


Radiation Fundamentals I

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AO/GBO single dish training school 2019

Outline

- Radiative transfer equation

- Thermal emission

Blackbody radiation

Free-free emission

- Non-thermal emission

Gyro radiation

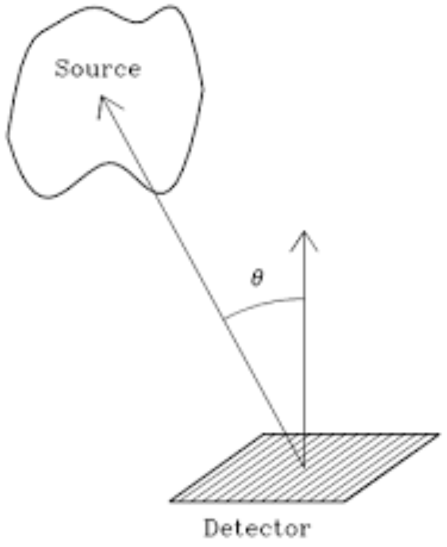
Synchrotron emission

Synchrotron self-Compton

Pulsar emission

- Spectral line emission (See Anish's talk)

Brightness and Flux density



- Brightness or Specific intensity

Total energy per unit area, per unit time, per unit frequency, per unit solid angle subtended by the source

$$I_\nu = \frac{dE}{dt\, d\sigma\, d\Omega\, d\nu\, cos\theta}$$

Units: $\mathrm{W\,m^{-2}\,Hz^{-1}\,Sr^{-1}}$

- Flux density

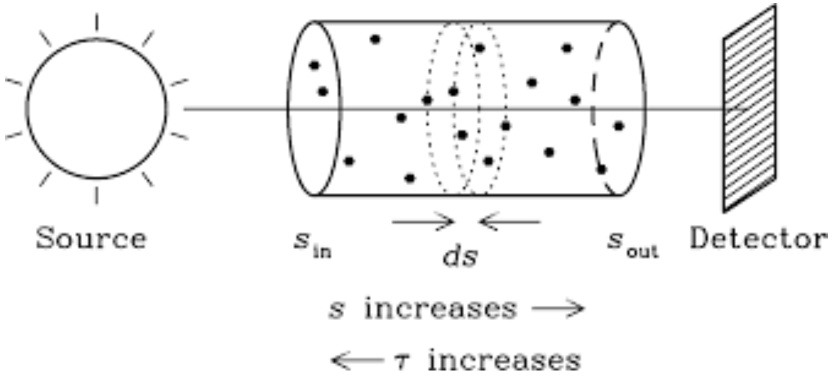
Spectral power received by the detector per unit projected area

$$S_\nu = \int_{source} I_\nu\, cos\theta\, d\Omega$$

Units: $\mathrm{W\,m^{-2}\,Hz^{-1}}$

$$1\mathrm{Jy} = 10^{-26}\,\mathrm{W\,m^{-2}\,Hz^{-1}}$$

Radiative transfer



$$\frac{dI_\nu}{ds} = -\alpha_\nu \, I_\nu + j_\nu$$

α_ν **-absorption coefficient**

j_ν **-emission coefficient**

- Solution $\quad I_\nu(\tau_\nu) = I_\nu(0)\, e^{-\tau} + S_\nu(1 - e^{-\tau_\nu})$
- Optical depth $\quad \tau_\nu(s) = \int_{s_0}^{s} \alpha_\nu(s')\, ds'$
- Source function $\quad S_\nu = \frac{j_\nu}{\alpha_\nu}$

- Optically thick $\quad \tau \gg 1 \quad I_\nu \approx S_\nu$
- Optically thin $\quad \tau \ll 1 \quad I_\nu \approx I_\nu(0) + S_\nu \, \tau_\nu$

Exercise 1

- The specific intensity of the sun at 10 GHz is

$$I_\nu = 1.78 \times 10^{-16}\,\mathrm{W\,m^{-2}\,Hz^{-1}\,Sr^{-1}}$$

Calculate flux density of the sun at this frequency.

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$$\Omega = \pi\,\theta^2 = 6.6 \times 10^{-5}\,\mathrm{Sr}$$
$$S_\nu = I_\nu\,\Omega = 1.2 \times 10^6\,\mathrm{Jy}$$

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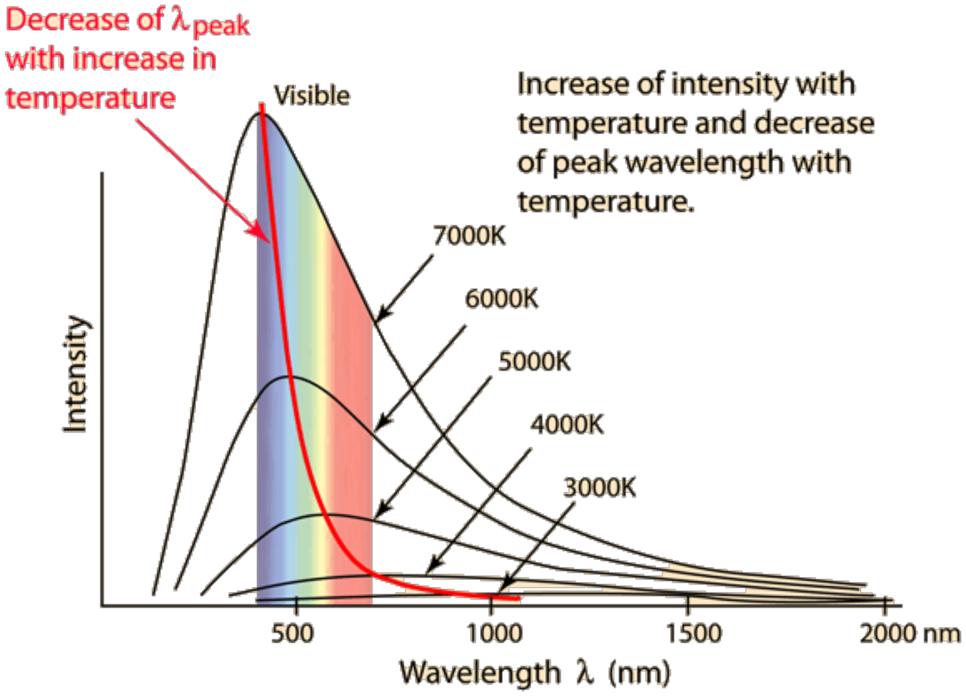
Thermal Emission

