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THE NOISE PROPERTIES OF HIGH ELECTRON MOBILITY TRANSISTORS

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ABSTRACT. A simple analytic model for the HEMT is developed which can be used to calculate the behavior of the transistor . The Pucel noise theory for GaAs MESFET's is modified to apply to the HEMT. Good agreement between calculation and experiment is found. The dependance of noise temperature on gate length and channel thickness is presented .

Introduction.

The High Electron Mobility Transistor is a new type of field effect transistor which has been shown to be capable of very low noise temperatures and high associated gain at microwave frequencies. Several papers have been published on the small signal properties of this device, notably by Shur and collaborators [1]. However, their analysis is confined to the portion of the I-V characterisics before the device goes into saturation ie. when the electrons achieve their peak velocity. This report describes a new method for calculating the small signal parameters in the saturated region and will seek to determine the noise properties of the device by modifying the Pucel [2] noise theory so that it applies to the HEMT.

The report is divided into 3 parts:

1) The calculation of the DC and small signal parameters,

2) The determination of the Noise parameters,

3) Instructions on how to use the program.

Of nescessity, the first 2 sections contain a lot of mathematics. Where possible this has been relegated to appendices, but some has been included in the text to show the outline of the calculation.

1.0 A Simple Analytical model of the High Electron Mobility Transistor.

The model proposed by Grebene and Ghandi [3] for the operation of the JFET in saturation will be modified to apply to the HEMT. As will be described, approximations made in this model are more appropriate in the case of a HEMT, so leading to a more accurate analysis. However, effects such as velocity overshoot, the formation of Gunn domains etc, which are so important in short gate devices cannot be included in this model, so some reservations must apply to the predictions. It is hoped that this comparitively simple model will allow the study of the changes in device properties caused by changing the material or geometrical parameters.

(1.1) The DC Model.

The Grebene and Ghandi model divides the device into two regions ; a linear region and a velocity saturated region. In region I, $0 < x < L_1$, Ohms law applies and the electron velocity is proportional to the electric field. In region II, $L_1 < x < L$ the carriers are assumed to travel at a fixed (saturated) velocity v_s . The boundary between the two is determined by satisfying the current continuity and field continuity equations.

The linear region of operation has been described in the literature by Delagebeaudeuf and Linh [4] and Lee et al [1]. Their models will be used to describe the operation of the

HEMT in this region. In the saturated region, the model proposed by Grebene and Ghandi for the operation of the JFET in saturation will be used to calculate the potential and field distribution in the channel.

(1.2) Current Flow in the Linear region.

Figure 1 shows the coordinate system and the device dimensions which are to be usedin the analysis. Figure 2 indicates the bias voltages applied, the electron velocity and the approximate field distribution in the HEMT. The problem to be solved is as follows. First, determine the current flowing in region I where the elecrons obey Ohms law and the electron density in the channel is roughly proportional to the applied gate voltage. Then, by calculating the current flowing in region II and equating this to that flowing in region I, and simultaneously matching the electric field at boundary, calculate the length of region I.

Using the model of Drummond et al,[5] the density of states in the channel is given by,

