Post-Pass Processing of Two-Way Timing Measurements

Larry R. D'Addario

4 February 1993; revised 8 July 1996

[Originally issued as OVLBI-ES Memo No. 38. This version is a significant revision of the original, which is now obsolete. The conclusions are qualitatively unchanged, but various details of derivations and some numerical results are corrected.]

1.0 Introduction

At each OVLBI earth station, the residual delay over the two-way path to and from the spacecraft will be measured frequently. These measurements must be processed so as to produce two different time series for later use: First, an estimate is needed of the timing error on the spacecraft induced by the imperfect compensation of the uplink signal for the uplink delay; we call this the uplink delay residual. It will be used during correlation to correct the time tags written on the signal tape, since the latter times are tied to the sampling times on the spacecraft. Second, an estimate is needed of the change in total delay that would be measured over successive intervals on a two-way link that did employ uplink compensation. That is, the effects of the uplink delay compensation and the downlink delay prediction should be removed. The result, known as the "integrated Doppler," simulates conventional Doppler tracking, making it convenient for use in orbit determination.

In this report, I first examine the nature of the available measurements (section 2); then I consider a rather simple estimator of the uplink timing residual, evaluate the errors in this estimator, and explore whether more sophisticated estimators might overcome some of the errors (section 3); and finally I consider the obvious estimator for the integrated Doppler and evaluate its inherent errors (section 4).

Background material on the overall objectives can be found in [1–2]. An earlier analysis by Linfield [3] led to results similar to those of section 3, but from a much different and more restricted viewpoint. Additional details on the transfer process and on some of the hardware designs are given by Springett in [4].

2.0 The Measurements

Consider the downlink signal being received at the earth station *now*. This signal was transponded at the spacecraft somewhat earlier, when it was tracking the uplink signal that was transmitted still earlier. The path is illustrated by the solid line in Figure 1, where the uplink path delay is A and the downlink path delay is B. Neglecting any delay within the spacecraft, the true two-way time is $A+B$. However, the earth station has knowledge only of the predicted orbit. From this it can compute a predicted signal path with predicted uplink time C and downlink time D. This path is shown as a dashed line in Figure 1. Note that it is based on the predicted spacecraft position at the predicted transponding time t_2, not the true transponding time t. But the signal being received now was actually transmitted $A+B$ earlier, not $C+D$ earlier; at the true transmission time, the predicted uplink and downlink times were C' and D', respectively, and were calculated from the predicted spacecraft position at a different predicted transponding time t_1.

The design of our OVLBI earth station involves compensation for the up- and downlink delays based on the predictions. This means that the uplink waveform is advanced in time by the amount of the predicted uplink delay.† The time of the received downlink waveform is then measured relative to a predicted waveform, resulting in the measured residual delay

$$\tau = (A - C') + (B - D) - \delta\tau_0 \tag{1}$$

where $\delta\tau_0$ accounts for any measurement ambiguities. In the present designs, the residual delay cannot be measured unambiguously because the nominal signals are periodic. In this situation, we adopt an

† In the present systems, the uplink and downlink signals are nominally sinusoids, so the time delays and advances are equivalent to phase shifts. Specifically, nominal signal $\sin 2\pi f t$ advanced by T becomes $\sin 2\pi f(t+T)$, which is a phase change of $2\pi f T$. Note that in the last expression f is the *nominal* frequency, not the instantaneous frequency $f(1+dT/dt)$. In this report, the discussion will be in terms of times rather than phases or frequencies, since it then applies equally well to the general case of non-sinusoidal signals.

Fig. 1: Schematic of the two way link geometry. Solid line is the true signal path, and dashed lines are various paths computed from the predicted orbit.

assumed value of τ (typically zero) at some initialization epoch; the error in this assumed value is then $\delta\tau_0$. This error, although unknown, remains constant for all subsequent measurements provided that they are made often enough to avoid introducing additional ambiguity ("cycle slip"). Notice that the present value of the predicted uplink delay C is not involved in (1).

