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# ACCURACY OF NOISE TEMPERATURE MEASUREMENT OF CRYOGENIC AMPLIFIERS

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## Table of Contents

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## 1. Introduction

This report addresses the issue of measurement errors encountered in the noise measurement of cryogenic amplifiers and systems. Two recent publications from Hewlett-Packard [3], [4] address the problem of accuracy of noise temperature measurement for room temperature applications. In one of these [3], the measurement uncertainty curves were given for an example of a room temperature measurement system. These curves were obtained by the Monte Carlo simulation of the measurement procedure and associated errors [3].

This report extends the Monte Carlo modeling approach to the measurement procedures and measurement systems used at NRAO for evaluation of cryogenic devices, amplifiers and receivers. The following sources of error encountered in the determination of noise temperature of a device under test (DUT) are taken into account:

- non-zero reflection coefficient of noise sources for "hot" and "cold" states, including changes in impedance of noise diode in "off" and "on" states,
- uncertainty in determination of temperatures and/or equivalent temperatures of noise sources,
- receiver nonlinearity,
- uncertainty in determination of a change in receiver gain between calibration and measurement,
- uncertainty in determination of a receiver noise temperature, and
- uncertainty due to a finite integration time for a given IF bandwidth.

Influence of these errors on the total measurement error could be strongly dependent on the noise and signal properties of a device (amplifier, system) under test. An obvious example would be the influence of DUT gain on the error caused by the uncertainty in determination of receiver noise temperature. A not so obvious, yet very important example, would be the influence of DUT input return loss on the error caused by the different reflection coefficient of "hot" and "cold" loads or the change in the noise diode impedance between "off" and "on" states.

Monte Carlo models of noise sources and their calibration procedures are described in Section 2 of this report. A typical measurement procedure of cryogenic amplifiers which uses noise diode-cold attenuator cascade as a noise source is discussed in Section 3. A measurement procedure employing two cold noise sources of different temperatures is discussed in Section 4. The Monte Carlo simulations of errors encountered in both procedures are given for an example of a cryogenic L-band amplifier with 25 dB of gain, noise temperature of 4 K and input return loss of 2 dB.

## 2. Characterization of Errors Connected with Noise Sources

### 2.1 Changes in Noise Diode Impedance in "Off" and "On" State

The L-band measurement system used at the NRAO CDL employs an HP-346-B noise diode with ENR of about 15.2 dB. This value of ENR for a typical amplifier would result in the ratio of noise powers (Y-factor) exceeding the dynamic range of the square-law detector and, therefore, 15 dB and 20 dB pads are used for room and cryogenic measurement, respectively.

Table I presents the results of the network analyzer measurement of the noise diode (SN/2037A00772) reflection coefficient in the "on" and "off" state for .5-2.5 GHz frequency range. The corresponding data for a noise diode with 15 dB and 20 dB attenuator are given in Table II.

The reflection coefficient for the noise diode varies significantly upon switching between "on" and "off" states. For the purpose of Monte Carlo simulations, this diode is modeled by two different reflection coefficients $\Gamma_{on}$ and $\Gamma_{off}$. The magnitude of $\Gamma_{on}$ is chosen as the worst case of measured values, and its phase is a random number with uniform distribution from $[0, 2\pi]$. That is:

$$\Gamma_{on} = |\Gamma_{max}| \, e^{-j\phi_{on}} \tag{1}$$

The data presented in Tables I and II indicate that the difference $|\Gamma_{on} - \Gamma_{off}|$ varies very little in the band for all noise sources. It is obviously much smaller for the "padded" noise diode and it differs from that of the diode alone by twice the attenuation of the pad. Therefore, the reflection coefficient, $\Gamma_{off}$, can be modeled by a sum of $\Gamma_{on}$ and a vector with magnitude equal to the worst case of the measured difference, and a random phase with uniform distribution from $[0, 2\pi]$. That is:

$$\Gamma_{off} = \Gamma_{on} + |\Gamma_{diff}| \exp(-j\phi_{diff}) \tag{2}$$

This model may be used only for noise diodes as there is strong correlation between the reflection coefficients in "on" and "off" states. It may not represent two different waveguide or coaxial loads, or two absorbers in front of a horn, having different physical temperatures. In this case, both reflection coefficient $\Gamma_h$ ("hot") and $\Gamma_c$ ("cold") are independent and can be modeled by the vectors with the worst case magnitude and uniformly distributed random phase (compare (1)).

TABLE I. Return Loss of Noise Diode in "On" and "Off" State.

FILE(ON) : NSODBON HEADER: NOISE SOURCE WHITH 0 dB ON
FILE(OFF): NSODBOFF HEADER: NOISE SOURCE WITH 0 dB AT. OFF

| | NOISE ON | | NOISE OFF | | DIFER. | |
|---|---|---|---|---|---|---|
| FREQ | MOD(dB) | PHASE | MOD(dB) | PHASE | MOD(dB) | PHASE |
| .500 | 28.41 | 102.60 | 41.82 | -85.06 | 26.74 | 101.25 |
| .700 | 28.01 | 74.85 | 42.76 | -133.45 | 26.69 | 70.57 |
| .900 | 27.77 | 48.98 | 39.89 | 170.69 | 26.56 | 38.41 |
| 1.100 | 27.69 | 25.24 | 36.54 | 126.00 | 26.67 | 6.85 |
| 1.300 | 27.92 | 1.79 | 33.87 | 95.22 | 26.73 | -24.24 |
| 1.500 | 28.38 | -21.48 | 31.75 | 68.88 | 26.71 | -55.54 |
| 1.700 | 29.13 | -45.29 | 30.26 | 43.23 | 26.76 | -87.22 |
| 1.900 | 30.29 | -69.44 | 29.29 | 19.25 | 26.85 | -118.49 |
| 2.100 | 31.81 | -95.87 | 28.76 | -3.99 | 26.88 | -149.48 |
| 2.300 | 33.75 | -125.92 | 28.59 | -27.70 | 26.94 | 179.16 |
| 2.500 | 35.88 | -161.18 | 28.83 | -50.51 | 27.04 | 149.25 |

TABLE II. Return Loss of Noise Diode with 15 dB and 20 dB Attenuator in "On" and "Off" State.

FILE(ON) : NS15DBON HEADER: NOISE SOURCE + 15 dB ON
FILE(OFF): NS15DBOFF HEADER: NOISE SOURCE +15 dB OFF

| | NOISE ON | | NOISE OFF | | DIFER. | |
|---|---|---|---|---|---|---|
| FREQ | MOD(dB) | PHASE | MOD(dB) | PHASE | MOD(dB) | PHASE |
| .500 | 36.12 | -10.86 | 36.37 | -16.56 | 55.94 | 60.04 |
| .700 | 35.83 | -19.89 | 36.58 | -23.38 | 55.67 | 13.25 |
| .900 | 35.55 | -29.76 | 36.43 | -29.30 | 55.88 | -34.08 |
| 1.100 | 35.79 | -39.98 | 36.43 | -35.99 | 55.95 | -81.10 |
| 1.300 | 35.68 | -45.88 | 35.73 | -40.28 | 55.90 | -129.95 |
| 1.500 | 36.14 | -52.82 | 35.65 | -48.23 | 56.07 | -175.76 |
| 1.700 | 36.58 | -55.65 | 35.76 | -54.56 | 56.56 | 136.36 |
| 1.900 | 36.67 | -58.68 | 35.91 | -61.67 | 56.15 | 88.83 |
| 2.100 | 36.63 | -62.89 | 36.37 | -68.53 | 56.26 | 41.39 |
| 2.300 | 36.51 | -66.93 | 36.91 | -72.41 | 56.18 | -5.61 |
| 2.500 | 36.19 | -72.20 | 37.00 | -74.60 | 56.41 | -49.12 |

FILE(ON) : NS20DBON HEADER: NOISE SOURCE + 20 dB ON
FILE(OFF): NS20DBOFF HEADER: NOISE SOURCE + 20 dB OFF

| | NOISE ON | | NOISE OFF | | DIFER. | |
|---|---|---|---|---|---|---|
| FREQ | MOD(dB) | PHASE | MOD(dB) | PHASE | MOD(dB) | PHASE |
| .500 | 65.20 | 65.17 | 81.26 | 90.49 | 66.51 | 60.68 |
| .700 | 75.15 | -88.84 | 65.38 | -152.67 | 66.25 | 8.53 |
| .900 | 64.31 | -97.14 | 65.27 | -144.92 | 66.54 | -38.10 |
| 1.100 | 59.48 | -124.03 | 61.49 | -148.68 | 66.75 | -74.19 |
| 1.300 | 59.70 | -104.10 | 63.70 | -84.36 | 66.47 | -131.75 |
| 1.500 | 59.52 | -131.95 | 61.57 | -106.99 | 66.69 | 178.45 |
| 1.700 | 65.65 | -126.49 | 62.96 | -92.49 | 67.88 | 133.77 |
| 1.900 | 69.41 | -115.43 | 62.31 | -100.41 | 66.96 | 90.87 |
| 2.100 | 76.92 | -61.09 | 65.90 | -122.43 | 66.82 | 41.64 |
| 2.300 | 70.37 | -7.86 | 75.25 | 177.61 | 66.46 | -5.87 |
| 2.500 | 73.93 | 1.97 | 70.84 | 92.83 | 69.05 | -52.43 |

## 2.2 Calibration of Noise Source Equivalent Temperatures

If the ENR of the noise diode is not known with sufficient accuracy, it is worthwhile to calibrate it against hot and cold load standards. Usually this is done with two matched terminations, one at the room temperature and the other at the $LN_2$ temperature. The noise output of the noise diode in "on" and "off" states is compared with the noise output of the standards with a precision receiver. The schematic view of the measurement procedure is presented in Figure 1.

The noise power measured by the receiver with each termination is [1], [3], [4]:

$$N_1 = kBG_o \frac{(1 - |\Gamma_c|^2)(1 - |\Gamma_R|^2)}{|1 - \Gamma_c \Gamma_R|^2} (T_R(\Gamma_c) + T_c) + \Delta N_1$$

$$N_2 = \left[ kBG_o \frac{(1 - |\Gamma_h|^2)(1 - |\Gamma_R|^2)}{|1 - \Gamma_h \Gamma_R|^2} (T_R(\Gamma_h) + T_h) + \Delta N_2 \right] L_{i2}$$

$$N_3 = kBG_o \frac{(1 - |\Gamma'_c|^2)(1 - |\Gamma_R|^2)}{|1 - \Gamma'_c \Gamma_R|^2} (T_R(\Gamma'_c) + T'_c) + \Delta N_3 \qquad (3)$$

$$N_4 = \left[ kBG_o \frac{(1 - |\Gamma'_h|^2)(1 - |\Gamma_R|^2)}{|1 - \Gamma'_h \Gamma_R|^2} (T_R(\Gamma'_h) + T'_h) + \Delta N_4 \right] L_{i4}$$

where k is the Boltzman constant, B is the noise bandwidth and $G_o$ is the power gain of the receiver (power measured/power delivered to the input). $T_h$ and $T_c$ are the temperatures of hot and cold standards, respectively. $T_h'$ and $T_c'$ are equivalent noise temperatures of the noise diode in "on" and "off" state, respectively. Each noise source has an associated reflection coefficient $\Gamma_h$, $\Gamma_c$, $\Gamma_h'$ and $\Gamma_c'$, respectively. The factors $L_{i2}$ and $L_{i4}$ represent receivers nonlinearity, while $\Delta N_1 \ldots \Delta N_4$ represent uncertainty resulting from a finite integration time.

