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DETECTOR LAW

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DETECTOR LAW

The relation between the input and output of a detector can be written

$$V_{out} = C_1 V_{in}^{\beta} \tag{1}$$

where β lies between 1 (linear detector) and 2 (square law detector). β is also dependent on the amplitude of the voltages, but over dynamic ranges normal in radio astronomy β can usually be regarded as constant.

For radiometer applications it is convenient to use a detector relation giving the power input (input temperature) as a function of the detector voltage output

$$T_{in} = C_2 V^{\alpha}_{out} \tag{2}$$

which means that

$$\alpha = \frac{2}{\beta}$$

compared to (1).

Thus, if $\alpha = 1$ we have a "square law detector", and if $\alpha = 2$ we have a "linear detector".

Let us apply equation (2) to the actual condition in a radiometer (figure 1):

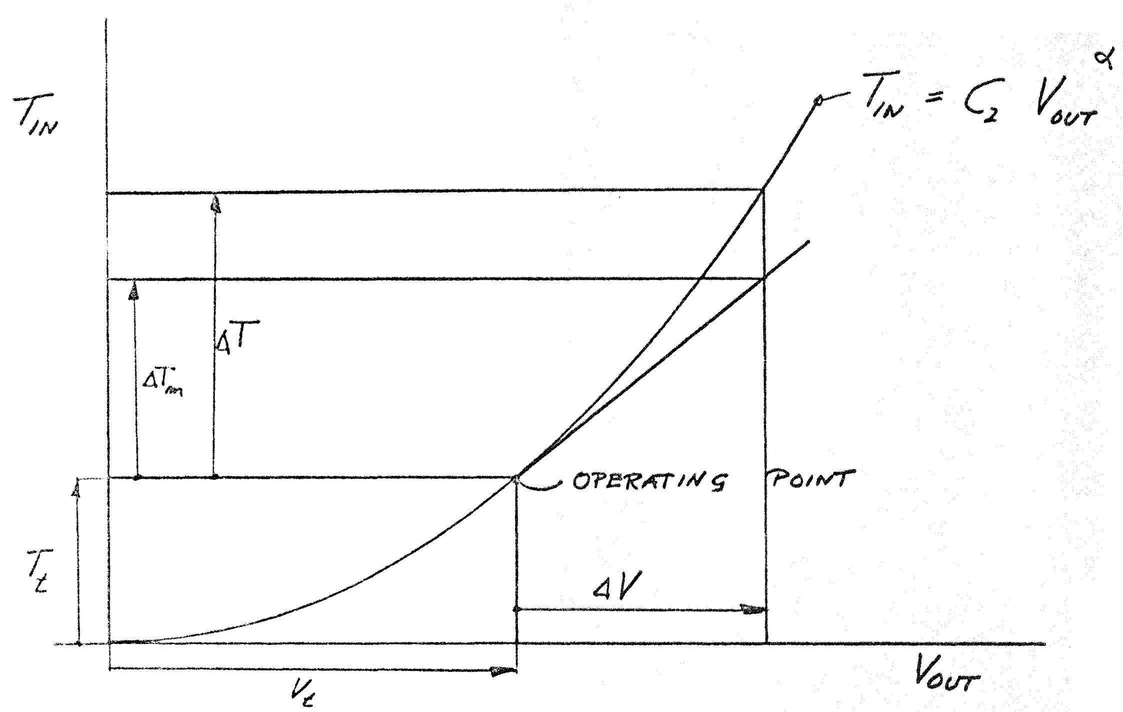


Fig. 1. -- Input-output relations of a detector

The operating point is determined by T_t and the corresponding output voltage V_t

T_t = System input noise temperature

V_t = Detector voltage at operating point

$$T_t = C_2 V_t^{\alpha} \tag{3}$$

according to (2).

