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ANTENNA FEED EFFICIENCY AND SPILLOVER CALCULATION PROGRAM

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ANTENNA FEED EFFICIENCY AND SPILLOVER CALCULATION PROGRAM

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A. Introduction

The program that is described in this report calculates the reduction in aperture efficiency of a paraboloidal antenna due to tapered illumination and spillover. The input of the program is the paraboloid F/D ratio and the feed pattern, specified in decibels relative to the maximum gain, at angular increments (typically 10°) from 0° to 180°. The output of the program is the taper efficiency, the spillover efficiency, the total efficiency (product of taper and spillover), and the spillover temperature at zenith, assuming a ground radiation temperature of 290 °K.

The program is intended as an easily used evaluation of feeds and does not predict the total antenna efficiency. This total efficiency will be reduced by feed support blockage, reflector phase errors, and usually to a much smaller extent by feed phase errors, cross polarization, and feed VSWR.

A one dimensional feed pattern is accepted by the program and the results are calculated assuming this pattern is circularly symmetric. Since the E- and H-plane patterns of a feed are usually different, the program should be used separately for each pattern and the results should then be averaged.

B. Theory

The taper efficiency, η_T, is defined as the ratio of the effective area of a paraboloid with tapered illumination to the physical projected area of the paraboloid. Various texts express η_T in terms of a radial distribution of electric field, E(r), over the aperture,

$$\eta_T = \frac{2\left|\int_0^1 E(r)rdr\right|^2}{\int_0^1 \left|E(r)\right|^2 rdr}$$

where r is normalized to the paraboloid radius and a uniform phase distribution is assumed. The field distribution is then expressed in terms of the feed power pattern,

$G(\Theta)$, using the relations,

$$E(r) = \sqrt{G(\Theta)}\ \cos^2 \Theta/2$$

$$r\,dr = 8\left(\frac{F}{D}\right)^2 \cdot \frac{\sin \Theta/2}{\cos^3 \Theta/2} \cdot d\Theta$$

where Θ is the angle with respect to the feed pattern maximum.

The taper efficiency can then be expressed as

$$\eta_T = 32\left(\frac{F}{D}\right)^2 \frac{\left[\int_0^{\Theta_0} G(\Theta) \tan \Theta/2\ d\Theta\right]^2}{\int_0^{\Theta_0} G(\Theta) \sin \Theta\ d\Theta}$$

where $\Theta_0 = 2 \tan^{-1} D/4F$ is the angle subtended by the edge of the paraboloid.

The spillover efficiency, η_s, is defined as the ratio of energy incident upon the reflector to the total energy emitted by the feed. It is calculated by integration of the feed radiation in rings of solid angle, $2\pi \sin \Theta\ d\Theta$, to give,

$$\eta_s = \frac{\int_0^{\Theta_0} G(\Theta) \sin \Theta\ d\Theta}{\int_0^{\pi} G(\Theta) \sin \Theta\ d\Theta}$$

The program computes and prints η_T, η_s, and the total efficiency, $\eta_T \eta_s$, all expressed as percentages.

The antenna temperature due to spillover with the antenna pointed at zenith is proportional to the fraction of feed radiation striking the ground, i.e., the radiation between Θ_0 and 90°. The zenith antenna temperature, T_z, is then given by

$$T_z = 290 \cdot \frac{\int_{\Theta_0}^{\pi/2} G(\Theta) \sin \Theta\ d\Theta}{\int_0^{\pi} G(\Theta) \sin \Theta\ d\Theta}$$

where a ground brightness temperature of 290 °K is assumed. This quantity is also computed and printed by the program.

The antenna temperature due to spillover with the antenna pointed at the horizon is given by 290 $(1 - \eta_s)/2$, where a 290 °K ground brightness temperature is assumed. This quantity is not printed by the program but is easily computed from η_s.

C. Program Description

A listing of the program is given in Figure 1. The program is written in Fortran IV for an IBM 360 Model 50 computer.

The integrations in the program are performed by the subroutine "QTFE" which is in the IBM subroutine library. This subroutine calculates integrals using a trapezoid area element between data points. An earlier version of the program used the subroutine "QSF" which calculates integrals by using a parabolic area element. This subroutine produced erroneous results because the integral value was slightly dependent on whether there was an odd or even number of points defining the integrand; the spillover calculation was critical to this error.

