Radiation Fundamentals II

D. Anish Roshi

Arecibo Observatory

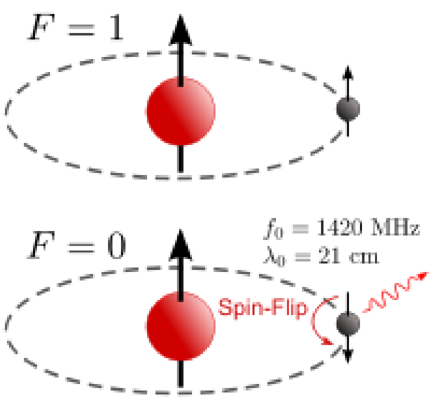
GBO/AO single dish workshop, Green Bank, August 19, 2019

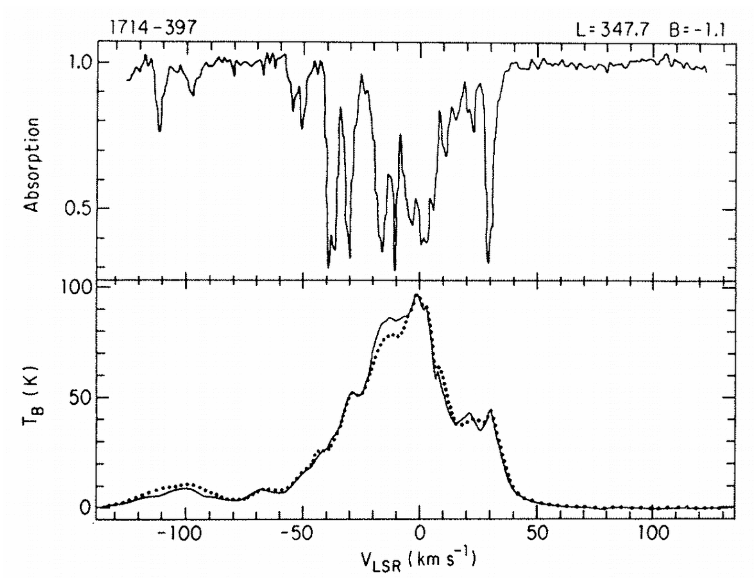
- Spectral lines of Astronomical interest
- Spectral line formation
- Polarization: application Faraday Rotation

Essential Radio Astronomy, Condon, J. & Ransom, S.

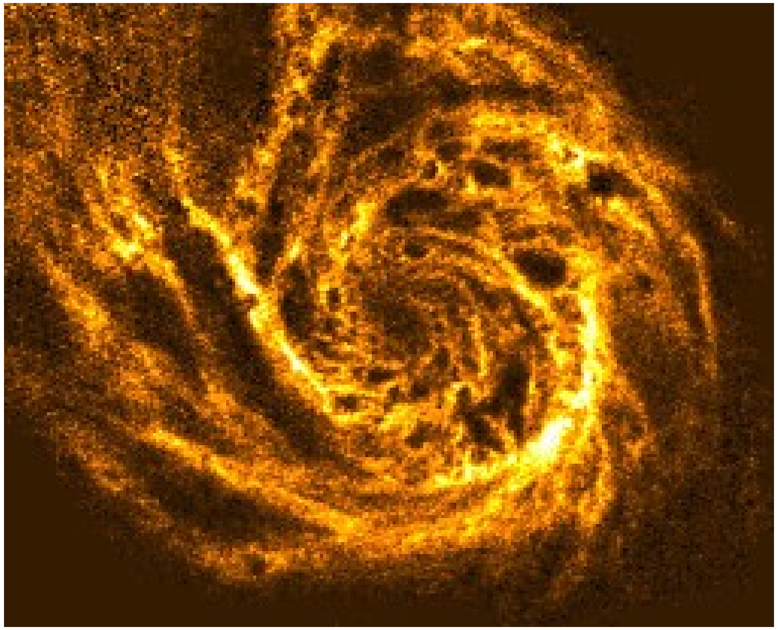
https://www.cv.nrao.edu/~sransom/web/xxx.html

Spectral Lines: HI 21cm line





HI spectrum from our galaxy

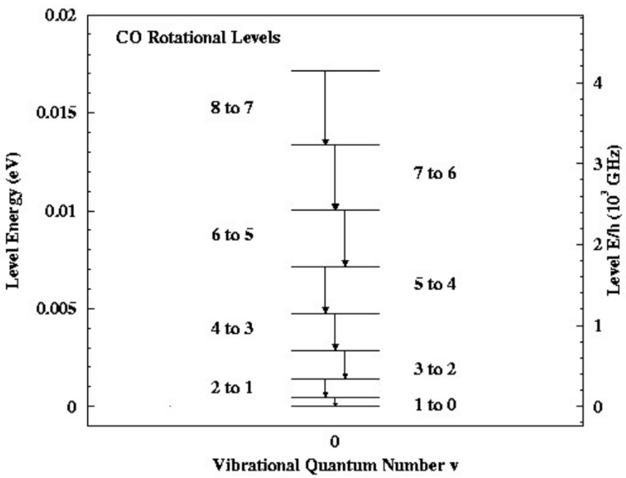


Pinwheel Galaxy (M101)

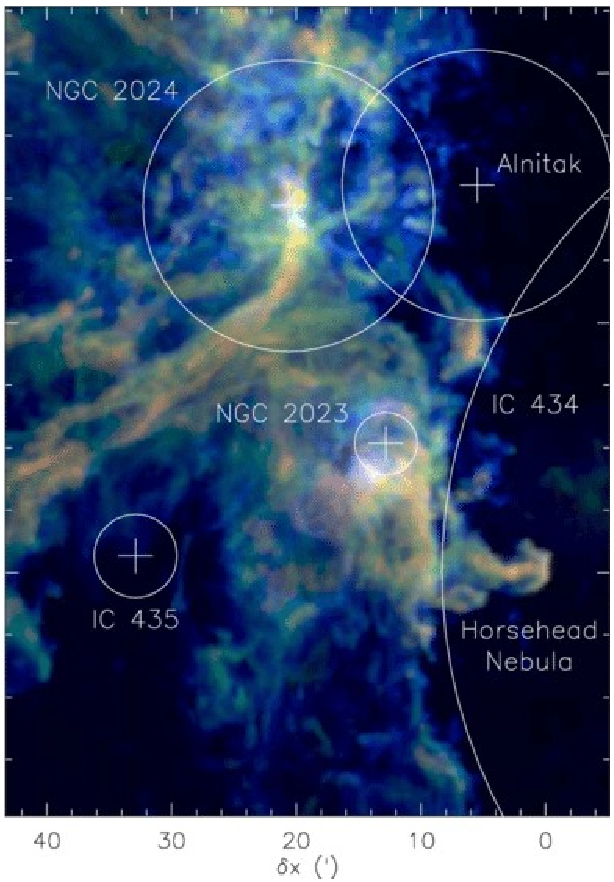
- Study of neutral component of ISM -- cold and warm; our Galaxy and external galaxy (cold – 60 K, 30 cm^{-3}, $5x10^{19}$ cm^{-2}; Warm – 8000 K, 0.2 cm^{-3}, 10^{18} cm^{-2})
- Kinematics of our Galaxy and external galaxy (eg. spiral galaxy)
- Large scale structure (nearby z<0.16; < 700 Mpc)

Spectral Lines: CO line

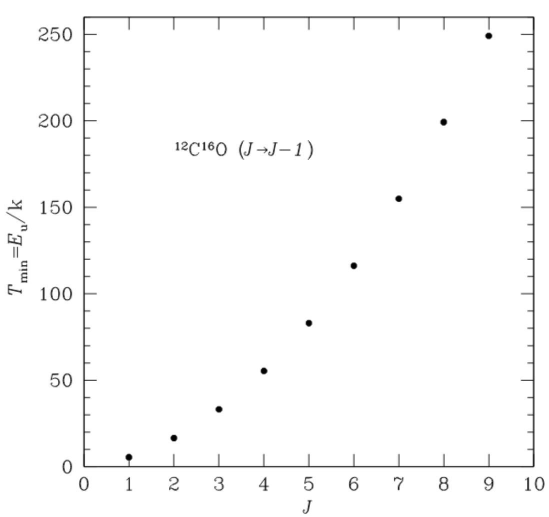
CO rotational ladder



$^{12}C^{16}O$ 1-0
115.27 GHz



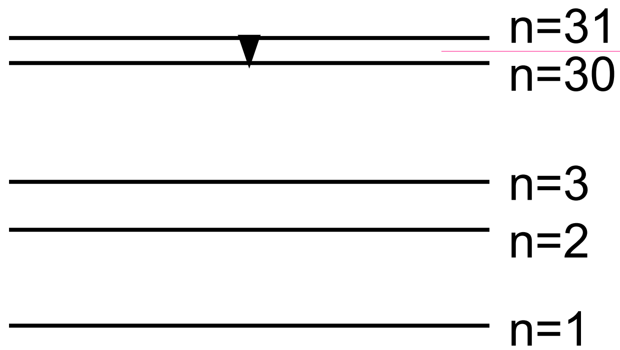
^{12}CO, ^{13}CO, $C^{18}O$ Image of Orion region



Study of cold molecular gas in ISM;
(10 K, 10^3 cm^{-3}, 20 pc)

Proxy for H_2; cloud properties (need to know X_{co}).

