clean optics of the GBT, and the rough accuracy of the calculator, using  $T_{MB}$  is justifiable for very extended sources.
- When using the  $T_R^*$  temperature scale, the calculator does not allow the source size to be below 0.2 FWHM as  $T_R^*$  is not defined for a point source.
- For  $T_A$  units, the calculator uses a source size of zero ( $x=0$ ) and  $\eta_S = \eta_A$ .
- For  $T_A^*$  units, the calculator adopts the estimated GBT forward efficiency of  $\eta_l = 0.99$ .

The calculator will report the aperture efficiency and the “extended source” efficiency in the results. The reported “extended source” efficiency depends on the adopted units as the following:

Units	Extended Source Efficiency Meaning
S(Jy)	$\eta_S$
$T_A$	$\eta_A$
$T_{mb}$	$\eta_{mb}$
$T_A^*$	$\eta_l$
$T_R^* (T_r)$	$\eta_l \eta_{fss}$

### E.5 Average Air Mass and Suggested Minimum Elevation

The calculator needs to determine a typical air mass under which one can expect an observation to be run. The air mass is taken as a weighted average over the expected elevation range of an observation. It is sufficiently accurate for the sensitivity calculator to use  $\csc(El)$  as an approximation for air mass.

From the source declination, the calculator determines the maximum elevation, which occurs at source transit (upper transit if the source is circumpolar). Using the algorithms in §3.4.2 of Condon & Basler (2011), the calculator derives a suggested minimum elevation that is based upon the source’s elevation at transit and the topocentric observing frequency. The user need not use the suggested minimum and can choose a minimum elevation as long as it is above  $5^\circ$  (the limit for the GBT) or, if observing a circumpolar source, the source’s elevation at lower transit.

The weighted average air mass is then calculated from the minimum and maximum elevations via:

$$AirMass = \frac{57.29 \cdot \ln\left(\frac{\tan(El_{Max}/2)}{\tan(El_{Min}/2)}\right)}{El_{Max} - El_{Min}}$$

### E.6 Opacity ( $\tau$ )

Note that opacity is never directly used by the calculator (only  $AtmosAtten$  is used directly) but a value for opacity is supplied to help for the user in planning observations. The calculator provides an estimate of the expected opacity for the typical weather and the specified range in elevations:

$$\tau = \frac{\ln(AtmosAtten)}{AirMass}$$

### E.7 $T_{SYS}$

$T_{SYS}$  is never directly used by the calculator but a value for  $T_{SYS}$  is supplied as a help for the user in planning observations. The calculator provides a typical  $T_{SYS}$  using the typical weather conditions:

$$T_{sys} = \frac{EST_{TS} \cdot \eta_{Track} \cdot \eta_{Surf}}{AtmosAtten}$$

### E.8 $K_1$ and $K_2$

Values for  $K_1$  and  $K_2$  are backend and backend mode dependent. For the backends currently in use, these parameters are both 1.0.

### E.9 $N_{Samp}^{Uncorr}$

$N_{Samp}^{Uncorr}$  represents the degree to which the data that are averaged together are uncorrelated. It describes how the sensitivity improves by averaging data from orthogonal polarizations or by use of different observing methods:

$$N_{Samp}^{Uncorr} = DualPol \cdot ObservingMethod$$

- DualPol = 2 if the user has specified they will be averaging polarizations, otherwise = 1.
- ObservingMethod = 2 if the user has specified in-band frequency switching and any of the ‘nodding’ observation types, otherwise = 1.

### E.10 FWHM beam size

Although the FWHM beam width of the GBT depends upon the feed illumination pattern and observing frequency (Goldsmith, 1987, 2002), for all receivers covered by the calculator, it is sufficient to use a feed taper of 13 dB and a 50-m dish radius. Therefore, using the formulae of Goldsmith:

$$FWHM = \frac{12.3 \cdot 10^3}{Frequency[MHz]} \text{ arc minutes}$$

### E.11 Topocentric frequency, frequency resolution, velocity resolution, and velocity coverage

The calculator provides values for the following quantities as a way to help observers plan their observations as well as to provide values it needs for various calculations.