$$n_s = \frac{\epsilon_2}{q(d + \Delta d)}(V_{sg} - V_{\text{off}}) \tag{1}$$

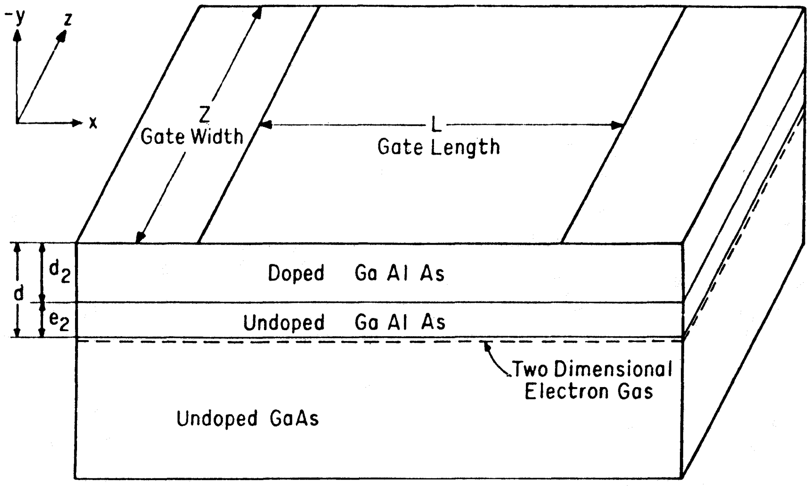

Figure one A schematic diagram of the HEMT showing the dimensions and the coordinate system used in the analysis.

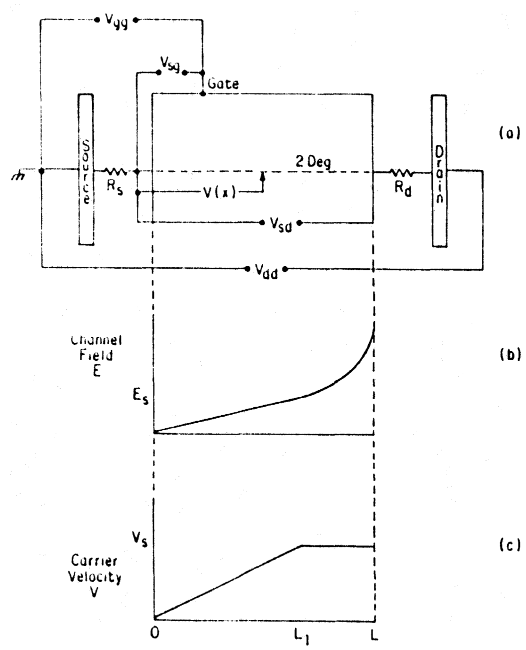

Figure two A cross section of the idealised HEMT showing the potentials used in the analysis.

a) The bias voltages applied to the HEMT. In the idealised model R_s and R_g are absent.

b) The field distribution in the 2D electron gas.

c) The electron velocity in the 2D electron gas . Below $x = L_1$,where velocity saturation occurs , constant mobility is assumed; beyond L_1 constant electron velocity is assumed.

where,

$V_{\text{off}} = \phi_b - \Delta E_c - V_p + \Delta E_{F0}$ is the voltage needed to destroy the 2D gas

$$\Delta E_{F0} = 0 \text{ at } 300\text{K}$$

$$= 0.025 \text{ below } 77\text{K}$$

$$\Delta d = \frac{\epsilon a}{q}$$

$$a = 0.125\,10^{-16}\,V m^{-2}$$

$$d = d_2 + e_2$$

$d_2 =$ thickness of doped AlGaAs

$e_2 =$ thickness of undoped buffer layer

$\epsilon_2 =$ dielectric constant of AlGaAs

$q =$ electronic charge

$$V_p = \frac{qN_2(d - e_2)^2}{2\epsilon_2}$$

$N_2 =$ doping density of the AlGaAs

$\Delta E_c =$ 2D Gas barrier height

Following Delagebeaudeuf [4], The charge in the channel at a point, x, is given by,

$$Q(x) = \frac{\epsilon_2}{(d + \Delta d)} \left[V_{\text{sg}} - V_{\text{off}} - V(x)\right] \tag{1}$$

where, $V(x)$, is the channel voltage at a point, x . To simplify the later analysis the following reduced potentials will be defined.

$$s = \frac{V_{sg} - V_{off}}{V_{off}}$$
$$w = \frac{V_{sg} - V_{off} - V(x)}{V_{off}}$$
$$p = \frac{V_{sg} - V_{off} - V(L_1)}{V_{off}}$$

where, L_1, is the point in the channel where the electrons reach their saturated velocity.

