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SINGLE SIDEBAND, DOUBLE SIDEBAND, OR MIXED INTERFEROMETER RECEIVERS

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Summary

SSB and DSB interferometer receivers are compared on the basis of their relative advantages and disadvantages.

A mixed system is proposed which has greater flexibility and has most of the advantages of both systems without some of their drawbacks. This new system might provide partial answers to present VLA problems, mainly in the areas of data handling and delay switching.

In the appendix a detailed calculation of the system behavior is given for the case of a mixed SSB/DSB receiver.

1. Comparison of SSB and DSB Receivers

1.1 The receiver output function.

By receiver output function we will understand the normalized response of an interferometer receiver to one source of radiation, as a function of the relative source position given by τ and the way the receiver operates on the antenna terminal voltages.

We first specify a receiver block diagram with its associated l. o. frequencies and passbands. (See figures 1 and 2.) The normalized receiver output functions then are:

$$R_{SSB} = \underbrace{\frac{\sin \pi B(\tau - \tau_d)}{\pi B(\tau - \tau_d)}}_{\text{envelope}} \underbrace{\cos (\omega_c \tau + \omega_{i1} \tau_d - \varphi_1 - \varphi_2)}_{\text{interference pattern}}$$

$$R_{DSB} = \underbrace{\frac{\sin \pi B(\tau - \tau_d)}{\pi B(\tau - \tau_d)} \cos \omega_i(\tau - \tau_d)}_{\text{envelope}} \underbrace{\cos(\omega_\ell \tau - \varphi)}_{\text{interference pattern}}$$

where $\omega = 2\pi f$.

1.2 Delay considerations.

Looking at the envelope function, we see that the envelope is wider, for equal total RF passbands, for the SSB situation. The limiting case is where in the DSB receiver f_i becomes equal to B/2. The envelope functions to compare in this limiting case become:

$$\text{SSB: } \frac{\sin 2\alpha}{2\alpha} \qquad \text{and} \qquad \text{DSB: } \frac{\sin \alpha}{\alpha} \cos \alpha$$

These expressions are equal to each other for each α.

With a small i.f. bandwidth compared to the sideband separation, however, the SSB envelope is considerably wider (up to 100 percent in practical cases) than the DSB envelope. Whether this is a desirable feature or not remains to be judged from case to case.

For the Westerbork Synthesis Telescope (referred to hereafter as WST) with only 4 MHz i.f. bandwidth, this has been one point in favor of SSB reception, allowing for example bigger delay steps.

Looking at the interference pattern functions, we notice that the receiver delay τ_d does not appear in the expression in the DSB case, but that it does, multiplied by the effective first intermediate frequency f_{ii}, in the SSB case. Consequently, in the SSB case one is limited in the choice of delay steps to multiples of one wavelength of the intermediate frequency, if the interference pattern phase is not to be changed when stepping the delay.

Moreover, every delay error (deviation from multiple of one wavelength) appears in the interference function. For the WST, for example, this leads to a 1.5×10^{-5} accuracy requirement per degree allowable phase error. Aiming at 1° phase accuracy in this part of the system requires already a delay cable temperature stability of ± 1.5 °C. In the DSB case we are completely free in the choice of a delay step. The accuracy requirement, again for the WST, would have been about one or two orders of magnitude less stringent.

1.3 Effective intermediate frequency.

To consider the importance of this quantity we rewrite the interference pattern expression for the SSB receiver as

$$\cos \left\{ \omega_{\ell_1} \tau - \omega_{i1} (\tau - \tau_d) - \varphi \right\}$$

If the receiver delay was always exactly right (i.e., $\tau_d = \tau$), then the effective intermediate frequency ω_{i1} would not influence the interference pattern expression. However, because the receiver τ_d has to be stepped in full wavelength increments, the ω_{i1} is important. Errors in this frequency cause phase errors. Again, for the WST this leads to a worst case i.f. accuracy requirement of 5×10^{-3} per degree phase error.[1]

In the DSB case the effective intermediate frequency does not appear in the interference pattern.

1.4 Local oscillator phases.

Both first and second l.o. phases appear in the interference pattern expression of the SSB receiver. In the DSB receiver the phase of the first l.o. is important but every phase error after the mixer causes a shift in the envelope only. This shift can be compensated for by an appropriate opposite delay shift, if necessary. In general the envelope is wide enough to tolerate phase errors up to some 20°. This, at first glance, seems to be in favor of the DSB system, but for this receiver as a consequence, intended phase swing can only be done at the first local oscillator. In general this is rather inconvenient. First, because it has to be done at a high frequency (around 3 GHz); second, because it has to be done at or near the focus of the individual interferometer elements, requiring long distance telecontrol; and third, because it might upset a phase lock system.

