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On Radiation Loading in Cryogenic Dewars and Emissivity Measurements for Radiation Shield Materials

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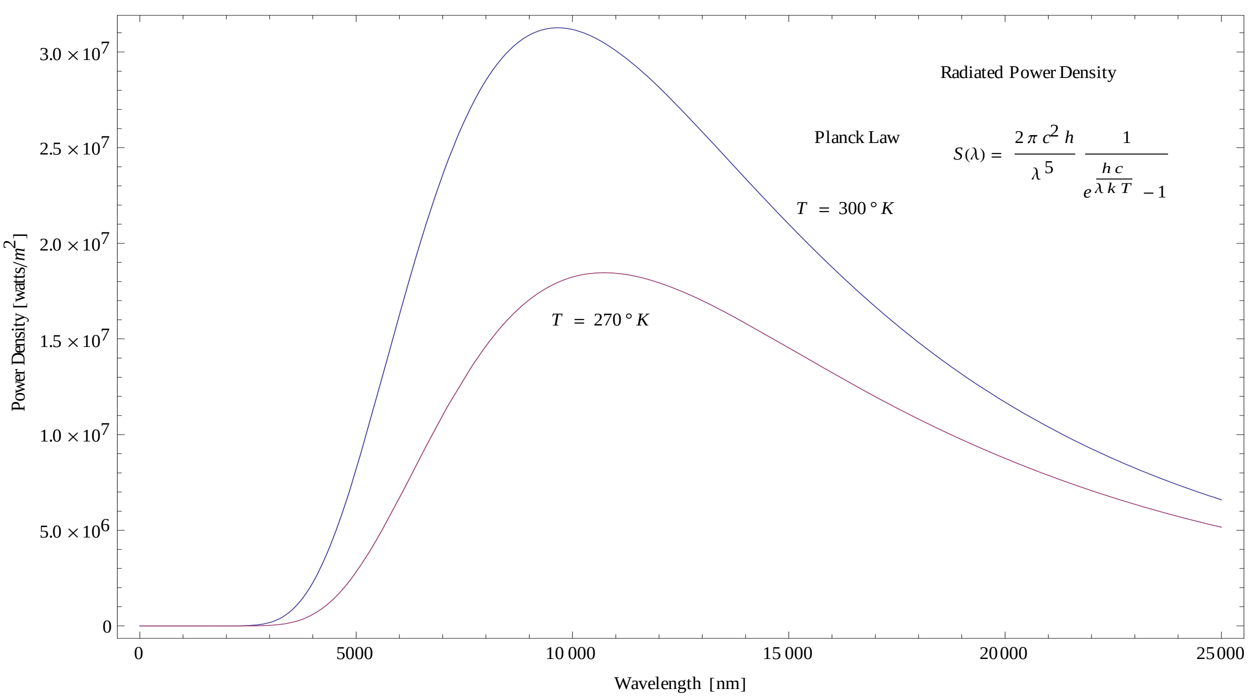
When estimating the refrigerator requirements for a cryogenic system, thermal radiation from the room temperature dewar walls to the internal surfaces is typically the largest heat load. A radiation shield is required that reduces the loading on the low-capacity stages of the refrigerator. Typically, the radiation shield is thermally connected to the high-temperature high-capacity stage of the refrigerator. This load is a function of the geometry of the outside dewar walls and radiation shield, along with the material properties of each. This EDIR will examine the equations involved in the calculations and present emissivity measurements on metals used for the radiation shields.

Many resources are available which develop the concepts for thermal radiation transfer. The relevant equations for this analysis are the black body heat transfer equations and the modified gray body equations.

<u>Black Body Equations:</u>

The warm surfaces emit thermal radiation in a black body spectrum at wavelengths that are a function of temperature, as shown in the plot. The spectrum of interest is shown in the plot below for a 300°K and 270°K black body. Material properties at these wavelengths are significant.



Radiation Heat Transfer Equation

The differential equation for the net radiation on surface ***j*** from an enclosed black body with ***N*** surfaces is:

$$dq_j = \sigma \sum_{k=1}^{N} (T_j^4 - T_k^4)\, dF_{dj-k} dA_j \qquad (1)$$

where σ is the Stefan-Boltzmann constant, differential dF_{dj-k} is the view factor between surfaces, and dA_j is the differential area (Rohsenow, Hartnett, & Cho, 1998). The view factor is a geometrical relation and is complicated to compute for all but the most basic relations. A table of computed values can be found: Link to View Geometries (Howell, 1982).