The current flowing is given by the following equation,

$$I_d = Q(x)\,\mu_0 Z \frac{dV}{dx}$$

which is equal to,

$$I_d = -\frac{\epsilon_2}{(d + \Delta d)} \mu_0 Z w V_{off}^2 \frac{dw}{dx}$$

when written in terms of the reduced potentials. Integrating this expression over the range $0 < x < L_1$ or alternatively $s < w < p$ gives,

$$I_d \int_0^{L_1} dx = -\frac{\epsilon_2}{(d + \Delta d)} \mu_0 Z V_{off}^2 \int_s^p w\,dw$$

So,

$$I_d L_1 = \frac{\epsilon_2}{2\,(d + \Delta d)} \mu_0 Z V_{off}^2 (s^2 - p^2) \tag{2}$$

In the saturated region, the current is simply given by the product of the number of charge carriers and their velocity, so

$$I_d = Q(L_1) Z v_s = \frac{\epsilon_2}{(d + \Delta d)} V_{off}\, p\, v_s \tag{3}$$

which must equal the current flowing in the unsaturated region. Equations (2) and (3) allow L1 to be calculated,

$$L_1 = \frac{(s^2 - p^2)}{2p} \frac{V_{\text{off}}}{E_s} \qquad (4)$$

where, $E_s = \frac{v_s}{\mu_0}$, is the field at which velocity saturation occurs.

(1.3)The Potential Distribution in the Channel.

The potential drop in the channel in the ohmic region is given by,

$$V_p = -(V(L_1) - V(0)) = V_{\text{off}}(s - p)$$

In region II, the potential drop must be obtained by the integration of the longitudonal electric field present over the range, $L_2 < x < L$, This field is determined only be the free electrons on the drain electrode.

Grebene and Ghandi [3] analysed this situaton for the JFET. A critical assumption in their analysis was that the thickness of the layer in which the current flowed was very much smaller than the thickness of the doped semiconducor (or alternatively the distance from the gate electode to the channel. This assumption is very much better satified in the case of the HEMT as the current flow is confined to a sheet.

Following their analysis, the potential in the channel may be approximated by the following equation,

$$\Phi(x) = \frac{2\,(d + \Delta d)}{\pi} E_s \sinh \frac{(\pi(x - L_1))}{2\,(d + \Delta d)}$$

so the voltage drop in region II is

$$V_{II} = \frac{2\,(d + \Delta d)}{\pi} E_s \sinh \left(\frac{\pi(L - L_1)}{2\,(d + \Delta d)} \right)$$

Adding this to the the drop in region I yields an expression for the source to drain voltage

$$V_{\mathrm{sd}} = V_{\mathrm{off}} \left(s - p + \frac{2\,(d + \Delta d)}{\pi L} \Psi \sinh \left(\frac{\pi L_2}{2\,(d + \Delta d)} \right) \right) \tag{5}$$

where, Ψ, is the saturation parameter defined as,

$$\Psi = \frac{E_{\mathrm{s}} L}{V_{\mathrm{off}}}$$

Combining equations (4) and (5) allows, L_1, to be eliminated and the reduced potentials, s, and, p, to be calculated for a given bias condition. The resulting nonlinear equations are most easily solved by computer.

(1.4) Small Signal Parameters.

The small signal parameters, r_{d}, and, g_{m}, can be evaluated by small purturbations of the applied bias. This could be done numerically, but more physical insight is provided if analytical expressions can be derived. These parameters are defined as follows,

$$r_{\mathrm{d}} = -\left.\frac{\partial V_{\mathrm{sd}}}{\partial I_{\mathrm{d}}}\right|_{V_{\mathrm{sg}}}$$

$$g_{\mathrm{m}} = -\left.\frac{\partial I_{\mathrm{d}}}{\partial V_{\mathrm{sg}}}\right|_{V_{\mathrm{sd}}}$$

As a similar method is used to derive both parameters, only the derivation of r_{d} will be described in detail.

(1.4.1) Drain Resistance.