3.0 Estimation of Uplink Residual

Since the timing of all signal processing on the spacecraft is tied to the received uplink signal, the error in this timing is the uplink residual

$$\tau_{\rm up} = A - C'. \tag{2}$$

Our task is to estimate this value from the available measurements of τ. Begin by re-writing (1) as

$$\tau = (A_g + A_i - C') + (B_g + B_i - D) - \delta\tau_0 \tag{3}$$

where A_g, B_g are the purely geometric (vacuum) parts of the delays and A_i, B_i are the additional delays due to the medium. Further re-arrangement gives

$$\tau = 2(A_g + A_i - C') - [(A_g - C) - (B_g - D)] - (A_i - B_i) - (C - C') - \delta\tau_0. \tag{4}$$

The first term is $2\tau_{\rm up}$ and the other terms may be regarded as errors. In that case, we can use the very simple estimate

$$\begin{aligned} \hat{\tau}_{\rm up} &= \tau/2 \\ &= \tau_{\rm up} - \epsilon_g - \epsilon_i - \epsilon_p - \delta\tau_0/2 \end{aligned} \tag{5}$$

where $\epsilon_g = \frac{1}{2}[(A_g - C') - (B_g - D)]$ is the geometrical error due to earth station motion during the round-trip time; $\epsilon_i = \frac{1}{2}(A_i - B_i)$ is the effect of non-reciprocity in the medium (such as ionospheric dispersion); and $\epsilon_p = \frac{1}{2}(C - C')$ is the effect of the error in predicted transponding time (i.e., the difference in predicted positions at the two predicted transponding times).

We now evaluate the approximate size of each of the error terms and consider whether any of them can be mitigated by using a more sophisticated estimator than (5).

3.1 Geometrical Error

The error ϵ_g arises because the position error $\vec{x}(t) - \vec{x}_p(t_2)$ (see Figure 1 for notation) is slightly different when projected onto the uplink and downlink paths, whereas the simple estimator assumes that the uplink and downlink time residuals are equal. Since earth station motion is due to earth rotation, the error is maximized when the spacecraft is on the local meridian. The transmitting and receiving locations then subtend an angle at the spacecraft of approximately

$$\phi = 2R_E\omega_E \cos l/c \tag{6}$$

where R_E, ω_E are the radius and angular velocity of the earth; l is the station latitude; and c is the speed of light. Since ϕ is normally small, the worst error occurs when the position error is nearly perpendicular to the propagation paths. Then

$$\begin{aligned} |\epsilon_g|_{\max} &\approx \frac{2}{c}|\vec{x}(t) - \vec{x}_p(t_2)| \sin\phi \\ &\approx \frac{4}{c^2} R_E\omega_E \cos l |\vec{x}(t) - \vec{x}_p(t_2)|. \end{aligned} \tag{7}$$

Taking the worst prediction error magnitude to be 1000 m and using $l = 38°$ (Green Bank), this gives

$$|\epsilon_g|_{\max} \approx 15.4\,\text{psec}.$$

It appears to be impossible to correct for this error using only the measured residuals. A correction may be possible in later processing if a substantially better orbit is then available (see 3.3 below for a discussion of similar calculations). However, since this is one of the smallest known errors, I do not consider it further here.

3.2 Medium Non-Reciprocity Error

Any difference between the true uplink and downlink delays beyond that due to the geometrical length difference is attributed to non-reciprocity in the medium. This can be due to traversing a different portion of the medium in the two directions, which in turn can be due to either local changes in the medium (turbulence) or relative motion of the medium and the earth-space vector during the round trip time. It can also be due to dispersion, since the signals are on different frequencies in the two directions.† The medium consists essentially of the troposphere and the ionosphere. Both are usually much closer to the earth station than to the spacecraft for the orbits of interest (except that VSOP's perigee of 1000 km approaches the outer edge of the ionosphere). Both are subject to turbulence, winds, and other time variability. Dispersion in the troposphere is negligible at microwave frequencies, but in the ionosphere it is significant for the frequencies being used for Radioastron and VSOP.

The longest round trip time currently of interest is 0.57 sec (Radioastron at 86,000 km range). Thus, medium variability on this and shorter time scales is of concern. The effects of turbulence are a bit too complicated to treat here. Data for the troposphere are quite sparce for the time scales of interest; the best data are from radio interferometers like the VLA, but little is available at sampling intervals less than 10 sec. Data for the ionosphere is extensive; nearly all is at low frequencies (<1 GHz), but extrapolation is possible since the frequency dependence is well known. Springett [4] has calculated the effects of both tropospheric and ionospheric turbulence on the coherence loss in a 300 sec integration at

† Dispersion is here attributed to the medium, mainly the ionosphere, but there can also be other causes of dispersion, such as multipath interference. All such effects are lumped together in this model. Future OVLBI spacecraft should be designed with the uplink and downlink frequencies essentially coincident.