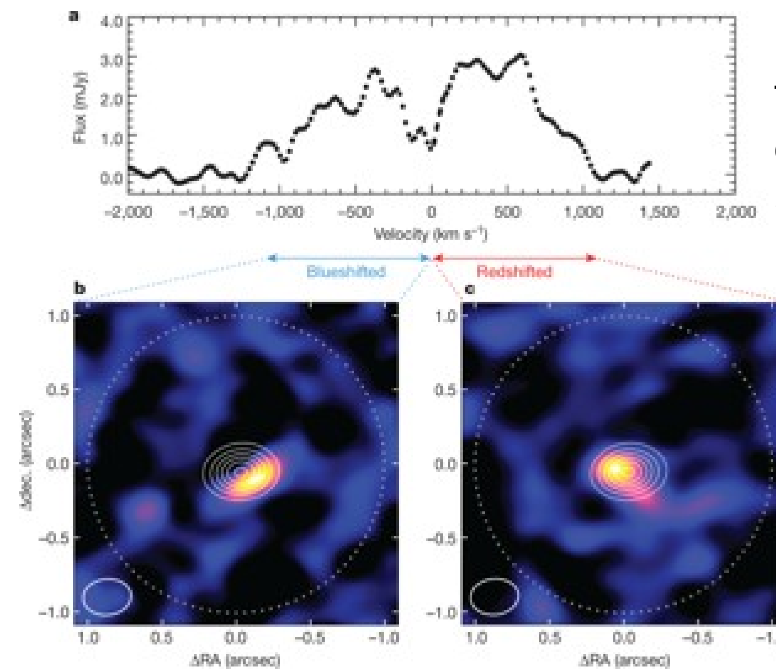
Spectral Lines: Radio Recombination Lines



Energy level of H atom

H30α (231.9 GHz)

Murchikova et al. (2019)



H30α line from the accretion disk around the Sgr A blackhole

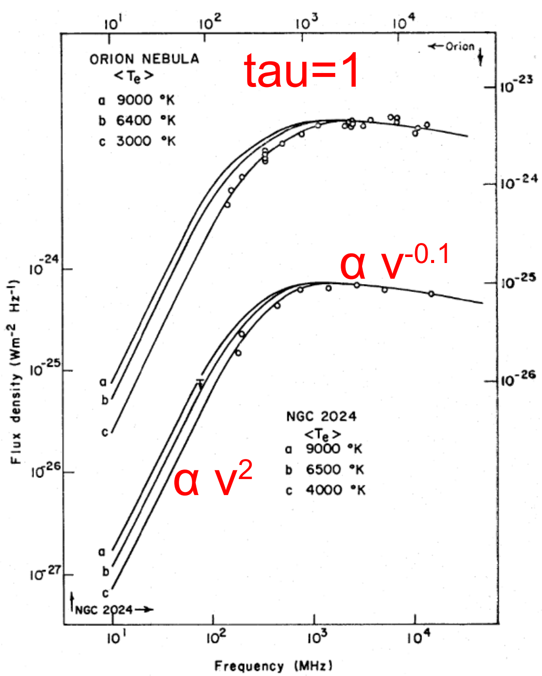


FIG. 3. The theoretically computed radio spectra for the Orion Nebula and NGC 2024, together with the observed flux densities.

Recombination lines: recombined electrons making transition at high quantum states

Study ionized component of ISM
(T_e~10^4 K, n_e ~ 0.1 cm^{-3} WIM; ~100 cm^{-3} HII)

Spectral Lines: Astronomical Masers

Thum et al. (1994)

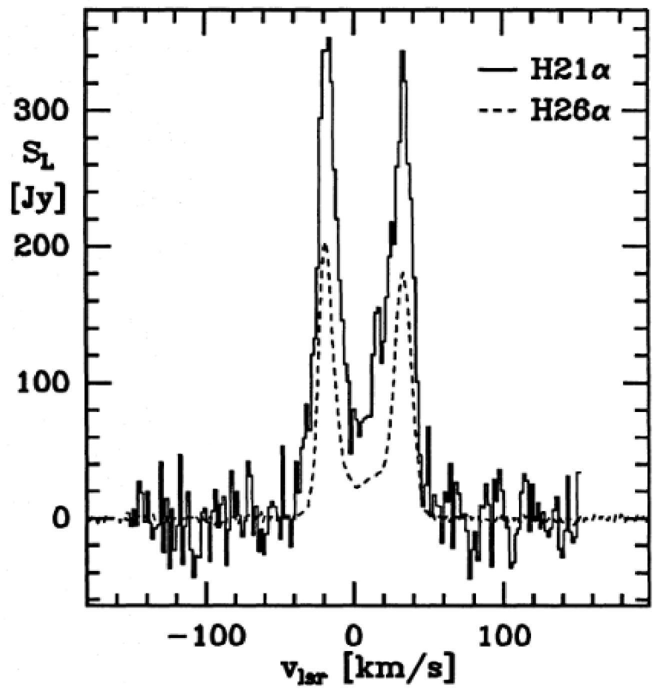
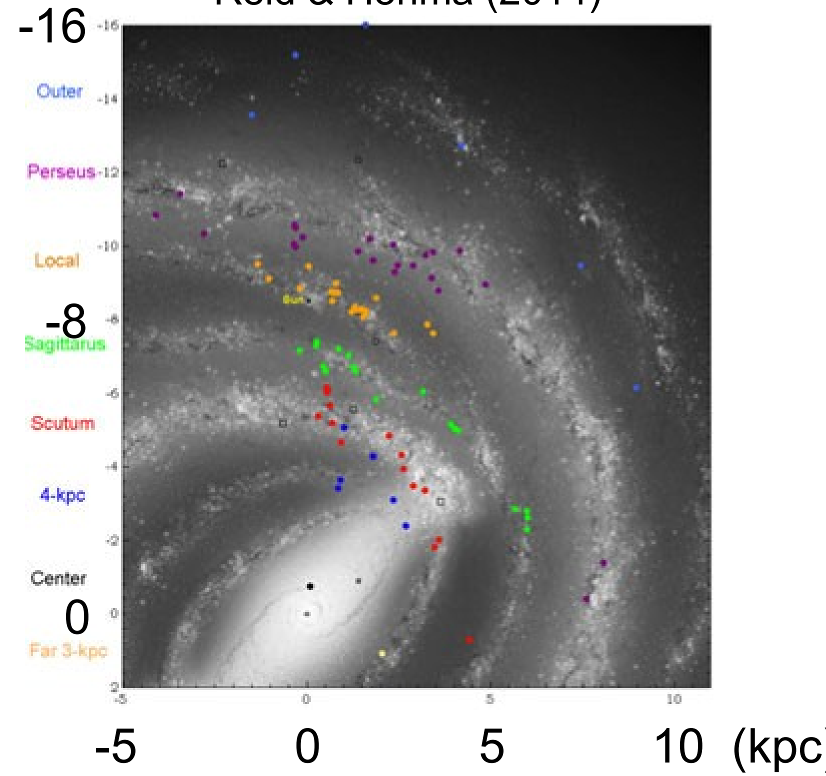


Fig. 1. H21α and H26α spectra toward MWC349, plotted on common velocity and flux scales. The spectra are smoothed to spectral resolutions of 1.0 (H26α) and 1.8 km s^{-1}(H21α). Linear baselines were removed from both spectra.