As,

$$r_{\mathrm{d}} = - \left. \frac{\partial V_{\mathrm{sd}}}{\partial I_{\mathrm{d}}} \right|_{V_{\mathrm{sg}}}$$

and,

$$V_{\mathrm{sd}} = V_{\mathrm{off}} \left[s - p + \frac{2\,(d + \Delta d)\,\Psi}{\pi L} \sinh \left[\frac{\pi L_2}{2\,(d + \Delta d)} \right] \right]$$

r_{d} can be written as follows,

$$-r_{\mathrm{d}} = \left[V_{\mathrm{off}} \frac{dp}{dI_{\mathrm{d}}} + \frac{\Psi}{L} \cosh \left(\frac{\pi L_2}{2\,(d + \Delta d)} \right) \frac{dL_2}{dI_{\mathrm{d}}} \right] \tag{6}$$

From equation (3),

$$p = \frac{\epsilon_2}{(d + \Delta d)} \frac{I_{\mathrm{d}}}{Z v_{\mathrm{s}} V_{\mathrm{off}}}$$

so that,

$$\frac{dp}{dI_{\mathrm{d}}} = \frac{(d + \Delta d)}{\epsilon_2 Z v_{\mathrm{s}} V_{\mathrm{off}}}$$

The relationship, $L = L_1 + L_2$,is used to determine the derivative, $\frac{dL_2}{dI_{\mathrm{d}}}$, and hence,

$$\frac{dL_2}{dI_{\mathrm{d}}} = -\frac{dL_1}{dI_{\mathrm{d}}} = -\frac{dL_1}{dp}\frac{dp}{dI_{\mathrm{d}}}$$

Differentiating equation (4) gives,

$$\frac{dL_1}{dp} = \frac{d}{dp}\left[\frac{s^2}{2p} - \frac{p}{2}\right]\frac{V_{\mathrm{off}}}{v_{\mathrm{s}}} = -\frac{V_{\mathrm{off}}}{2E_{\mathrm{s}}}\left[\frac{s^2 + p^2}{p^2}\right]$$

Substituting these values into the equation yields the result,

$$r_{\mathrm{d}} = \frac{(d + \Delta d)}{\epsilon_2 Z v_{\mathrm{s}}} \left[1 + \frac{s^2 - p^2}{2p^2} \cosh \left(\frac{\pi L_2}{2\,(d + \Delta d)} \right) \right]$$

and similarly,

$$g_{\mathrm{m}} = -\frac{\epsilon_2}{(d+\Delta d)} Z v_{\mathrm{s}} \frac{\left[1 - \frac{s}{p}\cosh\left(\frac{\pi L_2}{2(d+\Delta d)}\right)\right]}{\left[1 - \frac{s^2-p^2}{2p^2}\cosh\left(\frac{\pi L_2}{2(d+\Delta d)}\right)\right]}$$

(1.4.2) The Gate to Source Capacitance.

The final small signal parameter which needs to be determined is the gate to source capacitance, C_{sg} . This is defined as the rate of change of free charge on the gate with respect to the gate voltage with the source voltage held constant. To determine, C_{sg}, we need to calculate the charge stored in region I and then add the charge stored in region II. In short gate length microwave devices, there is an additional component to be added caused by the fringing capacitance at the edge of the gate electrode. This component can be dominant for devices with gates shorter then 0.5 microns. The charge stored under the gate in region I is,

$$Q_1 = \int_0^{L_1} Q(x)\,dx$$

$$= \frac{\epsilon_2}{(d+\Delta d)} \int_0^{L_1} (V_{\mathrm{sg}} - V_{\mathrm{off}} - V(x))\,dx \quad = \quad \frac{\epsilon_2}{(d+\Delta d)} V_{\mathrm{off}} \int_0^{L_1} w\,dx$$

But as, $dx = -Z\mu_0 \frac{\epsilon_2}{(d+\Delta d)} V_{\mathrm{off}}^2 \frac{w}{I_{\mathrm{d}}}\,dw$ then,

$$Q_1 = -Z\mu_0 \left(\frac{\epsilon_2}{(d+\Delta d)}\right)^2 \frac{V_{\mathrm{off}}^3}{I_{\mathrm{d}}} \int_s^p w^2\,dw$$