For the purpose of MC simulation, the noise source and noise standards reflection coefficients are modeled as described in previous

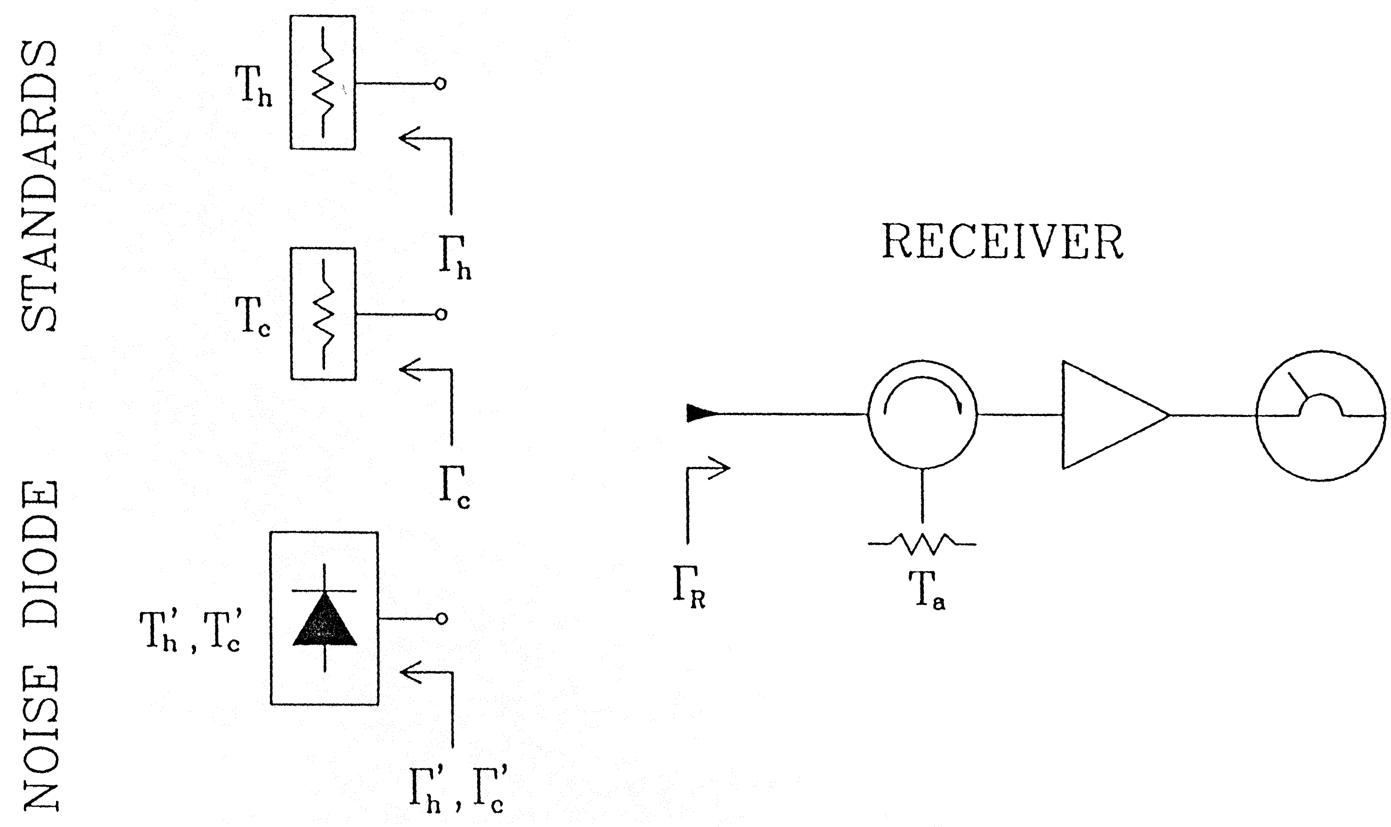



Fig. 1. A schematic view of the noise diode calibration procedure.

sections. The standards' temperatures $T_h$ and $T_c$ are random generated values having Gaussian distribution with:

$$\text{Mean } (T_h) = [T_h]_s \quad ; \quad 3\sigma(T_h) = \text{Error } [T_h]$$
$$\text{Mean } (T_c) = [T_c]_s \quad ; \quad 3\sigma(T_c) = \text{Error } [T_c] \qquad (4)$$

where $[\ ]_s$ indicates the value which is assumed for the standard.

The receiver reflection coefficient $\Gamma_R$ is modeled as a complex number with the magnitude equal to the worst case measured value and a random phase with uniform distribution. The model of receiver nonlinearity assumes for $L_{i2}$, $L_{i4}$ the uniform distribution (in dB) between $-L_{imax}$ and 0, where $L_{imax}$ is maximum receiver compression. $\Delta N_1$, $\Delta N_2$, $\Delta N_3$ and $\Delta N_4$ represent the uncertainty resulting from a finite integration time. In Monte Carlo model each one assumes a normal distribution with:

$$\text{Mean } (\Delta N_i) = 0 \qquad\qquad \sigma(\Delta N_i) = \frac{N_i}{\sqrt{Bt}}$$

where B is the noise bandwidth in Hz and t is the integration time in seconds.

The noise temperature of the receiver with isolator at the input is given by [2], [6]:

$$T_R(\Gamma_s) = T_{min} + (T_a + T_{min}) \frac{\left|\Gamma_s - \Gamma_R^*\right|^2}{(1 - \left|\Gamma_s\right|^2)(1 - \left|\Gamma_R\right|^2)} \qquad (6)$$

where $T_{min}$ is the minimum receiver noise temperature, $T_a$ is the physical temperature of the isolator and $\Gamma_s$ is the reflection coefficient of the source.

If in the measurement procedure of the equivalent noise temperature of the unknown source, the effects of reflection coefficients and non-linearities are neglected and $T_R$ is assumed to be constant, then the measured values with source "on" and "off" are given by:

$$[T_h']_M = [T_h]_s \frac{N_4 - N_1}{N_2 - N_1} + [T_c]_s \frac{N_2 - N_4}{N_2 - N_1} \qquad (7)$$

$$[T_c']_M = [T_h]_s \frac{N_3 - N_1}{N_2 - N_1} + [T_c]_s \frac{N_2 - N_3}{N_2 - N_1} \qquad (7)$$

where $[\ ]_s$ indicates the value which is assumed for the standard (not the random generated value), and $[\ ]_M$ indicates "measured value". In Monte Carlo simulation of the measurement procedure $N_1, \ldots N_4$ given by (3), assume random values according to models discussed in the preceding. Therefore, a set of possible "measurement" results can be computed from (7). A program "NCAL2D" was developed to perform the simulation for a sufficiently large number of "measurement" runs. As a result, the following parameters are computed:

a) Value of each parameter without error

b) MEAN: Most probable value of the measured parameter

c) SDEV: Standard deviation around the mean value

d) OFST: Difference between b) and a)

e) MAXIM: Maximum "measured" value in simulation

f) MDEV+: Difference between e) and a)

g) MINIM: Minimum "measured" value in simulation

h) MDEV-: Difference between a) and g)

as well as

$$T_x = T_h' - T_c' \qquad (8)$$

$$NdB = 10 \log \frac{T_x}{290}$$

The results of a simulation run are summarized in Table III for a measurement system with the following parameters:

<u>Noise Standards</u>:

$[T_h]_s$ = 304.5 K    Error $[T_h]$ = 1 K    $|\Gamma_h|$ = 30 dB

$[T_c]_s$ = 82 K    Error $[T_c]$ = 1 K    $|\Gamma_c|$ = 30 dB

TABLE III. Noise Diode Calibration Simulated by Monte Carlo Method.

CALIBRATION NOISE SOURCES:

| | | |
|---|---|---|
| 1) Th = 304.50 | 2) Err= 1.00 | 3) RL(dB)= 30.00 |
| 4) Tc = 82.00 | 5) Err= 1.00 | 6) RL(dB)= 30.00 |

UNKNOWN NOISE SOURCE:

| | |
|---|---|
| 7) Th' = 394.70 | 8) RL(dB)= 30.00 |
| 9) Tc' = 297.00 | 10) RC(dB)= 66.00 |

RECEIVER

| | |
|---|---|
| 11) Ta= 297.00 | 12) IRL(dB)= 20.00 |
| 13) Tr= 500.00 | 14) Lin(dB)= 0.00 |
| 15) BW(MHz)= 30.00 | 16) It(Sec)= .50 |

| | | | | | |
|---|---|---|---|---|---|
| Th' = | 394.70 | | | | |
| MEAN= | 394.81 | MAXIM= | 396.73 | MINIM= | 392.87 |
| SDEV= | .88 | MDEV+= | 1.92 | MDEV-= | 1.94 |
| OFST= | -.11 | | | | |
| Tc' = | 297.00 | | | | |
| MEAN= | 297.05 | MAXIM= | 299.27 | MINIM= | 295.49 |
| SDEV= | .48 | MDEV+= | 2.22 | MDEV-= | 1.57 |
| OFST= | -.05 | | | | |
| Tx= | 97.70 | | | | |
| MEAN= | 97.75 | MAXIM= | 99.47 | MINIM= | 96.08 |
| SDEV= | .72 | MDEV+= | 1.71 | MDEV-= | 1.67 |
| OFST= | -.05 | | | | |
| NdB= | -4.73 | | | | |
| MEAN= | -4.72 | MAXIM= | -4.65 | MINIM= | -4.80 |
| SDEV= | .03 | MDEV+= | .08 | MDEV-= | .07 |
| OFST= | -0.00 | | | | |

<u>Noise Diode with Attenuator</u>:

$[T_h']_s$ = 394.7 K    $|\Gamma_{max}|$ = 30 dB

$[T_c']_s$ = 297 K    $|\Gamma_{diff}|$ = 66 dB

<u>Receiver</u>:

$T_{min}$ = 500 K    $T_a$ = 297 K    $|\Gamma_R|$ = 20 dB

B = 30 MHz    t = 0.5 sec

These parameters are typical for the measurement system used for the L-band amplifier testing at the NRAO CDL. As a result, the <u>peak-to-peak</u> error ($3\sigma$) in the measured equivalent temperature of noise diode-attenuator tandem in a "hot" state is about 2.64 K. Examples of the dependence of this error on some parameters of the measurement system (with others kept constant) are shown in Figures 2, 3 and 4. The system is very sensitive to error in $[T_h]_s$ (hot standard temperature accuracy) but not very sensitive to $[T_c]_s$ (cold standard temperature accuracy). It is very important to keep the reflections of the standards as low as possible, but values better than 30 dB do not improve accuracy as the other error sources tend to dominate.

For cryogenic measurements the attenuator at the noise diode output is usually cooled to the same physical temperature as the amplifier under test. This is done to improve accuracy of measurement by greatly reducing the value of $T_c$ and removing uncertainty connected with the dewar transition. In fact, it becomes now a part of a noise source composed of the noise diode at room temperature, dewar transition and cold attenuator. The values of equivalent temperatures in the "on" and "off" state when the attenuator and noise diode are at different physical temperatures are given by:

$$T_{cold} = T_{off}/L + (1 - 1/L)\, T_{ATT}$$
$$T_{hot} = T_x \frac{L'}{L} + (1 - 1/L)\, T_{ATT} \qquad (9)$$

where:

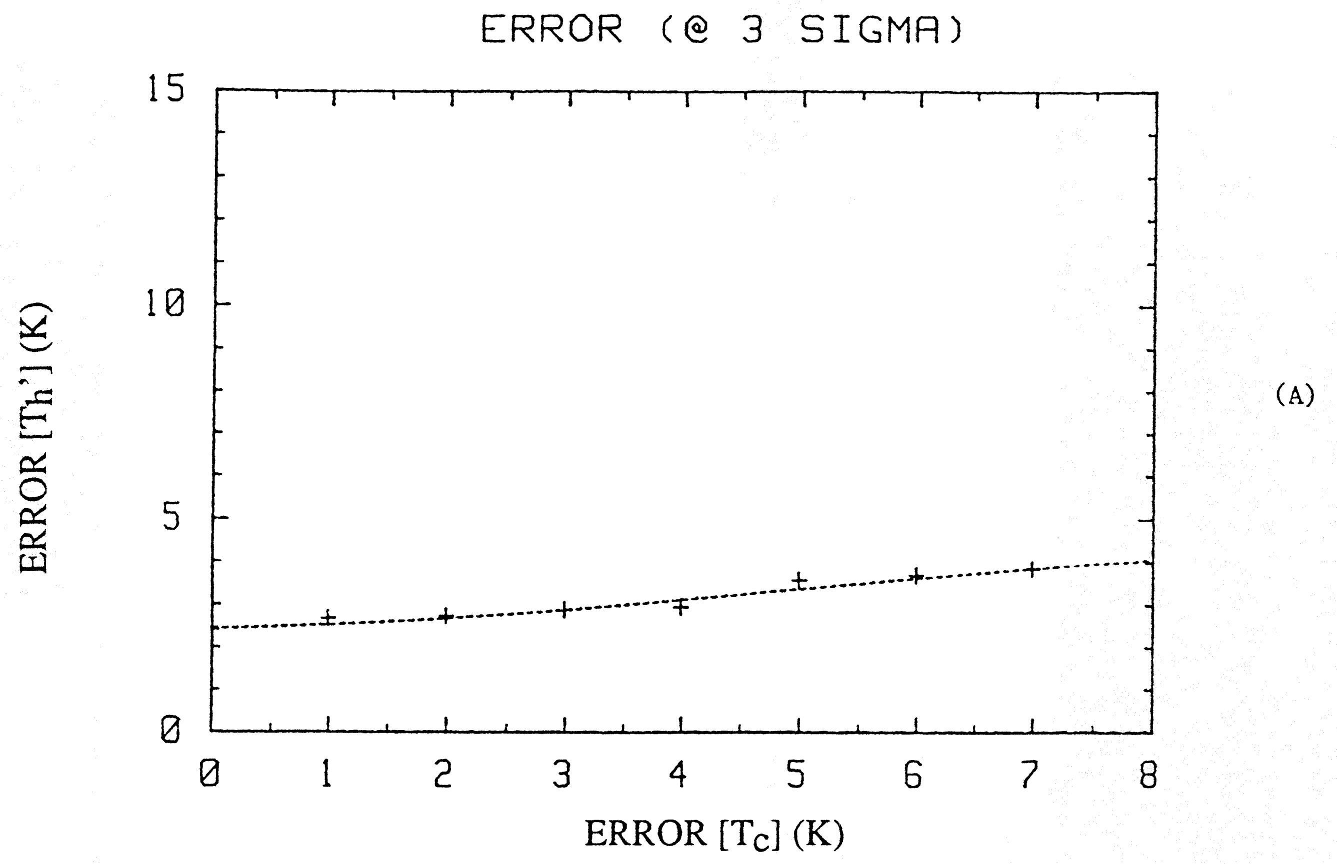
$T_{off}$ = ambient temperature of the noise diode

