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APERTURE EFFICIENCY FOR PRIME FOCUS PARABOLOIDAL REFLECTORS AND CASSEGRAIN ANTENNAS

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A program in basic by R. Fisher [1] on the HP9825A calculator has been in use for the calculation of prime focus efficiency, blockage, spillover and scattering on the 140-foot and 300-foot telescopes. In the antenna test range at Green Bank, radiation patterns of feeds were recorded on an X-Y plotter, which was the only form of output available. Values had to be read off these plots and input into the HP9825A calculator for the computation of aperture efficiency. This did not give the best accuracy, in addition to the time and effort involved.

Since then, a personal computer has been interfaced to the range for data acquisition [2]. The PC stores the amplitude and phase vs. azimuth information of the test feed on the hard disk. This report describes a Turbo Pascal efficiency program now running on the PC, which can read the values off the hard disk and compute the aperture efficiency readily.

Features of the Program

A list of features of the Turbo Pascal program are included hereunder.

1. Both the principal plane pattern values are used in combination as opposed to using them separately as in [1].

2. Crosspolarization efficiency is included.

3. Equations for taper, phase and crosspolarization efficiencies are taken from a paper by R. E. Collin [3].

4. The effect of non-coincident phase centers in the principal planes is accounted for.

5. Efficiency calculation for a Cassegrain antenna is included.

6. Blockage and scattering computations for a prime focus antenna is done using the same approach as in [1].

7. For a Cassegrain antenna, center blockage and subreflector edge diffraction effects as analyzed by P. S. Kildal [4] have been included. Also, subreflector struts blockage effect is accounted for.

8. Efficiencies on the 140-foot, 300-foot as well as the VLBA antennas can be computed. Since the taper efficiency is computed for a Cassegrain system, a word of caution, this efficiency for the VLBA antenna would be lower than the actual value, the reason being the VLBA antenna is a dual shaped reflector.

Prime Focus Reflector

The radiation pattern of a corrugated type of feed is given by

$$\overline{E}_f = \frac{e^{-jkr}}{r}\left[\hat{\Theta}\ e_\theta(\Theta)\ \sin\phi + \hat{\phi}\ e_\phi(\Theta)\ \cos\phi\right] \tag{1}$$

where Θ and ϕ are the polar angles in spherical coordinates as shown in Figure 1; e_Θ and e_ϕ are the far-zone field patterns in the principal planes and k is the free space wave number.

The aperture field of a parabolic reflector illuminated by the above feed, computed by ray optics is of the form

$$\overline{E}_a = \frac{e^{-2jkf}}{f}\cos^2\frac{\Theta}{2}\left[\hat{x}\left(e_\phi - e_\Theta\right)\sin\phi\cos\phi + \hat{y}\left(e_\phi\cos^2\phi + e_\Theta\sin^2\phi\right)\right] \tag{2}$$

where f is the focal length of the reflector.

Fourier transform of the aperture field in (2), gives the far-zone radiated field of the reflector. The co- and crosspolarized fields of the feed are proportional to $e_\Theta \sin^2\phi + e_\phi \cos^2\phi$ and $(e_\Theta - e_\phi)\sin\phi\cos\phi$, respectively, and these are translated into E_y and E_x on the aperture plane of the reflector as seen in (2).

The aperture efficiency of the reflector can be factored as

$$\eta_a = \eta_t\ \eta_{sp}\ \eta_x\ \eta_{ph}\ \cdots \tag{3}$$

where

η_t, η_{sp}, η_x and η_{ph} are the taper, spillover, cross-polarization and phase efficiencies. The other factors contributing to losses like blockage, random surface error, defocus, squint, etc., have not been shown.

Thomas in [5] defines the efficiencies in the following form:

$$\eta_t = 32\ (f/D)^2 \frac{\left[\int_0^{\Theta_0} \left(|e_\Theta| + |e_\phi| \right) \tan \frac{\Theta}{2}\, d\Theta \right]^2}{\int_0^{\Theta_0} \left(|e_\Theta| + |e_\phi| \right)^2 \sin \Theta\, d\Theta} \tag{4}$$

$$\eta_x = \frac{\int_0^{\Theta_0} \left(|e_\Theta| + |e_\phi| \right)^2 \sin \Theta\, d\Theta}{2 \int_0^{\Theta_0} \left(|e_\Theta|^2 + |e_\phi|^2 \right) \sin \Theta\, d\Theta} \tag{5}$$

$$\eta_{ph} = \frac{\left| \int_0^{\Theta_0} \left(e_\Theta + e_\phi \right) \tan \frac{\Theta}{2}\, d\Theta \right|^2}{\left[\int_0^{\Theta_0} \left(|e_\Theta| + |e_\phi| \right) \tan \frac{\Theta}{2}\, d\Theta \right]^2} \tag{6}$$

where f and D are the focal length and diameter of the reflector and Θ_0 is the half angle subtended by the paraboloid at the focus (Figure 2).