Reid & Honma (2014)



Plan view of Milky Way. Dots are the locations of newly formed OB-type stars determined from VLBI trigonometric parallaxes using associated H_2O or methanol maser emission. The Galactic center is denoted by the plus (+) sign at (0,0) kpc; the Sun is labeled in yellow at (0,8.4) kpc.

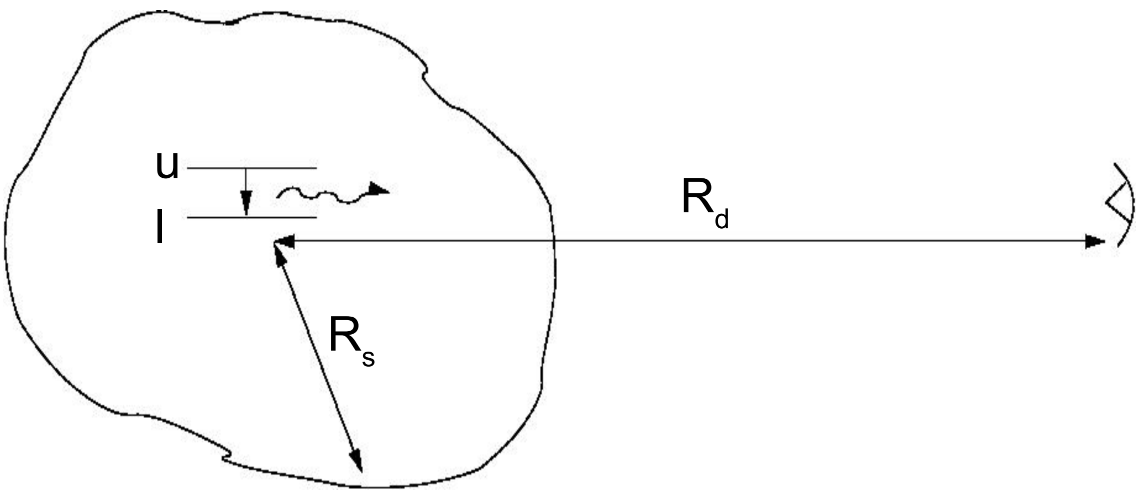
Stimulated microwave spectral line emission

OH, H_2O, CH_3OH, SiO, HRRL

Structure of Milky Way

Spectral line formation

Optically thin case; no background radiation



What we want?

n_d (cm^{-3}); $N \sim n_d\, 2R_s$ (cm^{-2})
T (K), R_s (pc)

Prediction for obs

$$u_\nu = \frac{1}{4\pi} N_u\, A_{ul}\, \phi(\nu)\, h\nu_0 \quad \mathrm{ergs/s/cm^3/Hz/strad}$$

Emissivity

$$P_\nu = 4\pi \frac{1}{4\pi} N_u\, A_{ul}\, \phi(\nu)\, h\nu_0\, \frac{4\pi}{3} R_s^3 \quad \mathrm{ergs/s/Hz}$$

Spectral density radiated by the source

$$S_L(\nu) = \frac{P_\nu}{4\pi R_d^2} \propto N_u\, A_{ul}\, 2R_s \left(\frac{R_s}{R_d}\right)^2 \phi(\nu)\, h\nu_0 \quad \mathrm{ergs/s/cm^2/Hz}$$

Flux density of the line

$$\propto N_u 2R_s A_{ul}\, \phi(\nu)\, h\nu_0\, \Omega_s \quad \mathrm{ergs/s/cm^2/Hz}$$

Column density of atoms in level u

Spectral line formation

Einstein A coefficient (spontaneous emission coefficient)

Classical approximation: A_{ul} is average spectral power emitted by a dipole divided by the photon energy.

HI 21cm line: $$A_{F=10} = 2.85 \times 10^{-15} \quad \mathrm{s}^{-1} \; (\sim 11 \text{ Myr half-life})$$

$^{12}C^{16}O$ rotational trans: $$A_{J=10} = 7.2 \times 10^{-8} \quad \mathrm{s}^{-1} \; (\sim 0.4 \text{ yr}; \propto \mathrm{J}^3)$$

RRL transitions: $$A_{n+1,n} = 5.3 \times 10^{9} \left(\frac{1}{n^5}\right) \quad \mathrm{s}^{-1} \; (\mathrm{n{=}100}; \sim 2 \text{ sec})$$

Spectral Line formation

Energy level population

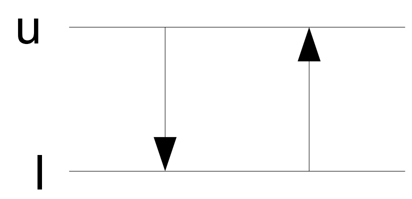
$$\frac{N_u}{N_l} = \frac{g_u}{g_l}\, e^{-\frac{E_{ul}}{k_b T}} \quad \text{T is a unique temperature in TD equilibrium}$$

Otherwise $T=T_{ex}$ (excitation temperature)

Statistical Equilibrium

For two level system

$$N_u A_{ul} + N_u C_{ul} + N_u B_{ul} \frac{I_\nu}{c} = N_l C_{lu} + N_l B_{lu} \frac{I_\nu}{c}$$



Multi-level system

$$N_i \sum_j R_{ij} = \sum_j N_j R_{ji}$$

Solution to this equation gives the level population; T_{ex} can be different for different levels

Spectral line formation

Energy level population: LTE approximation

$$\frac{N_u}{N_l} = \frac{g_u}{g_l}\, e^{-\frac{E_{ul}}{k_b T}} \qquad T = T_k \text{ the kinetic temperature}$$

(In general for LTE $T=T_{ex}$ is same of all level; first let us take $T=T_k$)

Condition: $C_{ul} \approx A_{ul}$

$$n^* \sigma v \approx A_{ul} \qquad n^* \text{ critical density (density of collision partner);}$$

σ cross section; v mean velocity

HI 21 cm line: $n^* << 1\ \mathrm{cm}^{-3}$

$^{12}C^{16}O$ J=1-0 line: $n^* \sim 10^3\ \mathrm{cm}^{-3}$ ($T_k \sim 20$ K)

RRLs: $n^* \sim 90\ \mathrm{cm}^{-3}$ ($T_k \sim 10^4$ K; n=100)

Spectral line formation

Energy level population: LTE approximation

$$\frac{N_u}{N_l} = \frac{g_u}{g_l}\, e^{-\frac{E_{ul}}{k_b T}} \qquad T = T_k \text{ same for all levels}$$

$$\frac{N_u}{n_d} = \frac{g_u\, e^{-\frac{E_u}{k_b T_k}}}{Q}$$

$$Q = \sum_i g_i\, e^{-\frac{E_i}{k_b T_k}} \qquad \text{Partition function}$$

HI 21cm line

$$Q \approx g_0 + g_1\, e^{-\frac{h\nu_{10}}{k_b T_k}} \qquad \frac{h\nu_{10}}{k_b} \sim 0.08 \text{ K}$$

$\approx 1 + 3 = 4$ (This is true for other values of T encountered in the ISM)

$$N_u \approx \frac{3}{4} n_d$$

Spectral line formation

Energy level population: LTE approximation

$^{12}C^{16}O$ J=1-0 line

$$Q_{rot} \approx \frac{2k_b T_k}{h\nu_{10}} = 0.36T_k$$

$$N_u \approx n_d \times \frac{3\, e^{-\frac{h\nu_{10}}{k_b T_k}}}{0.36T_k} \quad \text{for } \nu_{J=10} = 115.27 \text{ GHz transition}$$

- Q has contribution from rotation, vibration, ..
- Rotation – lowest energy state
- Q ~ Q_{rot}
- T_k ~ 20 K

(see Mangum & Shirley 2015; Turner 1991)