When only two surfaces are involved, equation one is simplified:

$$\dot{Q}_{1\to2} = A_1\sigma\, T_1^4 F_{12} - A_2\sigma\, T_2^4 F_{21} = A_1\sigma\, T_1^4 F_{12}(\, T_1^4 - T_2^4) \qquad (2)$$

where

$$A_1\, F_{12} = A_2\, F_{21}.$$

Gray Radiation Heat Exchange

To accurately model the system, the differential heat-transfer equation is modified to account for absorbed and reflected radiation. This relation assumes only diffuse radiation and doesn't account for specular reflections. The following derivation is taken from the reference, where a more detailed explanation can be found (Rohsenow, Hartnett, & Cho, 1998). Following the relevant derivation from the reference is helpful in understanding and applying the general concepts,

$$q_{ok} = \epsilon_k \sigma\, T_k^4 + \rho_k q_{ik} \qquad (3)$$

$$q_k = q_{ok} - q_{ik} \qquad (4)$$

where q_k is the net flux leaving a surface, k, q_{ok} is the flux-leaving object, q_{ik} is the flux impinging upon an object, and ρ_k is the reflection coefficient.

$$q_{ok} = \epsilon_k \sigma T_k^4 + (1 - \epsilon_k) q_{ik} \qquad (5)$$

$$q_k = \frac{\epsilon_k}{1-\epsilon_k}(\, \sigma\, T_k^4 - q_{ok}) \qquad (6)$$

And from the black body relation

$$q_k A_k = \sum_{j=1}^{N} q_{oj}\ F_{j-k} A_j \qquad (7)$$

$$= A_k \sum_{j=1}^{N} q_{oj}\ F_{k-j} \qquad (8)$$

Substituting equation 5 into 8, and equation 5 into 4, gives:

$$\sum_{j=1}^{N} (\gamma_{kj} - (1 - \epsilon_k) F_{k-j}) q_{oj} = \epsilon_k \sigma T_k^4 \qquad (9)$$

$$\sum_{j=1}^{N} (\gamma_{kj} - F_{k-j}) q_{oj} = q_k \qquad (10)$$

Since only the net flow of heat is needed, eliminating q_{ik} and q_{ok} in equation 10 with equation 6 allows calculation of the net heat flow from the simplified gray body equation when *T's* are known.

$$\sum_{j=1}^{N} \left(\frac{\gamma_{kj}}{\epsilon_j} - F_{k-j} \frac{1-\epsilon_j}{\epsilon_j}\right) q_j = \sum_{j=1}^{N} (\gamma_{kj} - F_{k-j}) \sigma T_k^4 \qquad (11)$$

γ_{kj} is the Kroneker delta function, which equals 1 when k = j, and zero otherwise. Given the emissivity and view factors for a set of *N* surfaces, the net heat transfer for each surface, and any geometry, can be calculated.

Planar Geometry or One Enclosed Surface

Equation 11 can be simplified for two surfaces and solved for the unknown parameters. For simple planar geometries, or convex geometry (where one surface completely encloses the other), the view factor is equivalent to the ratio of surface areas. This modifies the emissivity to give an effective emissivity, derived from the above general equations. A common geometry for cryogenic dewars is two concentric cylinders with radius r_1, and r_2, where $r_2 > r_1$. The heat transfer equation has two solutions; one for each surface. The net heat transfer is found by multiplying the solution of a particular surface by the area of that surface.

$$F_{11} = 0, F_{12} = 1, F_{21} = \frac{r_1}{r_2}, F_{22} = 1 - \frac{r_1}{r_2}$$

Solution for surface 1:

$$\widehat{\epsilon_1} = \frac{1}{1 + \left(\frac{1}{\epsilon_1} - 1\right)\frac{r_1}{r_2} + \left(\frac{1}{\epsilon_2} - 1\right)}$$

$$\widehat{\epsilon_1} = \frac{\epsilon_1 \epsilon_2}{\epsilon_2 + \epsilon_1(1 - \epsilon_2)\frac{r_1}{r_2}}$$