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[1] An error account for a SSB interferometer receiver may be found in BCAP: ITR 40-66.

Intended phase swinging makes lobe or fringe processing possible, which in turn can deliver a considerable data-system simplification. With the SSB receiver this processing can be done in the central control building at much more convenient second l.o. frequencies.

1.5 Sine-cosine reception.

Simultaneous sine and cosine reception gives a $\sqrt{2}$ improvement in sensitivity. It also gives a more accurate fringe-phase when the fringe frequency becomes zero.

Simultaneous S/C reception is possible for both SSB and DSB receivers, but in the DSB case, it needs pre-mixer signal and l.o. power division and separate i.f. return and delay cable systems. If in this case no r.f. amplifier is used, no improvement in sensitivity is obtained.

In the SSB case, splitting in sine and cosine channels can be done at the second mixer, requiring only an extra correlator channel. S/C reception may also be done on a time sharing basis, in both SSB and DSB receivers, but without the $\sqrt{2}$ sensitivity improvement. This requires a 90° phase switch in either the first or the second l.o. in the SSB receiver, and in the first l.o. line in the DSB receiver. Unless two i.f. return channels and separate mixers are used, this simple scheme might not always be possible in interconnected multi-interferometer systems. It is actually only possible in interferometer systems where interferometer elements from one group are correlated exclusively with elements from the other group and vice versa, and correlations between elements from one group are avoided. This is the case for the WST and the Cambridge one-mile radio telescope.

1.6 Tied array operation.

Tied array operation is said to occur if all element outputs of a system are added in phase (tied together), forming a beam of some shape at any moment. This operation is advantageous from the point of view of electronic system complexity. The new Mills Cross of the Sydney University operates in this mode.

With DSB receivers only one beam at the time can be formed, whereas with SSB receivers multiple beam operation is simple to implement, requiring only extra backends. Intermediate frequency return and delay cables are the same for every receiver (every beam). Are spectral line observations necessary at some moment in the future and is the number of independent correlators high, as in the VLA, then tied operation might be an answer.

1.7 Sensitivity to man-made interference and l.o. noise.

In DSB receivers, the width of the envelope asks for an as low as possible intermediate frequency. On the other hand, l.o. noise sidebands, mixer crystal noise and man-made interference ask for an as high as possible intermediate frequency. Because no image filter is present, straight through radiation of interference at i.f. is not unlikely and requires careful construction of the r.f. parts of the system. Furthermore, very good shielding and careful decoupling of power lines is necessary. The part of the cables leading to the focus, without precautions, acts as a $\lambda/2$ tuned aerial for medium wave broadcast. Although this probably mainly affects the total power component it might prove troublesome.

Because of better r.f. filtering and a higher i.f. the SSB receiver is better in this respect.

1.8 H-line reception.

In the detection of the line of neutral hydrogen and of other lines too, we essentially have to do with a non-uniform, asymmetrical spectrum of unknown position in the frequency domain.

DSB requires symmetrical passbands around the first l.o. For this reason only SSB reception is possible. This was another — the most important — consideration in the choice of SSB for the WST. See BCAP: ITR 35-65.

1.9 Résumé.

Reviewing the merits of both systems, the SSB system is clearly at best at the r.f. and first i.f. portions of the interferometer receiver system up to the second mixer located in the central control building.

It provides an easy way of obtaining S/C reception and tied array operation, it has relative freedom from interference, and lobe processing in the central control building is possible. Remember: Lobe stopping is at present planned or in operation on nearly all large interferometer systems.

Furthermore, SSB is the only possibility for H-line reception.

The DSB system is optimal in the delay switching portion of the receiver, and it does not require accurate i.f. passband control.

For interferometer systems of moderate complexity (with a reasonable number of correlator-data channels) and with maximum baselines of the order of 1 km both SSB and DSB receivers are entirely feasible. The choice becomes more or less a matter of taste, the SSB probably having a slight advantage over the DSB receiver because of its greater flexibility.

SSB is chosen for the new Cambridge one-mile radio telescope, the new Christiansen array, the new Mills Cross, the WST and the RRE interferometer, for example.

DSB receivers are on the Cal Tech, NRAO and CSIRO-Parkes interferometers.