The expressions in (4) and (6) have been used in [1] except that the field in only one plane, has been used at a time. It is noticed from (5) that when $|e_\Theta| = |e_\phi|$, the crosspolarization efficiency is unity, which is not always correct. Collin in [3] gives the radiated copolarized power intercepted by the reflector as proportional to

$$\frac{1}{16} \int_0^{\Theta_0} \left(2 |e_\Theta + e_\phi|^2 + |e_\Theta - e_\phi|^2 \right) \sin \Theta\, d\Theta \tag{7}$$

and total radiated power as proportional to

$$\frac{1}{4}\int_0^{\Theta_0}\left(|e_\Theta|^2 + |e_\phi|^2\right)\sin\Theta\, d\Theta \tag{8}$$

Using (7) and (8), the equations for taper, spillover, cross-polarization and phase efficiencies are given as:

$$\eta_t = 64\left(\frac{f}{D}\right)^2 \frac{\left(\int_0^{\Theta_0}\left|e_\Theta + e_\phi\right|\tan\frac{\Theta}{2}\, d\Theta\right)^2}{\int_0^{\Theta_0}\left(2\left|e_\Theta + e_\phi\right|^2 + \left|e_\Theta - e_\phi\right|^2\right)\sin\Theta\, d\Theta} \tag{9}$$

$$\eta_{sp} = \frac{\int_0^{\Theta_0}\left(2\left|e_\Theta + e_\phi\right|^2 + \left|e_\Theta - e_\phi\right|^2\right)\sin\Theta\, d\Theta}{\int_0^{\pi}\left(2\left|e_\Theta + e_\phi\right|^2 + \left|e_\Theta - e_\phi\right|^2\right)\sin\Theta\, d\Theta} \tag{10}$$

$$\eta_x = \frac{\int_0^{\Theta_0}\left(2\left|e_\Theta + e_\phi\right|^2 + \left|e_\Theta - e_\phi\right|^2\right)\sin\Theta\, d\Theta}{4\int_0^{\Theta_0}\left(\left|e_\Theta\right|^2 + \left|e_\phi\right|^2\right)\sin\Theta\, d\Theta} \tag{11}$$

$$\eta_{ph} = \frac{\left|\int_0^{\Theta_0}\left(e_\Theta + e_\phi\right)\tan\frac{\Theta}{2}\, d\Theta\right|^2}{\left(\int_0^{\Theta_0}\left|e_\Theta + e_\phi\right|\tan\frac{\Theta}{2}\, d\Theta\right)^2} \tag{12}$$

The effect of feed house blockage and scattered and spill-over radiation from the ground have been computed using the same methodology in [1]. Measured E and H plane field patterns of a prime focus feed are shown in Figure 3 and the computed efficiency in Figure 4. The scattered temperature P(J) in Figure 4 has three components, namely:

(1) Fraction of energy scattered to ground by the feed house assembly.

(2) Fraction scattered off the feed supports by the outgoing plane wave.

(3) Fraction scattered by the feed supports by the spherical wave from the feed.

The aperture efficiency with and without blockage are computed.

Cassegrain Antenna

For the Cassegrain system also, equations (9) through (12) have been used for the efficiency conputations with the following exceptions. The factor $64\ (f/D)^2$ in (9) is replaced with $64\ (fM/D)^2$, where M is the magnification of the Cassegrain system. Also, Θ_0 in the limits of integration is replaced by ψ_0, which is the half angle of an equivalent paraboloid from the secondary focus as shown in Figure 5.

Subreflector Diffraction and Blockage, and Struts Blockage Effects:

In the prime focus case the effect of blockage by the feed house and support struts is accounted for, by introducing a correction factor in the numerator of the taper efficiency integral, resulting in a taper efficiency with blockage [1]. Whereas, here, a separate interference efficiency η_{int} is computed and the aperture efficiency is given as

$$\eta_a = \eta_t \cdot \eta_{sp} \cdot \eta_x \cdot \eta_{ph} \cdot \eta_{int} \tag{13}$$

The interference efficiency as given in [4] is

$$\eta_{int} = \left|1 + \Delta_{cb} + \Delta_{sb} + \Delta_d\right|^2$$

where Δ_{cb}, Δ_{sb} and Δ_d are due to subreflector center blockage, feed support blockage and subreflector edge diffraction, respectively.