Hydrogen RRLs

Number density of atoms in quantum state n

$$N_n \propto n^2 \frac{n_p\, n_e}{T_e^{3/2}}$$

$$\propto n^2 \frac{n_e^2}{T_e^{3/2}} \qquad n_d \approx n_e$$

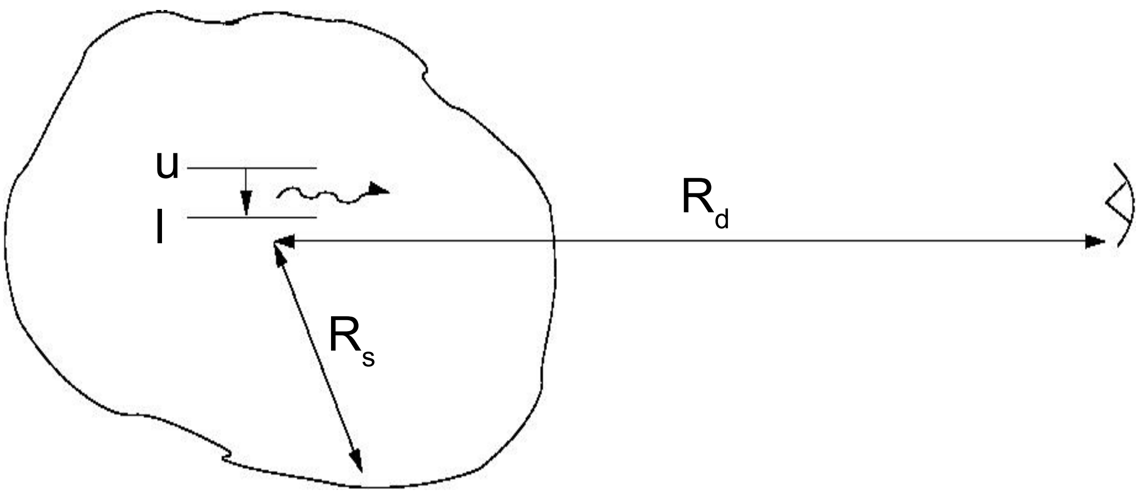
- Saha-Boltzmann equation in Q
- Q diverges for hydrogenic atoms
- For fulling ionized gas in ISM

  n_d ~ n_p the proton density ~ n_e

  T_k = T_e the electron temperature

Spectral line formation

Optically thin case; no background radiation



What we want?

n_d (cm^{-3}); N~n_d 2R_s (cm^{-2})
T (K), R_s (pc)

Prediction for obs

$$S_L(\nu) \;=\; \frac{P_\nu}{4\pi R_d^2} \propto N_u\; A_{ul}\; 2R_s\; \left(\frac{R_s}{R_d}\right)^2 \phi(\nu)\; h\nu_0 \quad \mathrm{ergs/s/cm^2/Hz}$$ Flux density of the line

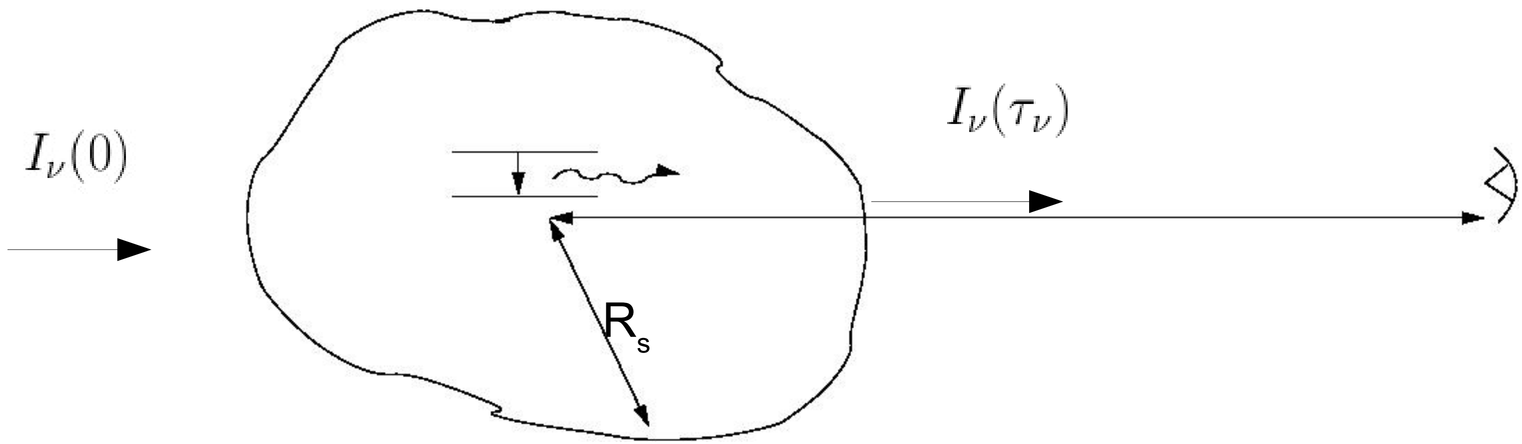
$$\propto\; N_u 2R_s\; A_{ul}\; \phi(\nu)\; h\nu_0\; \Omega_s \quad \mathrm{ergs/s/cm^2/Hz}$$

N_u in terms of the total density

Normalized Gaussian

Spectral line formation

Not optically thick; background radiation



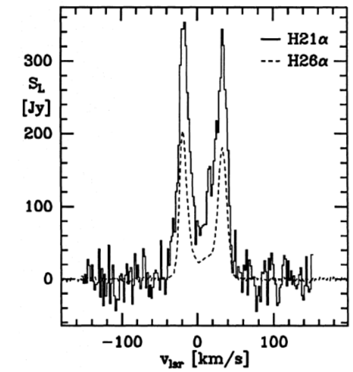


Fig. 1. H21α and H26α spectra toward MWC349, plotted on common velocity and flux scales. The spectra are smoothed to spectral resolutions of 1.0 (H26α) and 1.8 km s^{-1}(H21α). Linear baselines were removed from both spectra.

$$I_\nu(\tau_\nu) = I_\nu(0)\, e^{-\tau_\nu} + S_\nu(1 - e^{-\tau_\nu})$$

ergs/s/cm^2/Hz/strad

(background intensity can be approximately measured)

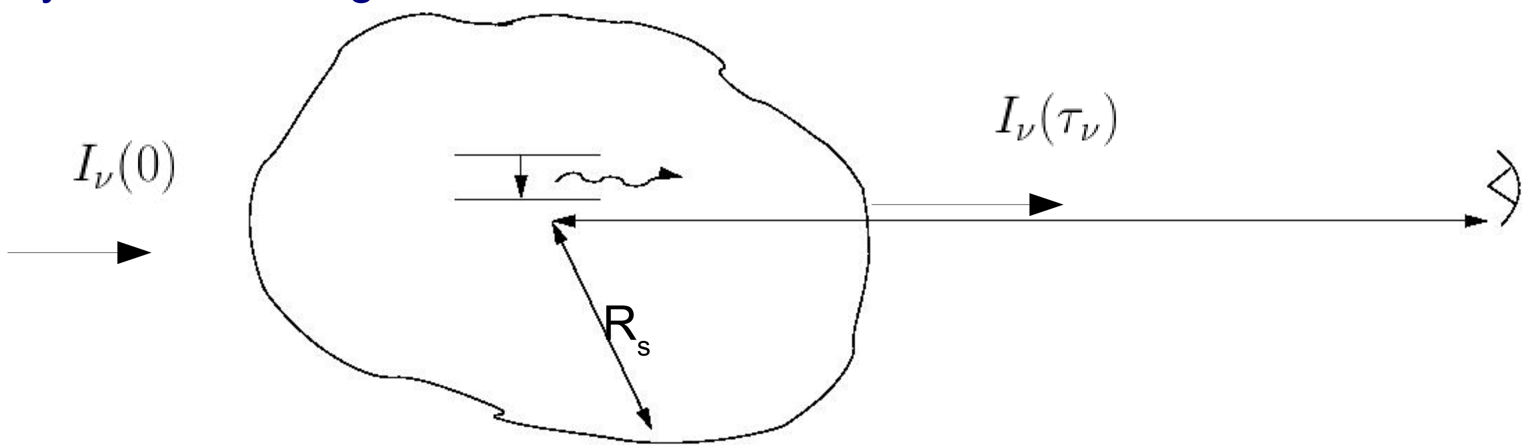
$$\tau_\nu = \int \kappa_\nu \mathrm{d}s \approx \kappa_\nu \times 2R_s \qquad \kappa_\nu \text{ absorption coefficient}$$

$$S_\nu = \frac{u_\nu}{\kappa_\nu}$$

ergs/s/cm^2/Hz/strad (Source function)

