TONE EXTRACTION USING FANNED-OUT DATA

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11 March 1996

1. THEORY

Consider a baseband signal of bandwidth B, sampled at rate $2B$, producing a sequence of N samples $\{s_0, s_1, \ldots, s_{N-1}\}$. We construct a synchronous detector or "tone extractor" for frequency ν, where $0 < \nu < B$, by calculating the complex amplitude

$$A(\nu) = \frac{1}{N}\sum_{n=0}^{N-1} e^{j2\pi n\nu/2B} s_n. \tag{1}$$

Now let the samples be "fanned out" by a factor F, producing F separate sequences $\{s_0, s_F, \ldots, s_{N-F}\}$, $\{s_1, s_{F+1}, \ldots, s_{N-F+1}\}$, ..., $\{s_{F-1}, s_{2F-1}, \ldots, s_{N-1}\}$. We can then re-arrange the terms of (1) to group them by the members of the separate sequences as

$$A(\nu) = \frac{1}{N}\sum_{f=0}^{F-1}\sum_{i=0}^{N/F-1} e^{j2\pi(Fi+f)\nu/2B} s_{Fi+f} \tag{2}$$

$$= \frac{1}{F}\sum_{f=0}^{F-1} e^{j2\pi f\nu/2B}\left[\frac{F}{N}\sum_{i=0}^{N/F-1} e^{j2\pi Fi\nu/2B} s_{Fi+f}\right]. \tag{3}$$

For simplicity we have taken N/F to be an integer, although the expressions could be extended to cover the more general case.

In (3), the expression in brackets may be recognized as being the same as the RHS of (1) with N replaced by N/F and B replaced by B/F. That is, it is the result of building a tone extractor that operates on only the fth fanout sequence. Replacing that expression with $A_f(\nu)$ then gives

$$A(\nu) = \frac{1}{N}\sum_{f=0}^{F-1} e^{j2\pi f\nu/2B} A_f(\nu). \tag{4}$$

Thus, the desired tone extractor may be built as a linear combination of the outputs of tone extractors for each fanout sequence. The coefficients of the linear combination are just complex exponentials, i.e., phase shifts. Note that this result holds for any value of the extracted frequency ν.

An attempt to construct an extractor using only one of the fanout sequences might be confused because the signal is then undersampled. Such an extractor would be sensitive not only to ν, but also to the $F-1$ other frequencies within $(0, B)$ that are "aliased" to ν. However, the combination (4) gives the desired result without aliasing because the set of F extractors, taken together, is able to process all samples.

This result may be extended to allow extraction of all the aliased frequencies separately and simultaneously. Those frequencies are given by

$$\nu_{\pm m} = (2mB/F \pm \nu) \bmod B, \qquad m = 0, \ldots, F/2-1 \tag{5}$$

including $\nu_{+0} = \nu$. By substituting $\nu_{\pm m}$ for ν in (1) and re-arranging terms in the same manner as (3), we find after some algebra that

$$A(\nu_{\pm m}) = \frac{1}{F}\sum_{f=0}^{F-1} e^{j2\pi f\nu_{\pm m}/2B} A_f(\nu). \tag{6}$$

Thus, each of the F frequencies may be extracted separately by summing the F outputs of the tone extractors for the separate fanout sequences after shifting the phase of each by an amount that depends on which tone is desired.

2. APPLICATION

In practice, fanout of high-speed signals is often used to reduce the clock speed required for processing, while requiring more circuits to operate in parallel. By using the above results, a group of relatively slow tone extractors can be made to operate like one fast extractor for the same frequency. Moreover, the same group of tone extractors, all set for the same frequency, can simultaneously detect signals at all the frequencies that would be aliased to that frequency if only one extractor were used.

The VLBA Formatter contains 8 tone extractors, but its design did not contemplate the possibility that the input signals might already be fanned out. At the U.S. OVLBI tracking stations, signals from VSOP will have been fanned out by a factor of $F = 4$ or 8 at the Formatter input (depending on mode); and signals from Radioastron will be fanned out by $F = 1$, 2, or 4. Each extractor can be connected only to one of the fanned-out signals. Nevertheless, by the method described here, it is possible to program the extractors to detect any frequency within the Nyquist bandwidth of any of the original signals, and to do so without aliasing. If desired, the same measurements can be used to compute separately the complex amplitudes at all the alias frequencies.

3. EXAMPLE

Consider the case of bandwidth $B = 16$ MHz and fanout factor $F = 4$, which applies to one mode of VSOP. Suppose that the signal is known to contain tones at 1 MHz, 2 MHz, ..., 15 MHz. By assigning one extractor to each of the 4 fanned-out signals, each at a clock speed of $2B/K = 8$ MHz, and programming each of them to $\nu = 1$ MHz, it is possible simultaneously to measure the complex amplitudes of the tones at 1 MHz, 7 MHz, 9 MHz, and 15 MHz. This is done by combining the measurements of the four extractors according to (6). Similarly, if they are all programmed to ν=2 MHz, then the tones at 2, 6, 10, and 14 MHz can all be measured.

In this case, although the effects of aliasing are completely avoided, other effects — especially quantization — cause some interaction among the tones during extraction. This was discussed in OVLBI-ES Memo No. 59.