The center blockage term is given by

$$\Delta_{cb} = - \frac{\int_0^{\psi_d} \left(e_\Theta(\psi) + e_\phi(\psi)\right) \tan\frac{\psi}{2}\, d\psi}{\int_0^{\psi_0} \left(e_\Theta(\psi) + e_\phi(\psi)\right) \tan\frac{\psi}{2}\, d\psi} \tag{14}$$

This includes the effect of optical shadowing by the subreflector weighted by the feed illumination taper.

The feed support blockage is given by

$$\Delta_{sb} = - \frac{\int_{\psi_d}^{\psi_{st}} \gamma(\psi) \left| e_\Theta + e_\phi \right| \tan \frac{\psi}{2} \, d\psi}{\int_0^{\psi_0} \left| e_\Theta + e_\phi \right| \tan \frac{\psi}{2} \, d\psi} \tag{15}$$

where

$$\gamma(\psi) = \frac{NW'}{4\pi f M \left(\frac{\sin \psi}{1 + \cos \psi} \right)}$$

N is the number of feed supports and W' is the width of each support, projected vertically on the reflector. The angles ψ_d, ψ_{st} and ψ_0 are shown in Figure 5.

Kildal in [4] shows that the effect of diffraction from the subreflector edge can be included in the aperture efficiency by adding a single line integral to the geometric optics aperture integral. He expresses the diffraction term by

$$\Delta_d = - (1 - j) \, C_d \sqrt{\frac{\lambda}{d}} \sqrt{1 - d/D} \; e_0 \tag{16}$$

where

$$e_0 = \frac{e_\phi(\psi_0) + e_\Theta(\psi_0)}{2 \, E(0)}$$

is the mean amplitude illumination at the edge of the sub-reflector and

$$C_d = \frac{1}{2\pi} \frac{\sin \psi_0}{\sqrt{\sin \theta_0}} \frac{2 \, E(0) \tan \frac{\psi_0}{2}}{\int_0^{\psi_0} \left| e_\theta + e_\phi \right| \tan \frac{\psi}{2} \, d\psi}$$

is the diffraction parameter.

Also, λ is the wavelength of operation, d and D are the subreflector and main reflector diameters, respectively. Measured principal plane patterns of a Cassegrain feed are given in Figure 6 and the computed efficiency on the VLBA antenna is shown in Figure 7.

The Turbo Pascal Program

The efficiency program is stored on the hard disk of the IBM PC at the antenna test range. The feed pattern data should be saved in data files in order to compute the efficiency. After the data is taken, the range software [2] returns to the main menu and by pressing the F4 function key the data can be saved in a file by giving it a name. When all the data is taken or at least the E and H plane patterns at a particular frequency have been taken and saved, exit the range software program by pressing F10 function key. Control returns to DOS and you are in the c:\patterns directory. Now type the command

scopeff

which executes the efficiency program.

The program lists all the files saved and asks you to select the two files, which have the E and H plane patterns, in that order. If you wish to calculate the aperture efficiency based on one plane only, just type the number of the particular file twice. You will then be asked to input the distance (in inches) between the phase centers in the two planes. Next you will be asked if you want to input the pattern level, past the azimuth boundary. If you said yes (y), then you will be asked to enter the level (in dbs). If not, the program takes the level at the azimuth boundary and assumes the same value for the rest of the patterns.

Caution: When the azimuth boundaries are less than ± 90° and if the pattern levels are high at the azimuth boundaries, spillover efficiency may be lower than actual and so it would be better to input the level past the boundary.

The phase patterns in the principal planes may be offset from one another, due to reasons like drift during the time interval between the measurements, position of the thumbwheel switches on the receiver, etc. The program takes into account this offset and corrects for it.

Other parameters like the telescope, frequency, type of focus, etc., which are required in the efficiency computations are read from the data file itself. So caution must be exercised in entering the correct parameters in the main menu of the data taking program.

REFERENCES

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[2] Will, W. A., Norrod, R. D., Srikanth, S., "Antenna Test Range Automation," Electronics Division Internal Report No. 272 (November 1987).