The program interpolates between data points at the reflector edge; i.e., integrals will be calculated to a reflector edge angle of 61° even though the pattern is specified only at 60° and 70°.

D. Preparation of Data Cards

There are two data cards for each pattern. Examples of data cards are shown in Figure 2. The format is as follows:

First Card

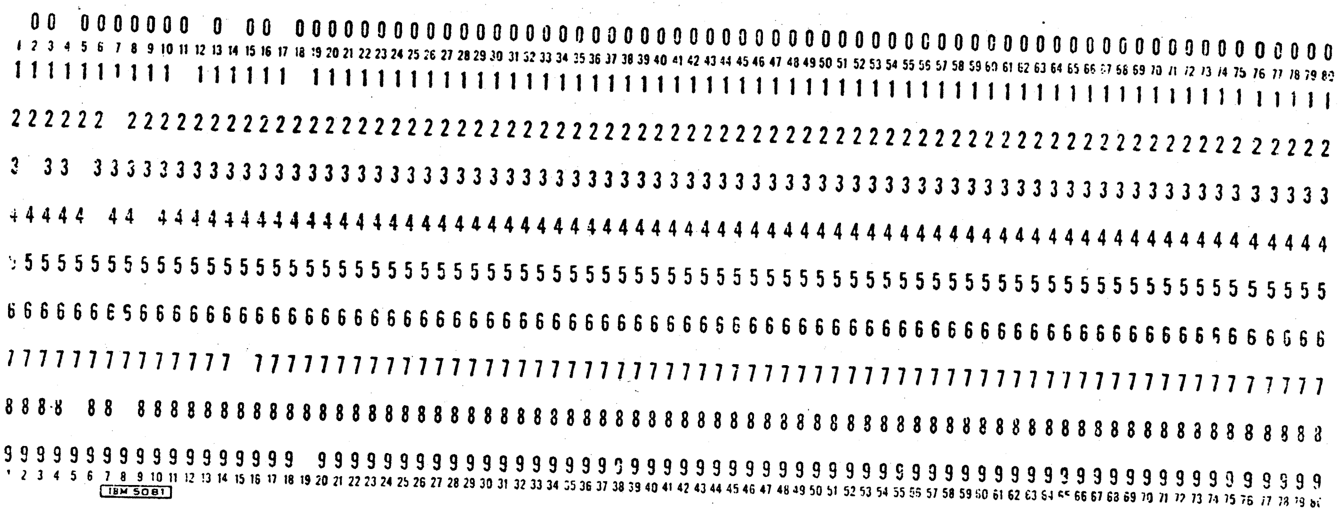
Columns	
1-2	A data identification number; this number will be printed with the result. Example: 03.
4-9	Paraboloid F/D ratio expressed with 4 decimal places. Example: 0.4284.
11-12	Incremental angle between pattern points. Example: 10.

```
FORTRAN IV G LEVEL  18              MAIN              DATE = 70164          12/06/13       PAGE 0001

0001            DIMENSION G(50),G2(50),G3(50),F2(50),F3(50)
0002            INTEGER*4A,B,C
0003            WRITE(6,3)
0004          3 FORMAT('1')
0005          6 READ(5,1,END=10)L,FD,H,A,N,(G(I),I=1,N)
0006          1 FORMAT(I2,1X,F6.4,1X,F2.0,1X,I2,1X,I3/(20F4.1))
0007            B=A+1
0008            C=A-1
0009            K=90./H
0010            K1=K+1
0011            K2=K+2
0012            HR=H*3.1415926535/180.
0013            TH=2*ATAN(1./(4.*FD))
0014            DO 20 I=1,N
0015         20 G2(I)=1./(10.**(G(I)/10.))*SIN(HR*FLOAT(I-1))
0016            CALL QTFE (HR,G2,F2,N)
0017            VAL1=F2(N)
0018            VAL2=F2(A)
0019            VAL12=F2(B)
0020            VAL30=F2(K1)
0021            VAL31=F2(K2)
0022            DO 30 I=1,B
0023         30 G3(I)=1./(10.**(G(I)/20.))*TAN(HR*FLOAT(I-1)/2.)
0024            CALL QTFE (HR,G3,F3,B)
0025            VAL3=F3(A)
0026            VAL13=F3(B)
0027            VAL22=VAL2+((VAL12-VAL2)*(TH-(HR*FLOAT(C)))/HR)
0028            VAL23=VAL3+((VAL13-VAL3)*(TH-(HR*FLOAT(C)))/HR)
0029            VAL33=VAL30+(VAL30-VAL31)*(90./H-FLOAT(K))
0030            VAL4=100.*32.*(FD*VAL23)**2/VAL22
0031            VAL5=100.*VAL22/VAL1
0032            VAL6=VAL4*VAL5/100.
0033            VAL7=290.*(VAL33-VAL22)/VAL1
0034            WRITE(6,2)L,VAL4,VAL5,VAL6,VAL7
0035          2 FORMAT(////,2X,I2,6X,'TAPER EFF',8X,F4.1//,10X,'SPILLOVER EFF',4X,
               1 F4.1//
               1 10X,'TOTAL EFF',8X,F4.1//,10X,'SPILLOVER TEMP',2X,F5.1)
0036            GO TO 6
0037         10 STOP
0038            END
```