1. To derive the **highest** topocentric frequency spacing the backend will allow, the calculator divides the bandpass width selected by the user by the maximum number of channels that the user’s configuration will allow.
2. To derive an approximate topocentric frequency from the user-specified rest frequency and velocity or redshift:

**Table 3**

Velocity Def.	Topocentric Freq.
Radio	$RestFreq \cdot (1 - V/c)$
Optical	$RestFreq/(1 + V/c)$
Redshift	$RestFreq/(1 + z)$

The calculator can provide only an approximate topocentric frequency -- it has no knowledge of when an observation will take place and cannot estimate the telescope motion with respect to a reference frame.

3. To convert a rest frame frequency resolution ( $\Delta f_{REST}$ ) to a topocentric frequency resolution:



Table 4

Velocity Def.	Topocentric Freq. Resolution
Radio	$\Delta f_{REST} \cdot (1 - V/c)$
Optical	$\Delta f_{REST} / (1 + V/c)$
Redshift	$\Delta f_{REST} / (1 + z)$

4. To convert a rest frame velocity resolutions ( $\Delta V_{REST}$ ) to a topocentric frequency resolution:

Table 7

Velocity Def.	Topocentric Freq. Resolution
Radio	$RestFreq \cdot \Delta V_{REST} / c$
Optical	$\frac{RestFreq \cdot \Delta V_{REST}}{c \cdot (1 + V/c)^2}$
Redshift	$\frac{RestFreq \cdot \Delta V_{REST}}{c \cdot (1 + z)^2}$

The calculator checks that the user's topocentric frequency resolution is not smaller than  $K2 \cdot$  highest topocentric frequency spacing the backend will allow, as calculated in step 1.

## F. Pulsar Observations

For pulsar observations, users can select the “duty cycle” of the pulsar (i.e., the ratio of the pulsar width to the period of the pulsar) and the effective bandwidth per window. The effective bandwidth selection provides users the flexibility to modify the bandwidth depending on the frequency overlap between windows and the amount of bandwidth lost to RFI. The calculator uses an internal parameter “Pulsar Factor” for its computations (Pulsar Factor =  $(bw/effective\_bandwidth) \cdot DC / (100 - DC)$ , where DC is the duty cycle percentage. This factor is 1.0 for non-pulsar options. For pulsar modes, this factor corrects for the effective bandwidth and for the amount of time the pulsar is in emission compared to the time not emitting. To avoid the effects of the input duty cycle while using the pulsar modes of the calculator (e.g., for the detection of a single pulse/burst), users can select a duty cycle of 50% so that  $DC / (100 - DC) = 1$ .

## References

- Baars, J.W.M, 1973, *IEEE Trans on Ant and Prop*, Vol AP-21, No. 4, p. 461.
- Condon, J.J. and Balser, D.S., 2011, “Dynamic Scheduling Algorithms, Metrics, and Simulations”, DS project Note 5.5, (<https://safe.nrao.edu/wiki/pub/GB/Dynamic/DynamicProjectNotes/dspn5.5.pdf>)
- Fraye, D. T., Ghigo, F., & Maddalena, R. J. 2018, “*The GBT Gain Curve at High Frequency*”, GBT Memo 301.
- Fraye, D. T., et al. 2019, “*Calibration of Argus and the 4mm Receiver on the GBT*”, GBT Memo 302.
- Goldsmith, P., 1987, *Int Journal of Infrared and Millimeter Waves*, Vol. 8, No. 7, p. 771.
- Goldsmith, P, 2002, in *Single-Dish Radio Astronomy: Techniques and Applications*, ed. Stanimorivic, Altschuler, Goldsmith, and Salter (ASP, Vol. 278), p. 45.
- Kutner, M.L. and Ulich, B.L., 1981, *Astrophysical Journal*, **250**, 341.
- Maddalena, R. J. 2010, “*Theoretical Ratio of Beam Efficiency to Aperture Efficiency*”, GBT Memo 276

Mason, B. 2013, “GBT Receivers' Continuum Sensitivity”, GBT Memo 282,  
([https://safe.nrao.edu/wiki/pub/GB/Knowledge/GBTMemos/GBT\\_Memo\\_282.pdf](https://safe.nrao.edu/wiki/pub/GB/Knowledge/GBTMemos/GBT_Memo_282.pdf)).