[3] Collin, R. E., "Aperture Efficiency for Paraboloidal Reflectors," IEEE Transactions on Antennas and Propagation, Vol. AP-32, No. 9 (September 1984).

[4] Kildal, P. S., "The Effects on Subreflector Diffraction on the Aperture Efficiency of a Conventional Cassegrain Antenna-An Analytical Approach," IEEE Transactions on Antennas and Propagation, Vol. AP-31, No. 6 (November 1983).

[5] Thomas, B. M., "Theoretical Performance of Prime Focus Paraboloids using Hybrid-Mode Feeds," Proc. IEE, Vol. 118, pp. 1539-1549 (1971).

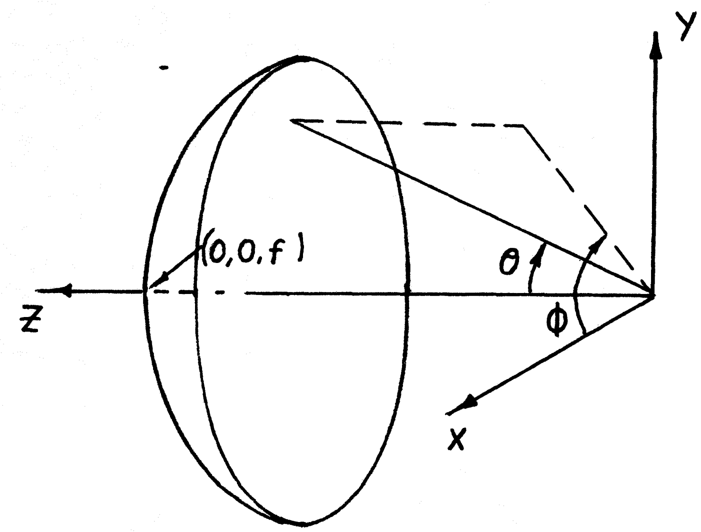


FIGURE 1. Paraboloidal reflector and the coordinate system.