$T_x$ = defined in (8), measured at room temperature

$T_{ATT}$ = physical temperature of the attenuator

$L'$ = attenuation value at room temperature

$L$ = attenuation value at $T_{ATT}$



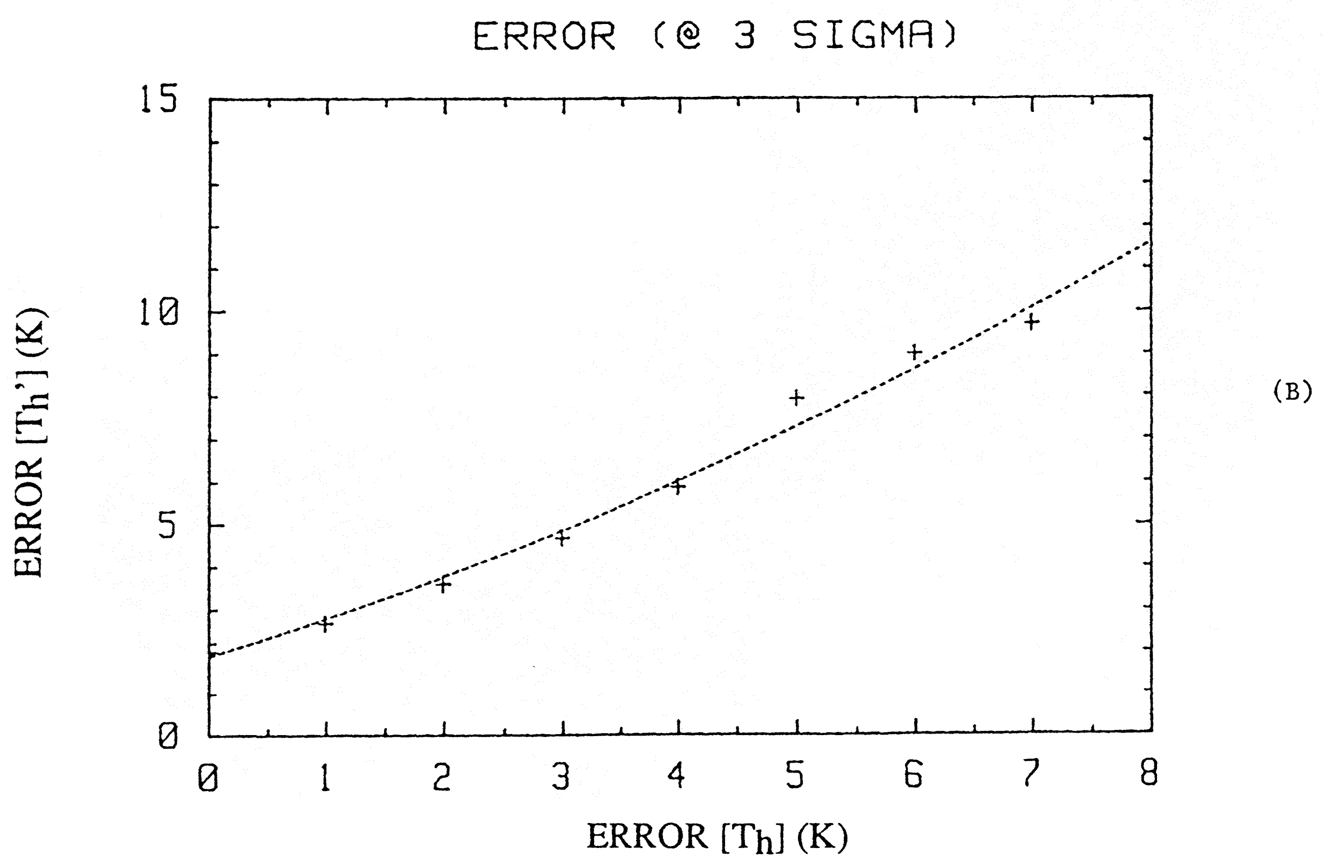
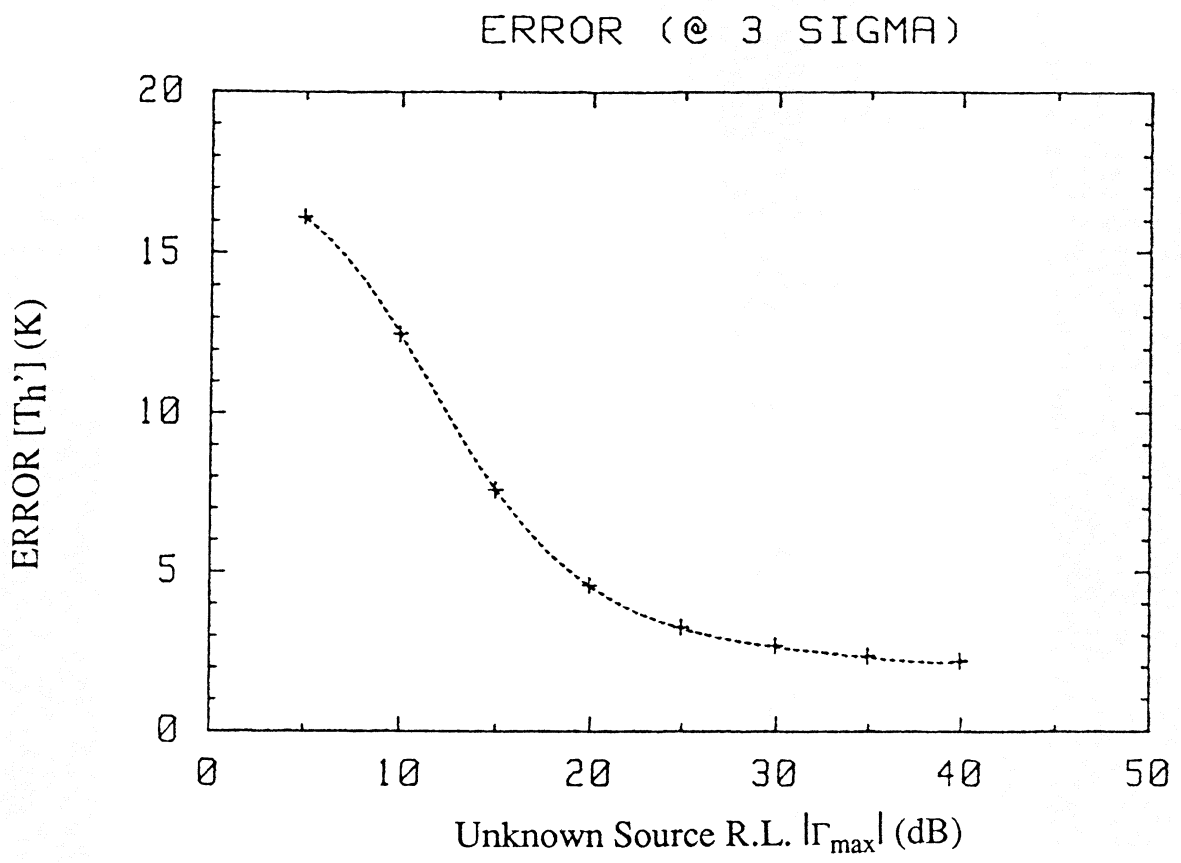
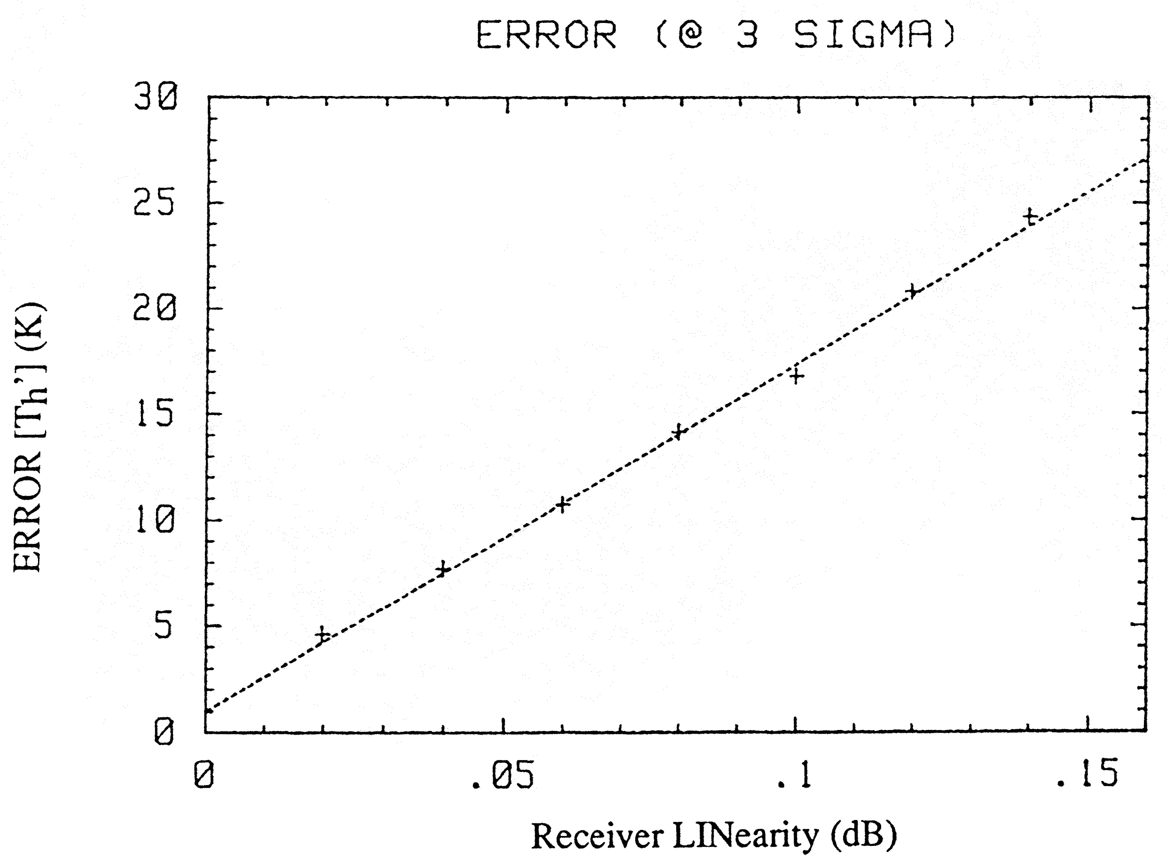




Fig. 2. Error in determination of the equivalent noise temperature for diode in "on" state as a function of accuracy of the cold (A) and hot (B) noise standards.

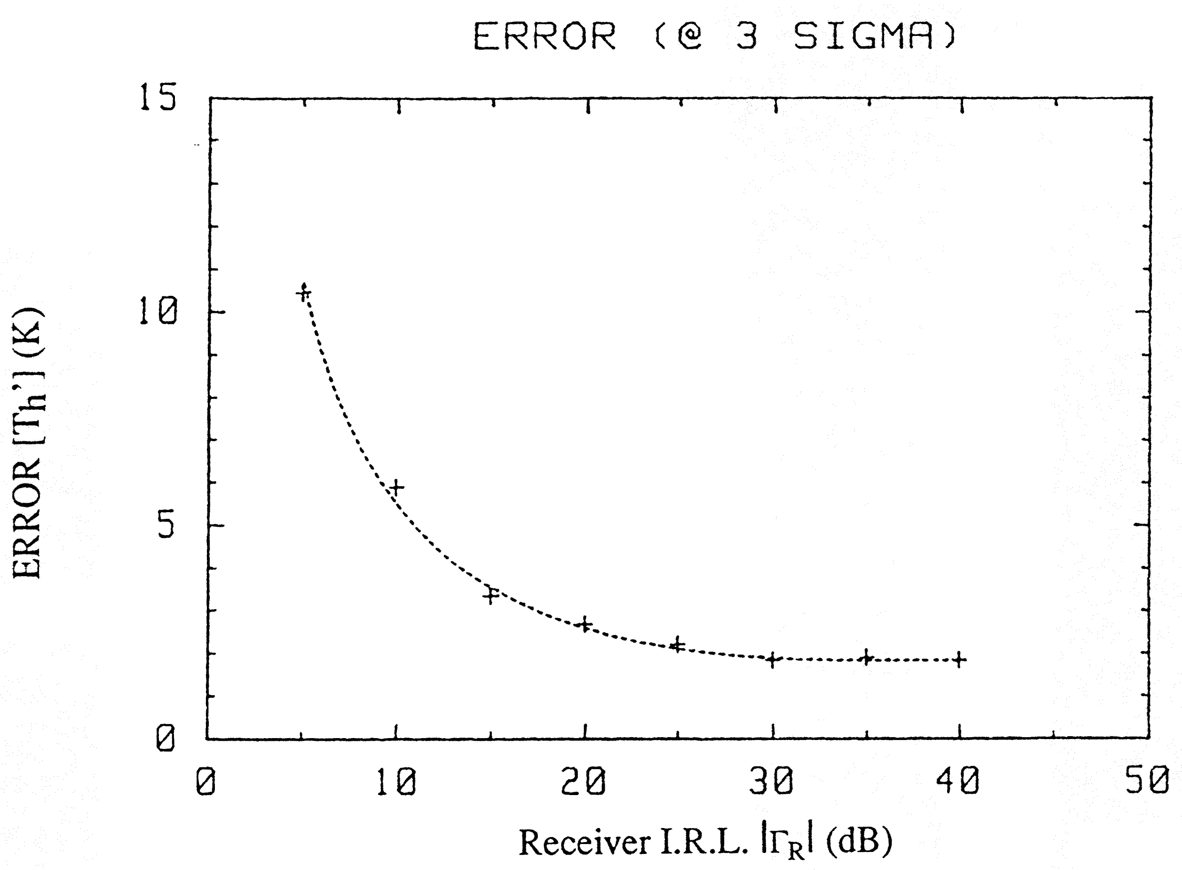



(A)