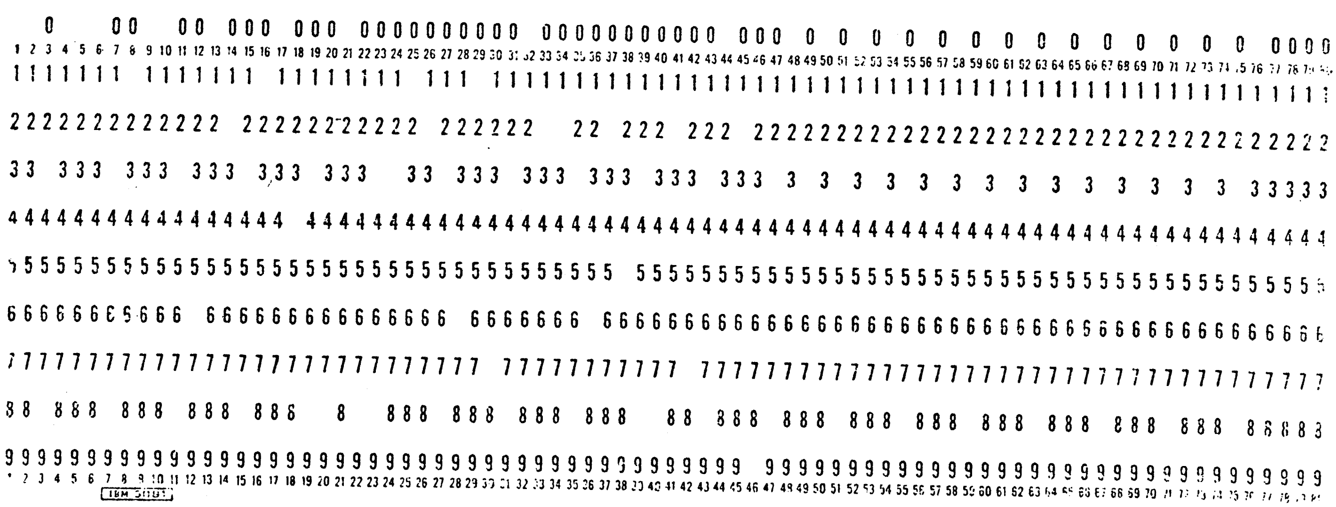
Figure 1 — Program Listing Written in Fortran IV for an IBM 360 Model 50 Computer.

03 0.4284 10 07 019



First Card

00.000.100.602.104.808.312.617.022.625.827.029.030.030.030.030.030.030.030.0



Second Card

Figure 2 — Examples of data cards. As many pairs of cards as desired may be run in one program.

First Card (continued):

Columns	
14-16	Number of pattern points falling on the reflector. Example: For F/D = 0.42 the reflector edge is 61° and hence for 10° increments this number is 7.
17-19	Number of pattern points to 180°. Example: For 10° increments this number is 19.