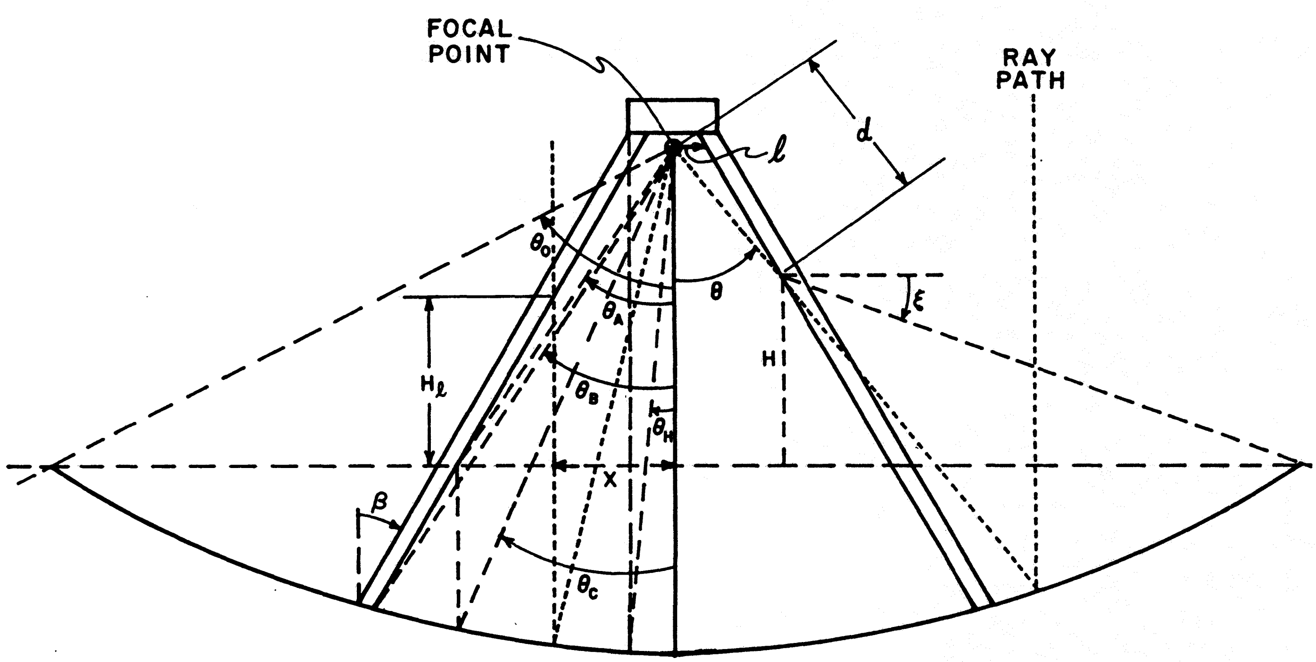
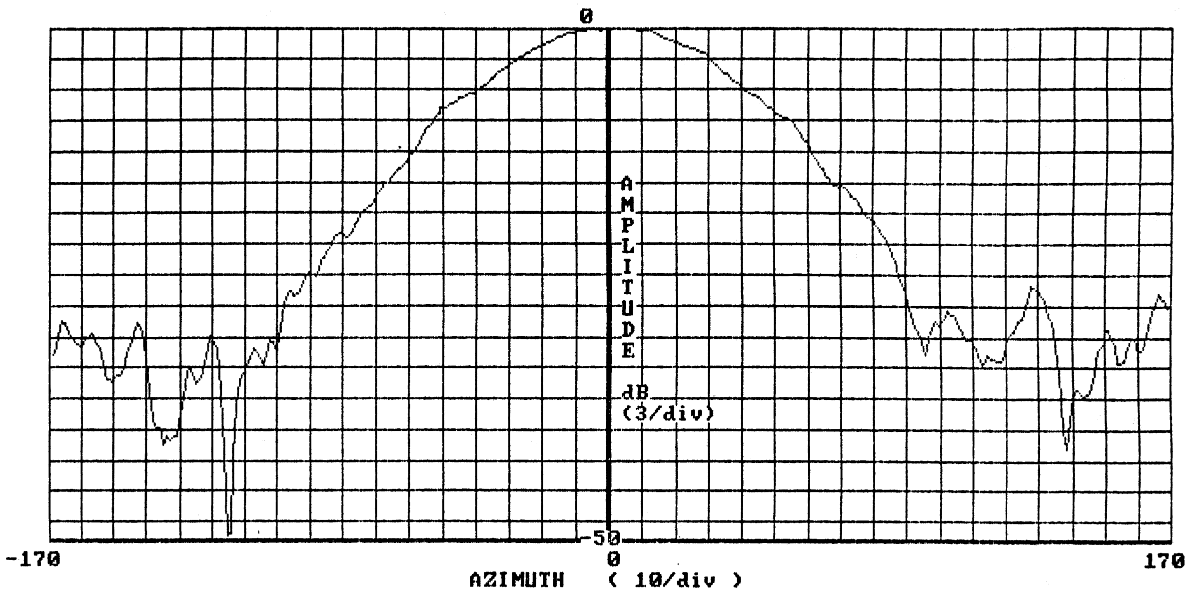
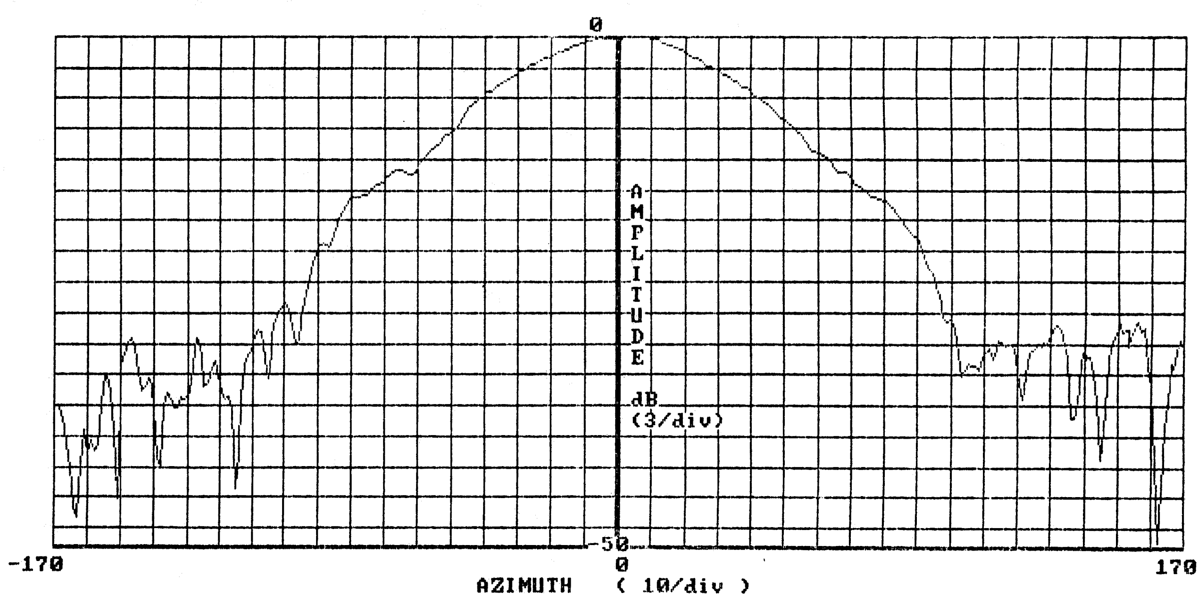


FIGURE 2. Prime focus geometry.