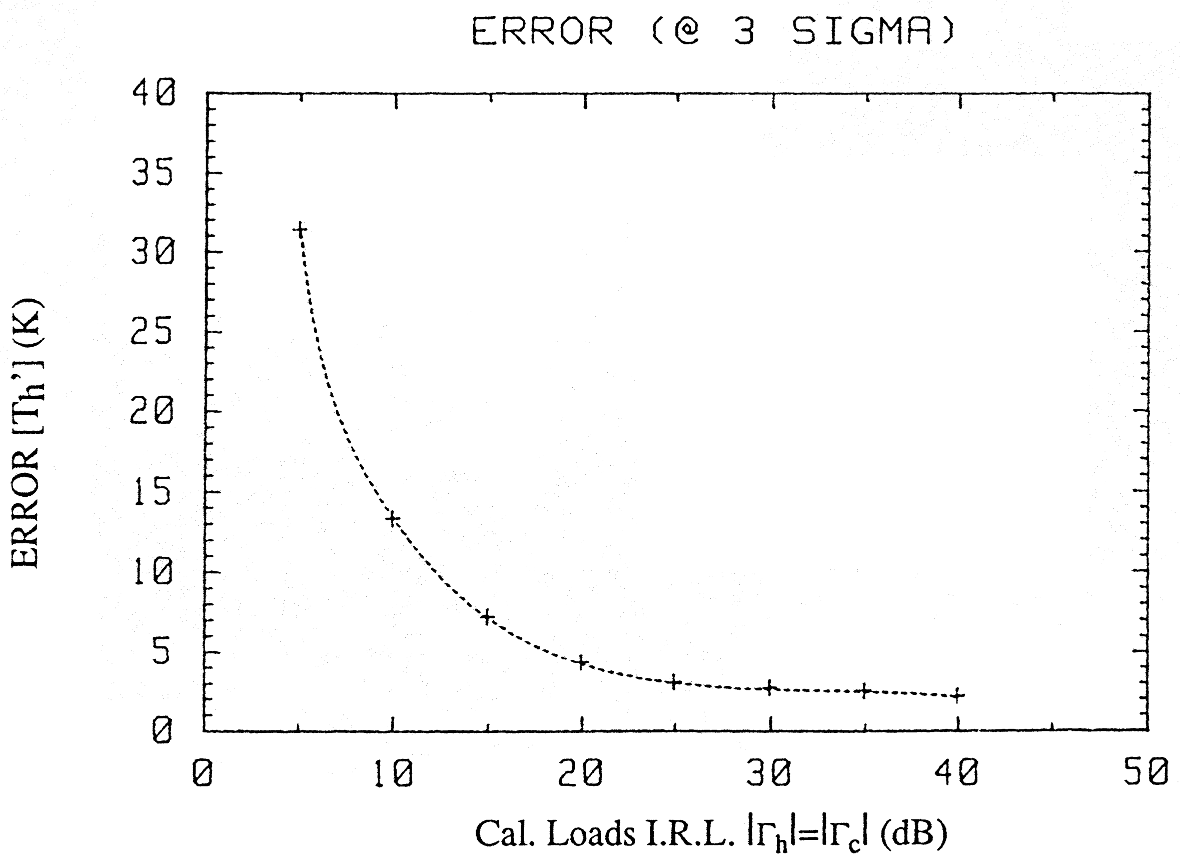



(B)

Fig. 3. Error in determination of equivalent noise temperature for diode in "on" state as a function of the return loss reflection of the noise diode (A) and receiver linearity (B).




(A)




(B)

Fig. 4. Error in determination of equivalent noise temperature for diode in "on" state as a function of input return loss of the receiver (A) and return loss of noise standards (B).

The additional uncertainty in calibrating this "composite" noise source is caused by a change in the attenuation of the attenuator and dewar transition lines upon cooling. The accuracy of $T_{cold}$ and $T_{hot}$ can be estimated in the same way as previously. Random values are generated for $T_{off}$, $T_x$, L, and $T_{ATT}$ with normal distribution. The uncertainty with which these parameters are known is assumed to be at $3\sigma$. The results of the Monte Carlo simulation for the L-band cryogenic measurement setup are summarized in Table IV. The dependence of the total error on the attenuation error is presented in Figure 5. The influence of that error on the $T_{cold}$ uncertainty is rather small, and it is quite strong on the uncertainty of $T_{hot}$.

3. Accuracy of Amplifier Measurement Using Noise Diode

A typical measurement of an amplifier noise temperature is usually done in two steps. First, the receiver is calibrated for noise and gain. Then, the amplifier is measured and the results are corrected for receiver contribution. Because of the inaccessibility of the cryogenic port for a receiver calibration, two different noise sources are used, one for calibration and the other for measurement. When an amplifier is inserted after receiver calibration had been performed, the receiver gain is lowered by about the value of amplifier gain with a precision attenuator. It introduces an additional error caused by an uncertainty in receiver gain determination.

A schematic view of the calibration procedure is shown in Figure 6. A comparison with Figure 1 demonstrates that this procedure is almost identical to that of a first step of the noise diode calibration procedure which was discussed in Section 2.2. Only the MC model for the reflection coefficient of the noise source is different in both cases, as discussed in Section 2.1.

A schematic view of the measurement procedure is shown in Figure 7. An amplifier under test is inserted and the total receiver gain is reduced by I.F. attenuator. The "cold attenuator" noise source is used at the amplifier input. The output noise power measurements in "on" and "off" state of the noise diode are given by:

$$N_2' = (K\ B\ G_o\ \Delta G_o\ G_{TDUT}(\Gamma_{sh}'))\ (T_h' + T_{sys}(\Gamma_{sh}')) + \Delta N_2')\ LIN'$$

$$N_1' = (K\ B\ G_o\ \Delta G_o\ G_{TDUT}(\Gamma_{sc}'))\ (T_h' + T_{sys}(\Gamma_{sc}')) + \Delta N_1') \qquad (9)$$

where $\Delta G_o$ is the receiver gain uncertainty and all other symbols possess their usual meaning.

The system noise temperature is given by:

TABLE IV. Monte Carlo Simulation of Calibration of "Cold Attenuator" Noise Source.

NOISE SOURCE:

| | | | |
|---|---|---|---|
| 1) Tx= | 97.70 | 2) Err= | 2.50 |
| 3) Tc= | 297.00 | 4) Err= | 1.00 |

ATENNUATOR:

| | | | |
|---|---|---|---|
| 5) L(dB)= | 20.00 | 6) Err= | .10 |

CRYO. TEMP.

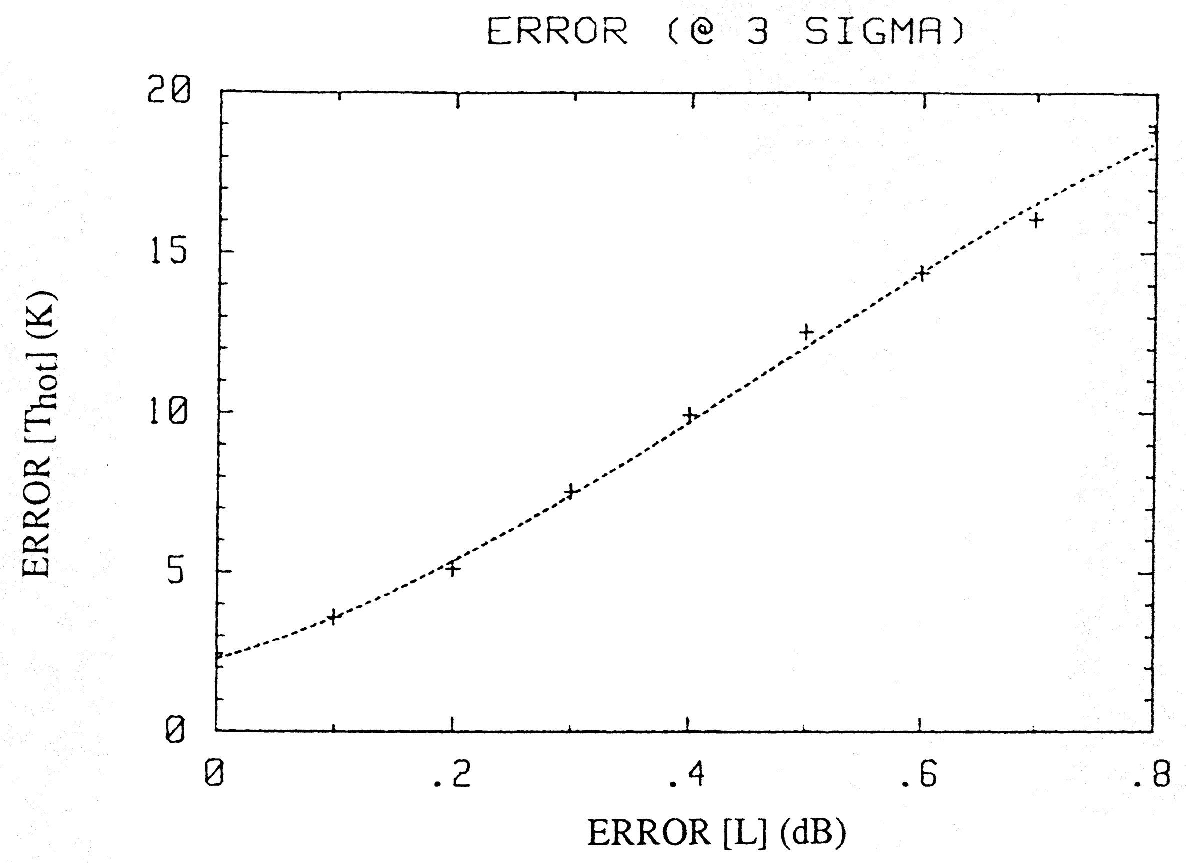
| | | | |
|---|---|---|---|
| 7) Ta= | 12.50 | 8) Err= | .50 |

Thot= 110.08

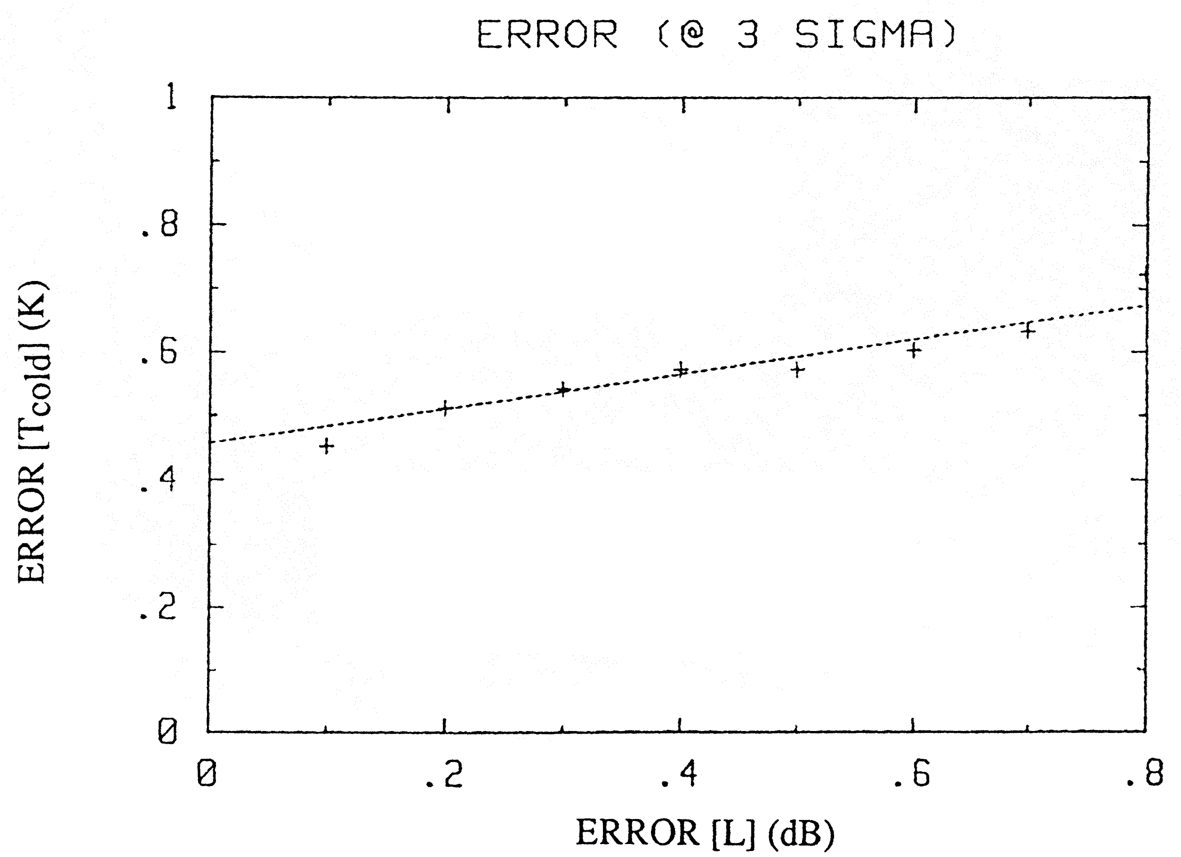
| | | | | | |
|---|---|---|---|---|---|
| MEAN= | 110.08 | MAXIM= | 113.39 | MINIM= | 106.11 |
| SDEV= | 1.12 | MDEV+= | 3.31 | MDEV-= | 3.97 |
| OFST= | -0.00 | | | | |

Tcold= 15.35

| | | | | | |
|---|---|---|---|---|---|
| MEAN= | 15.34 | MAXIM= | 15.96 | MINIM= | 14.88 |
| SDEV= | .17 | MDEV+= | .62 | MDEV-= | .46 |
| OFST= | 0.00 | | | | |




(A)




(B)

Fig. 5. Errors in hot and cold equivalent noise temperatures of "cold attenuator" noise source as a function of cryogenic attenuator uncertainty.