For complex systems and large baselines as in the case of the proposed NRAO-VLA two important considerations make the choice very difficult. On the one hand is DSB reception almost imperative from the point of delay tracking, on the other hand gives SSB the direct possibility of lobe processing (stopping or speeding up), S/C reception, etc., making SSB reception very desirable.

The mixed SSB/DSB system, now to be described, has SSB where it is mostly needed and DSB where that system is desirable.

2. Mixed SSB/DSB Interferometer Receiver

For a block diagram and passband and l.o. definitions, see figure 3.

The normalized receiver output function in this case becomes, as is derived in the appendix:

$$R_{SSB/DSB} = \underbrace{\frac{\sin \pi B(\tau - \tau_d)}{\pi B(\tau - \tau_d)} \cos \omega_{i2}(\tau - \tau_d)}_{\text{envelope}} \ \underbrace{\cos \left\{ (\omega_{\ell 1} + \omega_{\ell 2})\tau + \varphi \right\}}_{\text{interference pattern}}$$

From this expression we see that the envelope as well as the interference pattern are essentially DSB. However, the second l.o. phase appears in the interference pattern function. This system has the following characteristics and possibilities:

a. The system is SSB up to the second mixer in the central control building and second l.o. phase moves the interference pattern, thus S/C reception, lobe processing and H-line work are an easy possibility.

b. First intermediate frequency is high, giving added interference protection and relatively high insensitivity to first l.o. noise.

c. Because the first i.f. may be any frequency, it is possible to transmit more than one receiver output over the same i.f. return cable by frequency multiplex.

d. After the second mixer one may eventually choose SSB all the way (H-line).

e. The overall receiver behavior is DSB, giving easy delay clicking, lower delay cable loss and a l.o. determined interference pattern frequency.

f. The pre-second mixer passband has to be flat, and has to be wide enough to let the second i.f. passband determine the system passband.

g. Having both S and C receivers requires separate delay channels, but the same i.f. return is used and is needed only once. The delay channels, however, have to meet only DSB specifications.

Appendix

Mixed SSB/DSB Interferometer Receiver Performance Calculation

Figure 3 gives the assumed receiver bandpass, their location and shape and the position in the frequency domain of the local oscillators. Arbitrarily, a first l.o. frequency lower than the receiver passband is assumed. With minor changes a first l.o. frequency above the passband can be considered. All passbands are supposed to be rectangular, giving the simpler calculation. Other bandpass shapes are possible, effecting only the envelope of the receiver output — or correlation function. Figure 3 also gives the essential parts of the receiver block diagram. Because it is not intended here to include an error account, only perfect performance is considered. A starting point has been that the element terminal voltage for both of the interferometer halves is the same, but in general appears earlier on one element than on the other, depending on the relative source position. In other words, if the voltage on one terminal is U(t), the voltage on the other terminal is shifted in time over an amount τ seconds, giving $U(t + \tau)$. Voltage linear mixing and amplification is assumed throughout. No finite proportionality factors due to conversion loss, amplifier and antenna gains, etc., are taken into consideration. Normalization will be done at the end.

Following S. O. Rice (BSTJ, Vols. 23 and 24), a band limited noise voltage U(t) at the antenna terminals can be represented by a Fourier series of discrete frequencies:

$$U(t) = \lim_{\Delta f \to 0} \sum_{x = N_1}^{x = N_4} c_x \cos(\omega_x t + \varphi_x)$$

where: $\omega_x = 2\pi x \, \Delta f$

$c_x = [2g(f_x)\,\Delta f]^{1/2}$

$g(f_x)$ = noise power density spectrum

$(N_4 - N_1)\,\Delta f = f - f$

φ_x = a phase angle selected at random from the range $(0, 2\pi)$

If we consider the interferometer to operate over a finite time interval, we may dispose of taking the limit. The error in supposing the noise bandwidth still finite is negligibly small in any practical case. In our situation where the noise spectrum is rectangular, all c_x's are equal, and put to c. In the sum boundaries N_1 will correspond to f_1, N_2 with f_2, etc.