FOCUS = PRIME EDGE ILLUMINATION : RIGHT = -11.28 LEFT = -12.49
FEED = INTERFEROMETER/RCA,S/X FREQ = 2.7000 (GHz) PLANE = E
COMMENTS :
DATE : 01/28/88



FOCUS = PRIME EDGE ILLUMINATION : RIGHT = -11.40 LEFT = -12.71
FEED = INTERFEROMETER/RCA,S/X FREQ = 2.7000 (GHz) PLANE = H
COMMENTS :
DATE : 01/28/88

FIGURE 3. E and H plane patterns of a prime focus feed.

```
IMMMMMMMMMMMMMMMMMMMMMMMMMM EFFICIENCY & TEMPERATURES MMMMMMMMMMMMMMMMMMMMMMMMMM;
:                                                                               :
:                                                                               :
:   ON THE 140 FOOT PRIME FOCUS TELESCOPE                                       :
:   FREQUENCY =    2.700 Ghz                                                    :
:   FEED        : INTERFEROMETER/RCA,S/X                                        :
:                                                                               :
:                                                                               :
:   WITHOUT BLOCKAGE :                WITH BLOCKAGE :                           :
:   SPILLOVER EFF.    =    0.9234                                               :
:   TAPER EFF.        =    0.8128     TAPER EFF.         =    0.7142            :
:   CROSSPOLAR EFF.   =    0.9998                                               :
:   PHASE EFF.        =               SCAT. TEMPS. P(J) =                       :
:                          0.8157                             1.15              :
:                          0.9513                             0.85              :
:                          1.0000                             0.80              :
:                          0.9499                                               :
:                          0.8133     TOTAL SCAT. TEMP. =     2.80              :
:   SPILLOVER TEMP.   =   17.86       SPILLOVER TEMP.   =    16.02              :
:   APERTURE EFF.     =    0.7504     APERTURE EFF.     =     0.6594            :
:                                                                               :
:                                                                               :
:                                                                               :
HMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMM<
```

FIGURE 4. Efficiency of a prime focus feed.