Blackbody radiation



h -Planck constant

$6.62607004 \times 10^{-34}$ m^2 kg / s

c -speed of light

k -Boltzmann constant $1.38064852 \times 10^{-23}$ m^2 kg s^{-2} K^{-1}

ν -frequency of radiation

T -temperature

Planck's law

$$B_\nu = \frac{2\,h\,\nu^3}{c^2} \times \frac{1}{e^{h\nu/kT} - 1}$$

Rayleigh-Jeans law

$$h\nu \ll kT \qquad B_\nu = \frac{2\,\nu^2\,k\,T}{c^2}$$

Wien's law

$$h\nu \gg kT \qquad B_\nu = \frac{2\,h\,\nu^3}{c^2} \times e^{-h\nu/kT}$$

Blackbody radiation

- Stefan-Boltzmann law

$$B(T) = \int_0^\infty B_\nu d\nu \qquad B(T) = \frac{\sigma}{\pi} \times T^4$$

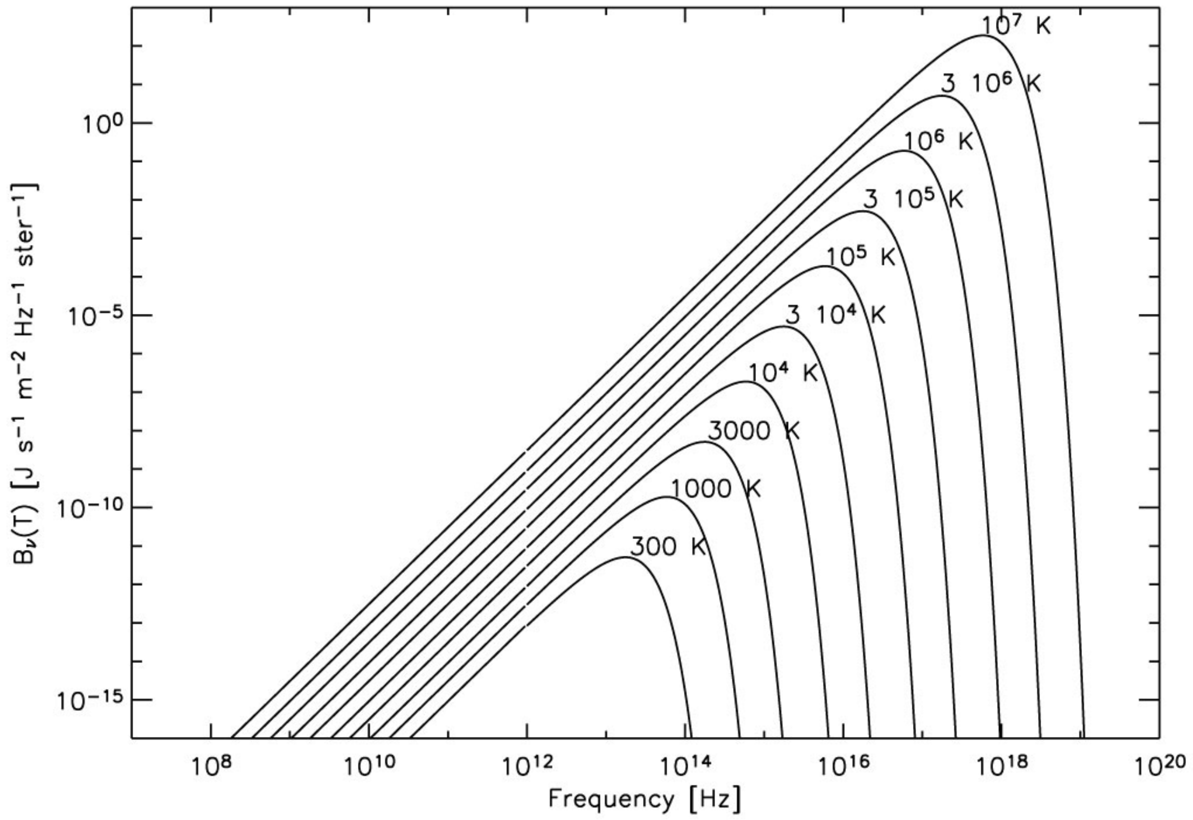
Total energy increases with T

- Wien's displacement law

$$\frac{dB_\nu}{d\nu} = 0$$

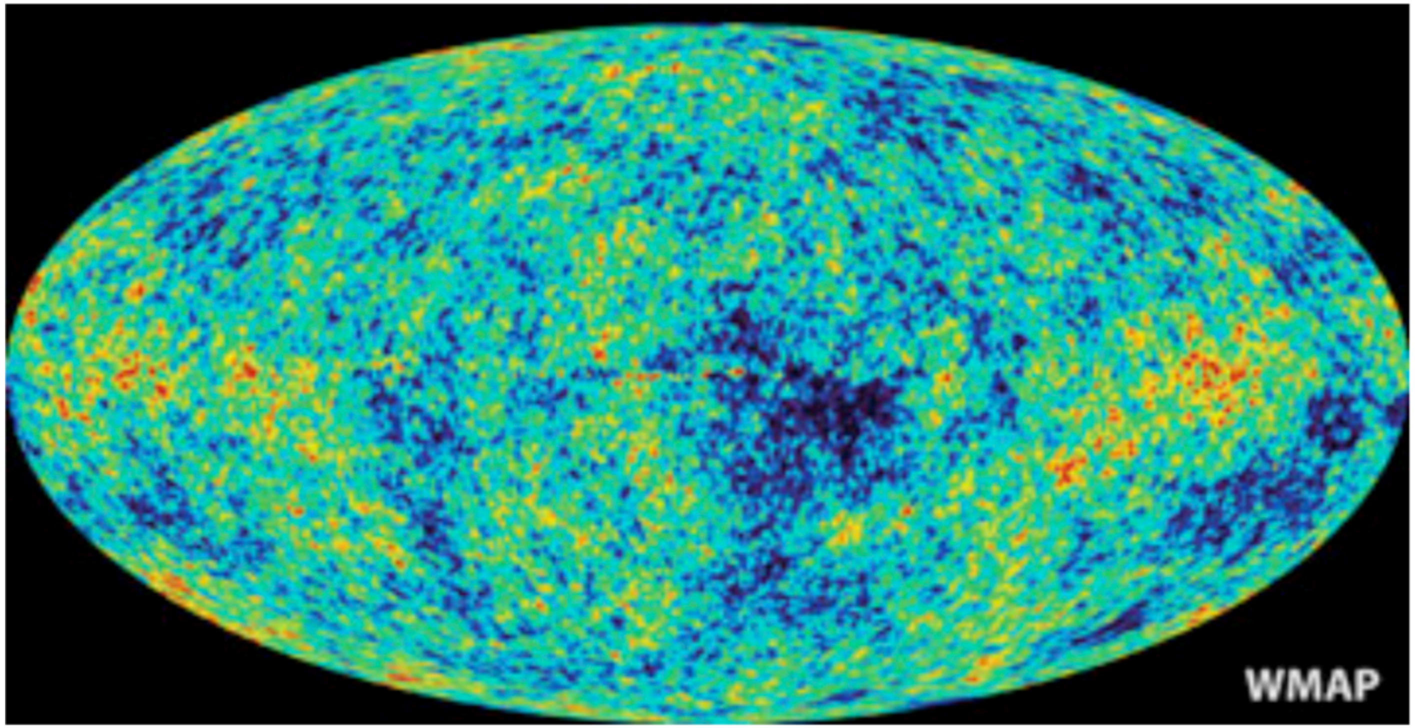
$$\nu_{max} = 59\,\mathrm{GHz} \times \left(\frac{T}{\mathrm{K}}\right)$$