In the A-channel the noise voltage U(t) is acted upon as follows:

1. Multiplicative mixing with $\cos \omega_{\ell_1} t$, $\omega_{\ell_1} < \omega_x$, results in:

$$\rightarrow \sum_{x=N_1}^{N_4} c \cos (\omega_x t + \varphi_x) \cos \omega_{\ell_1} t$$

2. After the first i.f. filter-amplifier we are left with the difference frequencies only:

$$\rightarrow \sum_{x=N_1}^{N_4} c \cos (\omega_x t + \varphi_x - \omega_{\ell_1} t)$$

3. Second multiplicative mixing with $\cos \omega_{\ell_2} t$ delivers:

$$\rightarrow \sum_{x=N_1}^{N_4} c \cos (\omega_x t + \varphi_x - \omega_{\ell_1} t) \cos \omega_{\ell_2} t$$

4. Filtering as indicated in fig. 4 in the second i.f. filter amplifier:

$$\rightarrow \sum_{x=N_1}^{N_2} c \cos (-\omega_x t - \varphi_x + \omega_{\ell_1} t + \omega_{\ell_2} t)$$

$$+ \sum_{x=N_3}^{N_4} c \cos (\omega_x t + \varphi_x - \omega_{\ell_1} t - \omega_{\ell_2} t)$$

5. The "folding over" process taking place in the second mixer and amplifier and essential in DSB operation is treated as follows:

Put $y = 2(\ell_1 + \ell_2) - x$; we may then write

$$\rightarrow \sum_{y=N_3}^{N_4} c \cos(\omega_y t - \omega_{\ell_1} t - \omega_{\ell_2} t - \varphi_{2\ell - y}) + \sum_{x=N_3}^{N_4} c \cos(\omega_x t + \varphi_x - \omega_{\ell_1} t - \omega_{\ell_2} t)$$

Now putting y = x again and summing we get:

$$\rightarrow \sum_{x=N_3}^{N_4} c \left[\cos(\omega_x t - \omega_{\ell_1} t - \omega_{\ell_2} t - \varphi_{2\ell - x}) + \cos(\omega_x t - \omega_{\ell_1} - \omega_{\ell_2} + \varphi_x)\right]$$

6. Delaying by zero seconds does not change the expression. Performing the same operations on the noise in the B-channel we get:

1. $$\sum_{y=N_1}^{N_4} c \cos\left\{\omega_y(t+\tau) + \varphi_y\right\} \cos \omega_{\ell_1} t$$

2. $$\sum_{y=N_1}^{N_4} c \cos\left\{\omega_y(t+\tau) - \omega_{\ell_1} t + \varphi_y\right\}$$

3. $$\sum_{y=N_1}^{N_4} c \cos\left\{\omega_y(t+\tau) - \omega_{\ell_1} t + \varphi_y\right\} \cos(\omega_{\ell_2} t - \varphi)$$

4. $$\sum_{y=N_1}^{N_2} c\cos\left\{-\omega_y(t+\tau)+\omega_{\ell_1}t+\omega_{\ell_2}t-\varphi_y-\varphi\right\}$$

$$+\sum_{y=N_3}^{N_4} c\cos\left\{\omega_y(t+\tau)-\omega_{\ell_1}t-\omega_{\ell_2}t+\varphi_y+\varphi\right\}$$

5. $$\sum_{y=N_3}^{N_4} c\left[\cos\left\{(\omega_y-\omega_{\ell_1}-\omega_{\ell_2})t-(2\omega_{\ell_1}+2\omega_{\ell_2}-\omega_y)\tau-\varphi_{2\ell-y}-\varphi\right\}+\right.$$

$$\left.+\cos\left\{(\omega_y-\omega_{\ell_1}-\omega_{\ell_2})t+\omega_y\tau+\varphi_y+\varphi\right\}\right]$$

6. Delay τ_d:

$$\sum_{y=N_3}^{N_4} c\left[\cos\left\{(\omega_y-\omega_{\ell_1}-\omega_{\ell_2})(t-\tau_d)-(2\omega_{\ell_1}+2\omega_{\ell_2}-\omega_y)\tau-\varphi_{2\ell-y}-\varphi\right\}+\right.$$

$$\left.+\cos\left\{(\omega_y-\omega_{\ell_1}-\omega_{\ell_2})(t-\tau_d)+\omega_y\tau+\varphi_y+\varphi\right\}\right]$$

The multiplier takes the product of the A- and B-channel noise output voltages. The following four products result:

α) $$c^2\sum_x\sum_y \cos\left\{(\omega_y-\omega_{\ell_1}-\omega_{\ell_2})(t-\tau_d)+(\omega_y-2\omega_{\ell_1}-2\omega_{\ell_2})\tau-\varphi_{2\ell-y}-\varphi\right\}$$

$$\times\cos\left\{(\omega_x-\omega_{\ell_1}-\omega_{\ell_2})t-\varphi_{2\ell-x}\right\}$$

$$\beta)\quad c^2 \sum_x \sum_y \cos\left\{(\omega_y - \omega_{\ell_1} - \omega_{\ell_2})(t-\tau_d) + (\omega_y - 2\omega_{\ell_1} - 2\omega_{\ell_2})\tau - \varphi_{2\ell - y} - \varphi\right\}$$

$$\times \cos\left\{(\omega_x - \omega_{\ell_1} - \omega_{\ell_2})\, t + \varphi_x\right\}$$

$$\gamma)\quad c^2 \sum_x \sum_y \cos\left\{(\omega_y - \omega_{\ell_1} - \omega_{\ell_2})(t-\tau_d) + \omega_y\tau + \varphi_y + \varphi\right\}$$

$$\times \cos\left\{(\omega_x - \omega_{\ell_1} - \omega_{\ell_2})\, t - \varphi_{2\ell - x}\right\}$$

$$\delta)\quad c^2 \sum_x \sum_y \cos\left\{(\omega_y - \omega_{\ell_1} - \omega_{\ell_2})(t-\tau_d) + \omega_y\tau + \varphi_y + \varphi\right\}$$

$$\times \cos\left\{(\omega_x - \omega_{\ell_1} - \omega_{\ell_2})\, t + \varphi_x\right\}$$

Working these expressions out into sum and difference frequencies, all the sum frequencies in these expressions contain t in their arguments, thus giving zero output after the integrator. So do the difference frequencies, except when $x = y$. The remains of the four double sums α), β), γ), δ) in that case are:

$$\text{from}\quad \alpha)\quad c^2 \sum_x \cos\left\{-(\omega_x - \omega_{\ell_1} - \omega_{\ell_2})\tau_d + (\omega_x - 2\omega_{\ell_1} - 2\omega_{\ell_2})\tau - \varphi\right\}$$

$$\beta)\quad c^2 \sum_x \cos\left\{-(\omega_x - \omega_{\ell_1} - \omega_{\ell_2})\tau_d + (\omega_x - 2\omega_{\ell_1} - 2\omega_{\ell_2})\tau - \varphi_{2\ell - x} - \varphi_x - \varphi\right\}$$

$$\gamma)\quad c^2 \sum_x \cos\left\{-(\omega_x - \omega_{\ell_1} - \omega_{\ell_2})\tau_d + \omega_x\tau + \varphi_x + \varphi_{2\ell - x} + \varphi\right\}$$

$$\delta)\quad c^2 \sum_x \cos\left\{-(\omega_x - \omega_{\ell_1} - \omega_{\ell_2})\tau_d + \omega_x\tau + \varphi\right\}$$

Taking α) and δ) together delivers:

$$\mathrm{I})\quad c^2 \sum_x \cos\left\{(\omega_{\ell_1} + \omega_{\ell_2})\tau + \varphi\right\} \cos\left\{(\omega_x - \omega_{\ell_1} - \omega_{\ell_2})(\tau - \tau_d)\right\}$$

β) and γ) together:

$$\text{II)} \qquad c^2 \sum_x \cos\left\{(\omega_{\ell_1} + \omega_{\ell_2})\,\tau + \varphi + \varphi_x + \varphi_{2\ell - x}\right\} \cos\left\{(\omega_x - \omega_{\ell_1} - \omega_{\ell_2})\,(\tau - \tau_d)\right\}$$

The second cosine terms in both sums are equal. However, in the first (I) sum all terms add up in phase; in the second sum (II) they add up with random phase. In the first case the total sum goes with N, where N is the total number of terms; in the second case it goes with $\sqrt{N}$. As we can make N as large as we like, we can make the second sum as small as we want, relative to the first sum. In the following we, for this reason, neglect the second sum (II).

I is a sum over a finite number of cosines and can be calculated exactly. The easiest way, however, is to let $\Delta f \to 0$. Where we sum between fixed frequencies f_3 and f_4 this gives us:

$$\int_{f_3}^{f_4} g \cos\left\{(\omega_{\ell_1} + \omega_{\ell_2})\,\tau + \varphi\right\} \cos\left\{(\omega - \omega_{\ell_1} - \omega_{\ell_2})\,(\tau - \tau_d)\right\}$$

with

$$c^2 = 2g\,\Delta f \xrightarrow{\Delta f \to 0} 2g\ \mathrm{d}f$$

Solving this integral we finally arrive at the following formula which represents the normalized receiver output function:

$$R_{SSB/DSB} = \frac{\sin \pi B(\tau - \tau_d)}{\pi B(\tau - \tau_d)} \cos \omega_{i2}(\tau - \tau_d) \cos\left\{(\omega_{\ell_1} + \omega_{\ell_2})\,\tau + \varphi\right\}$$

The extra B in the denominator is put there for the purpose of normalization. Definition of B and ω_{i2} as given in figure 3.