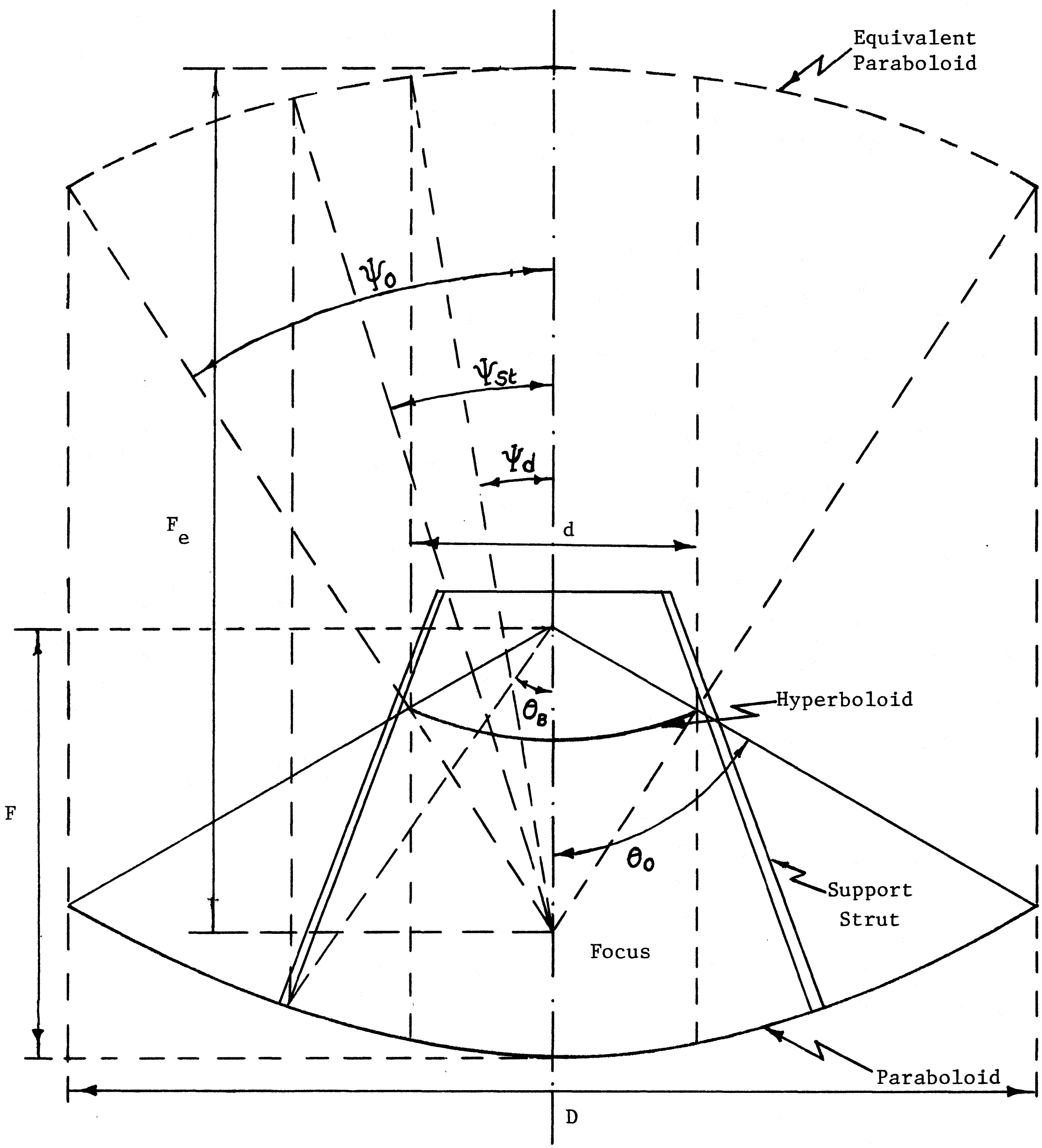
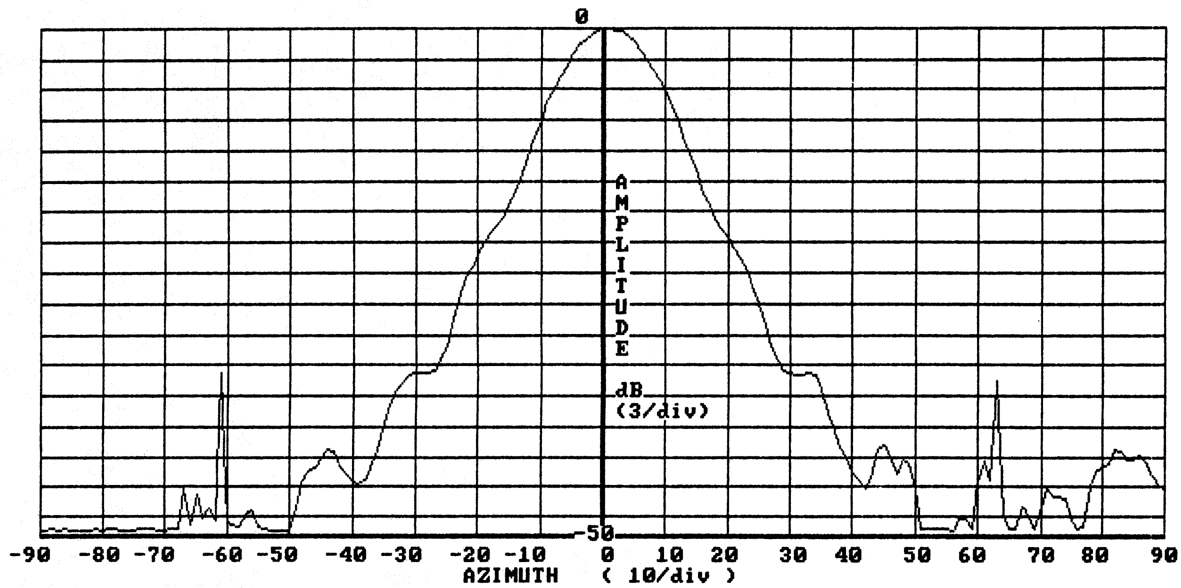
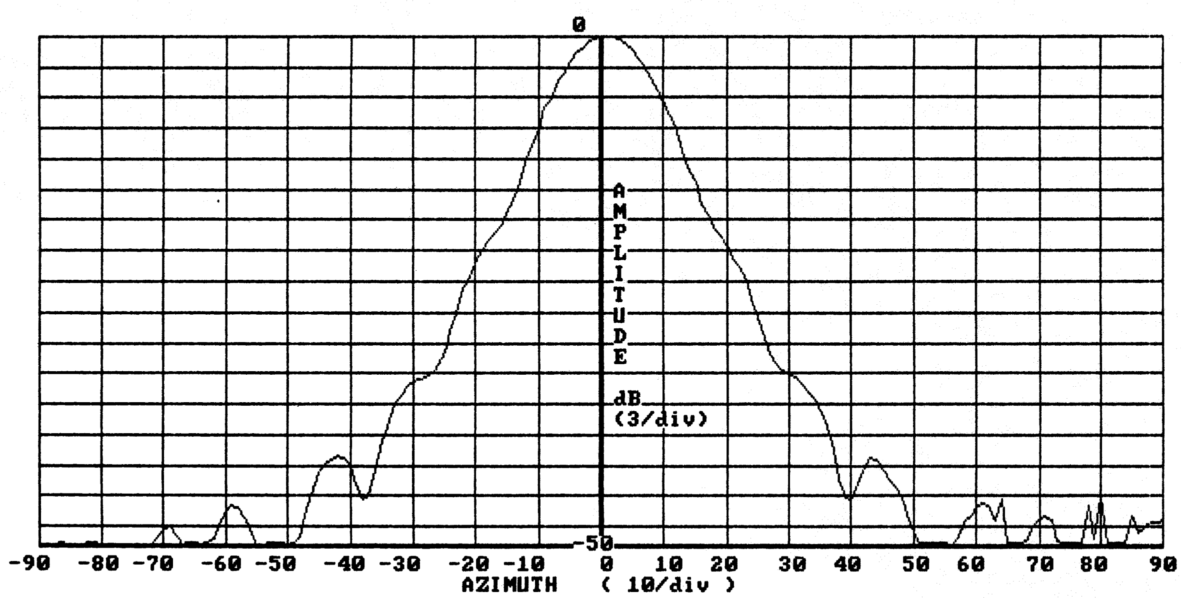