Thus, Q_1,is given by,

$$Q_1 = -Z\mu_0 \left(\frac{\epsilon_2}{(d+\Delta d)}\right)^2 \frac{V_{\mathrm{off}}^3}{I_{\mathrm{d}}} \frac{(p^3 - s^3)}{3}$$

Substituting for I_d gives

$$Q_1 = \frac{\epsilon_2}{(d+\Delta d)} V_{\text{off}} L_1 \frac{(p^3 - s^3)}{(p^2 - s^2)}$$

The charge stored in region II has to be added to this giving the final result for the charge stored under the gate,

$$Q = \frac{\epsilon_2}{(d+\Delta d)} V_{\text{off}} \left[L_1 \frac{2}{3} \frac{(p^3 - s^3)}{p^2 - s^2} + pL_2 \right]$$

The gate to source capacitance, C_{sg}, is defined to be,

$$C_{\text{sg}} = Z \frac{\partial Q}{\partial V_{\text{sg}}}$$

So, to evaluate the capacitance, the above expression has to be differentiated with respect to V_{sg} . Now,

$$\frac{\partial Q}{\partial V_{\text{sg}}} = \frac{\partial Q}{\partial p} \frac{dp}{dV_{\text{sg}}}$$

The derivative of p with respect to V_{sg} is simply $\frac{1}{V_{\text{off}}}$. Noting that L_1 and L_2 are both functions of p, (their derivatives were calculated in the previous section), enables the result to be obtained.

$$C_{\text{sg}} = \frac{\epsilon_2}{(d+\Delta d)} Z$$
$$\left[L_1 \left(\frac{2p^2}{p^2 - s^2} - \frac{4(p^3 - s^3)}{3(p^2 - s^2)^2} \right) + L_2 + \frac{V_{\text{off}}}{2E_s} \frac{s^2 - p^2}{p^2} \left[p - \frac{2(p^3 - s^3)}{3(p^2 - s^2)} \right] + 1.56 \right]$$

where the final numerical term is due to the gate fringing capacitance (Pucel et al [2,6]).

2.0 Noise Analysis of a HEMT.

The DC analysis described in section one has followed the analysis performed by Pucel, Haus and Statz [2,6] for the GaAs FET. In this section their noise analysis will be applied to the HEMT.

The analysis proceeded in 4 stages. In the first the drain circuit noise, assuming open circuit conditions, is calculated. Then the gate circuit noise with short circuit drain is determined and the correlation coefficient between these two noise sources calculated. Finally, using a simple equivalent circuit for the HEMT, the minimum noise figure and associated parameters are derived.

(2.1) Drain Circuit Noise in a HEMT.

The calculation of the mean square noise voltage at the drain, caused by thermal noise fluctuations in region I will, proceed as follows. The mean square magnitude of the noise voltage at a point, x_0, in the channel will be calculated, assuming that it is Johnson noise with a field dependant electron temperature. The noise voltage at the end of region I will then be determined by adding all these contributions together, assuming that they are incoherent. Finally the "amplification " of this noise voltage by region II will yield the mean square drain noise voltage, $\overline{|v_d^2|}$.

Van der Ziel [7] calculated the noise voltage in the drain circuit for the JFET. Pucel et al [2,6] enhanced this analysis to include both a field dependant noise temperature and the effects of region II on the noise voltage developed at the end of region I. Following the analysis of Pucel, modified to take account of the different mode of operation in the HEMT, it can be shown (appendix A) that the mean square noise voltage developed at the drain by thermal noise fluctuations in the channel is equal to,

$$\overline{|v_{\mathrm{d}}^2|} = 4kT_0\Delta f \frac{V_{\mathrm{off}}}{I_{\mathrm{d}}} \cosh^2\left(\frac{\pi L_2}{2\,(d+\Delta d)}\right)\{P_0 + P_\delta\} \tag{10}$$

where,

$$\begin{aligned} P_0 &= \frac{t}{3p^2}(p^3 - s^3) \quad \text{and,} \\ P_\delta &= \frac{\delta}{p}\ln\left(\frac{p}{s}\right) \end{aligned} \tag{10a}$$