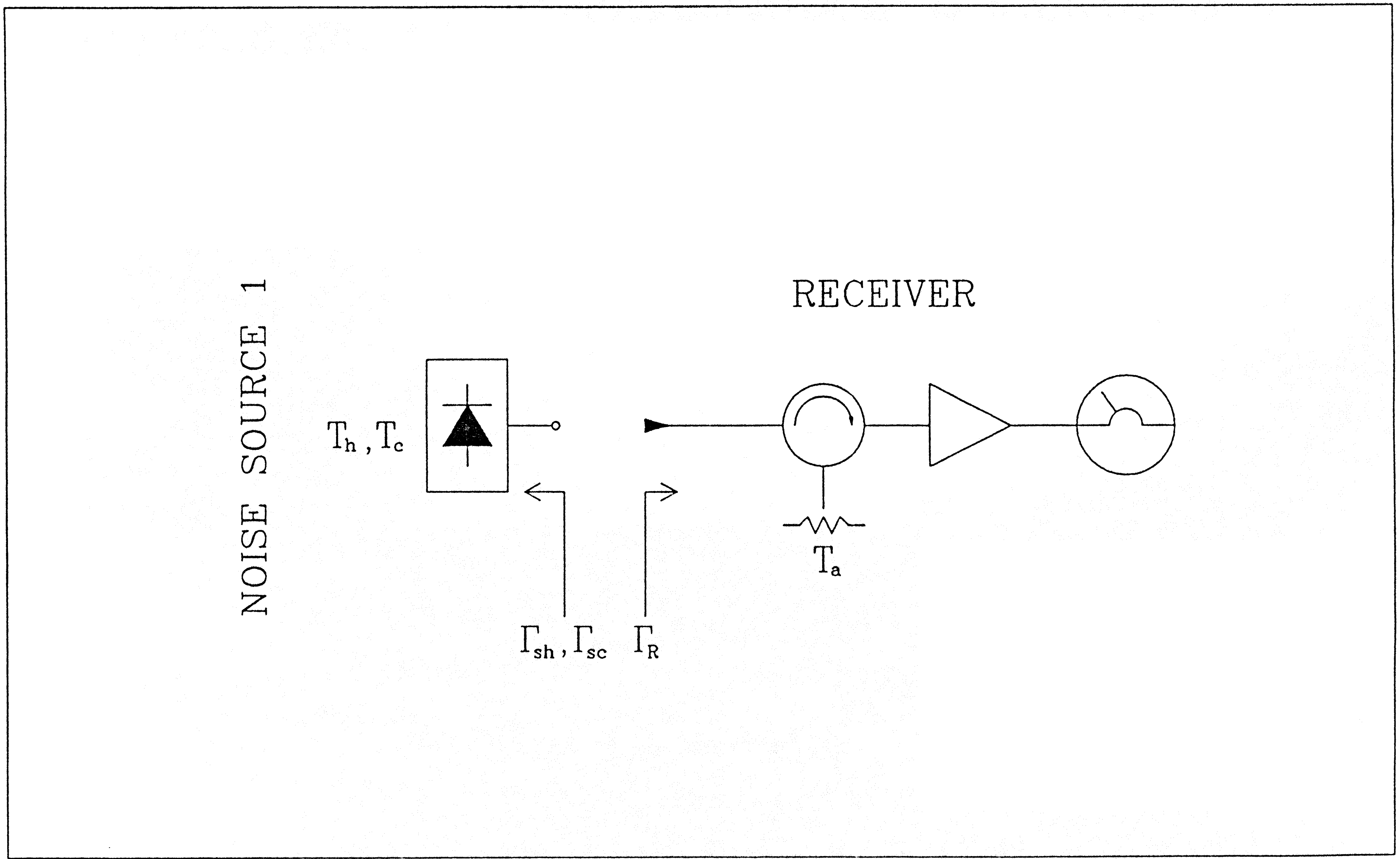



Fig. 6. A schematic view of the calibration procedure.

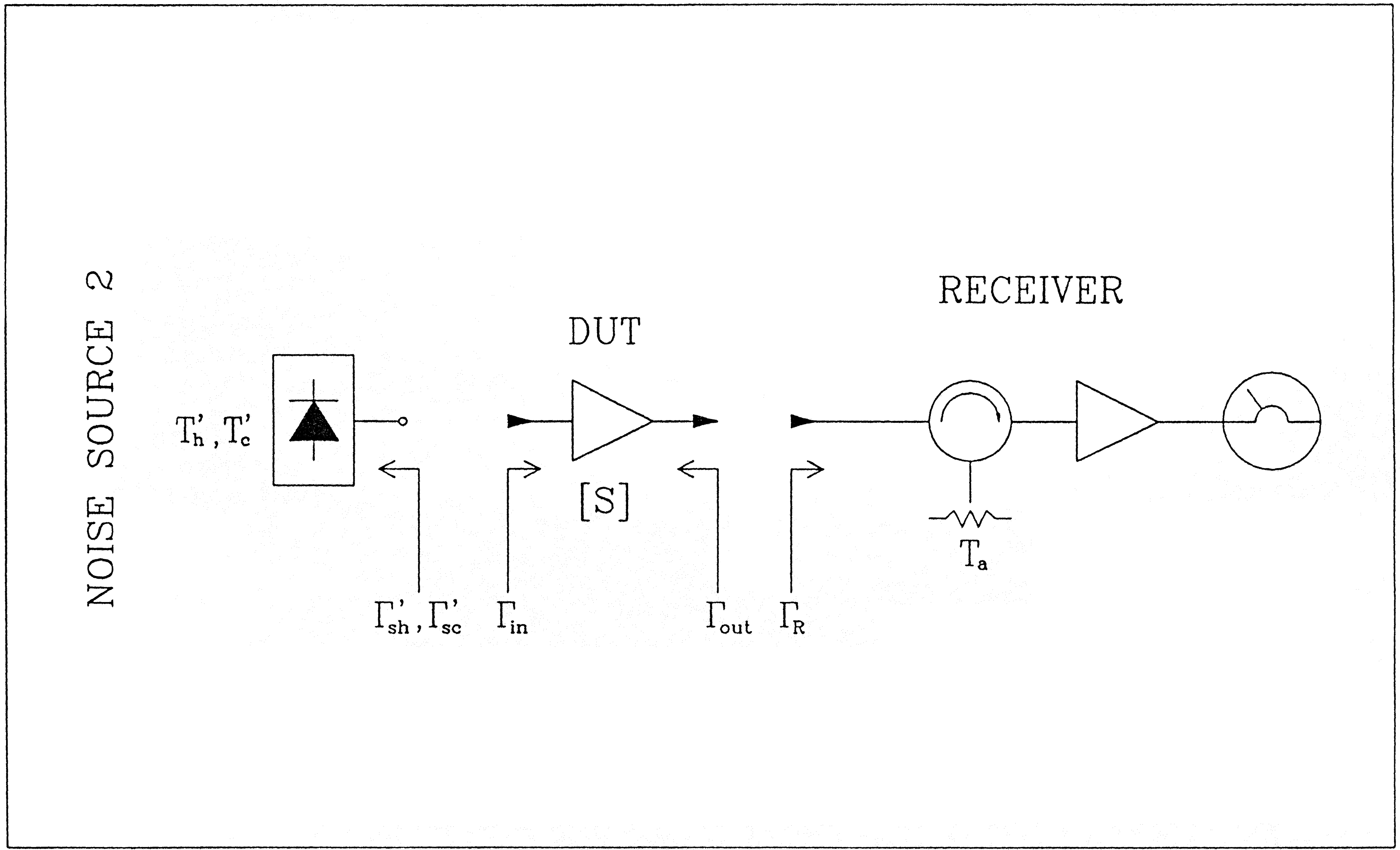



Fig. 7. A schematic view of the amplifier measurement procedure.

$$T_{sys}(\Gamma_s) = T_{DUT} + \frac{T_R(\Gamma_{out})}{G_{AVDUT}(\Gamma_s)} \qquad (10)$$

where $G_{AVDUT}(\Gamma_s)$ is the available gain of the device under test and may be expressed as:

$$G_{AVDUT}(\Gamma_s) = \frac{|S_{21}|^2 (1 - |\Gamma_s|^2)}{|1 - S_{11}\Gamma_s|^2 (1 - |\Gamma_{out}|^2)} \qquad (11)$$

where $S_{ij}$ are the scattering parameters of the device (amplifier) under test and $T_R(\Gamma_{out})$ is the receiver noise temperature.

During the process of calibration $T_R(\Gamma_s)$ is measured for a source impedance different than the output impedance $\Gamma_{out}$ of the device. As it is clear from expression (6), if $T_R(\Gamma_s)$ is known, then $T_R(\Gamma_{out})$ can be easily determined. Also, during the process of measurement, the transducer gain is directly measured and not the available gain. The relation between these two is:

$$G_{TDUT}(\Gamma_s) = G_{AVDUT}(\Gamma_s) \frac{(1 - |\Gamma_{out}|^2)(1 - |\Gamma_R|^2)}{|1 - \Gamma_{out}\Gamma_R|^2} \qquad (12)$$

where

$$\Gamma_{out} = S_{22} + \frac{S_{21} S_{12} \Gamma_s}{1 - S_{11}\Gamma_s} \qquad (13)$$

In the Monte Carlo simulation, the magnitudes of DUT S-parameters are assumed to be known while their phases are randomly chosen with the uniform distributions from $[0, 2\pi]$. Computations are performed according to the following formulas:

$$[G_{DUT}]_M = \frac{N_2 - N_1}{N_2' - N_1'}, \qquad (14)$$

$$[T_R]_M = \frac{[T_H]_s - y[T_c]_s}{y - 1} \quad ; \quad y = \frac{N_2}{N_1} \qquad (15)$$

$$[T_{sys}]_M = \frac{[T_H']_s - y'[T_c']_s}{y' - 1} \quad ; \quad y' = \frac{N_2'}{N_1'} \qquad (16)$$

$$[T_{DUT}]_M = [T_{sys}]_M - \frac{[T_R]_M}{[G_{DUT}]_M} \qquad (17)$$

where $[\ ]_M$ indicates the "measured" value as a result of MC simulation, while $[\ ]_s$ denotes the assumed value of a given parameter.

A computer program NERR2D was developed for the simulation of this measurement procedure. The results of the simulation for the L-band measurement system used at the CDL are summarized in Table V. In this example, the amplifier input return loss is assumed to be rather small (IRL = 2 dB) as it is often the case in low noise amplifiers. The errors assumed for the noise sources are that obtained in the simulation described in Section 2.2 (compare Tables III and IV).

The examples of the dependence of the peak-to-peak error in noise temperature measurement on several parameters of the measurement system (with others kept constant as given in Table V) are shown in Figures 8 through 11. It is worth observing that the error is not dependent on the DUT or noise source reflection coefficient, as a result of a very small change in the impedance of the "composite" noise source between the "on" and "off" state. An equally interesting, although more obvious, result is the strong dependence of the error on the amplifier (DUT) gain (Figure 11).

## 4. Accuracy of Amplifier Measurement Using "Hot" and "Cold" Loads

In measurements of cryogenic amplifiers, it is sometimes possible to attach loads to different stages of the refrigerator and to use these loads as noise standards. The question is whether this method is more accurate than the "cold attenuator" method. The approach described in Section 3 can be used for this analysis. The only difference is the removal of the restriction on the variation of the reflection coefficient between the "hot" and "cold" state of the noise source, as in this method the noise sources are independent.