Peak frequency depends on T



CMB

Black body at T=2.725 K



Exercise 2

- (RL) A supernova remnant has an angular diameter of 4.3 arcmins and a flux density of 1.6e-19 erg cm^-2 s^-1 Hz^-1 at 100 MHz. Calculate the brightness temperature.
- At what frequency will the emission be a maximum (assuming BB)?

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$$\Omega = \pi\,\theta^2$$

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$$\nu_{max} = 59\,\mathrm{GHz} \times \left(\frac{T}{\mathrm{K}}\right)$$ = 2.4x 10^18 Hz

Radiation from an accelerated charge

- Electric field

$$E_r = \frac{q}{4\,\pi\,\epsilon\,r^2}$$

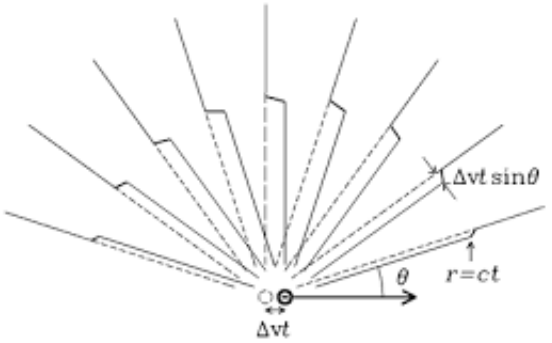
- Poynting flux (power per unit area)

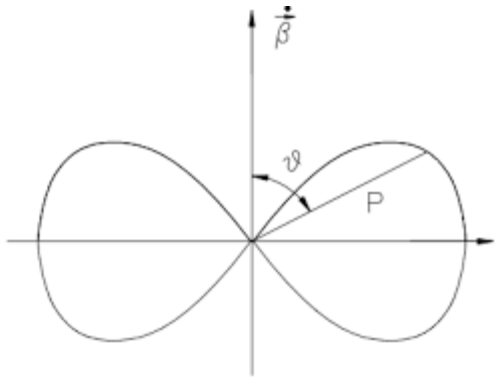
$$S = \frac{q^2\dot{v}^2 sin^2\theta}{16\pi^2\epsilon_0 c^3 r^2}$$