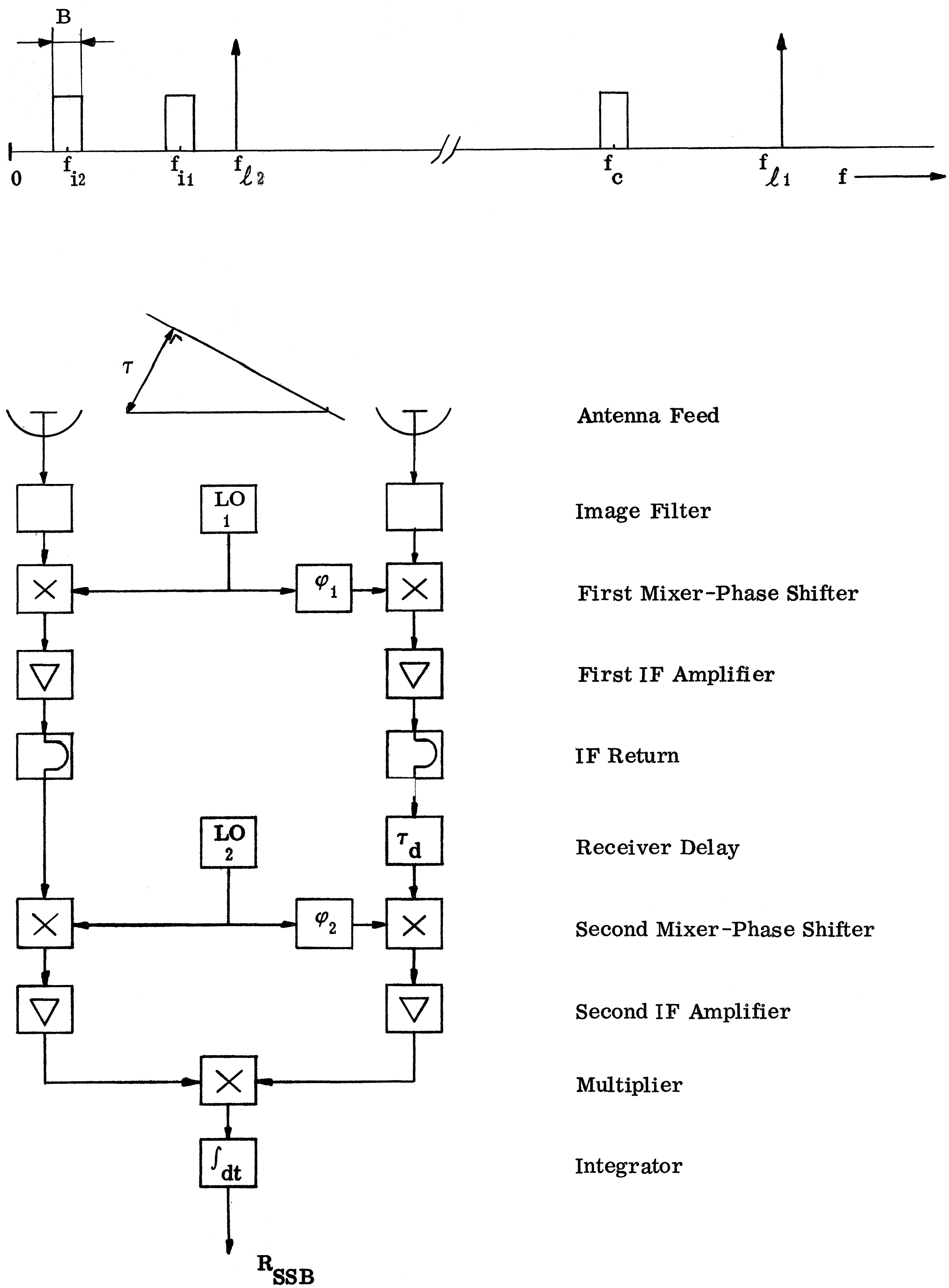
B
0
f_{i2}
f_{i1}
$f_{\ell 2}$
f_c
$f_{\ell 1}$
f
τ
LO_1
φ_1
LO_2
τ_d
φ_2
$\int dt$
R_{SSB}
Antenna Feed
Image Filter
First Mixer-Phase Shifter
First IF Amplifier
IF Return
Receiver Delay
Second Mixer-Phase Shifter
Second IF Amplifier
Multiplier
Integrator