Spectral line formation

Optically thin; no background case



$$I_\nu(\tau_\nu) = I_\nu(0)\, e^{-\tau_\nu} + S_\nu(1 - e^{-\tau_\nu})$$

Line flux density $S_L(\nu) = S_\nu\, \tau_\nu\, \Omega_s$ For $I_\nu(0) = 0; \tau_\nu << 1$

$$= u_\nu\, 2R_s\, \Omega_s$$ ergs/s/Hz/cm^2

Spectral line formation

Source function

$$S_\nu \;=\; B(T) \approx \frac{2k_bT}{\lambda^2}$$

In TD equilibrium by Kirchhoff's law

LTE approximation

$$\frac{N_u}{N_l} \;=\; \frac{g_u}{g_l}\, e^{-\frac{E_{ul}}{k_bT}} \qquad T = T_{ex}$$

$$S_\nu \;\approx\; B(T_k) \approx \frac{2k_bT_k}{\lambda^2} \qquad T_{ex} = T_k \text{ same for all levels}$$

$$S_\nu \;\approx\; B(T_{ex}) \approx \frac{2k_bT_{ex}}{\lambda^2} \qquad T_{ex} \text{ same for all levels}$$

Spectral line formation

Solution to RT in temperature

$$I_\nu(\tau_\nu) \;=\; I_\nu(0)\, e^{-\tau_\nu} + S_\nu(1-e^{-\tau_\nu})$$

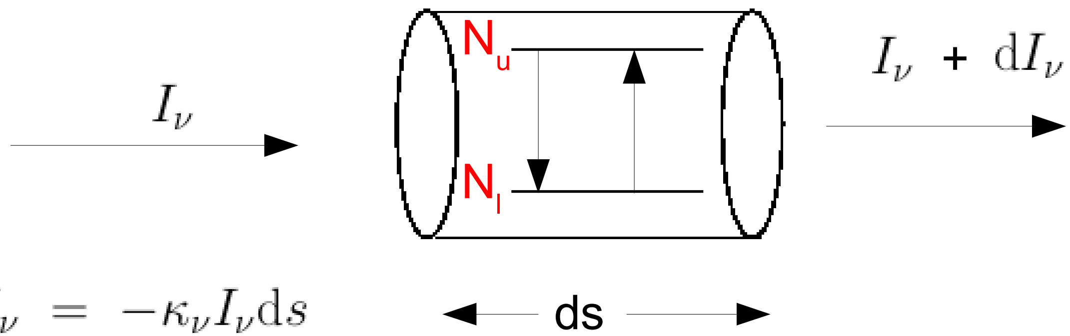
$$S_\nu \;\approx\; B(T_{ex}) \approx \frac{2k_b T_{ex}}{\lambda^2} \qquad T_{ex} \text{ same for all levels}$$

$$I_\nu(0) \;=\; \frac{2k_b T_{\nu,b}}{\lambda^2} \qquad T_{\nu,b} \text{ brightness temperature}$$

$$T_\nu(\tau_\nu) \;=\; T_{\nu,b}\, e^{-\tau_\nu} + T_{ex}(1-e^{-\tau_\nu})$$ K

Spectral line formation

Line optical depth:LTE case



$$dI_\nu = -\kappa_\nu I_\nu ds$$

Stimulated emission

Absorption

$$= h\nu_0 \, N_u \, B_{ul} \, \frac{I_\nu}{c} ds \, \phi(\nu) - h\nu_0 \, N_l \, B_{lu} \, \frac{I_\nu}{c} ds \, \phi(\nu)$$

$$\kappa_\nu = \frac{h\nu_0}{c} \left(N_l \, B_{lu} - N_u \, B_{ul} \right) \phi(\nu)$$

$$= \frac{c^2}{8\pi\nu_0^2} N_u \, A_{ul} \, (e^{\frac{h\nu_0}{k_b T_{ex}}} - 1) \, \phi(\nu)$$

Using the relationship between B and A coefficients

$$\tau_\nu = \int \kappa_\nu ds \propto N_u \, 2R_s \, A_{ul} \, \frac{1}{\nu_0 T_{ex}} \phi(\nu) \quad \text{For } \frac{h\nu_0}{k_b T_{ex}} << 1$$

Spectral line formation

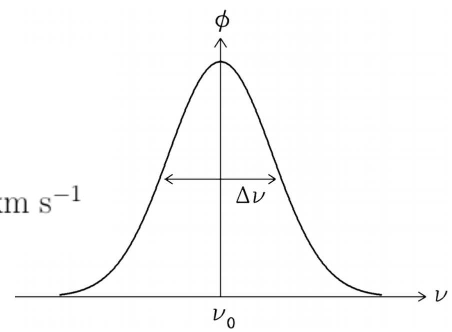
Peak line optical depth: LTE case

$$\tau_\nu \propto N_u \, 2R_s \, A_{ul} \frac{1}{\nu_0 T_{ex}} \phi(\nu)$$

$$\phi(\nu_0) = \frac{0.94}{\Delta\nu} \quad \Delta\nu \text{ is FWHM}$$

$$\frac{\Delta\nu}{\nu_0} = \frac{\Delta v}{c} \quad \Delta v \text{ is FWHM (usually) km s}^{-1}$$

Gaussian profile function



HI 21cm line

N_u = ¾ n_d; A_{ul} is const
Freq = 1420.405 MHz

$$\tau_\nu(\nu_0) = 5.1335 \times 10^{-19} \frac{N_H(\text{cm}^{-2})}{T_s(\text{K})\Delta v(\text{km s}^{-1})}$$

N_H column density of HI, $T_{ex} = T_s$ spin temperature

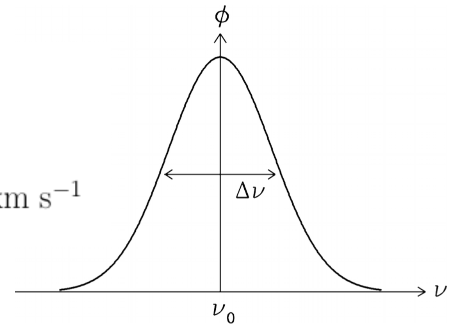
Spectral line formation

Peak line optical depth: LTE case

$$\tau_\nu \propto N_u \, 2R_s \, A_{ul} \frac{1}{\nu_0 T_{ex}} \phi(\nu)$$

$$\phi(\nu_0) = \frac{0.94}{\Delta\nu} \quad \Delta\nu \text{ is FWHM}$$

$$\frac{\Delta\nu}{\nu_0} = \frac{\Delta v}{c} \quad \Delta v \text{ is FWHM (usually) km s}^{-1}$$

Gaussian profile function 

CO J=1-0 transition

$$\tau_{\nu_{1,0}}(\nu_0) = 3.95 \times 10^{-15} \left(1 - e^{-\frac{h\nu_{1,0}}{k_b T_{ex}}}\right) \frac{N_{CO}(\text{cm}^{-2})}{T_{ex}(\text{K}) \;\; \Delta v(\text{km s}^{-1})}$$

N_{CO} column density of CO, T_{ex} excitation temperature

(did not use $h\nu/k_b T_{ex} << 1$ approximation; $T_{ex} \sim 20$ K)

Spectral line formation

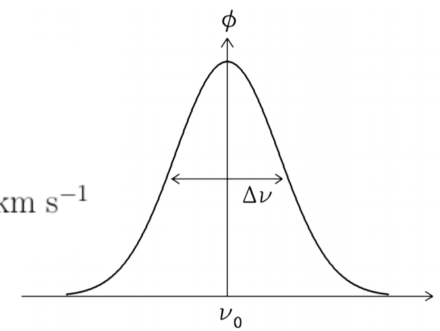
Peak line optical depth: LTE case

$$\tau_\nu \;\propto\; N_u \, 2R_s \, A_{ul} \, \frac{1}{\nu_0 T_{ex}} \, \phi(\nu)$$

$$\phi(\nu_0) = \frac{0.94}{\Delta\nu} \quad \Delta\nu \text{ is FWHM}$$

$$\frac{\Delta\nu}{\nu_0} = \frac{\Delta v}{c} \quad \Delta v \text{ is FWHM (usually) km s}^{-1}$$