Second Card

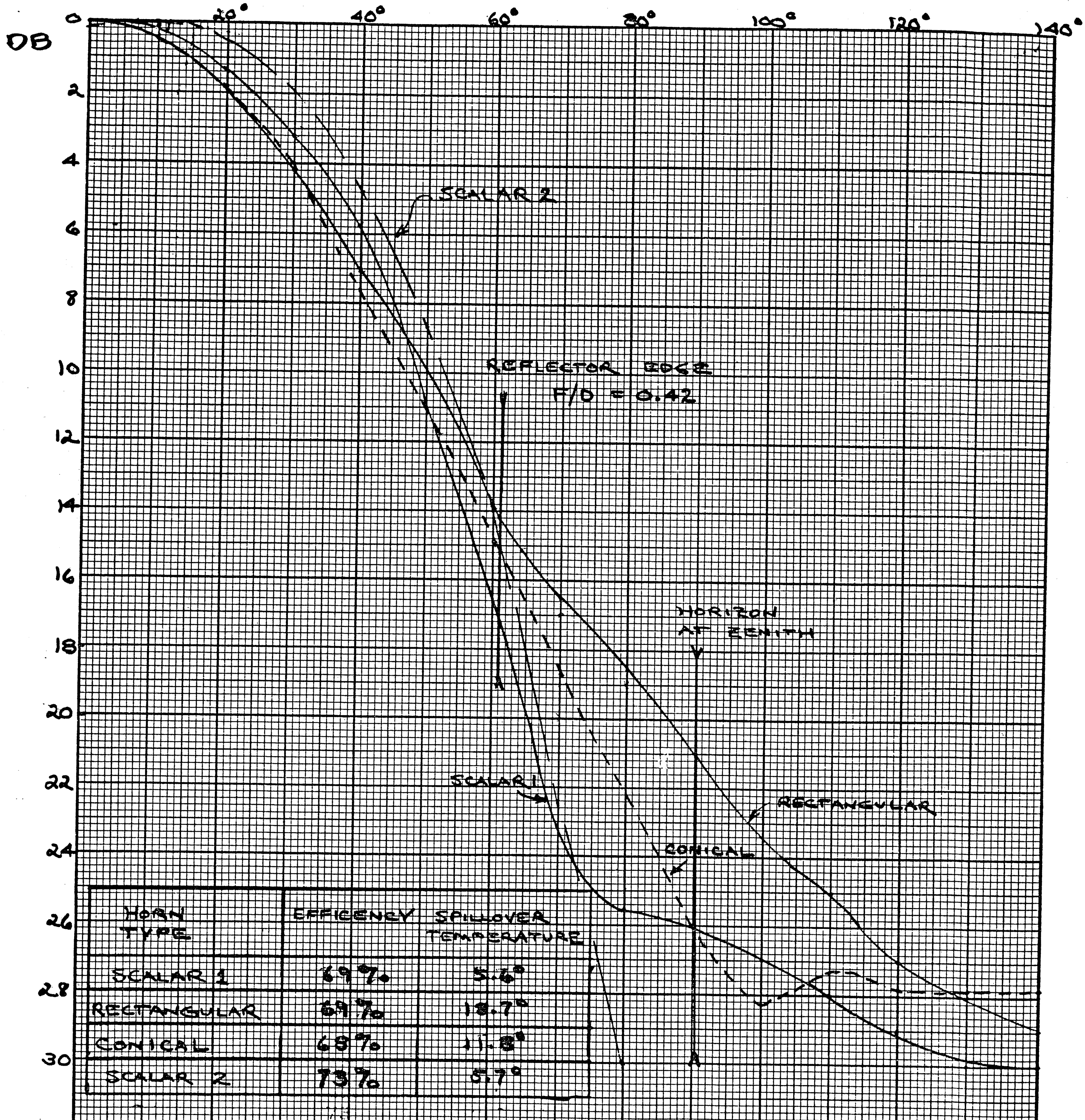
This card has the feed gain in dB (positive values) starting at the beam center and going to 180°. The value is given by two digits, decimal point, and another digit. There is no space between values. Example:

Columns	
1-4	00.0
5-8	00.1
9-12	02.1
⋮	⋮
73-76	30.0

E. Typical Results

The program output for various types of feeds is given in Table 1. Patterns, total efficiency, and spillover temperature (all average of E- and H-plane values) for four of these feeds are shown in Figure 3.