A computer program NERR1 was developed to perform this analysis and the results are summarized in Table VI. Noise sources of equivalent temperatures 82 K and 4 K were used which had the same accuracy and return loss as the standard used in an example of Section 2 (Table III). The resulting uncertainty of measurement for $[T_{DUT}]_M$ is about 1.5 K ($3\sigma$) which is worse than in the cold attenuator case. This surprising result is caused by the uncertainties in phases of the reflection coefficients of the amplifier and noise standards. The dependence of the accuracy of noise measurement on the return losses of the noise standards and amplifier are presented in Figure 12. It should be stressed that in this case it is very important to assure that the standards have very low reflections (compare Figure 9) if measurements are to be performed on highly reflective amplifiers. As expected, this method offers better accuracy of measurement for relatively well-matched amplifiers as a result of better inherent accuracy of equivalent temperatures of "hot" and "cold" loads.

TABLE V. An Example of Simulation of the Accuracy of Amplifier Noise Measurements for the Measurement Procedure Schematically Presented in Figure 7.

CALIBRATION NOISE SOURCE:

| | | |
|---|---|---|
| 1) Th= 635.00 | 2) Err= 7.00 | 3) RL(dB)= 30.00 |
| 4) Tc= 297.00 | 5) Err= 1.00 | 6) CH(dB)= 56.00 |

MEASUREMENT NOISE SOURCE:

| | | |
|---|---|---|
| 7) Th' = 110.00 | 8) Err= 3.00 | 9) RL(dB)= 30.00 |
| 10) Tc' = 15.35 | 11) Err= .50 | 12) CH(dB)= 66.00 |

RECEIVER

| | | |
|---|---|---|
| 13) Ta= 297.00 | 14) Tr= 500.00 | 15) IRL(dB)= 20.00 |
| 16) Gerr(dB)= .30 | 17) Lin(dB)= 0.00 | |
| 18) BW(MHz)= 30.00 | 19) It(Sec)= .50 | |

D.U.T.

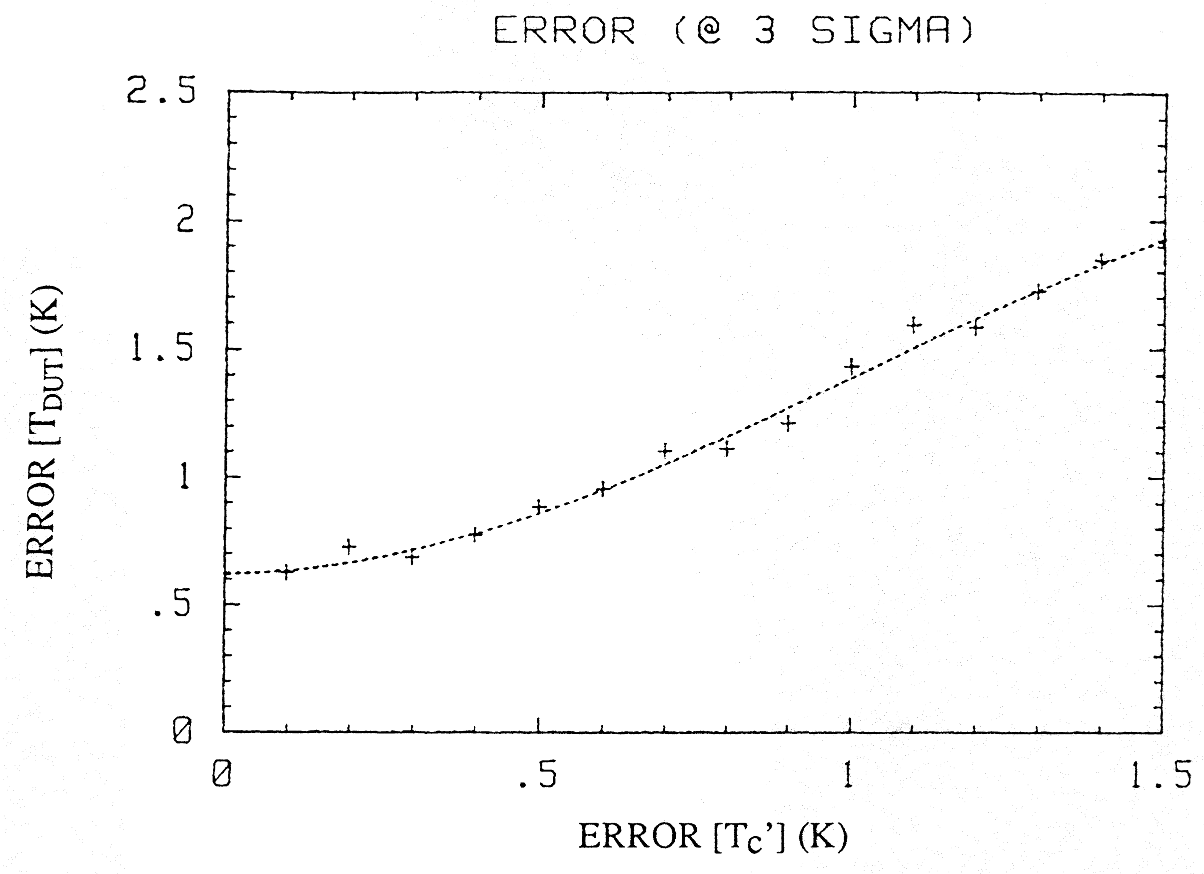
| | |
|---|---|
| 20) [S11](dB)= -2.00 | 21) [S12](dB)=-50.00 |
| 22) [S21](dB)= 25.00 | 23) [S22](dB)=-15.00 |
| 24) Tdut= 4.00 | |

Tdut= 4.00

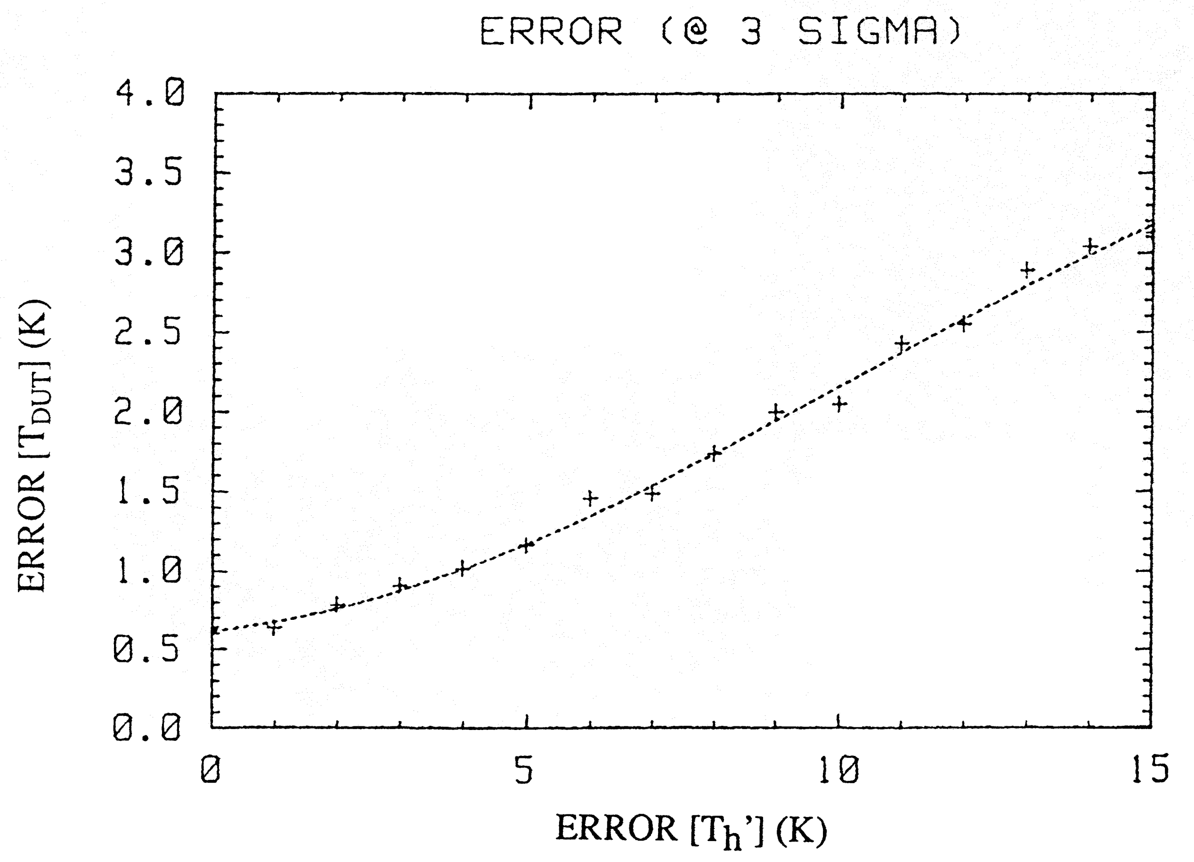
| | | | | | |
|---|---|---|---|---|---|
| MEAN= | 4.02 | MAXIM= | 4.92 | MINIM= | 3.37 |
| SDEV= | .28 | MDEV+= | .90 | MDEV-= | .65 |
| OFST= | -.02 | | | | |

Gdut= 25.00

| | | | | | |
|---|---|---|---|---|---|
| MEAN= | 25.01 | MAXIM= | 25.51 | MINIM= | 24.47 |
| SDEV= | .22 | MDEV+= | .50 | MDEV-= | .54 |
| OFST= | -.01 | | | | |

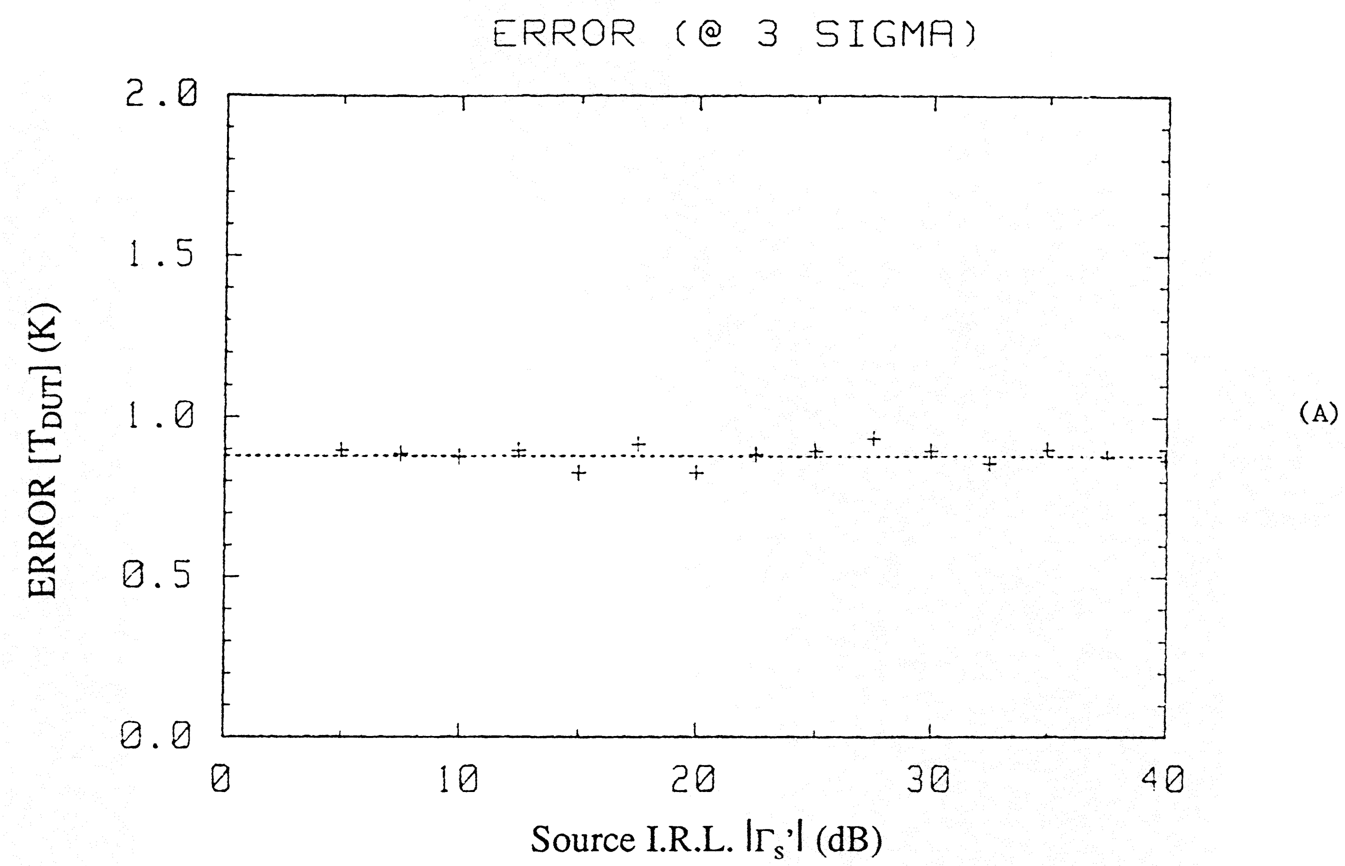



(A)




(B)

Fig. 8. Accuracy of noise temperature measurement as a function of noise source "hot" and "cold" temperature uncertainty.