Gaussian profile function



RRL alpha transition

For ionized gas $T_{ex}=T_k=T_e$

N_u α n^2 $1/T_e^{3/2}$ n_e^2

A_{ul} α $1/n^5$

RRL alpha freq α $1/n^3$

$$\tau_\nu(\nu_0) = 1.92 \times 10^3 \frac{EM(\text{pc cm}^{-6})}{T_e^{5/2}(\text{K})\Delta\nu(\text{KHz})}$$

$$EM = \int n_e^2 \text{d}s \quad \text{Emission measure}$$

Knowing source function and optical depth we can get S_L from RT solution

Spectral line formation

Excitation temperature T_{ex}

Consider two level system

$$N_u A_{ul} + N_u C_{ul} + N_u B_{ul}\frac{I_\nu}{c} = N_l C_{lu} + N_l B_{lu}\frac{I_\nu}{c}$$ Statistical equilibirum

T_{ex} definition

$$\frac{N_u}{N_l} = \frac{g_u}{g_l}\, e^{-\frac{E_{ul}}{k_b T_{ex}}} = \frac{C_{lu} + B_{lu}\frac{I_\nu}{c}}{A_{ul} + C_{ul} + B_{ul}\frac{I_\nu}{c}}$$

Radiation dominates

$$\frac{B_{lu}\frac{I_\nu}{c}}{A_{ul} + B_{ul}\frac{I_\nu}{c}}$$

Collision dominates

$$\frac{C_{lu}}{C_{ul}} = \frac{g_u}{g_l}\, e^{-\frac{E_{ul}}{k_b T_k}}$$

From detail balance

Critical density
$C_{ul} \sim A_{ul}$

$$T_{ex} \rightarrow T_b$$

$$T_{ex} \rightarrow T_k$$

Spectral line formation

Non-LTE effects

$$\frac{N_u}{N_l} = \frac{g_u}{g_l}\, e^{-\frac{E_{ul}}{k_b T_{ex}}}$$

T_{ex} different for different level

$$= \frac{b_u}{b_l}\frac{g_u}{g_l}\, e^{-\frac{E_{ul}}{k_b T_k}}$$

LTE level population with $T=T_k$

b_u, b_l are departure coefficient

Solve for b_u, b_l using statistical equilibrium equation.

Maser emission

$$N_u > N_l$$

Upper level population greater than lower level

$$\frac{N_u}{N_l} = \frac{g_u}{g_l}\, e^{\frac{E_{ul}}{k_b T_{ex}}}$$

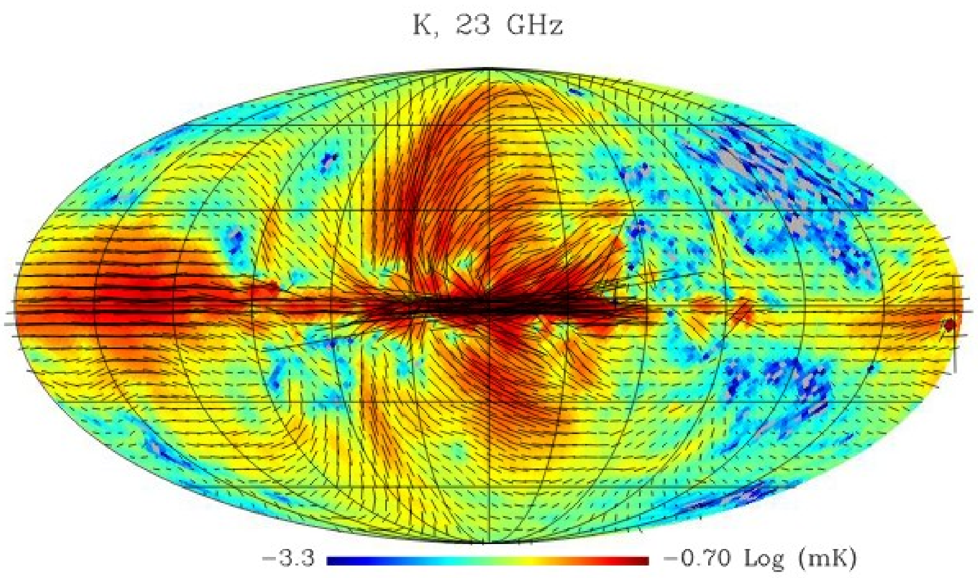
T_{ex} is negative !!

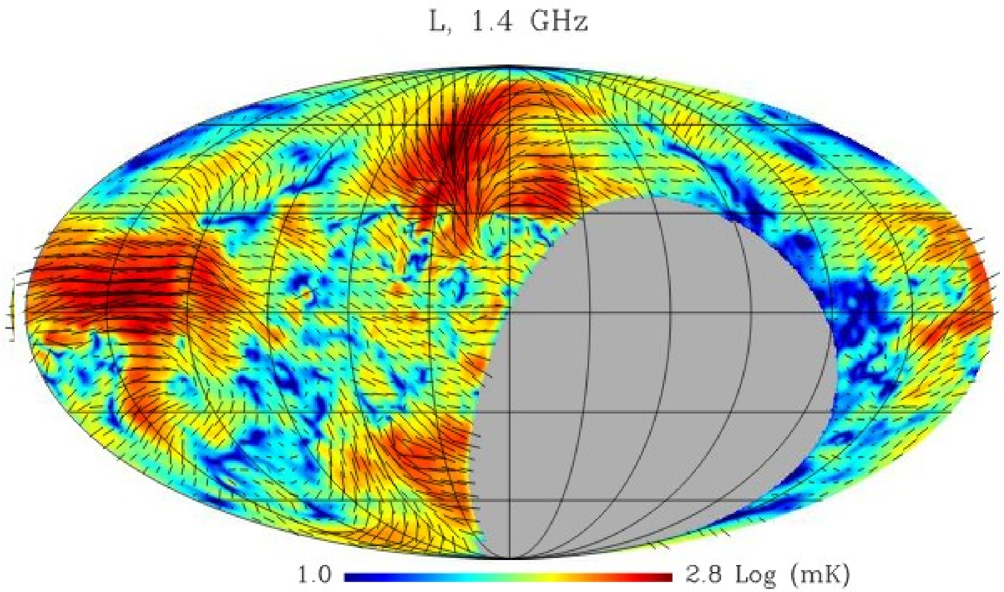
$$I_\nu(\tau_\nu) \approx I_\nu(0)\, e^{\tau_\nu}$$

Optical depth is negative → exponential amp.

Polarization

Polarization of galactic background emission



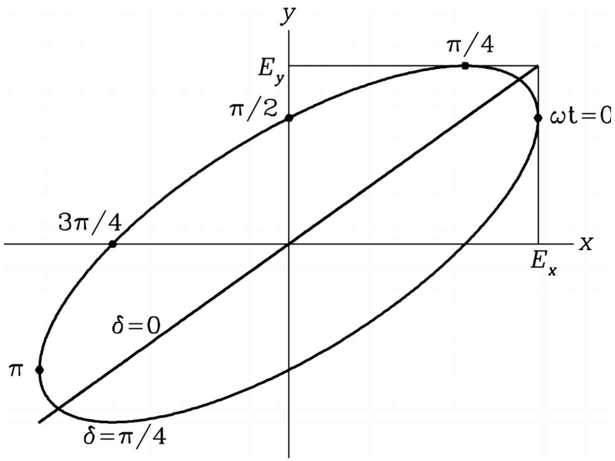


WMAP image of polarized galactic synchrotron emission at 3° scale

DRAO image of galactic polarized synchrotron Emsission smoothed to 3° scale. The difference in emission structure is dominated by Faraday depolarization.