Results on the square horn are in good agreement with results calculated by a program developed by Scientific-Atlanta, Inc. The NRAO program gives an E-H average of 86.2% and 86.8% for taper and spillover efficiency, whereas the alternate program gives 84.8% and 89.1%. The small differences may be due to the 10° pattern increments and E-H plane averaging in the NRAO program.



HORN TYPE	EFFICIENCY	SPILLOVER TEMPERATURE
SCALAR 1	69%	5.6°
RECTANGULAR	69%	18.7°
CONICAL	68%	11.8°
SCALAR 2	73%	5.7°

Figure 3 — The patterns, total efficiency, and spillover temperature for 4 feeds is shown. The plotted pattern is the average of E- and H-plane patterns and the efficiency and spillover temperature are the average results of two computations assuming the E- and H-plane patterns are circularly symmetric. "Scalar 1" is a 10.69 GHz, 6.55 λ scalar feed while "Scalar 2" is a 1.40 GHz, 2.85 λ sealar feed. The rectangular horn was 1.16 λ by 0.85 λ, and the conical horn had 1.0 λ inside diameter.

TABLE 1

Results of Efficiency and Spillover Temperature Calculations for Various Feed Horns

Data Number	Feed Description	Plane	Taper Efficiency %	Spillover Efficiency %	Total Efficiency %	Spillover Temperature °K
1	Variable Polarization 1.67 GHz Scalar	E	76.3	95.1	72.6	11.6
2		H	77.6	95.7	74.3	10.9
22	10.69 GHz Scalar 6.55 λ Diameter	E	68.7	97.0	66.6	5.5
21		H	73.7	97.3	71.7	5.7
32	1.67 GHz Scalar 7.55 λ	E	74.9	96.6	72.3	7.2
31		H	78.2	96.5	75.5	7.9
29	1.4 GHz Scalar 2.85 λ Diameter	E	72.4	97.9	70.8	5.0
30		H	77.2	97.4	75.2	6.4
24	10.69 GHz Rectangular Horn, 1.16 λ x 0.85 λ	E	76.1	90.0	68.5	22.0
23		H	75.5	93.2	70.3	15.4
27	Square Horn 0.9 λ Square	E	85.7	88.1	75.5	28.0
28		H	86.8	85.6	74.3	34.9
26	Conical Horn 1.0 λ Inside Diameter	E	73.4	94.2	69.1	11.7
25		H	70.5	95.1	67.0	11.9

F. Some Useful Approximations

In order to develop some feeling for the relation between the feed pattern and spill-over, some approximate formulas and examples will be given.

The fraction of power, ϵ , transmitted by an antenna of gain, $G(\Omega)$, into a solid angle, Ω_0, is given by

$$\epsilon = \frac{1}{4\pi} \int^{\Omega_0} G(\Omega)\, d\Omega$$

If $G(\Omega)$ is assumed constant over Ω_0, we have,

$$\epsilon = \frac{\Omega_0}{4\pi} G(\Omega_0)$$

The feed gain can be expressed as the ratio of the on-axis feed gain, G_0, to the pattern function, P. For an F/D = 0.42 paraboloid, $10 \log G_0$ is approximately 10 dB. Thus the fraction of power in Ω_0 can be expressed as

$$\epsilon = \frac{\Omega_0}{4\pi} \cdot \frac{10}{P}$$

where $P = 10^{P_{dB}/10}$ is the average pattern in Ω_0.

Example 1

Suppose Ω_0 is the complete half-sphere extending 90° to 180° from the feed axis. Thus $\Omega_0 = 2\pi$ and we can make the following table:

P_{dB}	$100 \cdot \epsilon$	Spillover Temperature at Horizon
20 dB	5.0 %	7.5°
25 dB	1.7 %	2.5°
30 dB	0.5 %	0.7°

Example 1 (continued):

The conclusion is that patterns greater than 30 dB down have negligible effect on efficiency or antenna temperature. The feed pattern can be measured on a pattern range with 30 dB dynamic range.

Example 2

Suppose Ω_0 is the segment extending from 60° to 90° from the feed axis. This is the solid angle which the ground subtends for an F/D = 0.42 paraboloid pointed at zenith. The value of Ω_0 is π and we have:

P_{dB}	$100 \cdot \epsilon$	Spillover Temperature at Zenith
15 dB	7.5 %	21.8°
20 dB	2.5 %	7.3°
25 dB	0.7 %	2.2°
30 dB	0.2 %	0.7°

Thus the spillover is very critical to average patterns of 15 dB (poor) to 20 dB (good) in this solid angle.