Thus, the net heat flow from surface one to surface two can be calculated from

$$\dot{Q}_{2\to 1} = A_1 \hat{\epsilon}_1 \sigma(\mathrm{T}_2^4 - \mathrm{T}_1^4)$$

Radiation Resistance

From equations of gray body radiation, the concept of radiation resistance is derived. This method is useful when analyzing heat flow for a limited number of surfaces, which are completely enclosed. This is also useful for a series of flat or convex surfaces where no self radiation is exists. The net heat flow from a surface, *1,* is the difference in the black body radiation and the radiosity, *J*, divided by the radiation resistance. This is the circuit analogy to heat transfer.

$$\dot{\mathrm{Q}}_\mathrm{i} = \frac{(\sigma \mathrm{T}^4 - \mathrm{J_i})}{\mathrm{R_i}}$$

$$\mathrm{R_i} = \frac{1-\epsilon_i}{\mathrm{A_i}\epsilon_i}$$

Also, the geometry of the surfaces must be included with the addition of a view factor,

$$\frac{1}{\mathrm{A_i} F_{i,i+1}}$$

giving at total resistance

$$R = \frac{1-\epsilon_1}{A_1\epsilon_1} + \frac{1}{F_{1,2}A_2}.$$

For a series of parallel plates, the total resistance is

$$\sum_{i=1}^{N} R_i = \frac{1-\epsilon_1}{A_1\epsilon_1} + \frac{1}{A_1 F_{12}} + \frac{1-\epsilon_2}{A_2\epsilon_2} + \cdots$$

with $F_{12} = \frac{A_2}{A_1} A_1$ and $A_1 > A_2$

$$R = \frac{1-\epsilon_1}{A_1\epsilon_1} + \frac{1}{A_2} + \frac{1-\epsilon_2}{A_2\epsilon_2} + \frac{1}{A_3} + \cdots$$

Radiation Heat Transfer Equation for Super Insulation

Layers of super insulation covering the radiation shield reduces the loading dependent upon the number of layers of insulation and the emissivity of the material. The amount of reduction in loading can be calculated from the last term in the equation below (Chen & Yu, 2008).

$$R = \frac{1}{A_c}\left[1 + \left(\frac{1}{\epsilon_h} - 1\right)\frac{A_c}{A_h} + \sum_{i=1}^{N} \frac{A_C}{A_i}\frac{1}{\epsilon_r}\right]$$

The cited paper includes the emissivity in the space resistance $\frac{1}{\epsilon_i F_{i,i+1} A_i}$, rather than $\frac{1}{F_{i,i+1} A_i}$, so that Equation 10 differs from the reference. Heat transfer from the hot surface to the cold surface is:

$$\dot{Q}_{h \to c} = \frac{\sigma\left(T_h^4 - T_c^4\right)}{R}$$

where A_c and ε_c are the area and emissivity of the cold surface, A_h and ε_h are the area and emissivity of the hot surface, A_j and ε_r are the area and emissivity of each insulating sheet. Consider a cylindrical geometry, with $A_i = A_c = A_h = 1$ and $\varepsilon_r = \varepsilon_h = \varepsilon_c = 0.1$. Adding 10 layers of super insulation increases the radiation resistance from 10 to 110 and reduces the loading factor 11.

Emissivity Measurement

The challenge is to accurately determine the emissivity of metals. As the measurement emissivity depends upon the surface roughness, wavelength and polarization, a wide range of published values exist for this property. Eight samples, as indicated in Table 1 of metals typical for use in radiation shields, were submitted to the NIST laboratory for reflectance testing (Hanssen & Whilthan, 2010).

Sample	Material	Surface Preparation	Finish
1	OFHC Copper	1200 Grit Micromesh	Bright Gold
2	OFHC Copper	400 Grit Wet/Dry	Standard Gold
3	3003 Aluminum	600 Grit Wet/Dry	Electroless Nickel
4	6061-T6 Aluminum	1200 Grit Micromesh	Electroless Nickel
5	6061-T6 Aluminum	600 Grit Wet/Dry	Unplated
6	304L Stainless Steel	400 Grit Wet/Dry	Electropolish
7	304L Stainless Steel	1200 Grit Micromesh	Electropolish
8	Brass	Acid Copper Process	Copper

Table 1: Metal samples for reflectance testing

Results shown for the samples are shown in Figures 1 and 2. It is concluded that a gold finish is necessary for an emissivity of 0.05 or below. Most published results for the other materials indicate much lower emissivity than measured by NIST, thus using the published values will underestimate the calculated radiation load. Finishes other than gold improved the reflectance over the unfinished sample, but were less than the published values.