To calculate the noise produced by sources in region II, a different method is employed. Noting that the DC and small signal analysis of region II employed in this model is exactly the same as that used by Pucel et al. [2] for the GaAs FET, their analysis can be followed directly. The mean square noise voltage, $\overline{|v_{d_2}^2|}$, at the drain caused by noise sources in region II is,

$$\overline{|v_{d_2}^2|} = \frac{32\,(d+\Delta d)^2}{\pi^4 v_s^2}\frac{q^2 D N \Delta f}{\epsilon_2^2 b_p Z}\sinh^2\left(\frac{\pi b_p}{2\,(d+\Delta d)}\right)$$
$$\left\{\exp\left[\frac{\pi}{2\,(d+\Delta d)}(L - x_0)\right] - 1\right\}^2 dx_0 \tag{11}$$

where, D, is the high field diffusion coefficient and, b_p, the thickness of the conducting layer.

In the case of the HEMT, b_p is small, so taking the limit of equation (11) as b_p tends to zero allows the noise voltage to be calculated. The limit of this expression is zero, implying that no noise is generated in region II.

A possible explanation for this suprising result procedes as follows. In the GaAS FET, the dipole produced by the spontaneous fluctuations in the channel consists of two line charges of length, b_p, separated be a small distance Δx . The field produced by such a charge distribution decays as the log of the distance. For the HEMT, where the conducting layer is assumed to be infinitessimally thin, the field of a dipole decays as 1/distance. The noise fluctations induced on the drain are proportional to the field, so as the dipole field decays much more rapidly for the HEMT, the noise will be much smaller. Thus, is seems justified to ignore the contributions from region II in this analysis.

Hence, the mean square noise voltage on the drain is given by equation (10). The noise current flowing may then be obtained by simple circuit analysis and is given by

$$\overline{|i_{\mathrm{g}}^2|} = \frac{\overline{|v_{\mathrm{d}}^2|}}{r_{\mathrm{d}}^2} = 4kT_0\Delta f\frac{V_{\mathrm{off}}}{I_{\mathrm{d}}r_{\mathrm{d}}^2}\cosh^2\left(\frac{\pi L_2}{2\left(d+\Delta d\right)}\right)\{P_0+P_\delta\} \tag{12}$$

(2.2) Gate Circuit Noise.

A different procedure is followed to calculate the noise current induced on the gate by the noise fluctuations in the channel. The calculation is decribed in detail in appendix B, so only a short outline will be presented here.

The previous calculations have shown that there are noise voltage fluctuations along the channel, which will induce noise charges on the gate since it is capacitvely coupled to the channel. As these charges are time dependant, noise currents will flow.Pucel et al [2,6], determined these noise currents for the GaAs FET. A similar analysis, performed for the HEMT, shows that the induced charge per unit gate width, Z, caused by an elementary thermal noise fluctuation at a point, x_0, in the channel is (equation B4),

$$\Delta q_r = \frac{\epsilon_2}{(d+\Delta d)}L_1\frac{\Delta i_{\mathrm{d}}}{I_{\mathrm{d}}}V_{\mathrm{off}}\left[\kappa-\gamma(p-w)+\frac{L_2}{L_1}p\right]$$

where,

$$\gamma = \frac{Z\epsilon_2\mu_0 r_d p V_{\text{off}}}{L_1 (d+\Delta d) \cosh\left(\frac{\pi L_2}{2(d+\Delta d)}\right)}$$