An input signal ΔT will give a change in output voltage

$$(\Delta T + T_t) = C_2(\Delta V + V_t)^{\alpha} \tag{4}$$

Combining (3) and (4) gives

$$\frac{\Delta T}{T_t} = \left(1 + \frac{\Delta V}{V_t}\right)^{\alpha} - 1 \tag{5}$$

Small changes in input temperature ∂T from the operating point (T_t, V_t) gives

$$\partial T = \alpha C_2 V_y^{\alpha - 1} \partial V \tag{6}$$

and together with (3)

$$\frac{\partial T}{T_t} = \alpha \frac{\partial V}{V_t} \tag{7}$$

Assuming that this linear relation between input temperature and output voltage is correct also for large changes in output voltage we get an apparent measured input temperature ΔT_m

$$\frac{\Delta T_m}{T_t} = \alpha \frac{\Delta V}{V_t} \tag{8}$$

(See Figure 1.)

We now define a correction μ which has to be applied to the apparent measured input temperature ΔT_m to get the true input temperature ΔT

$$\Delta T = \mu \Delta T_m \tag{9}$$

$$\mu = \frac{\frac{\Delta T}{T_t}}{\frac{\Delta T_m}{T_t}} = \frac{\left(1 + \frac{\Delta V}{V_t}\right)^{\alpha} - 1}{\alpha \frac{\Delta V}{V_t}}$$

$$\boxed{\mu = \frac{\left(1 + \frac{\Delta V}{V_t}\right)^{\alpha} - 1}{\alpha \frac{\Delta V}{V_t}}} \tag{10}$$

Curves giving μ [or $(\mu - 1)$] as a function of $\frac{\Delta V}{V_t}$ for different values of α between $\alpha = 1$ (square law detector) and $\alpha' = 2$ (linear detector) are shown in Figure 2. (Normally, $1.6 < \alpha < 1.7$ for the NRAO standard receiver.)

It may also be of interest to be able to estimate the approximate value of μ prior to an observation knowing the approximate expected value of ΔT and the system noise temperature T_t.

We rewrite equation (8) with the use of (9)

$$\frac{\Delta T}{\alpha \mu T_t} = \frac{\Delta V}{V_t} \tag{11}$$

and substitution into (10) gives

$$\boxed{\mu = \frac{\frac{1}{\alpha} \frac{\Delta T}{T_t}}{\left(1 + \frac{\Delta T}{T_t}\right)^{1/\alpha} - 1}} \tag{12}$$

Curves describing this equation are shown in Figure 3.

The important equations derived are:

$$\boxed{\begin{aligned} T &= C\,V^{\alpha} && (1) \\ \Delta T &= \mu \Delta T_m && (9) \\ \mu &= \frac{\left(1 + \frac{\Delta V}{V_t}\right)^{\alpha} - 1}{\alpha \frac{\Delta V}{V_t}} && (10) \\ \mu &= \frac{\frac{1}{\alpha}\frac{\Delta T}{T_t}}{\left(1 + \frac{\Delta T}{T_t}\right)^{1/\alpha} - 1} && (12) \end{aligned}}$$

In order to be able to make the necessary corrections the following quantities have to be measured in addition to ΔV (and ΔT_m, which follows directly from the thermal calibration):

1. Detector law α
2. Operating point V_t

1. Detector law. The detector law exponent α is measured by introducing known changes of input noise power to the detector and measuring the corresponding detector output voltages. Plotting output voltage against input noise power on log-log paper gives a straight line for constant α. The operating V_t is later chosen to give a maximum dynamic signal range with constant α. (See Figure 4.)

α may change with time and has to be remeasured occasionally. Replacement of the detector diode and/or components in the detector circuit may also change α.

2. Operating point V_t. The detector current is a measure of the operating point when the receiver is switched to signal position (or is balanced in the switched mode). In order to find the operating level measured in units of recorder deflection and/or digital print-out, the gain between detector and output system must be measured. This is done by introducing a known square wave voltage, in phase with the switch frequency, into the audio-phase-detector system. The source impedance of this square-wave calibration signal should be identical to the detector output impedance.

If the number of output units for this calibration voltage V_{cal} is N_{cal} then

$$N_t = \frac{V_t}{V_{cal}} N_{cal}$$

where N_t is the operating point referred to the output of the radiometer.