Fig. 1

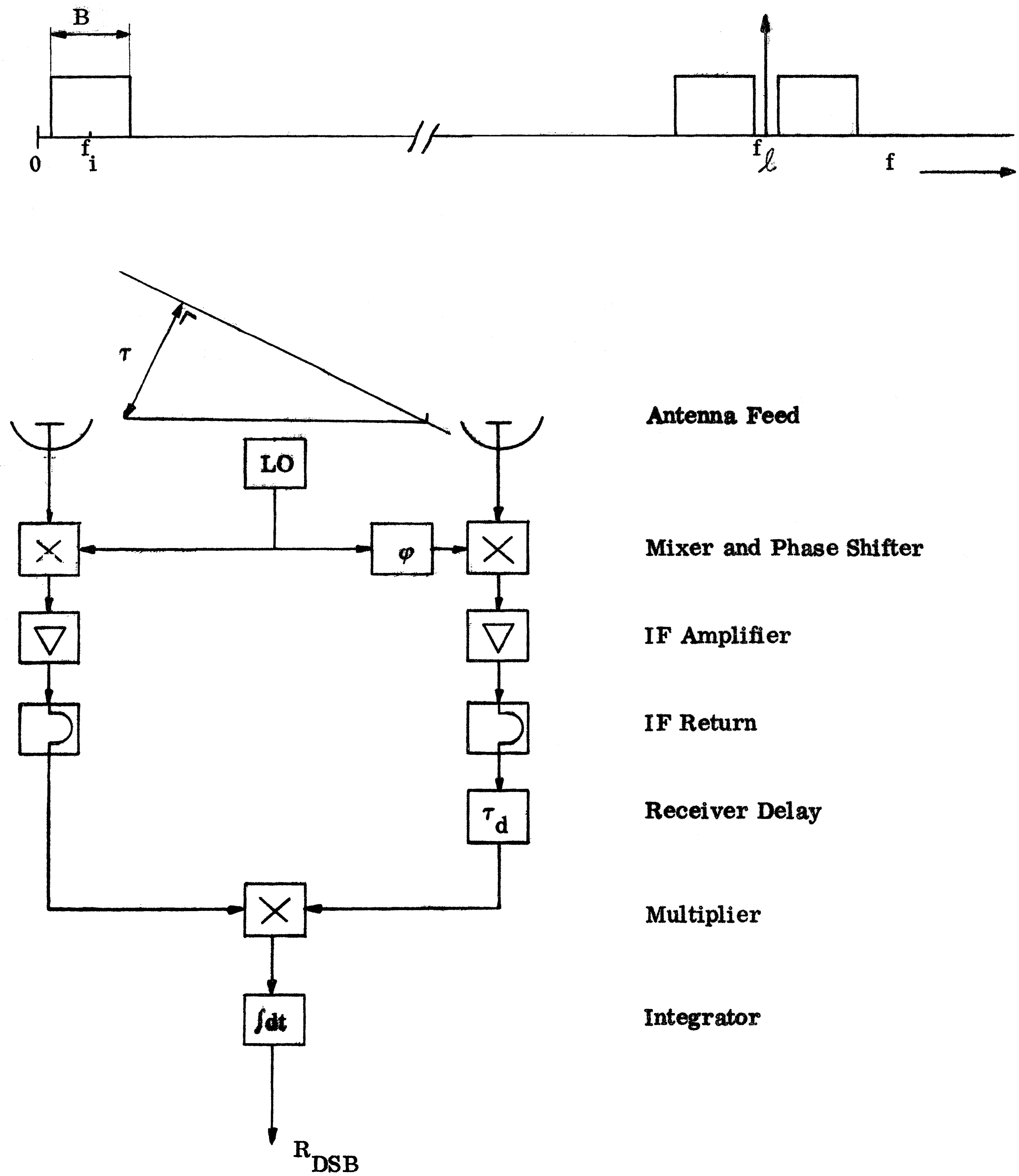
B
0
f_i
f_ℓ
f
τ
Antenna Feed
LO
φ
Mixer and Phase Shifter
IF Amplifier
IF Return
τ_d
Receiver Delay
Multiplier
$\int dt$
Integrator
R_{DSB}