$$\kappa = \frac{1}{(s^2 - p^2)}\left[s^2(p-s) - \frac{(p^3 - s^3)}{3}\right]$$

By integration this expression along the channel so as to sum up the contributions from all points, and accounting for the elevated electron temperature, the mean square value of the fluctuating charge can be determined. The spectral components of the noise currents can then be obtained by multiplying the spectral components of the fluctating charge calculated above by the angular frequency. Doing this, one arrives at the following expression for the mean square of the induced gate noise,

$$\overline{|i_g^2|} = \omega^2 D\,[R_0 + R_\delta] \tag{13}$$

where,

$$R_0 = \frac{(p^3 - s^3)}{3p^2}\left[\kappa'^2 + \gamma^2 p^2 - 2\kappa'\gamma p\right] + \frac{(p^4 - s^4)}{2p^2}\left[\gamma^2 p - \kappa'\gamma\right] + \frac{(p^5 - s^5)}{5p^2}\gamma^2$$

$$R_\delta = \delta p \ln\left(\frac{p}{s}\right)\left[\kappa'^2 + \gamma^2 p^2 - 2\kappa' p\right] + 2p(p-s)\left[\gamma^2 p - \kappa'\gamma\right] + \frac{(p^2 - s^2)}{2}p\gamma^2$$

$$D = 4kT_0\frac{\Delta f}{I_d}\frac{V_{\text{off}}}{r_d^2}\cosh^2\left(\frac{\pi L_2}{2(d+\Delta d)}\right)B$$

$$B = \left[\frac{\epsilon_2 L_1 V_{\text{off}}}{(d+\Delta d) I_d}\right]^2$$

$$\kappa' = \kappa - \frac{L_2}{L_1}$$

The terms, R_0, and, R_δ, refer to the noise produced by normal thermal noise and the hot electron noise.

Using similar arguments to those in the previous section, the noise current induced in the gate circuit by the sources in region II, can be shown to be negligable. Thus, the strength of the noise sources in a HEMT have been determined.

It remains to calculate the correlation between the gate and drain circiut noise. Since they arrise from the same fundemental noise mechanism, they are strongly correlated. Assuming that the correlation coefficient is purely imaginary as it is in the case case of the JFET (Klassan [8]), it can be calculated using the methods of Pucel et al [2,6], and Van der Ziel [5]. The calculation ,detailed in appendix B, shows that,

$$C = \frac{\overline{i_g^* i_d}}{\sqrt{|\overline{i_g^2}||\overline{i_d^2}|}} = \frac{\frac{1}{p^2}\left[S_0 + S_\delta\right]}{\sqrt{[P_0 + P_\delta][R_0 + R_\delta]}} \tag{14}$$

where,

$$S_0 = \frac{1}{p^2}\left[\frac{(\kappa' - \gamma p)(p^5 - s^5)}{5} + \frac{\gamma(p^6 - s^6)}{6}\right]$$

and,

$$S_\delta = \delta\frac{1}{p^2}\left[\frac{(\kappa' - \gamma p)(p^2 - s^2)}{2} + \frac{\gamma(p^3 - s^3)}{3}\right]$$

(2.3) Noise coefficients.

The following set of dimensionless noise coefficients are normally defined,

$$P = \frac{|\overline{i_d^2}|}{4kT_0\Delta f g_m}$$

$$R = \frac{|\overline{i_g^2}| g_m}{4kT_0\omega^2 C_{sg}^2}$$

and, C, the correlation coefficient between them. These parameters can be defined simply in terms of the quantities just calculated. They have the following values:

$$P = \frac{2(d+\Delta d)}{r_{\mathrm{d}}^2 \epsilon_2} \frac{L_1}{Z\mu_0 V_{\mathrm{off}} g_{\mathrm{m}} (s^2 - p^2)} \cosh^2\left(\frac{\pi L_2}{2(d+\Delta d)}\right) [P_0 + P_\delta]$$

$$R = Z^2 \frac{V_{\mathrm{off}}}{I_{\mathrm{d}}} \frac{B}{C_{\mathrm{sg}}^2} g_{\mathrm{m}} [R_0 + R_\delta]$$

$$C = \frac{\frac{1}{p^2}[S_0 + S_\delta]}{\sqrt{[P_0 + P_\delta][R_0 + R_\delta]}}$$

