



Brief Introduction to the Radiometer Equation

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$$\sigma_S = \frac{T_{sys}}{G \sqrt{n_p t \Delta f}}$$

Terms and Concepts

σ_S

Standard Deviation of Flux Density

T_{sys}

System Temperature

G

Gain

n_p

Number of Polarization

t

Time

Δf

Bandwidth

Text books on Radio Astronomy

- Essential Radio Astronomy
- <https://science.nrao.edu/opportunities/courses/era>

Essential Radio Astronomy



[James J. Condon](#), [Scott M. Ransom](#)

Princeton University Press, Apr 5, 2016 - [Science](#) - 376 pages

Essential Radio Astronomy is the only textbook on the subject specifically designed for a one-semester introductory course for advanced undergraduates or graduate students in [More »](#)

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Tools of Radio Astronomy

Authors: **Wilson**, Thomas, **Rohlfs**, Kristen, **Huettemeister**, Susanne

Presents the 6th edition of a leading textbook on radio astronomy to include state-of-the-art descriptions of instrumentation and new observations

σS

Intrinsic Power P (Watts)

Distance R (meters)

Aperture A (sq.m.)

Flux = Power/Area

Flux Density (S) =

Power/Area/bandwidth

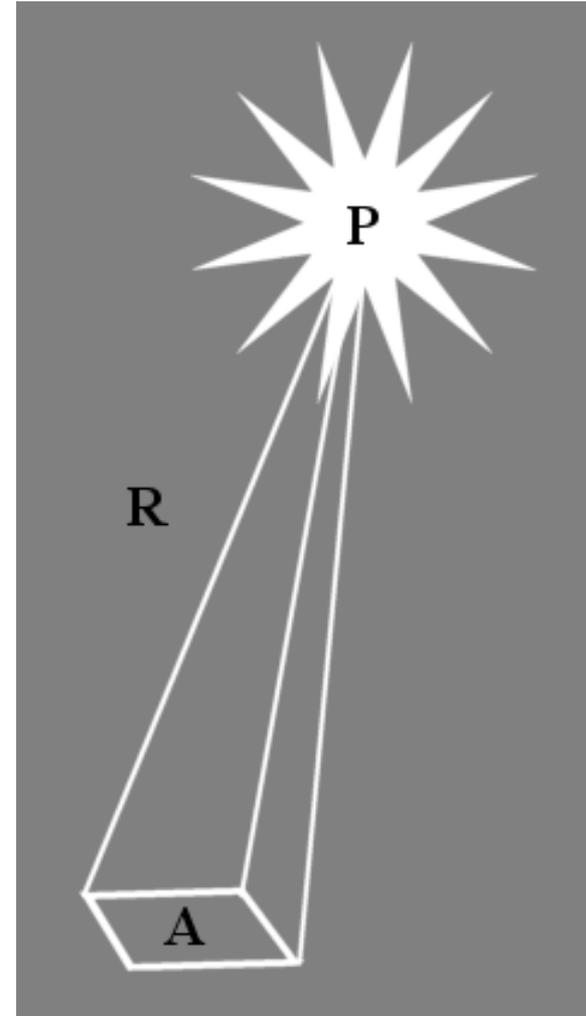
Bandwidth (Δf)