22 GHz, based on reasonable extrapolations of published data, and finds that the results are acceptable for Radioastron and VSOP. I will not consider turbulence effects any further here.

Even if the medium is static in an earth-rotating frame, motion of the earth-space vector through the medium during the round trip time can produce significant non-reciprocity. When the elevation angle at the earth station is low, the path length within the medium varies rapidly with elevation, and this effect is worst if the spacecraft is at it highest speed (perigee) and will pass overhead. In that case, and assuming that $A_i = A_{i0}/\sin e$ for elevation e and zenith excess delay A_{i0}, geometrical calculations give

$$\frac{dA_i}{dt}(A+B) = -A_{i0}\frac{\cos e}{\sin^2 e}\frac{2v}{c}\sqrt{1+R_E/(R_E+h)} \tag{8}$$

where v is the spacecraft velocity and h is its altitude. At $e = 5\,\mathrm{deg}$, $v = 9.1\,\mathrm{km/sec}$, and $A_{i0} = 8.3\,\mathrm{nsec}$ (typical sea level value), this is always less than 70 psec. With local meteorological data, the actual value could be known to within 20%, allowing a correction to $\hat{\tau}_{\mathrm{up}}$ in post processing with a residual error of about 14 psec.

Next, consider dispersion in the ionosphere. The phase delay on an earth-space path at frequency f is given to good accuracy by

$$A_i = -\frac{K}{cf^2}N \tag{9}$$

where $K = 40.3\,\mathrm{m-Hz}^2(\mathrm{e/m}^2)^{-1}$, and N is the integrated electron density or "total electron content" (TEC) along the line of sight. Because of the f^{-2} dependence, the effect is greater for Radioastron's timing links than for VSOP's; using $f = 7.215\,\mathrm{GHz}$ for the uplink and 8.472 GHz for the downlink, we find

$$\epsilon_i = \tfrac{1}{2}(A_i - B_i) = (7.09\times 10^{-28}\,\mathrm{sec\text{-}m}^2)\,N. \tag{10}$$

The zenith TEC at any fixed earth station is highly variable, with a typical diurnal change of a factor of six at mid latitudes and additional variations with season and the sunspot cycle. The TEC at the horizon is typically three times the zenith value. Furthermore, short term variations (time scale of minutes to hours) occur because of traveling ionospheric disturbances (TIDs). The overall long-term variability is thus several orders of magnitude, and is larger near the geomagnetic equator and poles. At mid latitudes, the maximum value (horizon) is about $10^{18}\,\mathrm{e/m}^2$ [6-7], in which case Radioastron's ϵ_i is 709 psec. This is fairly large, and it can vary rapidly as the earth-space vector crosses the day-night terminator in the ionosphere. Nevertheless, the behavior is rather well understood and can be accurately modeled. Although the variation can be rapid on a moving path, the electron density distribution is stable for long periods of time (many days) when viewed in a frame fixed to the earth-sun vector. Exceptions to the latter statement occur after solar flares and in the presence of TIDs. Still, even at solar maximum, the daily variation from the monthly average for a fixed location and time of day is rarely more than 10^{17} [7] (Radioastron $\epsilon_i = 71\,\mathrm{psec}$), allowing correction to this level based on model parameters updated no better than weekly. In fact, such model parameters are broadcast by the GPS satellites in order to allow their users to make range corrections [8] and are updated about every few days.

Local observations of the ionosphere at the OVLBI earth stations would allow creation of improved models. Two-frequency observations of a GPS satellite's carrier phases (on 1.2276 and 1.57542 GHz) allow determination of TEC in that satellite's direction to about 4×10^{16} or $\epsilon_i = 28\,\mathrm{psec}$ absolute [9], but since the errors are predominantly systematic calibration effects the measurement of variations over several hours is at least an order of magnitude better. By observing many satellites at once, a spatial model can be derived that allows estimation of the TEC in any direction of interest; errors in the spatial model are not well known, but seem to be a few times 10^{16}. GPS receivers capable of making the necessary measurements are commercially available [10].