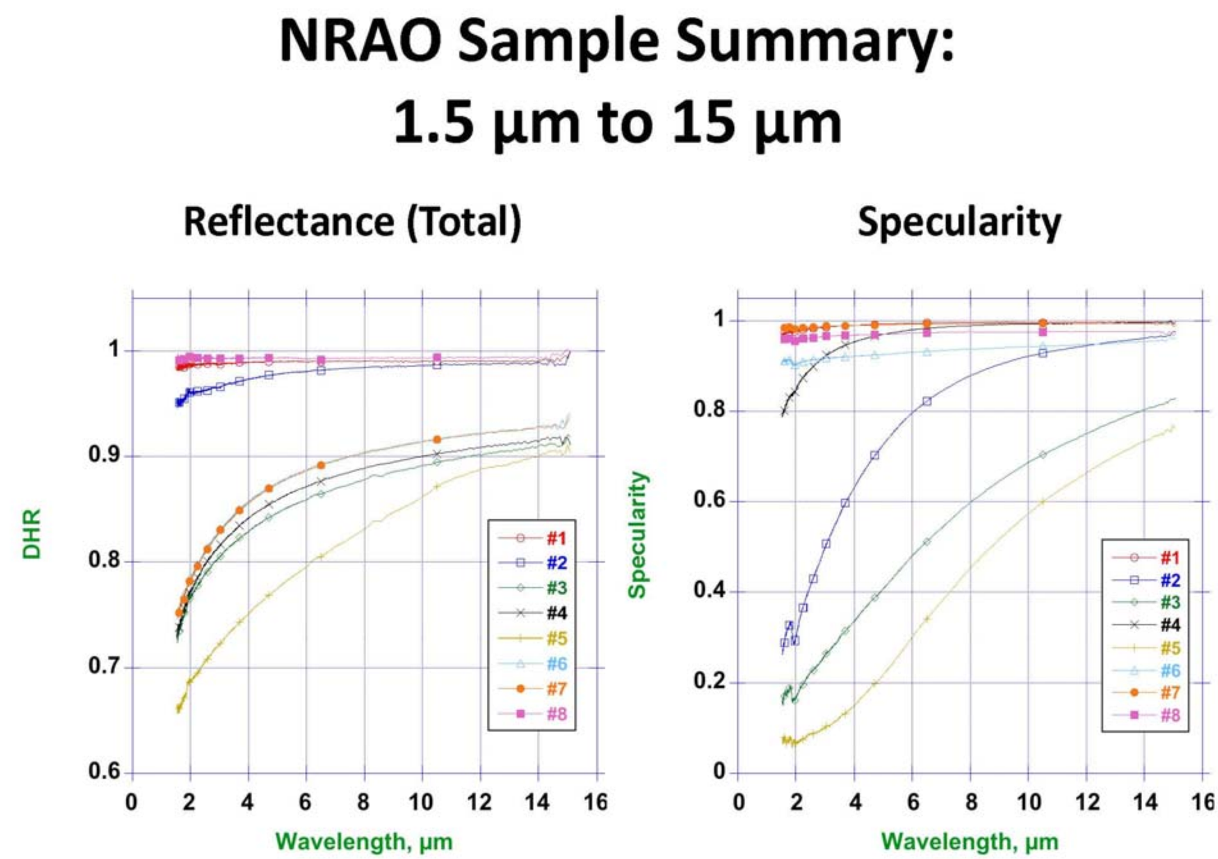


Figure 1: Results of reflectance measurements from materials of Table 1.