Fig. 2

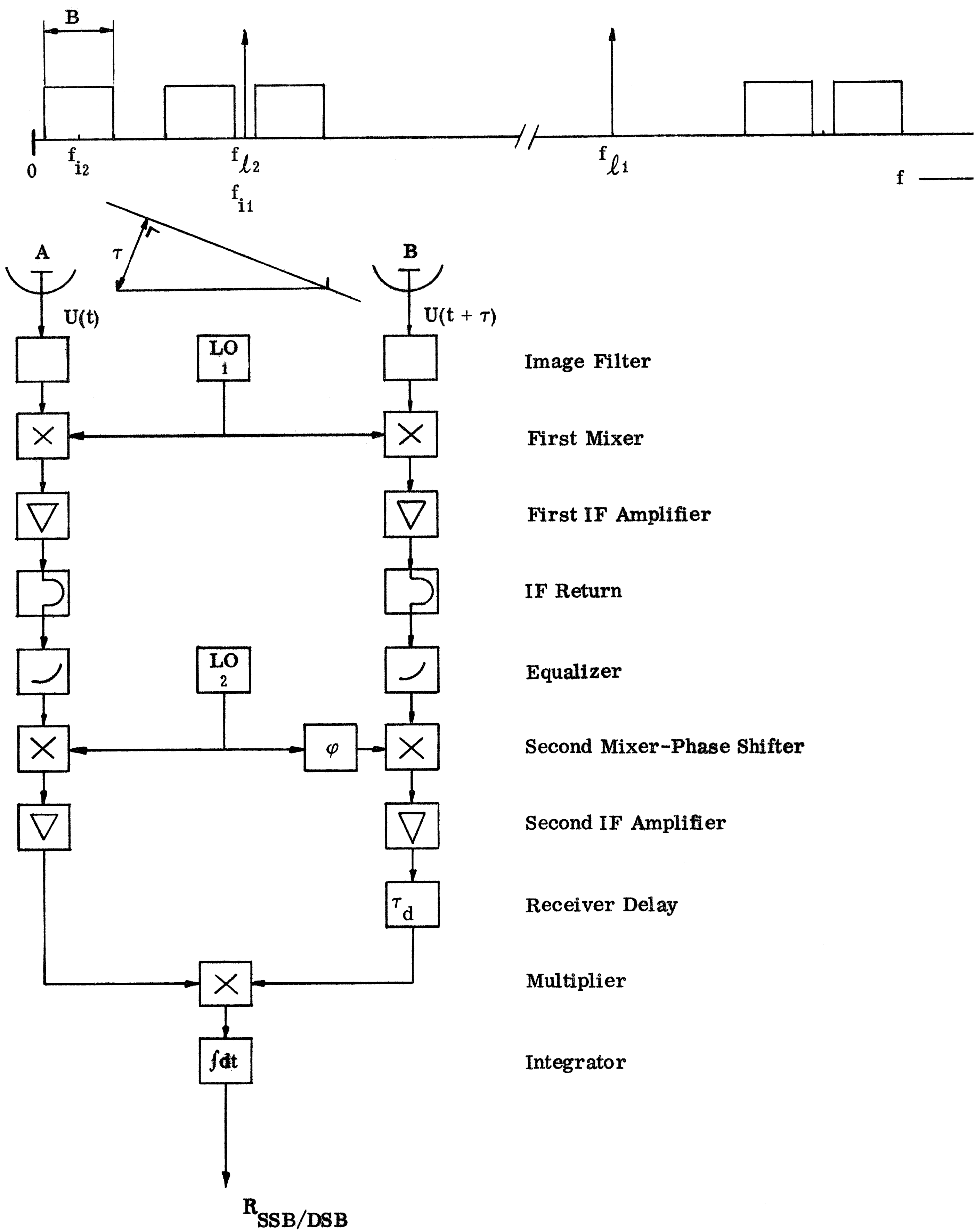
B
0
f_{i2}
$f_{\ell 2}$
f_{i1}
$f_{\ell 1}$
f
A
B
τ
U(t)
U(t + τ)
LO 1
LO 2
φ
τ_d
$\int dt$
$R_{SSB/DSB}$
Image Filter
First Mixer
First IF Amplifier
IF Return
Equalizer
Second Mixer-Phase Shifter
Second IF Amplifier
Receiver Delay
Multiplier
Integrator

Fig. 3