Title: DECLINATION CORRECTIONS FOR THE 300-FT TELESCOPE

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To: George Seielstad and Harry Payne  
From: Jim Condon  
Subject: Declination corrections for the 300-foot telescope

A simple geometrical/physical model for the 300-foot telescope declination pointing corrections indicates that 1) N-S motion of the reflector focus caused by “gliding rotation” has no effect on pointing and 2) most of the pointing correction \( \Delta \delta \) can be explained by a gravitational deformation of the feed support structure (e.g., sagging feed legs). Declination pointing errors measured with a number of receivers indicate that the magnitude of this sag is about 0.04 inches per degree of zenith angle. The model suggests that the declination pointing correction is well represented by the power series \( \Delta \delta = C_1 + C_2(\delta - \delta_z) + C_3(\delta - \delta_z)^2 + C_4(\delta - \delta_z)^3 \), where \( \delta_z \approx 38.5^\circ \) is the zenith declination; symmetry implies that the quadratic coefficient \( C_3 \) should be nearly zero. Drift-scan declination measurements were made with the 7-feed 6-cm receiver and -0.29 inches/degree N-S focus correction. The observed declination corrections can be fit by \( \Delta \delta = +23 \) arcsec-29.5 arcsec/degree \( (\delta - \delta_z) \) over the declination range \(-6^\circ \leq \delta \leq +51^\circ\); the rms residual is only 9 arcsec. The series \( \Delta \delta = C_1 + C_2\delta + C_3\delta^2 + C_4\delta^3 \) currently used to express the declination correction is numerically unstable and should be changed. Finally, the N-S focus correction has such a large effect on pointing that the indicated N-S correction should be incorporated into the indicated declination recorded on the telescope tape.

I. POINTING CORRECTION MODEL

Figure 1 shows the N-S cross section of the 300-foot telescope. The \((x, y)\) coordinate origin is at its vertex, and the focal plane is at \( y = F = 1525 \) inches. In the focal plane, gravity may pull the feed house center at \( h \) downhill to the position \( x_h \). The amount of sagging should be roughly proportional to the component of gravity perpendicular to the telescope axis, or \( \sin(\delta - \delta_z) \). Replacing \( \sin(\delta - \delta_z) \) by the first term in \( \sin(\delta - \delta_z) \approx (\delta - \delta_z) - (\delta - \delta_z)^3/3! + ... \) allows the sag to be approximated by

\[
x_h = \eta(\delta - \delta_z), \tag{1}
\]

where \( \eta > 0 \). The best-fit paraboloid makes a “gliding rotation” of angle \( \theta \ll 1 \) radian about a point at \( y = (2 + q)F \) slightly above the vertex center of curvature at \( y = 2F \) (von Hoerner, S. 1980, IEEE Transactions, AP-28, p. 652), so the focus \( f \) shifts uphill to the position

\[
x_f = \theta(1 + q)F. \tag{2}
\]

The \( x \)-separation of the focus and feed-house center has been measured (Fisher, J., and Payne, H. 1982, Engineering Memo 148); it is

\[
x_f - x_h = -(\alpha + \eta)(\delta - \delta_z) + C, \tag{3}
\]
where \((\alpha + \eta) \approx +0.29\) inches/degree and \(C \approx +0.44\) inches. In the simplest theory, this separation should also go as \(\sin(\delta - \delta_z)\), but the Fisher–Payne data indicate a linear or even faster variation at large \((\delta - \delta_z)\). The position \(x_\phi\) of the central feed \(\phi\) can be displaced from the feed-house center by the N-S focus corrector:

\[
x_\phi - x_h = -\gamma(\delta - \delta_z) + C;
\]

tracking constants \(\gamma = (\alpha + \eta) = +0.29\) inches/degree and \(C = +0.44\) inches are needed to keep the feed at the focus. From Figure 1 it is clear that the beam declination \(\delta_b\) will deviate an angle \(-\theta + \beta \epsilon\) (\(\beta\) is the beam deflection factor) from the telescope vertex declination \(\delta\) measured by the inductosyn, so a pointing correction

\[
\Delta \delta \equiv \delta - \delta_b = \theta - \beta \epsilon
\]

is required. Combining the above equations and eliminating \(\epsilon = (x_f - x_\phi)/F \ll 1\) radian yields

\[
\Delta \delta = \frac{\beta(\delta - \delta_z)}{F} \left[ \alpha \left(1 - \frac{1}{\beta(1 + q)}\right) + \eta - \gamma \right] + C_1,
\]

where \(C_1\) is a constant offset to be determined experimentally. The three terms in the brackets represent corrections for gliding rotation, feed support sag, and the N-S motion of the receiver box in the feed house, respectively. This equation indicates that most of the pointing correction \(\Delta \delta\) is linear in \((\delta - \delta_z)\), although a cubic term \(\propto (\delta - \delta_z)^3\) may be required. The symmetry of the problem suggests that any possible terms proportional to even powers of \((\delta - \delta_z)\) should be quite small.