From the above results, it seems clear that the post processing should include the calculation of a correction for the ionospheric dispersion based on a model that yields the TEC along the earth station to satellite path as a function of time. At least the GPS broadcast model [8] can be used, but it should be possible to do somewhat better, especially if local data is available. The improved estimate of the uplink delay residual then becomes

$$\hat{\tau}_{\mathrm{up}} = \tfrac{1}{2}\tau + \hat{\epsilon}_i \tag{11}$$

where $\hat{\epsilon}_i$ is the estimate of ϵ_i derived from the model.

3.3 Effect of Discrepency in Predicted Transponding Times

The error term $\epsilon_p = \frac{1}{2}(C - C')$ arises because the uplink delay compensation and downlink delay prediction were computed for different positions along the predicted orbit, corresponding to the predicted transponding times at transmission and reception, respectively (see Figure 1). The worst error occurs when the true position $\vec{x}(t)$ and the two predicted positions $\vec{x}_p(t_1), \vec{x}_p(t_2)$ are all along the line of sight from the earth station. (This situation occurs in practice when the spacecraft is near the horizon.) In that case,

$$|\epsilon_p| = \frac{1}{2c}|t_2 - t_1| \left|\frac{d\vec{x}}{dt}\right|,$$

and since $t_2 - t_1 = A - C' + B - D$, we have

$$|\epsilon_p|_{\max} = \frac{1}{2c}|A - C' + B - D|_{\max} \left|\frac{d\vec{x}}{dt}\right|_{\max}. \tag{12}$$

Again taking the worst range error to be $c(A - C') = c(B - D) = 1000\,\mathrm{m}$, and using $|d\vec{x}/dt|_{\max} = 9.1\,\mathrm{km/sec}$ (VSOP at perigee), we find

$$|\epsilon_p|_{\max} = 101\,\mathrm{psec}.$$

Although this is fairly small, it is not necessarily negligible; it is more than 2 cycles at 22 GHz. The significance of this error depends on the time scale of its variation, as well as on the phase stability needed for the astronomical observations being done. The rate of change of ϵ_p is to first order proportional to the error in predicted orbital velocity; if $|\frac{d}{dt}(\vec{x} - \vec{x}_p)| < 1\,\mathrm{m/sec}$, then $|d\epsilon_p/dt| < 0.1\,\mathrm{psec/sec}$. The error *variation* then corresponds to 1 radian at 22 GHz after about 70 sec of integration.

Estimators which avoid or mitigate this error are therefore worth studying. In an earlier report [5], I devised a scheme that I believed would in fact avoid the error. It involved using a slightly different method of calculating the downlink delay prediction, thus modifying both D and the measured residual τ. Unfortunately, that memo was incorrect; whatever downlink delay prediction is used in real time can be undone later, so the final error in estimating τ_{up} is unaffected by it. Only the estimation algorithm is ultimately important.

The only approach so far discovered for reducing this error requires estimating it from the position error $\vec{x} - \vec{x}_p$. This might be possible in post-pass processing if the improved, reconstructed orbit is then available, even though it was not available in real time. In such post-pass processing, it would be necessary to reconstruct precisely the predictions that were applied in real time and then to recompute them in accordance with the improved orbit. Specifically, the error involves C', the uplink prediction actually used, and C, the uplink prediction appropriate to the reception time. In the post-processing, we assume that certain data from the real time operation is available, including the predicted orbit actually used and the algorithm actually used to compute the uplink predictions. Then C can be accurately recomputed (or was recorded in real time) and C' could be computed if the true transponding time t were known. Using the reconstructed orbit $\vec{x}_r(t)$, we compute an estimate of the true transponding time from

$$\hat{t} = t_{\mathrm{dn}} - \frac{1}{c}|\vec{x}_r(\hat{t}) - \vec{g}(t_{\mathrm{dn}})| - \hat{B}_i \tag{13}$$

where $t_{\mathrm{dn}} = t_2 + D$ is the true reception time, $\vec{g}$ is the earth station position, and $\hat{B}_i$ is an estimate of the medium delay on the downlink (although neglecting the latter will have a very small effect). Notice that (13) must be solved recursively for $\hat{t}$. From this we can estimate the true transmission time $\hat{t} - \hat{A}$ and then the corresponding predicted uplink time $\hat{C}'$, finally yielding an estimate of the error

$$\hat{\epsilon}_p = \tfrac{1}{2}(C - \hat{C}'). \tag{14}$$

We can apply the latter as a correction to the uplink residual estimate (5). The final uplink residual estimate is

$$\hat{\tau}_{\mathrm{up}} = \tfrac{1}{2}\tau + \hat{\epsilon}_i + \hat{\epsilon}_p. \tag{15}$$