Example 3

Considering the range of 60° to 70° from the feed axis, $\Omega_0 = .316\ \pi$, and we obtain:

P_{dB}	$100 \cdot \epsilon$	Spillover Temperature at Zenith
13 dB	4.0 %	11.6°
14 dB	3.2 %	9.3°
15 dB	2.5 %	7.3°
16 dB	2.0 %	5.8°
17 dB	1.6 %	4.6°
18 dB	1.3 %	3.7°
20 dB	0.8 %	2.3°
23 dB	0.4 %	1.2°

Example 3 (continued):

(Note that P_{dB} refers to pattern measurements; do not include 3 dB for space attenuation.) Since Ω_0 will be approximately the same for 70° to 80° and 80° to 90° the above table can also be used for these increments. The zenith spillover temperature can then be evaluated by use of the table. For example, the rectangular horn pattern plotted in Figure 1 gives the following results:

Angle Range	Average Pattern Value	Spillover Temperature at Zenith
60° - 70°	15	7.3°
70° - 80°	20	2.3°
80° - 90°	23	1.2°
Total	-------------------------	10.8°

This is in good agreement with the 11.8° value computed by the program.

12/21/71

ADDITIONS TO ELECTRONICS DIVISION INTERNAL REPORT NO. 93
ANTENNA FEED EFFICIENCY AND SPILLOVER CALCULATION PROGRAM FOR A CASSEGRAIN ANTENNA

by Joseph Greenberg

The title of this program is CSPILTAP. Read the 'Antenna Feed Efficiency and Spillover Calculation Program', Electronics Division Internal Report No. 93, which refers to a parabolic reflector. The theory and mathematics are the same; however, the input-output procedure is different.

The data cards are as follows:

See the first data card:

Columns	
1-5	This represents the angle of the feed's incident radiation measured from beam center. It is in F5.2 format (Ex. 19.23, 09.00) and measured in degrees
6-9	This represents the incremental angle between pattern points. It is in F4.1 format(Ex. 11.1, 01.0). Do not use values less than 00.2.
11-12	This represents the number of pattern points entered. The points start at zero degrees and progress at the above increment. The points beyond the number specified are assumed sixty db. by the program
13-80	Any comment or title desired. This is printed above each output pattern

See the second and third data cards:

These cards have the feed gain in db. (positive values), starting at beam center and going to the position specified on card one. The values are in F4.1 format with no spaces between them (ex. 12.7). Use as many cards as needed for the specified number of points. Have 20 points per card.

The output is as shown. For each set of the first card and the data points, one pattern is produced. The program computes the values for 90%, 100%, and 110% of the edge angle given.

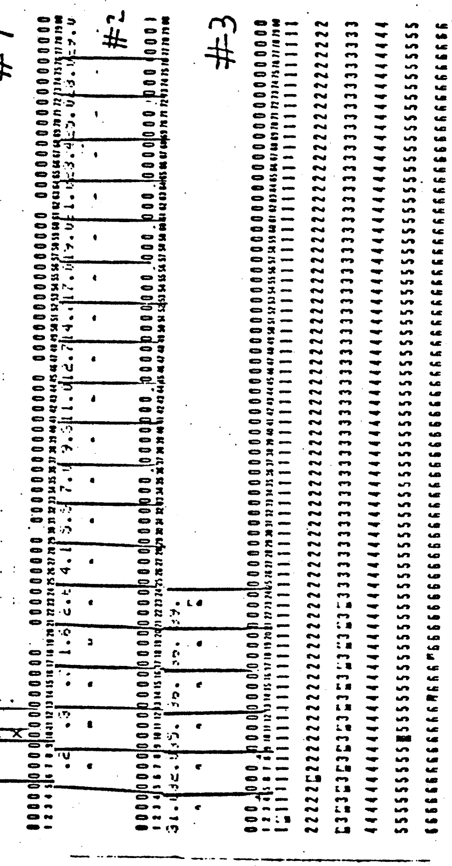
RCA CASSEGRAIN ANTENNA FEED REPORT FREQ=2.695 GHZ