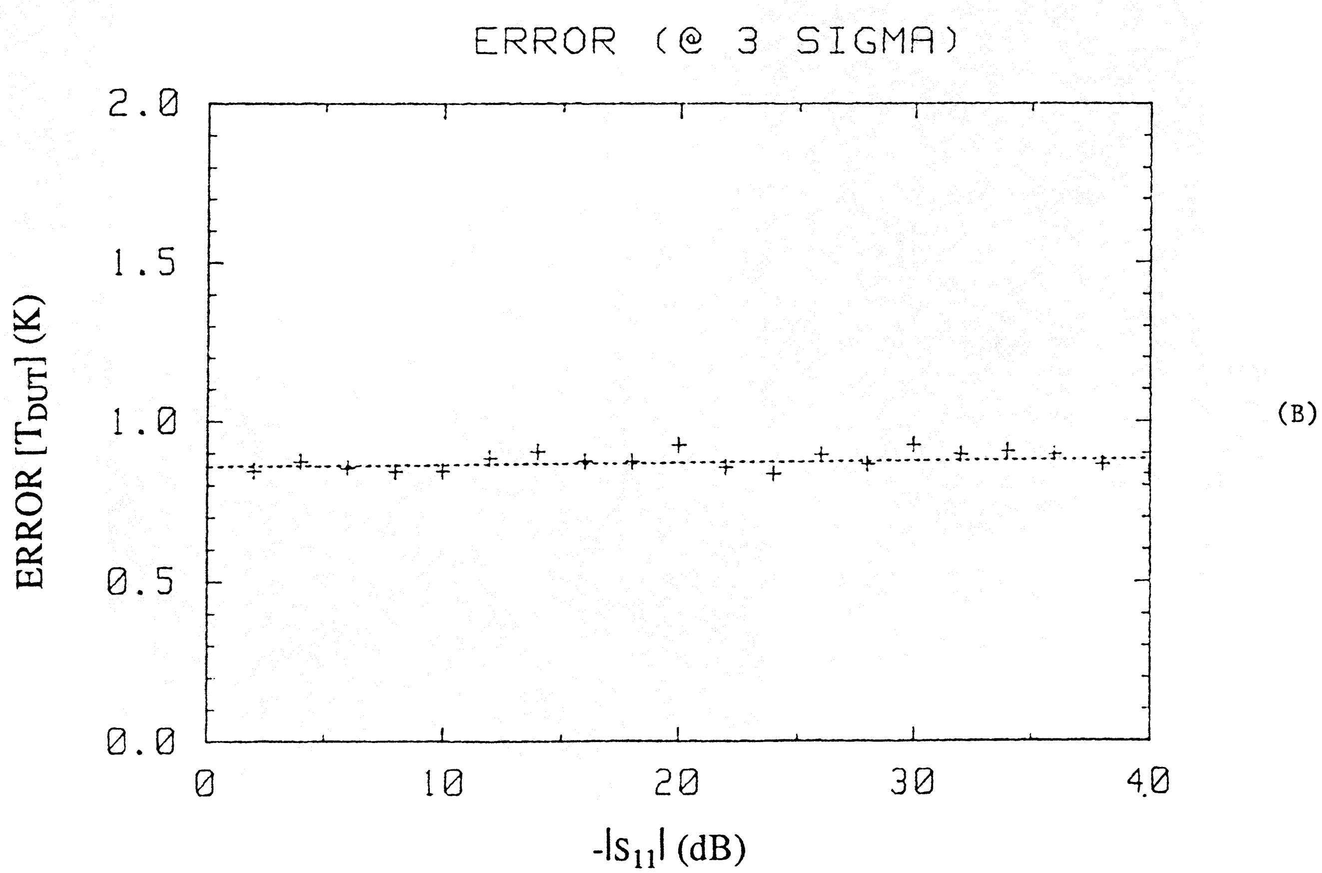
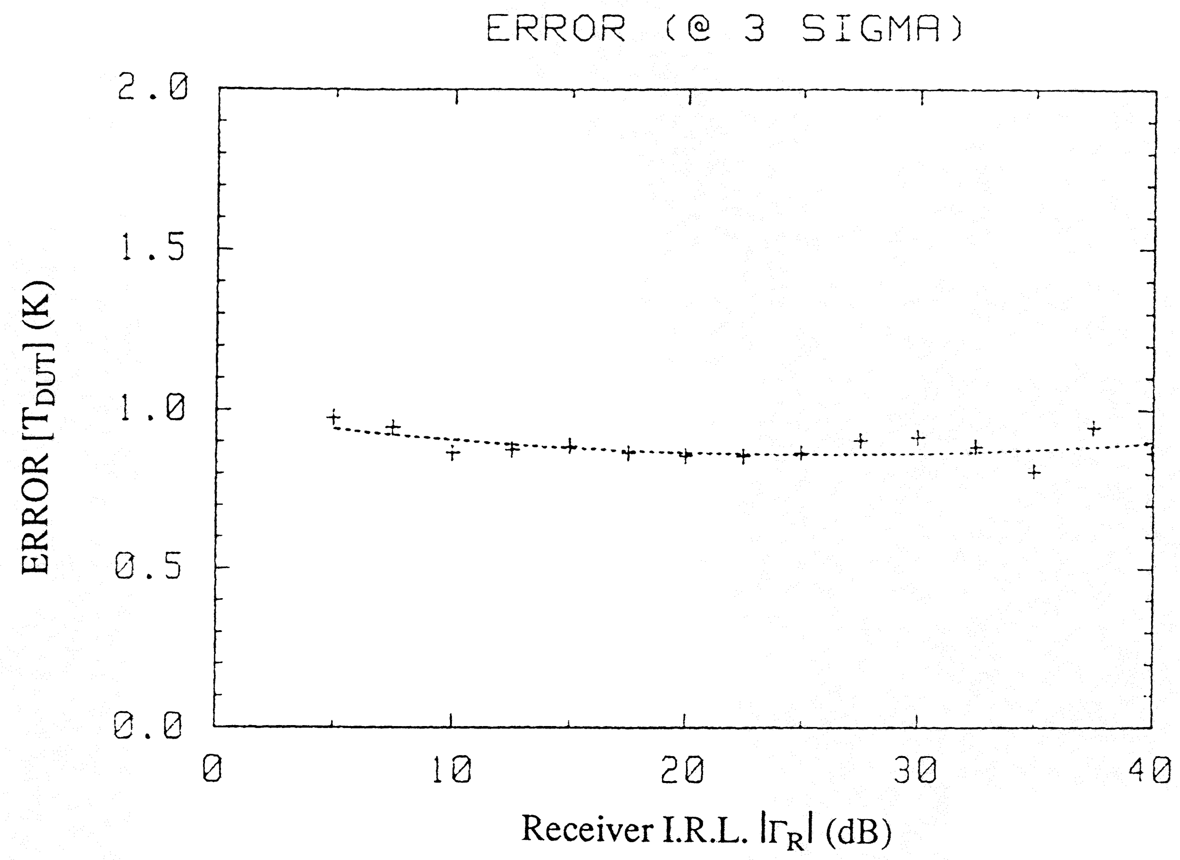
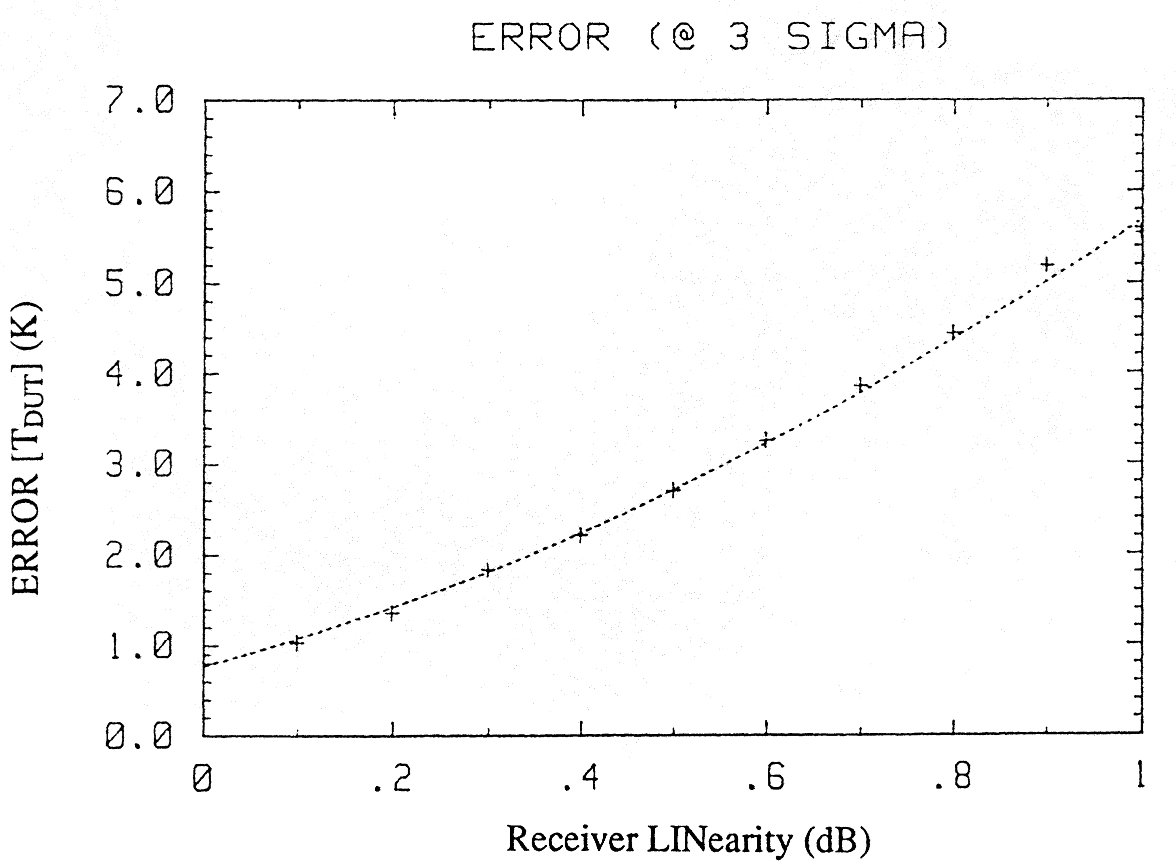




Fig. 9. Accuracy of noise temperature measurement as a function of noise source reflection (A) and $|S_{11}|$ of DUT (B).

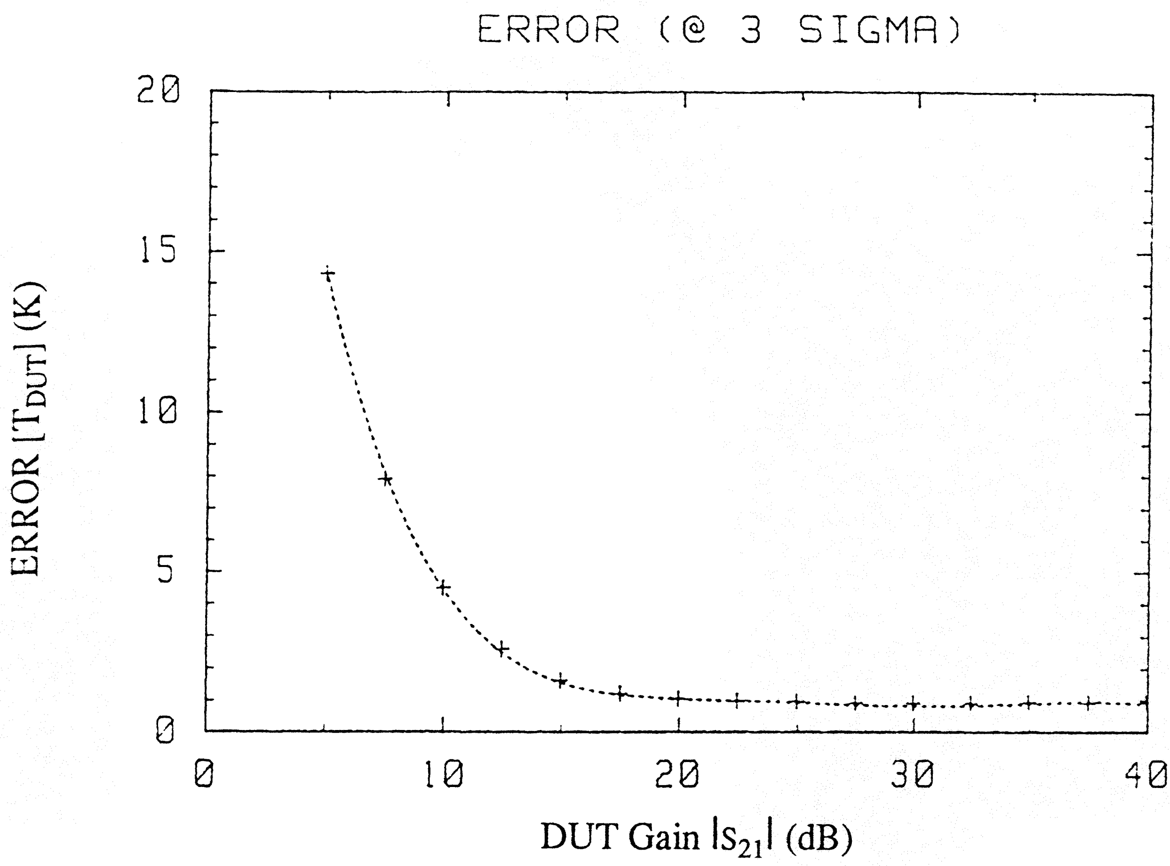



(A)