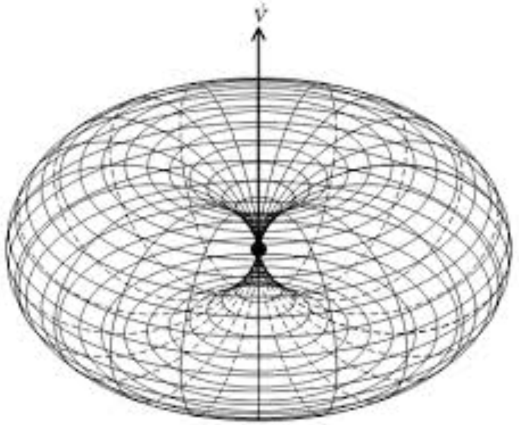
- Total power emitted

$$P = \frac{q^2\dot{v}^2}{6\pi\epsilon_0 c^3}$$

Larmor formula

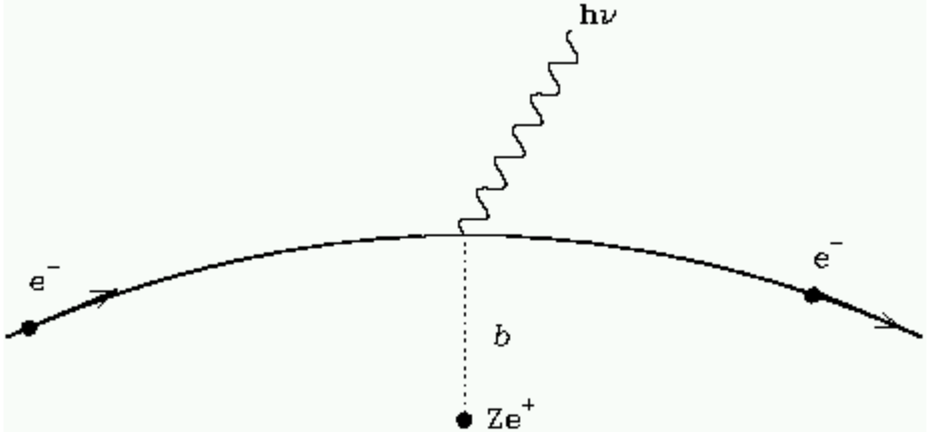






Free-free radiation

- Electron accelerated by encounter with a charged particle

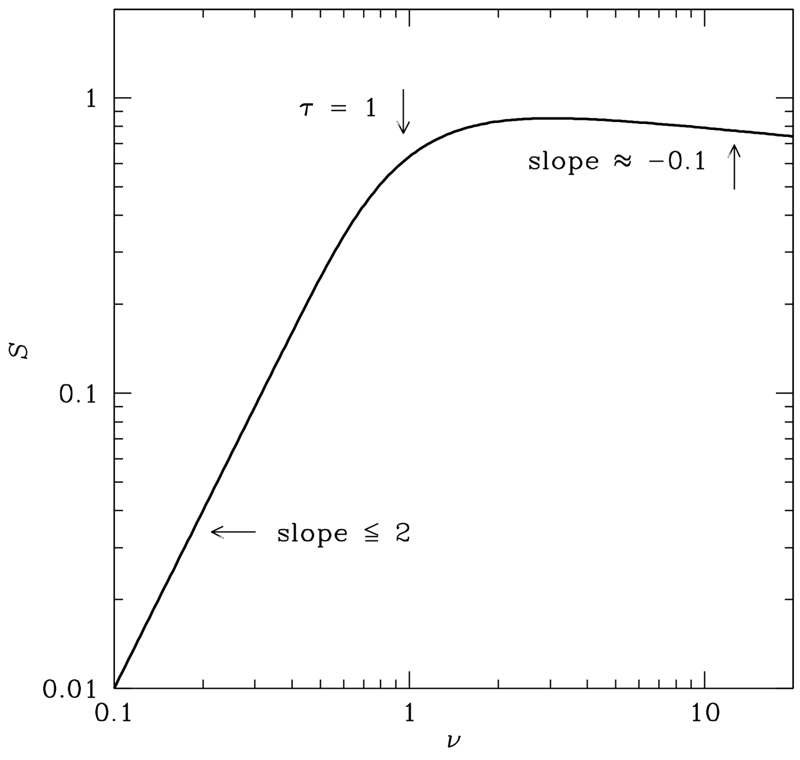


$$P = \frac{q^2 \dot{v}^2}{6\pi\epsilon_0 c^3}$$

Spectrum of an HII region

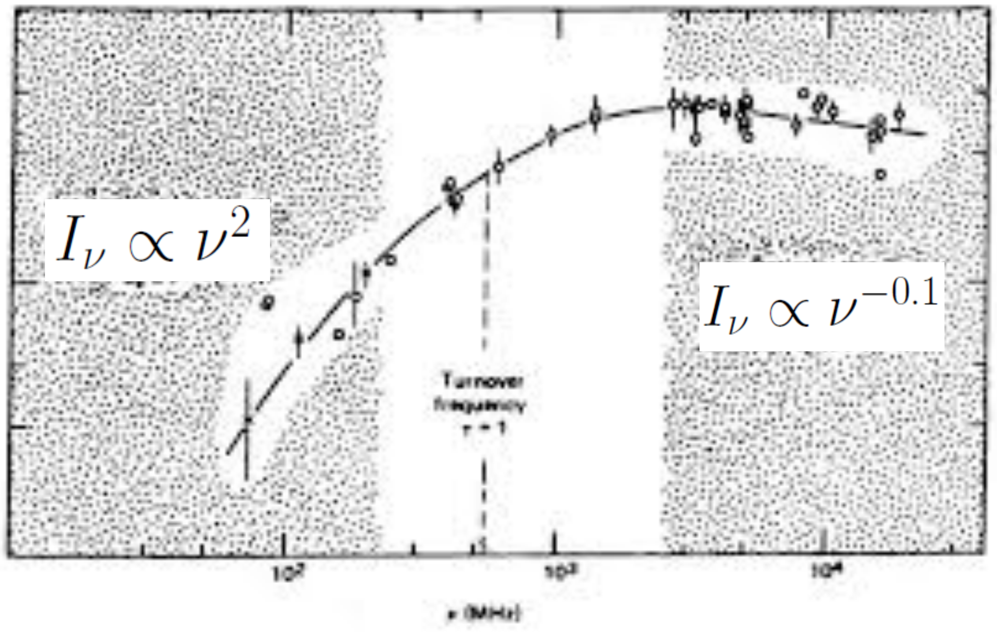
- Maxwellian distribution of electron speeds

$$f(v) \propto v^2 T^{-3/2} e^{-mv^2/2kT}$$



HII regions

Orion Nebula: well studied region with ongoing star formation.



Radio spectrum of the Orion Nebula

HST image of the Orion Nebula

Exercise 3

- (ERA) An HII region has a flux density of 0.1 Jy at 0.3 GHz, where emission is optically thick. The flux is 0.8 Jy at 10 GHz. Calculate the optical depth at 10 GHz.

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Rayleigh-Jeans extrapolated flux density

$$S_{10\,\mathrm{GHz}} = 0.1\,\mathrm{Jy} \times \left(\frac{10\,\mathrm{GHz}}{0.3\,\mathrm{GHz}}\right)^2 = 111\,\mathrm{Jy}$$

$$I_\nu \approx S_\nu \tau$$

$$\tau = 0.8/111 = 0.007$$

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Non-thermal Emission

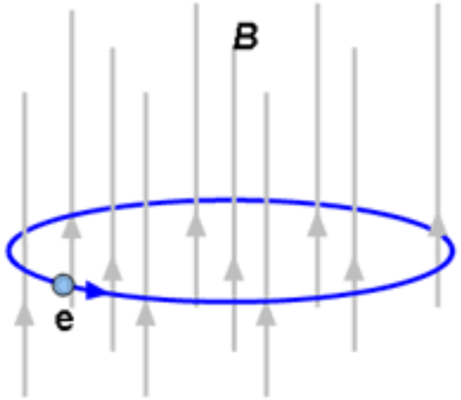
Non Thermal radiation

Electrons accelerated by magnetic field

- **Gyro radiation** (non-relativistic)
- **Cyclotron radiation** (relativistic)
- **Synchrotron radiation** (ultra-relativistic)