A “Jansky” is a unit of flux density

$10^{-26} \text{ Watts} / \text{m}^2 / \text{Hz}$

$$P = AS\Delta f$$

$$P = 10^{-26} 4\pi R^2 S\Delta f$$

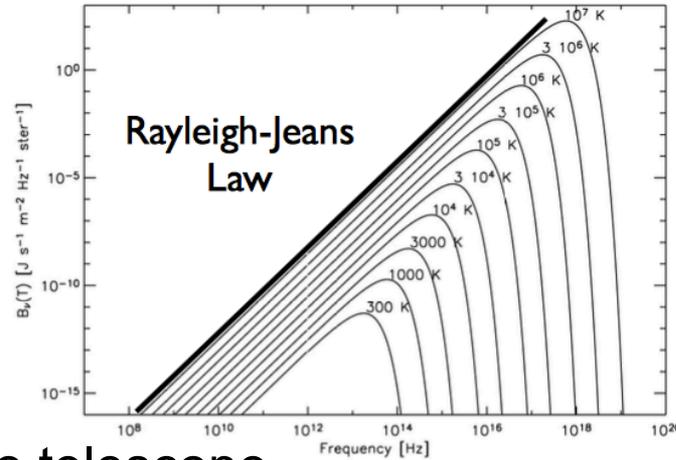
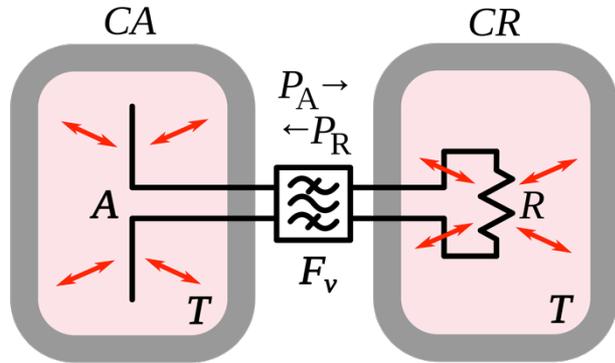


$$T_{sys}$$

Detected power (P, watts) from a resistor R at temperature T (kelvin) over bandwidth Δf (Hz)

$$P_R = \langle V^2 \rangle / R = kT \Delta f$$

$$P_A = \frac{1}{2} A_e S \Delta f$$



$$S = \frac{2kT_A}{A_e}$$

$$B_f = \frac{2kT}{\lambda^2}$$

Power P_A detected in a radio telescope
Due to a source of flux density S

$$P_A = 2\pi A_e B_f \Delta f$$

power as equivalent temperature.

Antenna Temperature T_A

Effective Aperture A_e

$$A_e = \frac{\lambda^2}{4\pi} = \frac{\lambda^2}{\Omega_A}$$

$$T_{sys}$$

Physical temperature vs antenna temperature

For an extended object with source solid angle Ω_s ,
And physical temperature T_s , then

$$\text{for } \Omega_s < \Omega_A \quad T_A = \frac{\Omega_s}{\Omega_A} T_s$$

$$\text{for } \Omega_s > \Omega_A \quad T_A = T_s$$

$$\text{In general : } T_A = \frac{1}{\Omega_A} \iint_{\text{source}} P_n(\theta, \phi) T_s(\theta, \phi) d\Omega$$

T_{sys}

System Temperature

= total noise power detected, a result of many contributions

$$T_{sys} = T_{ant} + T_{rcvr} + T_{atm} (1 - e^{-\tau a}) + T_{spill} + T_{CMB} + \dots$$

Table 2.2: Properties of the Prime Focus and Gregorian Focus Receivers.

Name	ν Range (GHz)	Polarization	Beams	Polns/Beam	T_{rec} (K)	T_{sys} (K)
— Prime Focus Receivers —						
PF1 Rcvr_342	0.290-0.395	Lin/Circ	1	2	12	46
PF1 Rcvr_450	0.385-0.520	Lin/Circ	1	2	22	43
PF1 Rcvr_600	0.510-0.690	Lin/Circ	1	2	12	22
PF1 Rcvr_800	0.680-0.920	Lin/Circ	1	2	21	29
PF2 Rcvr_1070	0.910-1.230	Lin/Circ	1	2	10	17
— Gregorian Focus Receivers —						
L-band Rcvr1_2	1.15-1.73	Lin/Circ	1	2	6	20
S-band Rcvr2_3	1.73-2.60	Lin/Circ	1	2	8-12	22
C-band Rcvr4_6	3.95-8.0	Lin/Circ	1	2	5	18
X-band Rcvr8_10	8.00-10.1	Circ	1	2	13	27
Ku-band Rcvr12_18	12.0-15.4	Circ	2	2	14	30
KFPA RcvrArray18_26	18.0-26.5	Circ	7	2	15-25	30-45
Ka-band Rcvr26_40 (MM-F1)	26.0-31.0	Lin	2	1	20	35
Ka-band Rcvr26_40 (MM-F2)	30.5-37.0	Lin	2	1	20	30
Ka-band Rcvr26_40 (MM-F3)	36.0-39.5	Lin	2	1	20	45
Q-band Rcvr40_52	38.2-49.8	Circ	2	2	40-70	67-134
W-band Rcvr68_92 (FL1)	67-74	Lin/Circ	2	2	50	160
W-band Rcvr68_92 (FL2)	73-80	Lin/Circ	2	2	50	120
W-band Rcvr68_92 (FL3)	79-86	Lin/Circ	2	2	50	100
W-band Rcvr68_92 (FL4)	85-92	Lin/Circ	2	2	60	110