FIGURE 5. Cassegrain geometry.



FOCUS = CASSEGRAIN EDGE ILLUMINATION : RIGHT = -11.27 LEFT = -14.19
FEED = VLBA 2.3 GHZ FREQ = 2.1500 (GHz) PLANE = E
COMMENTS : COPOLAR
DATE : 12/30/87



FOCUS = CASSEGRAIN EDGE ILLUMINATION : RIGHT = -11.28 LEFT = -14.23
FEED = VLBA 2.3 GHZ FREQ = 2.1500 (GHz) PLANE = H
COMMENTS : COPOLAR
DATE : 12/30/87

FIGURE 6. E and H plane patterns of a Cassegrain feed.

```
IMMMMMMMMMMMMMMMMMMMMMMMMM EFFICIENCY & TEMPERATURES MMMMMMMMMMMMMMMMMMMMMMMMMM;
:                                                                              :
:                                                                              :
:                                                                              :
:   ON THE 82 FOOT CASSEGRAIN TELESCOPE                                        :
:   FREQUENCY =    2.150 Ghz                                                   :
:   FEED      = VLBA 2.3 GHZ                                                   :
:                                                                              :
:                                                                              :
:   SPILLOVER EFF.                  =   0.9095                                 :
:   TAPER EFFICIENCY                =   0.8410                                 :
:   CROSSPOLAR EFF.                 =   1.0000                                 :
:   PHASE EFFICIENCY                =   1.0000                                 :
:   APERTURE EFF.(W/O BLOCKAGE)     =   0.7649                                 :
:   CENTER BLOCKAGE TERM            =  -2.6423884267E-02 -2.6956767146E-05     :
:   STRUTS BLOCKAGE TERM            =  -2.1501264165E-02 -2.1212624423E-05     :
:   DIFFRACTION TERM                =  -2.8624935190E-02  2.8787906920E-02     :
:   INTERFER. EFFICIENCY            =   0.8536                                 :
:   APERTURE EFF.(WITH BLOCKAGE)    =   0.6529                                 :
:                                                                              :
:                                                                              :
:                                                                              :
:                                                                              :
HMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMMM<
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FIGURE 7. Efficiency of a Cassegrain feed.