Gyro radiation

- The magnetic force on the particle



$$F = q\,(v \times B)$$

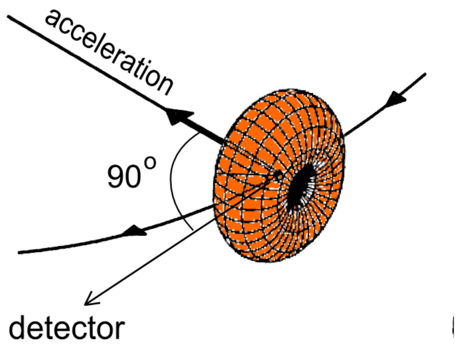
- Gyro frequency

$$\omega = q\,B/m$$

Synchrotron radiation

- Synchrotron power in the electron's rest frame

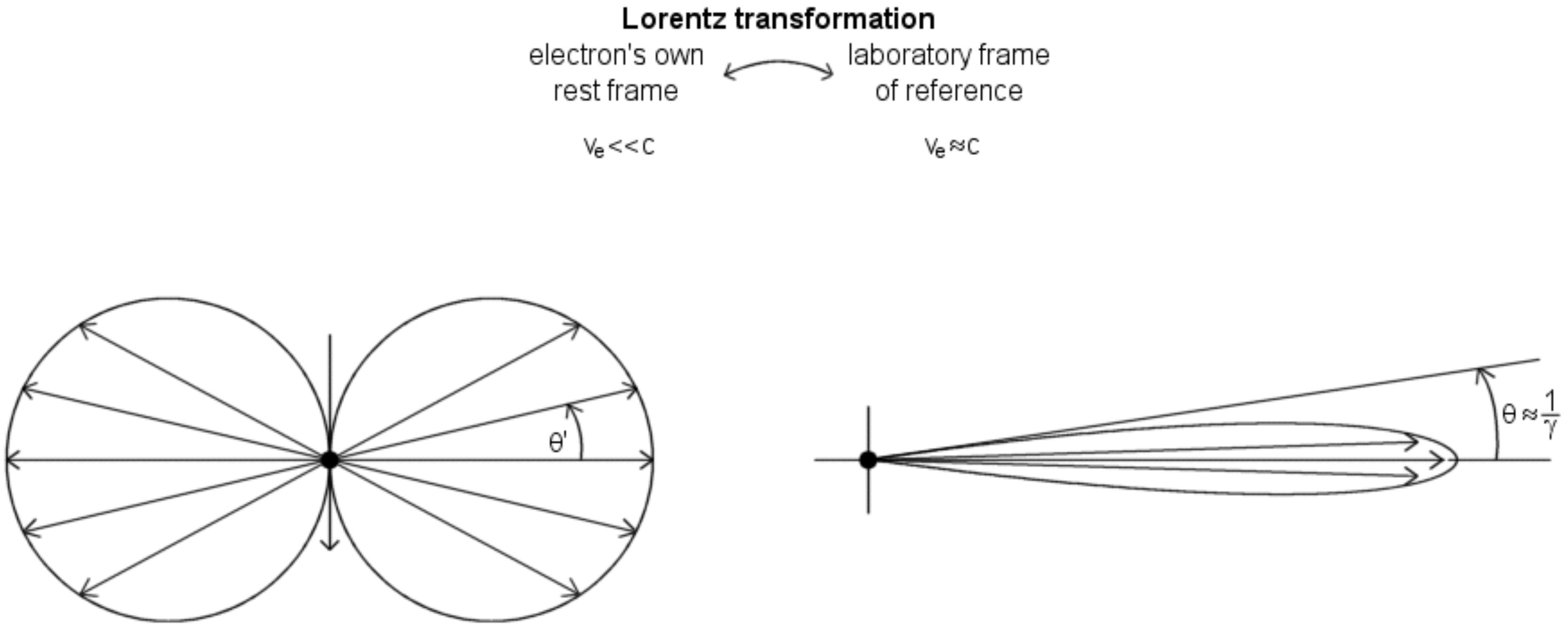
$$P' = \frac{q^2 \dot{v}'^2}{6\pi\epsilon_0 c^3}$$



- Power in the observer frame

$$P = \frac{q^2\,\gamma^4\,\dot{v}^2}{6\,\pi\,\epsilon_0\,c^3}$$

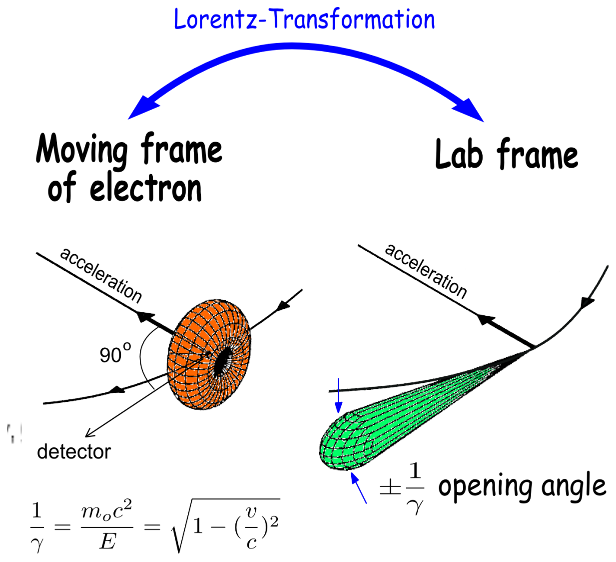
Relativistic beaming of synchrotron power



$$\tan\theta = \frac{\sin\theta'}{\gamma(\beta + \cos\theta')}$$

$$\theta \approx \frac{1}{\gamma}$$

half-opening angle in radians



Synchrotron spectrum

- The critical frequency

$$\nu_c \sim \gamma^2 \nu_G$$

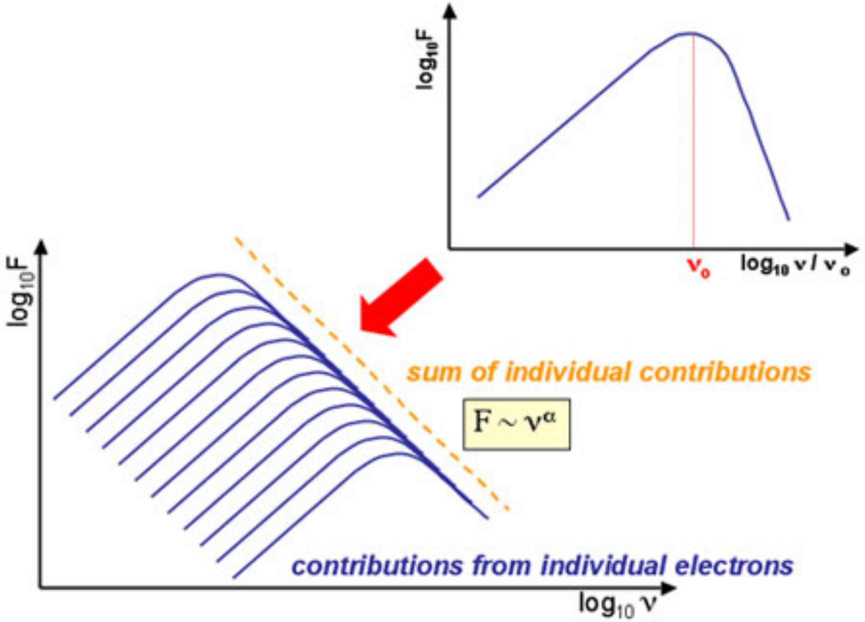
For an ensemble of synchrotron electrons:

- Power law distribution of particle energies

$$N(E) \propto E^{-p}$$

- Spectral index of radiation

$$S_\nu \propto \nu^{-(p-1)/2}$$



Exercise 4

- (ERA) Interstellar magnetic field strength in a normal spiral galaxy like ours is $B \sim 10 \mu G$. Calculate the electron gyro frequency.