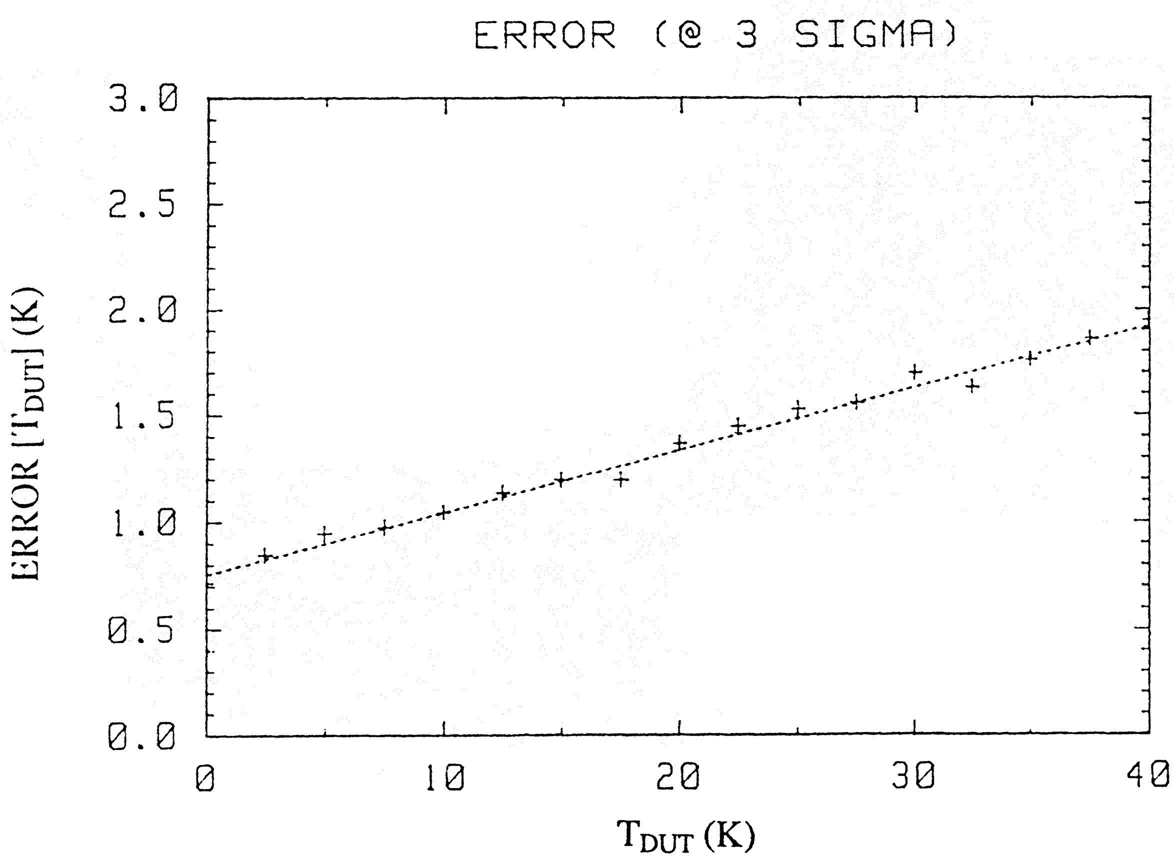



(B)

Fig. 10. Accuracy of noise temperature measurement as a function of receiver input return loss (A) and linearity (B).




(A)




(B)

Fig. 11. Accuracy of noise temperature measurement as a function of an amplifier (DUT) gain (A) and noise temperature (B).

TABLE VI. An Example of Simulation of the Accuracy of Amplifier Noise Measurements Using Cryogenic Loads as Noise Sources.

CALIBRATION NOISE SOURCE:

| | | |
|---|---|---|
| 1) Th= 593.00 | 2) Err= 7.00 | 3) RL(dB)= 30.00 |
| 4) Tc= 298.00 | 5) Err= 1.00 | 6) RL(dB)= 30.00 |

MEASUREMENT NOISE SOURCE:

| | | |
|---|---|---|
| 7) Th' = 82.00 | 8) Err= 1.00 | 9) RL(dB)= 30.00 |
| 10) Tc' = 4.00 | 11) Err= .50 | 12) RL(dB)= 30.00 |

RECEIVER

| | | |
|---|---|---|
| 13) Ta= 297.00 | 14) Tr= 500.00 | 15) IRL(dB)= 20.00 |
| 16) Gerr(dB)= .30 | | 17) Lin(dB)= 0.00 |
| 18) BW(MHz)= 30.00 | | 19) It(Sec)= .50 |

D.U.T.

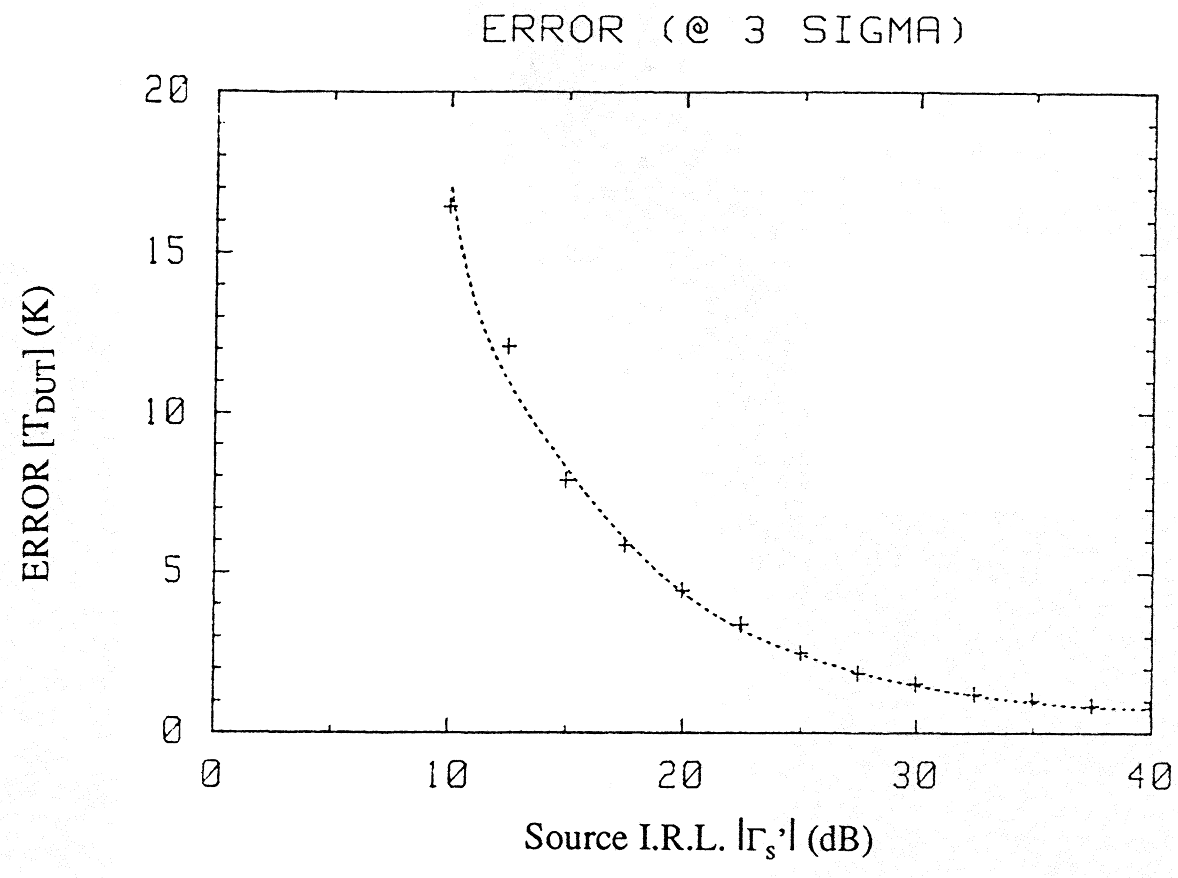
| | |
|---|---|
| 20) [S11](dB)= -2.00 | 21) [S12](dB)=-50.00 |
| 22) [S21](dB)= 25.00 | 23) [S22](dB)=-15.00 |
| 24) Tdut 4.00 | |

Tdut= 4.00

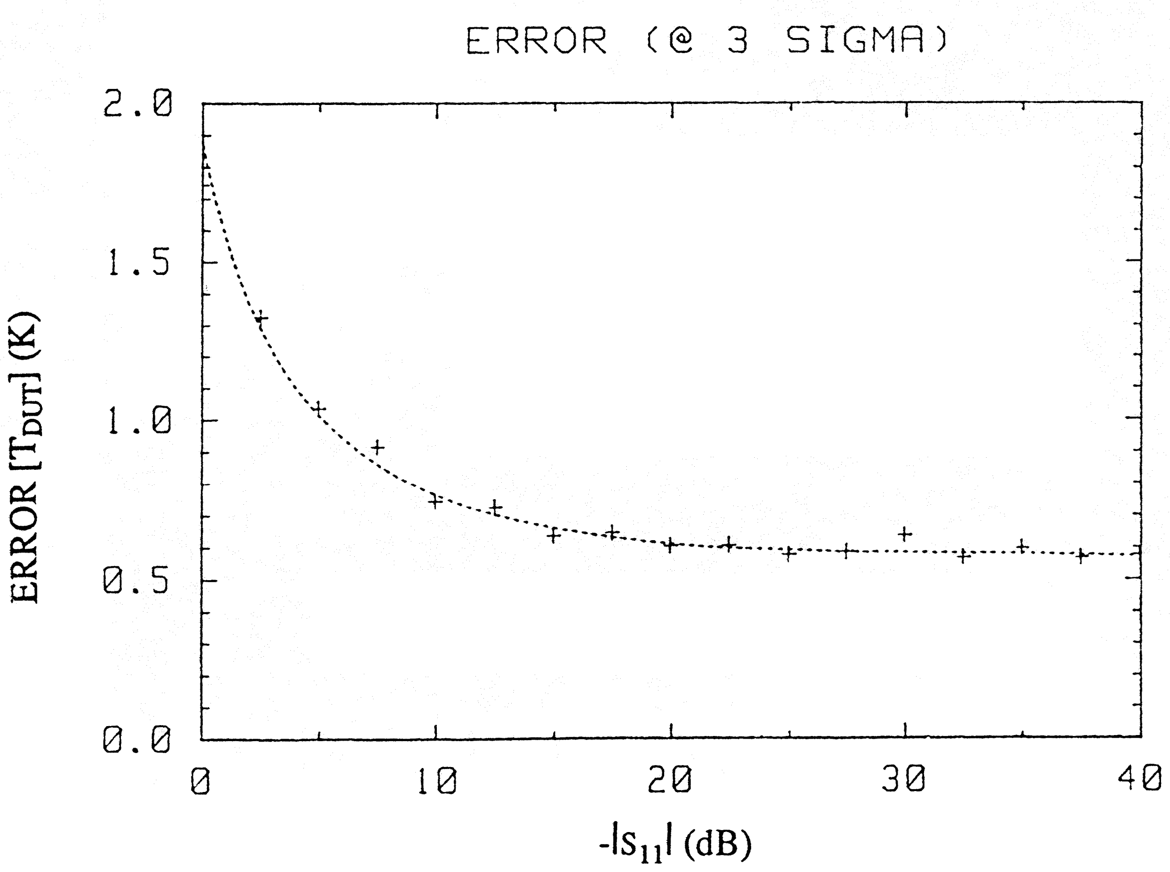
| | | | | | |
|---|---|---|---|---|---|
| MEAN= | 4.05 | MAXIM= | 5.26 | MINIM= | 2.86 |
| SDEV= | .51 | MDEV+= | 1.21 | MDEV-= | 1.19 |
| OFST= | -.05 | | | | |

Gdut= 25.00

| | | | | | |
|---|---|---|---|---|---|
| MEAN= | 25.02 | MAXIM= | 25.53 | MINIM= | 24.40 |
| SDEV= | .24 | MDEV+= | .50 | MDEV-= | .62 |
| OFST= | -.02 | | | | |




(A)




(B)

Fig. 12. Accuracy of noise temperature measurement with two cryogenic loads as a function of their reflection (A) and $|S_{11}|$ of DUT (B).

## 5. Conclusions

The Monte Carlo simulation of measurement errors encountered in typical noise temperature measurement systems of cryogenic amplifiers and devices was described. It revealed a strong dependence of noise temperature measurement accuracy on the factors which importance is sometimes neglected, like, for instance, the input return loss of an amplifier under test or residual return losses of noise sources. The described procedure and associated computer routines should be very useful in assessing the accuracy of measurement of different noise measurement systems.


## Acknowledgement

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