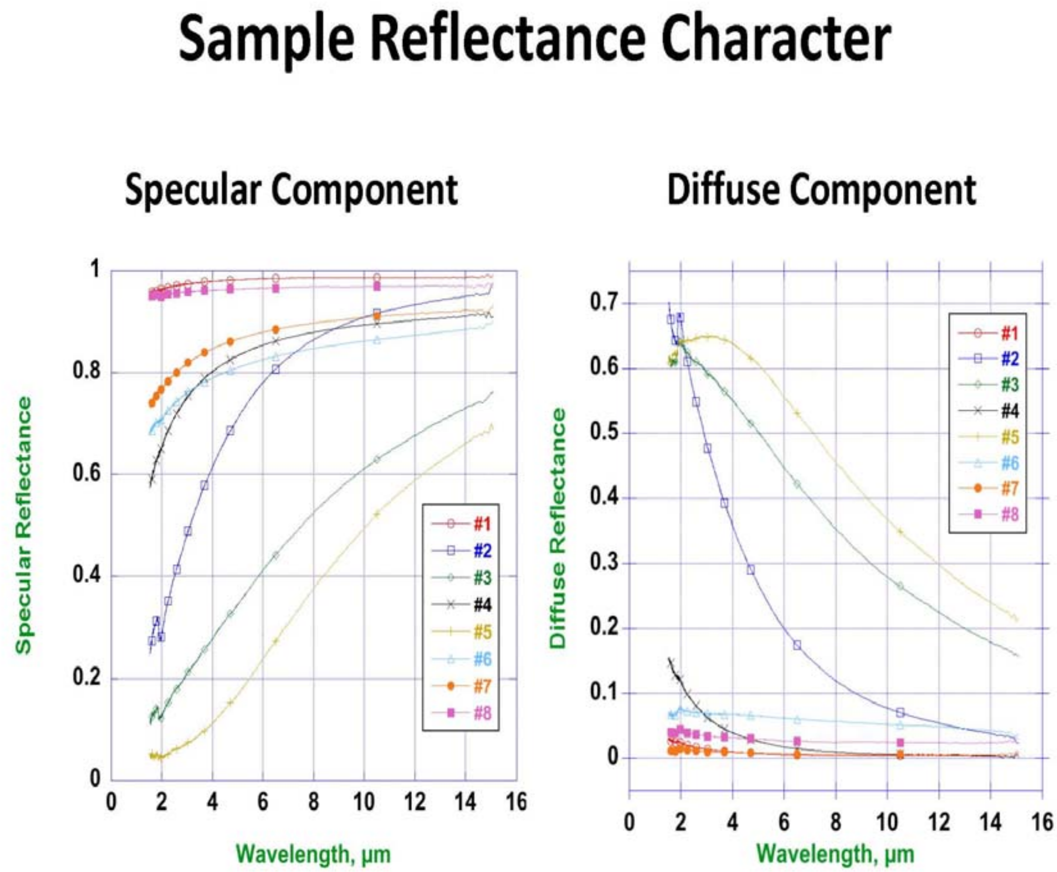


Figure 2: Measured reflectance characteristics from materials of Table 1.

Example Calculations

Model W-band Feed as Cone for Infrared Radiation Transfer.

View Factor for Cone gives the view factor for a right circular cone (feed) to base (window).

$$\mathrm{F_{c\to b}} = \frac{1}{\sqrt{1+(\frac{\mathrm{h}}{\mathrm{r}})^2}}$$

$$\mathrm{F_{c\to c}} = 1 - \mathrm{F_{c\to b}}$$

$$\mathrm{F_{c\to b}} = \frac{\mathrm{A_{cone}}}{\mathrm{A_{base}}}\mathrm{F_{b\to c}}$$

W-band horn dimensions as modeled by cone and

l=1.68 “

h=1.84 "

r_{ap}=0.406 " aperture radius

r_{wg}=0.0355 " waveguide radius

α =0.217 rad. cone half angle.