- Assuming that Galactic synchrotron radiation at frequency 1 GHz is produced by electrons in this magnetic field, calculate the Lorentz factor of these electrons.

Exercise 4

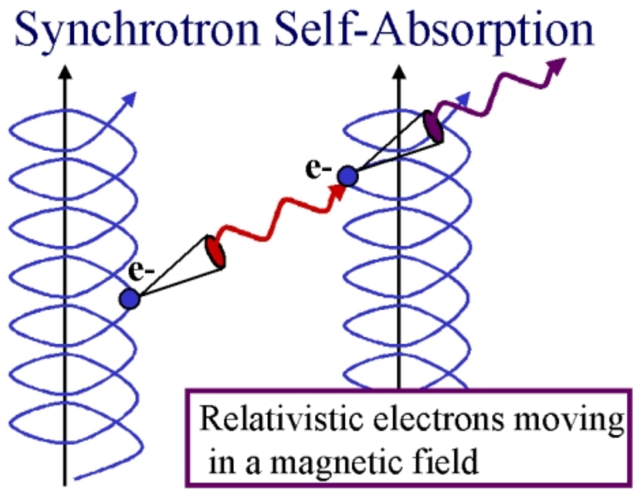
- (ERA) Interstellar magnetic field strength in a normal spiral galaxy like ours is B~10μG. Calculate the electron gyro frequency.

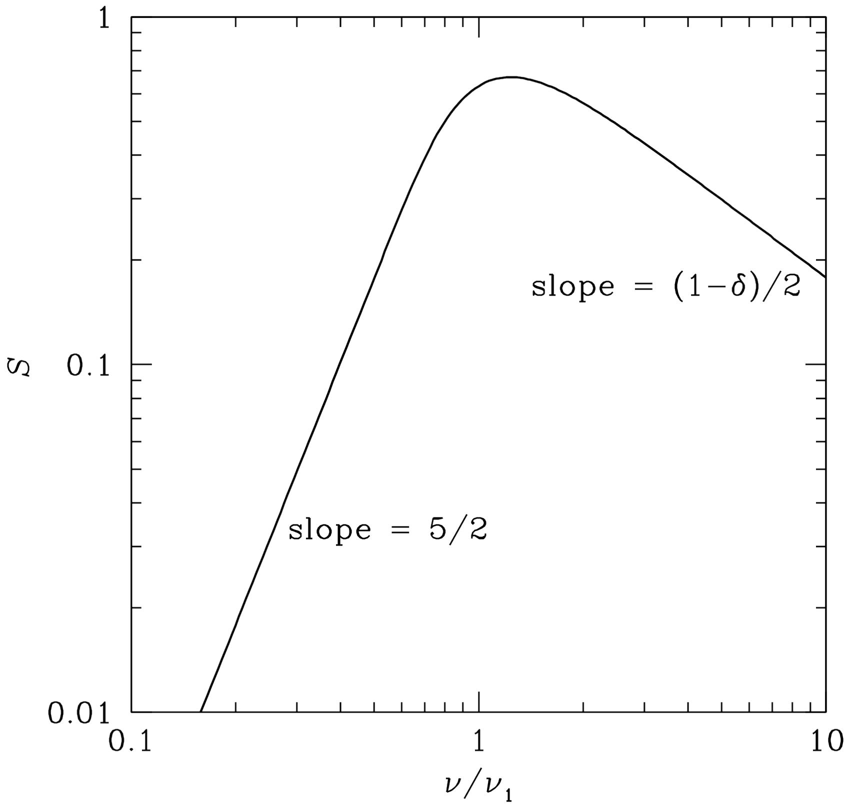
$$\nu_G = e\,B/m = 1.6 \times 10^{-19} \times 10 \times 10^{-10}\mathrm{T}/(2\pi 9.1 \times 10^{-31}) = \mathbf{28}\ \ \mathrm{Hz}$$

Assuming that Galactic synchrotron radiation at frequency 1 GHz is produced by electrons in this magnetic field, calculate the Lorentz factor of these electrons.