	EDGE ANGLE	TAPER EFFICIENCY	SPILLOVER EFFICIENCY	TOTAL EFFICIENCY
90% OF EDGE ANGLE	8.10 DEGREES	93.8%	79.0%	74.1%
100% OF EDGE ANGLE	9.00 DEGREES	91.0%	84.8%	77.2%
110% OF EDGE ANGLE	9.90 DEGREES	87.6%	88.9%	77.9%

RCA CASSEGRAIN ANTENNA FEED REPORT FREQ=8.085 GHZ

	EDGE ANGLE	TAPER EFFICIENCY	SPILLOVER EFFICIENCY	TOTAL EFFICIENCY
90% OF EDGE ANGLE	8.10 DEGREES	93.0%	86.5%	80.5%
100% OF EDGE ANGLE	9.00 DEGREES	89.6%	91.9%	82.3%
110% OF EDGE ANGLE	9.90 DEGREES	85.3%	95.4%	81.4%

9. 1. RCA CASSEGRAIN ANTENNA FEED REPORT FREQ=2.695 GHZ



#1

#2

#3

Revised version of page 6 of 'The Antenna Feed Efficiency and Spillover Calculation Program'

First Card (continued):

Columns		
14-15	Number of pattern points falling on the reflector. Example: For F/D = 0.42 the reflector edge is 61° and hence for 10° increments this number is 7.	If these are left blank, they will be computed automatically.
17-19	Number of pattern points to 180°. Example: For 10° increments this number is 19.	
21-80	Any comment or title desired. It will be printed with the output. This can be left blank.	

Second Card

This card has the feed gain in dB (positive values) starting at the beam center and going to 180°. The value is given by two digits, decimal point, and another digit. There is no space between values. Example:

Columns	
1-4	00.0
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DATE = 71355 09/38/07 PAGE 0001

```
 1 FORMAT(F5.2,F4.1,1X,I2,17A4/(20F4.1))
 2 FORMAT(1H0,I2,'0% OF EDGE ANGLE', 4X,F5.2,' DEGREES',8X,F5.1,'%',
  12(17X,F5.1,'%'))
 3 FORMAT(1H0/30X,17A4//24X,'EDGE ANGLE      TAPER EFFICIENCY     SPIL
  1LOVER EFFICIENCY     TOTAL EFFICIENCY')
   DIMENSION G(999),G2(999),G3(999),F2(999),F3(999),CAS(17)
   INTEGER*4A,B,C
 6 READ(5,1,END=10)TTH,H,N0,CAS,(G(I),I=1,N0)
            N=(180./H)+1.
   WRITE(6,3) CAS
   KW=N0+1
   DO 12 JM=KW,N
12 G(JM)=60.
   DO 5 LA=9,11
   STH=LA/10.*TTH
   TH=STH*.01745
   FD=1./(4.*TAN(TH/2.))
   A=STH/H+1
   B=A+1
   C=A-1
   K=90./H
   K1=K+1
   K2=K+2
   HR=H*.01745
   DO 20 I=1,N
20 G2(I)=1./(10.**(G(I)/10.))*SIN(HR*FLOAT(I-1))
   CALL QTFE (HR,G2,F2,N)
   VAL1=F2(N)
   VAL2=F2(A)
   VAL12=F2(B)
   VAL30=F2(K1)
   VAL31=F2(K2)
   DO 30 I=1,B
30 G3(I)=1./(10.**(G(I)/20.))*TAN(HR*FLOAT(I-1)/2.)
   CALL QTFE (HR,G3,F3,B)
   VAL3=F3(A)
   VAL13=F3(B)
   VAL22=VAL2+((VAL12-VAL2)*(TH-(HR*FLOAT(C)))/HR)
   VAL23=VAL3+((VAL13-VAL3)*(TH-(HR*FLOAT(C)))/HR)
   VAL33=VAL30+(VAL30-VAL31)*(90./H-FLOAT(K))
   VAL4=100.*32.*(FD*VAL23)**2/VAL22
   VAL5=100.*VAL22/VAL1
   VAL6=VAL4*VAL5/100.
 5 WRITE(6,2) LA,STH,VAL4,VAL5,VAL6
   GO TO 6
10 STOP
   END
```