Polarization

Measurement

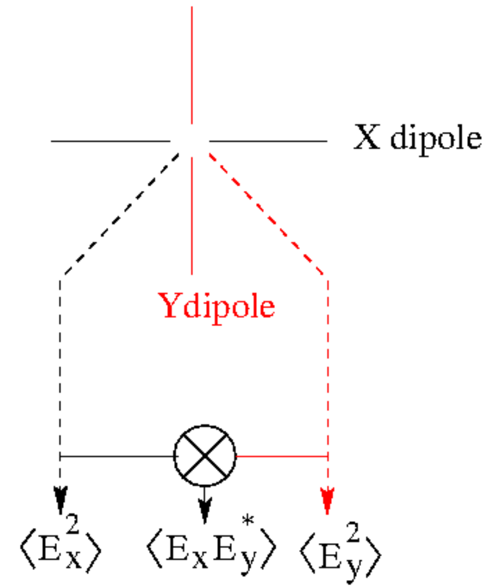


General elliptical pol from two (orthogonal) linear pol

$$\vec{E} = \left(E_x\, e^{j\phi_x}\, \hat{x} + E_y\, e^{j\phi_y}\, \hat{y}\right) e^{j\omega t}$$

$$E_x, E_y, \delta = \phi_x - \phi_y$$

p, fractional pol



Stokes parameters (incoherent source)

$$I = < E_x^2 > + < E_y^2 >$$

$$Q = < E_x^2 > - < E_y^2 >$$

$$U = 2\,\mathrm{Re}\ < E_x\, E_y^* > = 2 < E_x\, E_y\, cos(\delta) >$$

$$V = 2\,\mathrm{Im}\ < E_x\, E_y^* > = 2 < E_x\, E_y\, sin(\delta) >$$

Polarization

Measurement

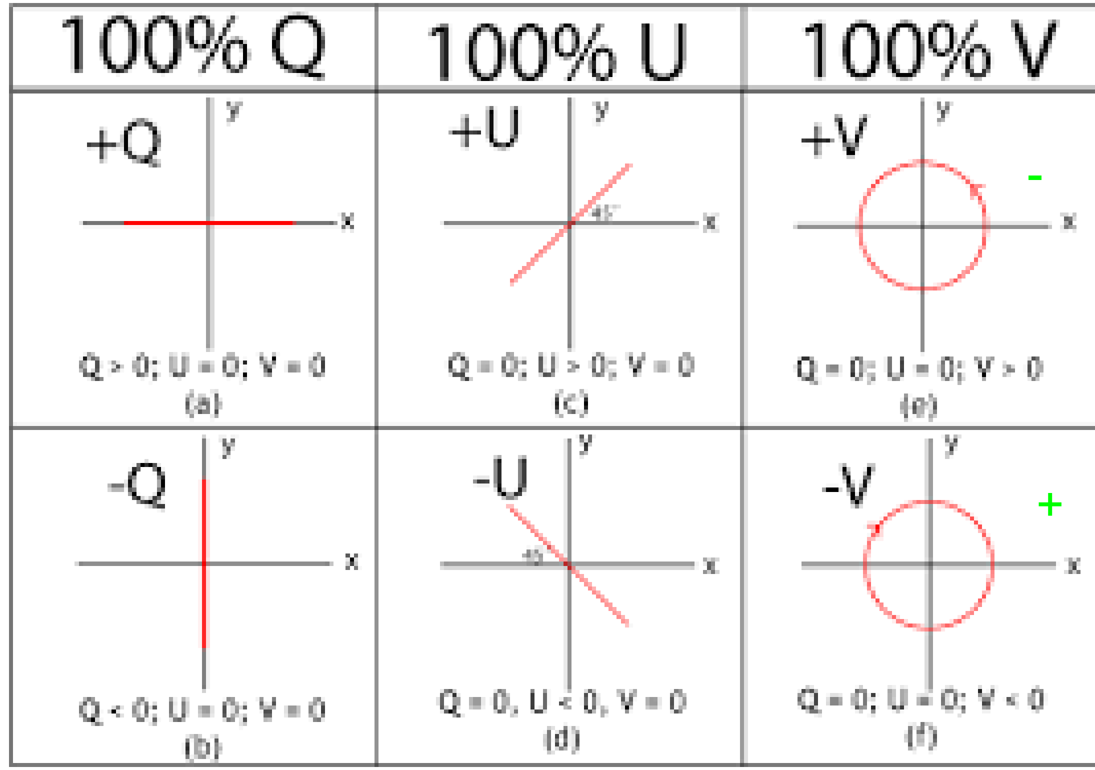
Stokes parameters (incoherent source)

$$I = <E_x^2> + <E_y^2>$$

$$Q = <E_x^2> - <E_y^2>$$

$$U = <E_{45}^2> - <E_{-45}^2>$$

$$V = <E_l^2> - <E_r^2>$$



Receiving sense

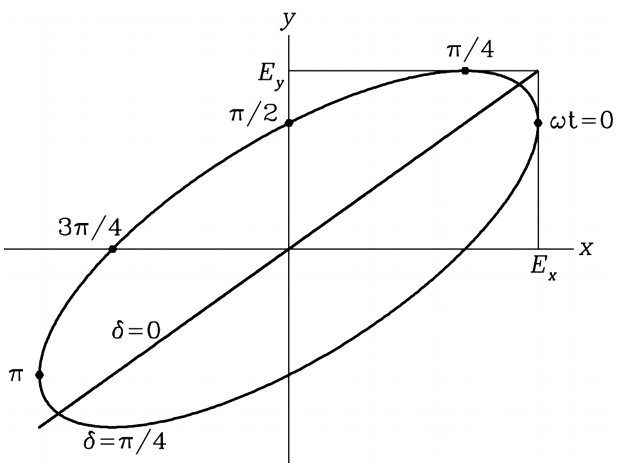
Hamaker & Bregman (1996)

$$\chi = \frac{1}{2}\tan^{-1}\left(\frac{U}{Q}\right)$$ Angle of linear polarization

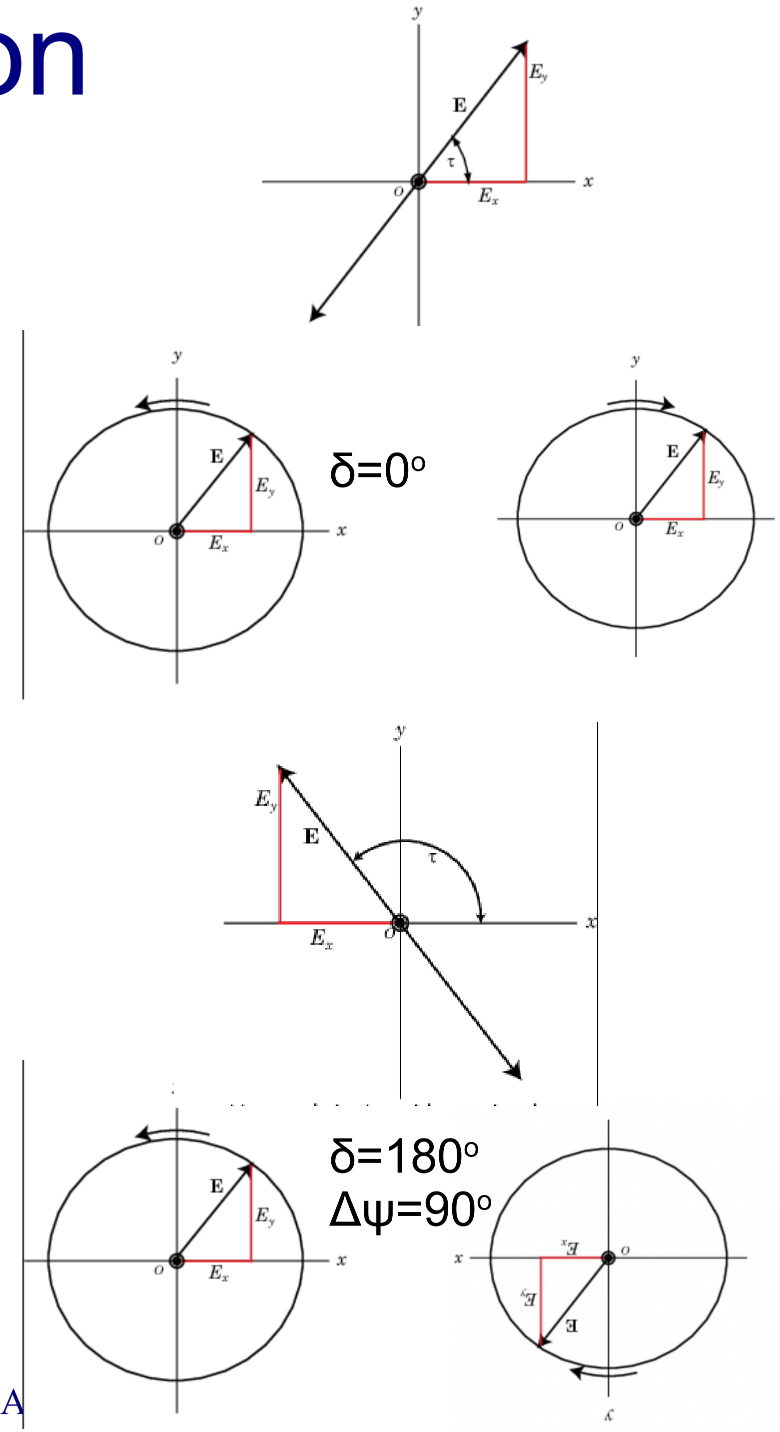
$$P = Q + iU = |P|\, e^{2i\chi}$$ Linear polarization in complex notation; |P| lin. pol flux density