$$\nu_c \approx \gamma^2 \nu_G$$
$$\gamma = \sqrt{(10^9/28)} \sim 10^4$$

Synchrotron self absorption

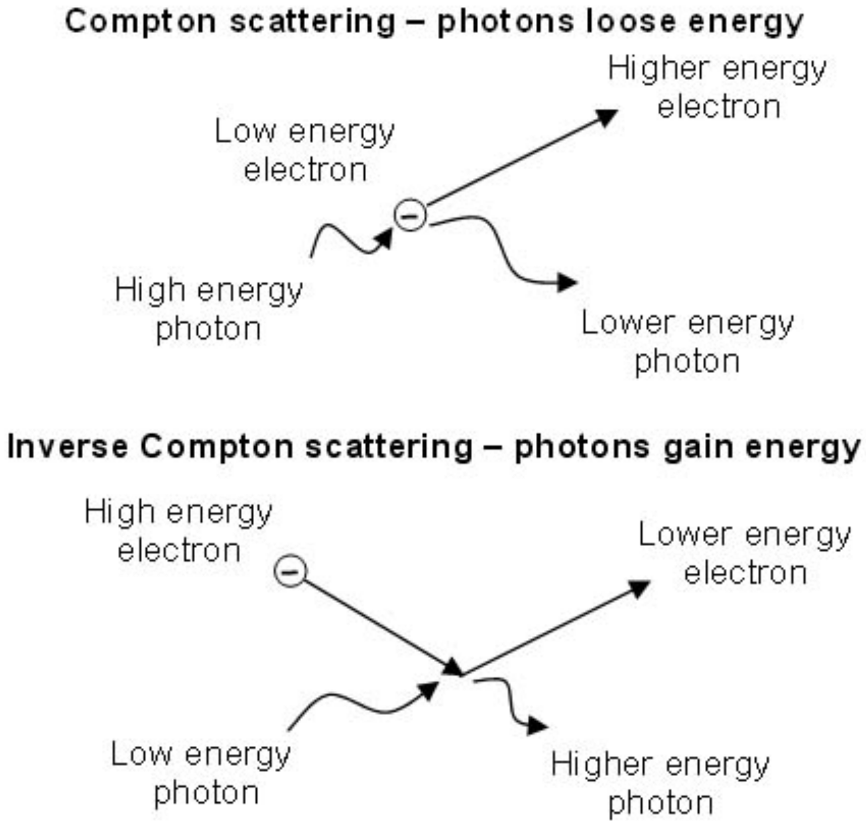




- In the Rayleigh-Jeans limit

$$I_\nu \propto \nu^{5/2}$$

Synchrotron self-compton

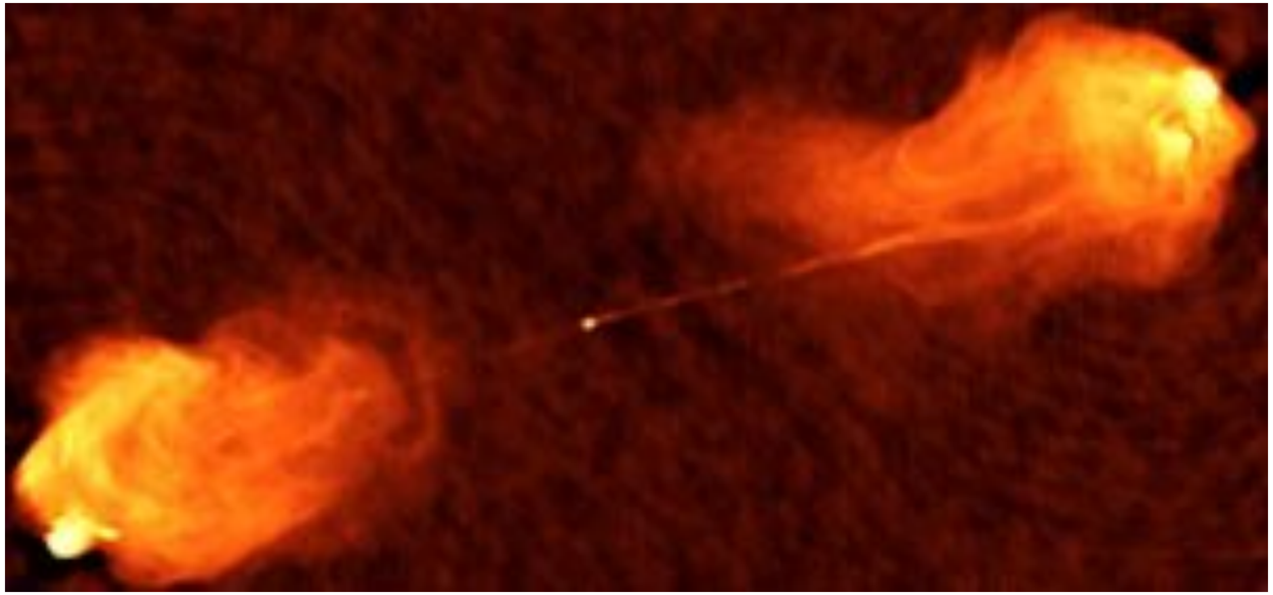


- Photon energy increased by γ^2
- 1 GHz radio photons -> X-ray for gamma = 10^4

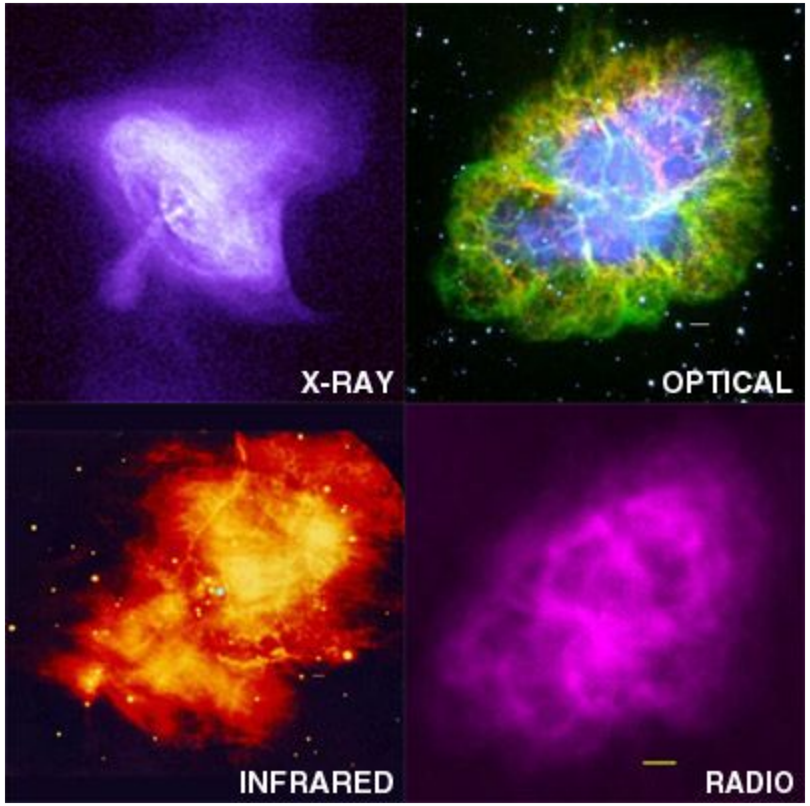
Exercise 5

- SN PTF11qcj has a flux density of 6 mJy at 1.5 GHz. Assuming optically thin emission all the way, calculate the expected flux density at 15 GHz. Assume p=3.