If we call a signal Δ, then

$$\frac{\Delta N}{N_t} = \frac{\Delta V}{V_t}$$

and this ratio may be used in the detector law correction (fig. 2). (Note that any DC offset introduced between phase-detector and recorder must be subtracting when reading N_{cal} and ΔN.

$$\mu = \frac{\left(1+\frac{\Delta V}{V}\right)^{\alpha} - 1}{\alpha \frac{\Delta V}{V}}$$

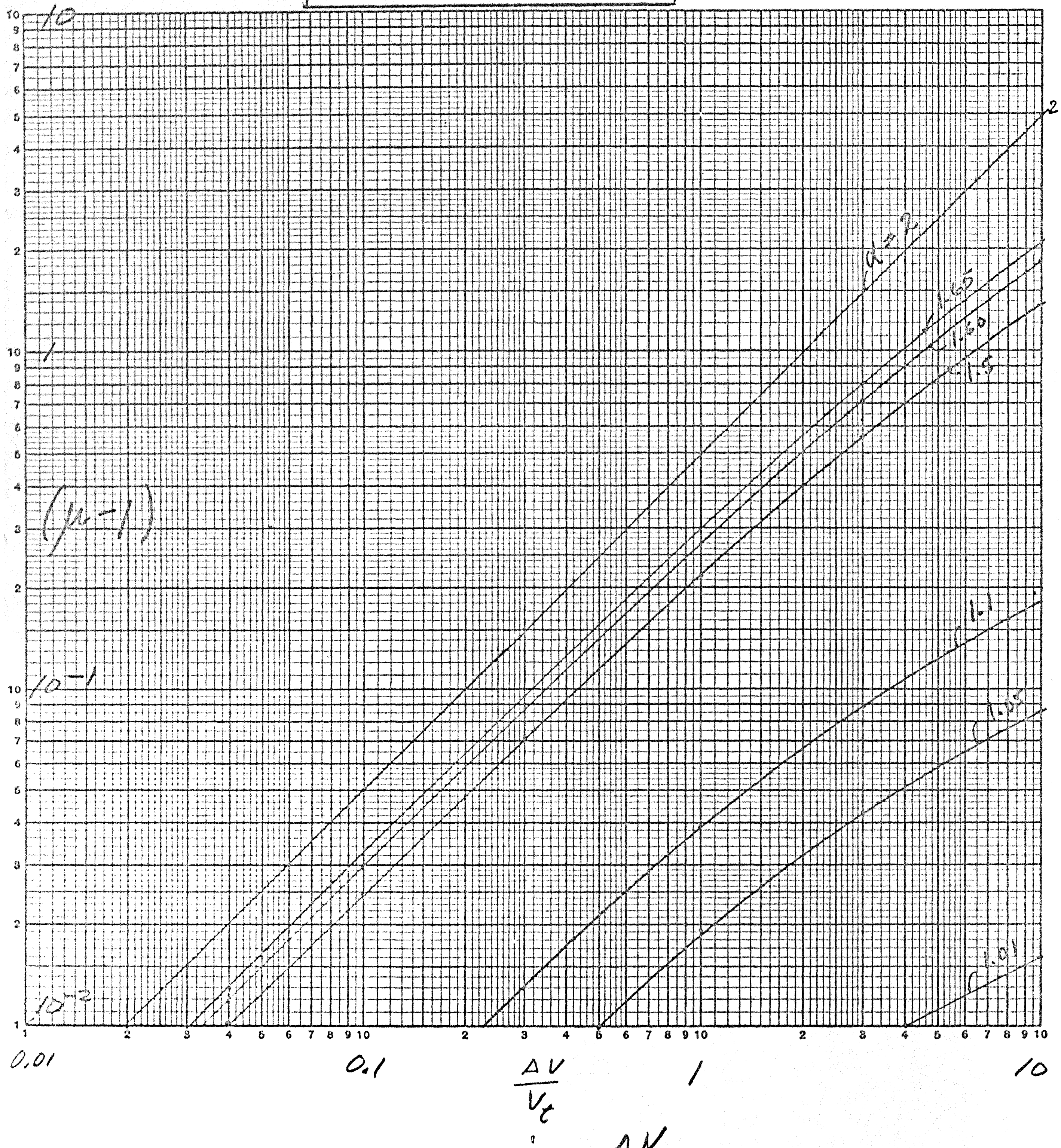


Fig 2

$$\mu = \frac{\frac{1}{\alpha}\,\frac{\Delta T}{T_t}}{\left(1+\frac{\Delta T}{T_t}\right)^{1/\alpha}-1}$$