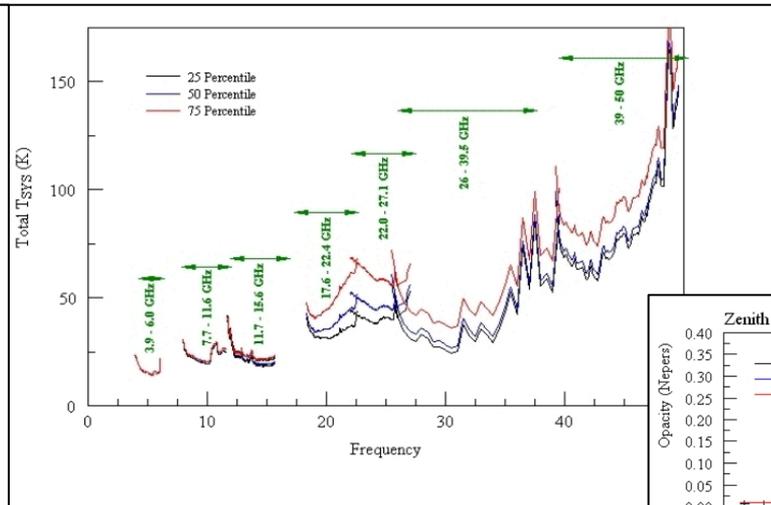
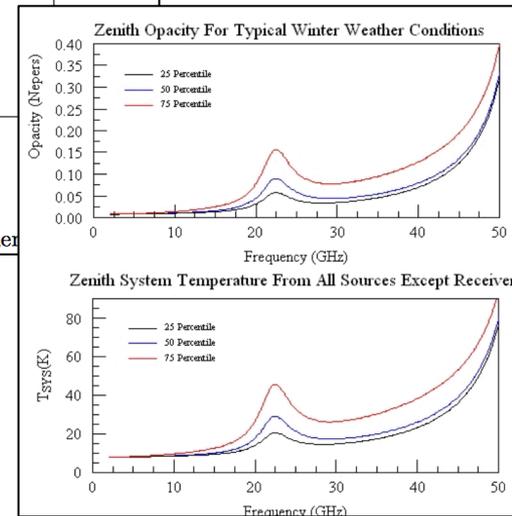


Figure 16.3: The zenith system temperatures for typical weather



T_{sys}

System Temperature

= total noise power detected, a result of many contributions

$$T_{sys} = T_{ant} + T_{rcvr} + T_{atm} (1 - e^{-\tau a}) + T_{spill} + T_{CMB} + \dots$$

$$\sigma_{T_{sys}} = \frac{T_{sys}}{\sqrt{n_p t \Delta f}}$$

Receiver Gain Instability

What happens:

$$P = gT_{sys}(k\Delta f) \quad \Delta P = \Delta g T_{sys}(k\Delta f)$$

Indistinguishable from:

$$\Delta P = g\Delta T_{sys}(k\Delta f)$$
$$\Delta T_{sys} = T_{sys} \left(\frac{\Delta g}{g} \right)$$

$$\sigma_{T_{sys}} = T_{sys} \left[\frac{1}{t\Delta f} + \left(\frac{\Delta g}{g} \right)^2 \right]^{\frac{1}{2}}$$

Flux Density Noise Level

$$S = \frac{2kT_{sys}}{A_e} = \frac{T_{sys}}{G} = S.E.F.D.$$

$$G = \frac{A_e}{2k} \quad \sigma_S = \frac{\sigma_{T_{sys}}}{G}$$

$$\sigma_S = \frac{T_{sys}}{G \sqrt{n_p t \Delta f}}$$



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