Synchrotron sources



VLA image of Cygnus A



The crab nebula

X-ray emission condensed near the central pulsar and in jets

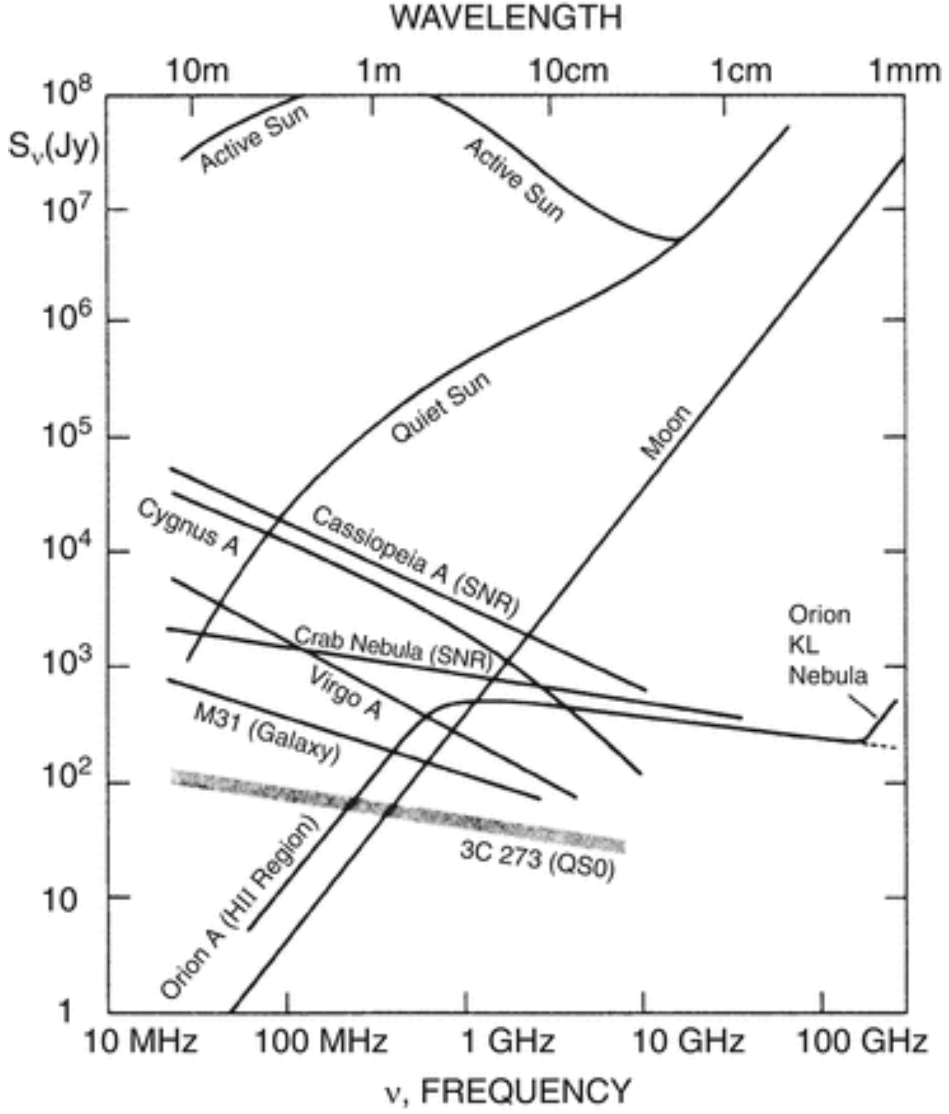
Thermal/non-thermal

Synchrotron

$$\alpha \sim -0.8$$

Blackbody

$$\alpha \sim +2$$



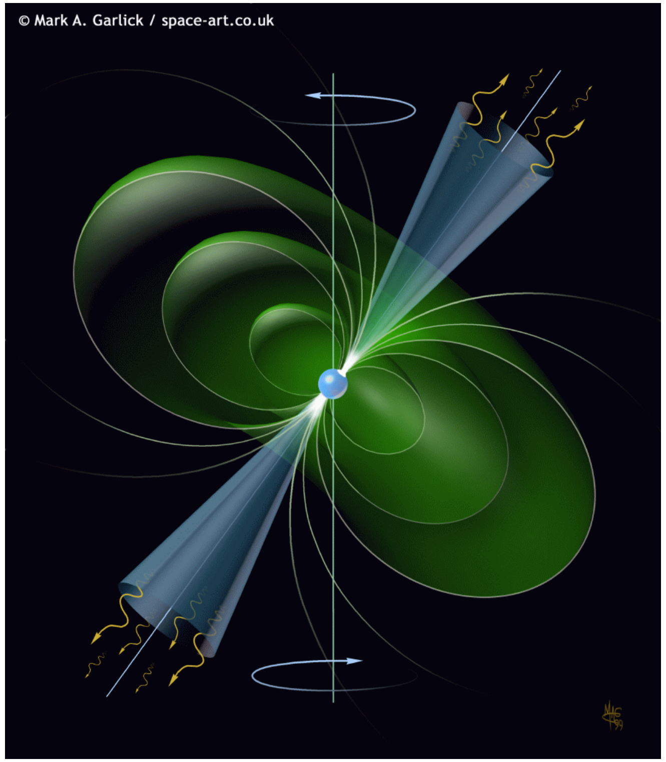
Pulsar emission

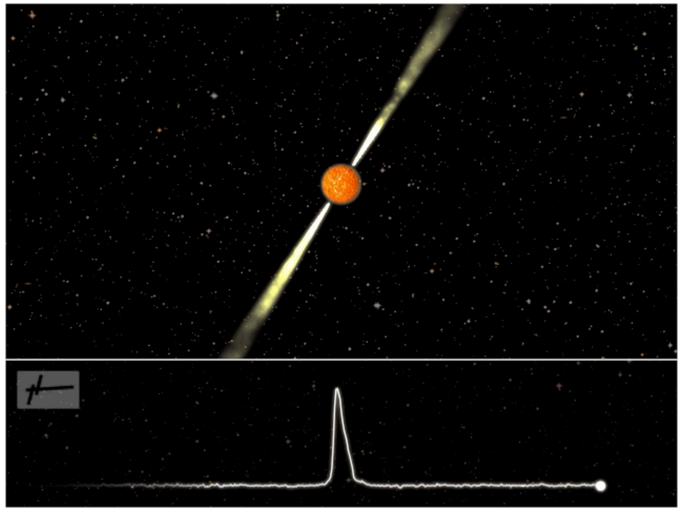
Extreme objects

- Mass ~ 1.25 – 2M_{sun}
- Radius ~ 10-12 km
- Magnetic fields ~ 10^8-10^14 G

- Brightness temperatures: 10^25 – 10^30 K

- Exact emission mechanism still unknown
- Coherent curvature radiation from charged bunches





Thank you!