T_w= 300 ° K

T_c= 50° K

Surface Area of Cone = 0.0015 m^2 from $\pi r_{op}\left[\sqrt{(h^2+r_{ap}^2)} - \sqrt{(h^2-l^2)-r_{wg}^2}\right]$

Surface Area of Window =0.00033 m^2, ε= 0.2 for the dewar wall and ε= 0.1 for the radiation shield, and ε= 1 for the window gives:

$$\dot{Q}_{w\to c} = 0.082\, Watts$$

Modeling as a hemispherical hole underestimates the contribution from the window:

$$\dot{Q}_{w\to c} = 0.055\, Watts$$

As does modeling as parallel plates:

$$\dot{Q}_{w\to c} = 0.030\, Watts$$

K-band Focal Plane Array (KFPA) Dewar

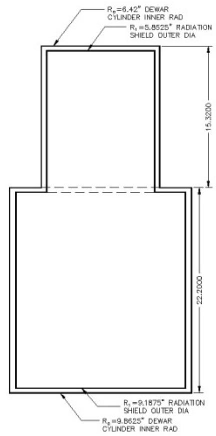


Figure 3. Model of KFPA dewar: two concentric cylinders, where the smaller cylinder is atop the larger one.

$$\hat{\epsilon_i} = \frac{\epsilon_i \epsilon_o}{\epsilon_o + \epsilon_i(1 \; - \epsilon_o)\dfrac{r_i}{r_o}}$$

$$\dot{Q}_{o\to i} = A_i \hat{\epsilon}_i \sigma(\ \mathrm{T_o^4} - \ \mathrm{T_i^4})$$

T_w= 300 ° K

T_c= 50° K

And ε_i= 0.11 for the dewar wall (sample #6 material) and ε_o= 0.11 for the radiation shield (sample #3 material). The end plates add ~7 watts, giving the total:

$$\dot{Q}_{o\to i} = 33\, Watts$$

This exceeds the first stage capacity of the Model 350 refrigerator of 25 Watts. Adding four layers of super insulation, which consists of an aluminized mylar bonded with polyester scrim. Nominal thickness is 3.5 mil (1/2 mil for the mylar + 3 mil scrim).

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The radiation resistance with $\varepsilon = 0.1$ increases from 0.021 in^{-2} to 0.12 in^{-2}. Indeed, the initial cool-down of the KFPA was unsuccessful, until four layers of super insulation were added to the radiation shield, reducing the load to

$$\dot{Q}_{o \to \iota} = 6\, Watts$$

Conclusion

Modeling of the radiation loading in cryogenic dewars with programs such as Mathematica accurately estimate the heat load for different geometries. Care must be taken in selecting view factors that best describe the geometries involved. With the proper selections, a series of equations are solved for the net heat flows, given the temperature for each surface. The greatest error is from the unknown reflectance (emissivity) of the materials. Measurements of this property indicate that the gold plated surfaces are near published values over the wavelength of interest. The other coatings are typically less reflective than the published values.