Polarization

Measurement

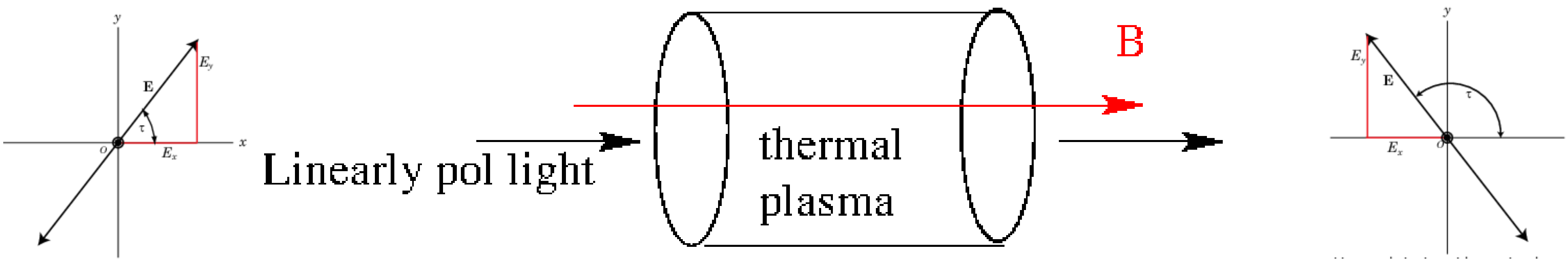


General elliptical pol from two (orthogonal) circular pol



Polarization

Faraday Rotation



Linear polarization with positive slope

- Linear pol light → sum of right and left circular pol

- Right and left circular pol velocities are different
(due to coupling with the cyclotron motion of thermal electrons)

$$2\Delta\psi = \left(\frac{2\pi}{\lambda_r} - \frac{2\pi}{\lambda_l}\right) \Delta s$$

ΔΨ is pol rotation angle

$$\Delta\psi \approx \lambda^2(\mathrm{m}^2) \times 0.8125 \int_{obs}^{source} B_{||}(\mu\mathrm{G}) n_e(\mathrm{cm}^{-3}) \mathrm{d}s(\mathrm{pc})$$

$$\text{For } \omega >> \omega_p \;\&\; \omega >> \omega_c$$

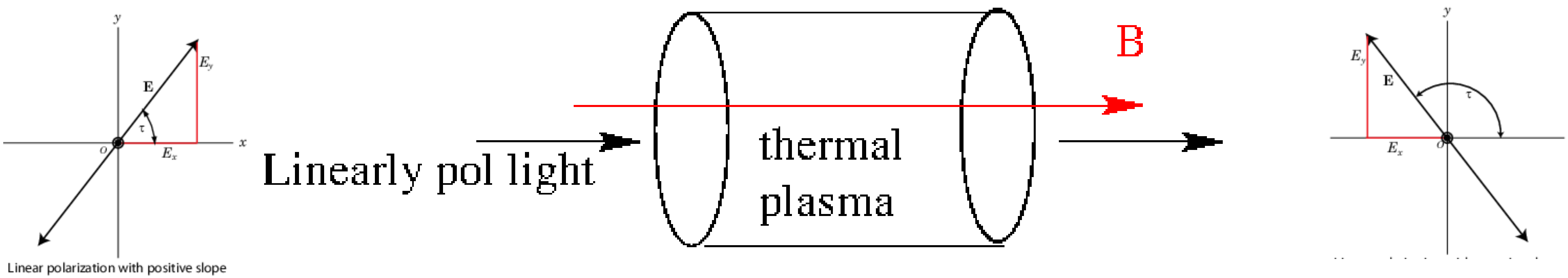
B ~ 1μG f_c ~ 2.8 Hz
n_e ~ 0.1 cm^{-3} f_p ~ 2.8 KHz

$$\approx \lambda^2(\mathrm{m}^2)\mathrm{RM}\ (\mathrm{rad\ m}^{-2})$$

RM is the rotation measure

Polarization

RM measurement



$$\chi = \frac{1}{2}\tan^{-1}\left(\frac{U}{Q}\right)$$

Angle of the measured linear pol

$$P = Q + iU = |P|\, e^{2i\chi}$$

|P| is the flux density of linear polarization

$$\Delta\psi \approx \lambda^2(\mathrm{m}^2)\mathrm{RM}\ (\mathrm{rad\ m}^{-2})$$

Diff between the incident and emerged pol angle

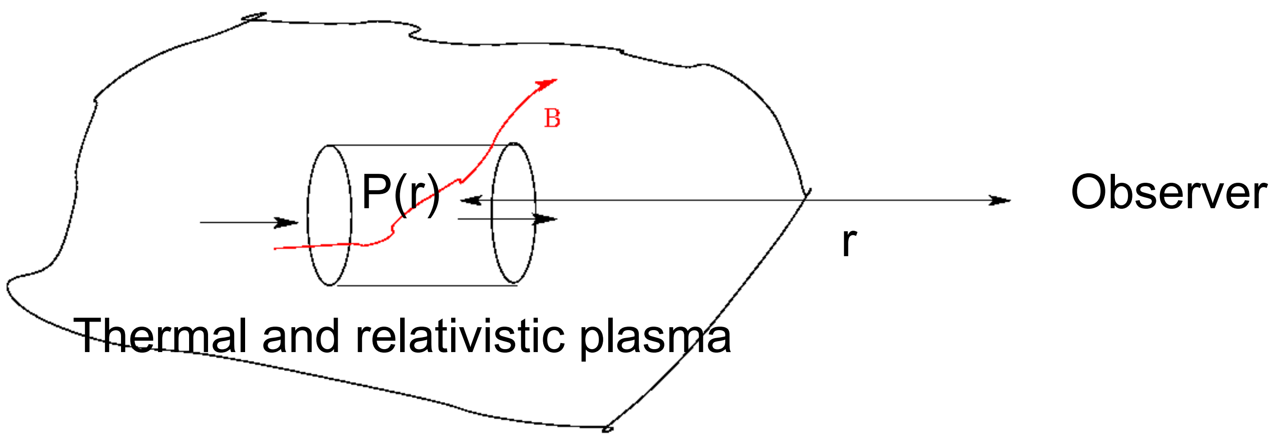
$$\mathrm{RM} = \frac{\mathrm{d}\chi}{\mathrm{d}\lambda^2}$$

Combining RM and DM one could get $B_{||}$ field (Haverkorn et al 2015)

Polarization

Faraday tomography or RM synthesis

Burn (1966)
Brentjens & de Bruyn (2005)



$$\phi(r) = 0.8125 \int_{obs}^{r} B_{||} \, n_e \mathrm{d}s$$ Faraday depth (not RM; RM is the integral over the whole source)

$$\Delta\psi(r) = \lambda^2 \phi(r)$$ Change in pol angle due to Faraday depth

$$P_{obs}(\lambda^2) = \int P(r) \, e^{2i\phi(r)\lambda^2} \mathrm{d}\phi(r)$$ Fourier transform (Φ, λ^2)

$Q_{obs} + iU_{obs}$

Q(r) + iU(r) linear pol at r average over the source

Thank You