The pointing correction for gliding rotation is very nearly zero because \(\beta(1 + q) \approx 1\) (cf. von Hoerner 1980), so I will ignore it. (In fact, it may be exactly zero for any feed illumination pattern, as suggested by Figure 1. The height \(y = 2F + q\) of the pivot point puts it at the center of average curvature of the illuminated reflector, to minimize the rms phase error over the best-fit paraboloid. Thus the same height should be the pivot for the beam produced by an off-axis feed. It would be nice if some engineer could verify this conjecture.)

The value of \(\eta\) can be estimated from pointing curves measured before the N-S focus corrector was installed \((\gamma = 0)\). Linear fits to \(\Delta \delta\) curves calculated from the old pointing coefficients given in the 300-foot telescope Observer’s Manual were made with an assumed \(\beta = 0.865\). They result in sag rates \(\eta \approx 0.047, 0.038, 0.030,\) and \(0.036\) inches/degree for the old 6, 9, 11, and 21-cm receivers, respectively. Declinations were recently measured with the 7-feed 6-cm receiver and \(\gamma = +0.29\) inches/degree N-S focus correction. The best linear fit \(\Delta \delta = 23\) arcsec–29.5 arcsec/degree \((\delta - \delta_z)\) implies \((\eta - \gamma) = -0.248\) inches/degree of focus motion caused by gliding rotation \((\beta = 0.878)\) and \(\eta = 0.042\) inches per degree, consistent with the earlier results. The fit residuals \(\text{observed } \Delta \delta \text{ minus fit } \Delta \delta\) are only 9 arcsec rms for 30 sources between \(\delta = -6^\circ\) and \(\delta = +51^\circ\) (Figure 2).

II. POINTING CORRECTION EQUATION

The model described above suggests that a “natural” form for the pointing correction equation is \(\Delta \delta \approx C_1 + C_2(\delta - \delta_z) + C_3(\delta - \delta_z)^2 + C_4(\delta - \delta_z)^3\) with \(C_3\) nearly zero. The old form \(\Delta \delta \approx C_1 + C_2 \delta + C_3 \delta^2 + C_4 \delta^3\) is quite inappropriate for this model. Also, its terms are so highly correlated \(\text{not orthogonal over the declination range covered by the 300-foot telescope) that the coefficients } C_i \text{ cannot be well determined from pointing data and their values obscure, rather than reflect, the physical processes underlying the pointing errors.
As an illustration of this problem, consider the old 6-cm E-W pointing equation $\Delta \delta = -171.60 + 6.35\delta - 0.0374\delta^2 + 0.00034\delta^3$ plotted in Figure 3. It is very closely approximated (1 arcsec rms) by $\Delta \delta = +39 + 5.0(\delta - \delta_2) + 0.0(\delta - \delta_2)^2 + 0.00033(\delta - \delta_2)^3$. To express nonlinearities in $\Delta \delta$ the old form requires large, nearly cancelling quadratic and cubic terms (also shown in Fig. 3 as solid curves); the new form needs only a small cubic term (cf. dashed lines in Fig. 3).

Finally, I think that it is dangerous to hide the huge pointing shifts produced by the N-S focus corrector in the pointing corrections. If the actual amount of focus correction is not exactly what the observer expected (e.g., due to a bug in setup program, failure of the N-S focus corrector to keep up with the commanded position because it can't move fast enough or gets caught in a limit, or a difference between the commanded and indicated declination during slew-rate tracking — all of these problems have already occurred in the C238 6-cm 7-feed survey program), the positions recorded on the telescope tape will be seriously (and silently) in error. It would be much safer to use the indicated offset of the N-S focus corrector to make automatic declination corrections, and let the pointing correction $\Delta \delta$ handle only those small residual errors that are fairly time- and setup-independent.