Acknowledgements

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Works Cited

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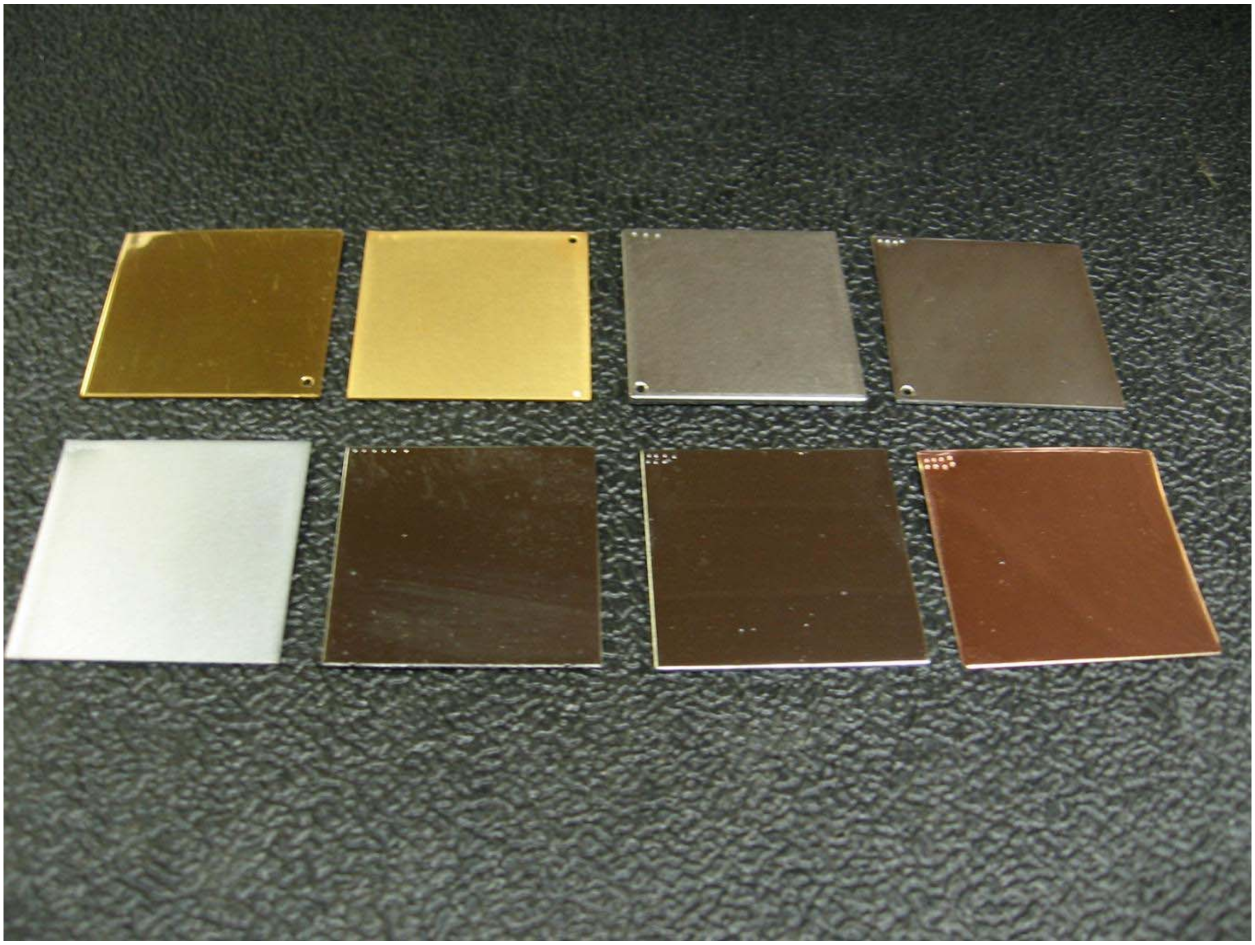
Rohsenow, W. M., Hartnett, J. R., & Cho, Y. I. (1998). *Handbook of Heat Transfer (3rd Edition).* McGraw Hill.


Picture of Emissivity Measurement Samples. Sample 1 is top left, going in order, with sample 8 to the bottom right.

Mathematic Code for Solving Heat Transfer Equation

$\mathrm{ClearAll}[\mathrm{eq}, q, \epsilon, q, T, \mathrm{hc}, r, \sigma, \mathrm{FV}]$

$\mathrm{FV}_{1,2} = 1$; where 1 is the inner wall of a cylinder and 2 is the outer wall;

$\mathrm{FV}_{1,1} = 1 - \mathrm{FV}_{1,2};$

$\mathrm{FV}_{2,1} = \frac{r_1}{r_2};$

$\mathrm{FV}_{2,2} = 1 - \mathrm{FV}_{2,1};$

$$\mathrm{For}[k = 1, k < 3, k++, \mathrm{eq}_k = \sum_{j=1}^{2} ((\mathrm{KroneckerDelta}[k, j]/\epsilon_j - \mathrm{FV}_{k,j} \frac{1 - \epsilon_j}{\epsilon_j}) q_j - (\mathrm{KroneckerDelta}[k, j] - \mathrm{FV}_{k,j}) \sigma T_j{}^4)]$$

$$\mathrm{heatsolutions} = \mathrm{Solve}[\{\mathrm{eq}_1 == 0, \mathrm{eq}_2 == 0\}, \{q_1, q_2\}]$$

$$\{\{q_1 \to -\frac{\sigma r_2 T_1^4 \epsilon_1 \epsilon_2 - \sigma r_2 T_2^4 \epsilon_1 \epsilon_2}{-r_1 \epsilon_1 - r_2 \epsilon_2 + r_1 \epsilon_1 \epsilon_2}, q_2 \to \frac{(\sigma r_1 T_1^4 - \sigma r_1 T_2^4) \epsilon_1 \epsilon_2}{-r_1 \epsilon_1 - r_2 \epsilon_2 + r_1 \epsilon_1 \epsilon_2}\}\}$$