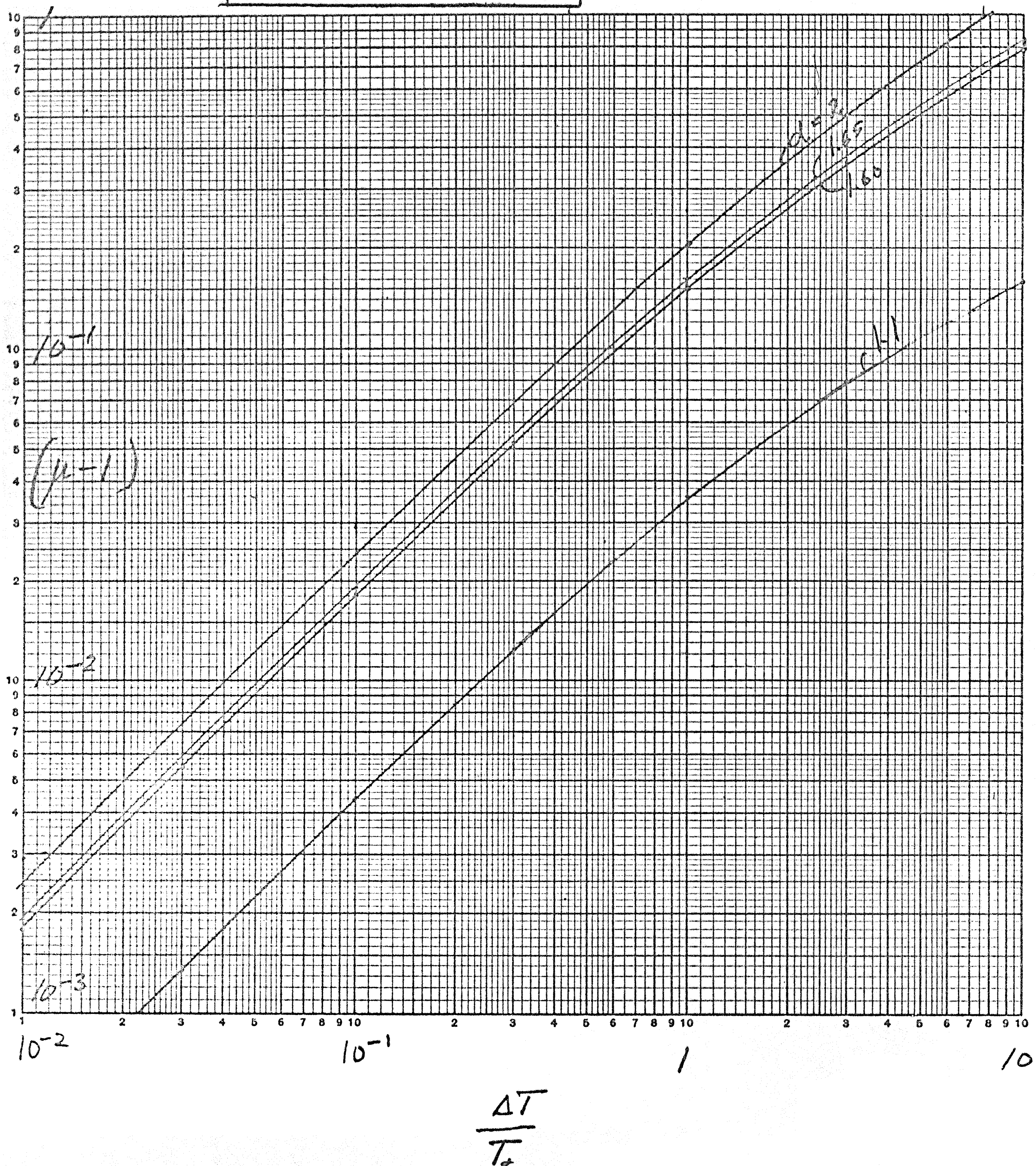


Fig 3

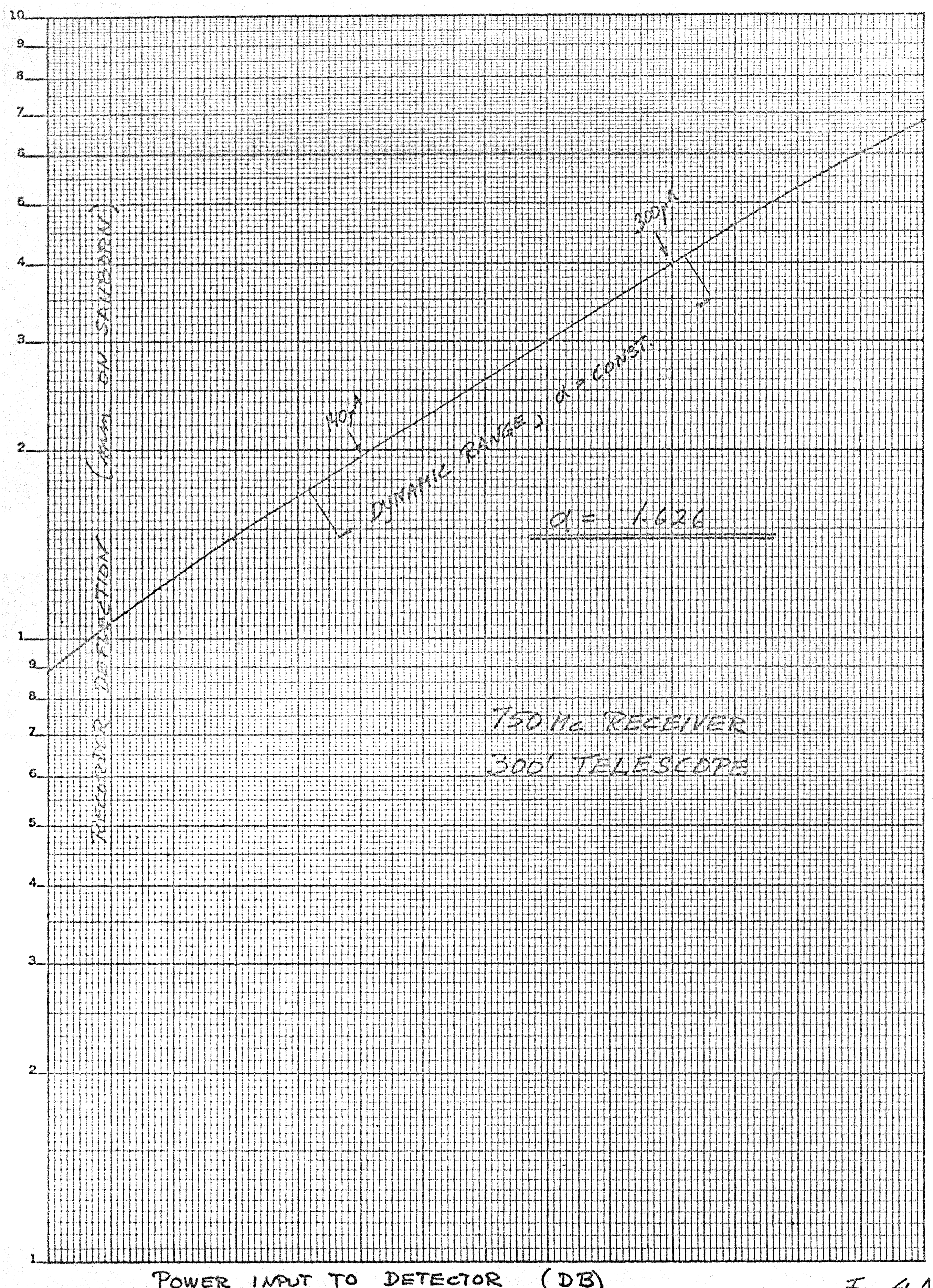
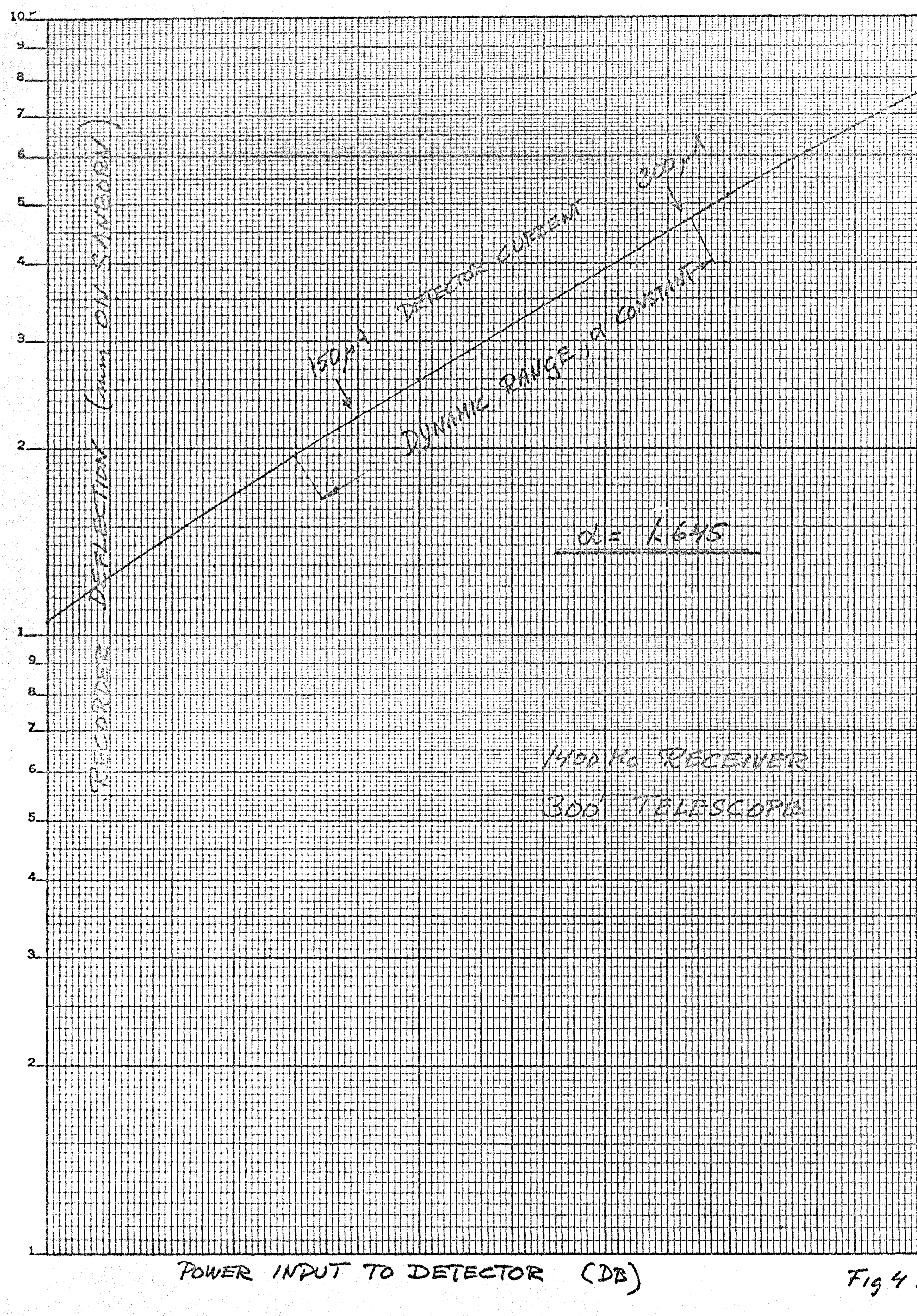

RECORDER DEFLECTION (MM ON SANBORN)
140μA
300μA
DYNAMIC RANGE, α = CONST.
α = 1.626
750 Mc RECEIVER
300' TELESCOPE
POWER INPUT TO DETECTOR (DB)

Fig 4A


10
9
8
7
6
5
4
3
2
1
9
8
7
6
5
4
3
2
1
RECORDER DEFLECTION (mm ON SANBORN)
DETECTOR CURRENT
300 μA
150 μA
DYNAMIC RANGE IN CONSTANT
α = 1.645
1400 Mc RECEIVER
300' TELESCOPE
POWER INPUT TO DETECTOR (DB)
Fig 4 B