4.0 Estimation of Integrated Doppler

The two way integrated Doppler over the time interval $[t_a, t_b]$ is defined as

$$I = [A(t_b) + B(t_b) - A(t_a) - B(t_a)]f_d \tag{16}$$

where f_d is a fixed reference frequency, typically the nominal uplink frequency; that is, I is the change in two-way propagation time over the interval, measured in cycles of the reference frequency. An obvious and straightforward estimator of this is obtained by letting $T = \tau + C + D$, and taking

$$\hat{I} = T(t_b) - T(t_a). \tag{17}$$

Then, from (1),

$$\hat{I} = I + [C(t_b) - C'(t_b)] - [C(t_a) - C'(t_a)], \tag{18}$$

so that this estimator is subject to the same type of error as was discussed in section 3.3, namely that induced by the difference in predicted position at the two predicted transponding times. This error arises only because of the need to undo the uplink compensation, and does not arise in conventional Doppler tracking in which no such compensation is used. Although the error gets arbitrarily small for a sufficiently small interval $t_b - t_a$, this is not helpful because the estimate is required at successive intervals covering as much of the orbit as possible. By an analysis similar to that in section 3.3, we conclude that the peak-to-peak error over an orbit with maximum predicted position error 1000 m and maximum spacecraft velocity 9.1 km/sec is

$$|I - \hat{I}|_{\max} \approx (204\,\mathrm{psec})f_d. \tag{19}$$

This corresponds to a light distance of 61 mm that should be included in the error budget of any orbit determination process based on this data. Of course, (19) gives the worst-case error; a statistical analysis of the distribution of such errors over particular orbits is beyond the scope of this report.

Computation of the estimator (17) requires knowledge of the compensation and prediction delays C, D actually used in real time. Therefore, the post-processing software must be able to re-compute these accurately (using an algorithm implemented in precisely the same way as used for the real time system), or the values actually used must have been recorded.

References

[1] D'Addario, L. R., 1991, "Time Synchronization in Orbiting VLBI," *IEEE Trans. on Instrumentation & Meas.*, vol. 40, pp. 584–590, June 1991 (reprints available from the author).

[2] Linfield, R. and J. Springett, 1991, "The Need for Doppler Compensation in the Phase Uplink for VSOP and Radioastron," OVLBI Earth Station Memo No. 9, 10 Jan 1991.

[3] Linfield, R., "An analytical model of the space VLBI phase transfer process." JPL IOM 335.6-92-002, 28 Jan 1992.

[4] Springett, J. C., "Phase stable frequency transfer for orbiting VLBI radio telescopes." NeoComm Systems Inc. report, prepared for JPL under contract 958183, 15 Oct 1992.

[5] D'Addario, L. R., "Two-way phase transfer with uplink compensation," OVLBI Earth Station Memo No. 11, 4 April 1991 (now obsolete).

[6] Evans, J. V., and T. Hagfors, *Radar Astronomy.* McGraw-Hill, New York, 1968.

[7] Donatelli, D. E., and R. S. Allen, "Time cells for adaptive prediction of total electron content." *Radio Science*, vol. 16, pp. 261–269, 1981.

[8] Klobuchar, J. A., "Ionospheric time delay algorithm for single-frequency GPS users." *IEEE Trans. Aerospace & Elec. Syst.*, vol. AES-23, pp. 325–331, 1987.

[9] Lanyi, G. E., and T. Roth, "A comparison of mapped and measured total ionospheric electron content using global positioning system and beacon satellite observations." *Radio Science*, vol. 23, pp. 483–492, 1988.

[10] Allen Osborne Associates, sales literature on "SNR Rogue Series" of GPS receivers; 756 Lakefield Rd, Westlake Village, CA 91361; 805/495-8420.