The expressions for the noise parameters for the HEMT presented in the next section can be simplified if the following functions of P, R, and C are used.

$$K_g = P\left(\left(1 - C\sqrt{\frac{R}{P}}\right)^2 + \left(1 - C^2\right)\frac{R}{P}\right)$$

$$K_c = \frac{1 - C\sqrt{\frac{R}{P}}}{\left(\left(1 - C\sqrt{\frac{R}{P}}\right)^2 + \left(1 - C^2\right)\frac{R}{P}\right)}$$

$$K_r = \frac{R\left(1 - C^2\right)}{\left(\left(1 - C\sqrt{\frac{R}{P}}\right)^2 + \left(1 - C^2\right)\frac{R}{P}\right)}$$

These coefficients will be used in the following section to calculate the noise figure and associated parameters for the HEMT.

Results.

The noise temperature predicted by this theory is proportional to frequency, so the analysis is restricted to high frequencies where the contribution to the noise from traps and other low frequency noise generators is negligable. The validity of any theory can only be tested by comparing its predictions to experiment. This requires a detailed knowlege of the device dimensions and the associated parasitic elements. A simple equlvalent circuit for the HEMT will suffice to determine the noise temperature, the small puturbations introduced by the other circuit elements can be introduced if necessary [2].

The equivalent circuit chosen, shown in figure 3, is identical that of Pucel et al [2], so the derivation of the noise parameters follows theirs exactly. Thus, the noise parameters are given by :

$$r_n = (R_g + R_s) + K_r\left(\frac{1+\omega^2 {C_{sg}}^2 R_i^2}{g_m}\right)$$

$$g_n = K_g\frac{\omega^2 C_{sg}^2}{g_m}$$

$$Z_{opt} = (R_s + R_g) + K_g\left(R_i + \frac{1}{j\omega C_{sg}}\right)$$

$$T_{min} = 2T_0 g_n\left[R_c + R_{opt}\right]$$

The noise temperature is then,

$$T_n = T_{min} + \frac{T_0 g_n}{R_{sou}}\left[(R_{sou} - R_{opt})^2 + (X_{sou} - X_{opt})^2\right]$$

where Z_{sou} is the source impedance .

In measurements on an actual transistor, it is very difficult to measure all the noise parameters, particulaly if it is packaged and the measured noise parameters have to be de-embedded to arrive at the "chip" noise parameters. For this reason, the comparison between

Table I The Quantum Well HEMT

Parameter		Value
Gate length	L	$0.35\mu m$
Undoped epithickness	e_2	$0.035\mu m$
Doped epithickness	d_2	$0.035\mu m$
Doping Density	N_d	$3.8 \times 10^{17} cm^{-3}$
Mobility	μ_0	$4500 cm^2 \ V^{-1} s^{-1}$
Saturation Field	E_s	$2900 V cm^{-1}$
Barrier Height	ΔE_c	0V
Source Resistance	R_s	5Ω
Gate Resistance	R_g	5Ω
Drain Resistance	R_d	3Ω
Intrinsic Resistance	R_i	1Ω

Table II The TRW HEMT

Parameter		Value
Gate length	L	$0.35\mu m$
Undoped epithickness	e_2	$0.002\mu m$
Doped epithickness	d_2	$0.068\mu m$
Doping Density	N_d	$2.8 \times 10^{17} cm^{-3}$
Mobility	μ_0	$4500 cm^2 \ V^{-1} s^{-1}$
Saturation Field	E_s	$2900 V cm^{-1}$
Barrier Height	ΔE_c	$0V$
Source Resistance	R_s	3Ω
Gate Resistance	R_g	3Ω
Drain Resistance	R_d	3Ω
Intrinsic Resistance	R_i	1Ω

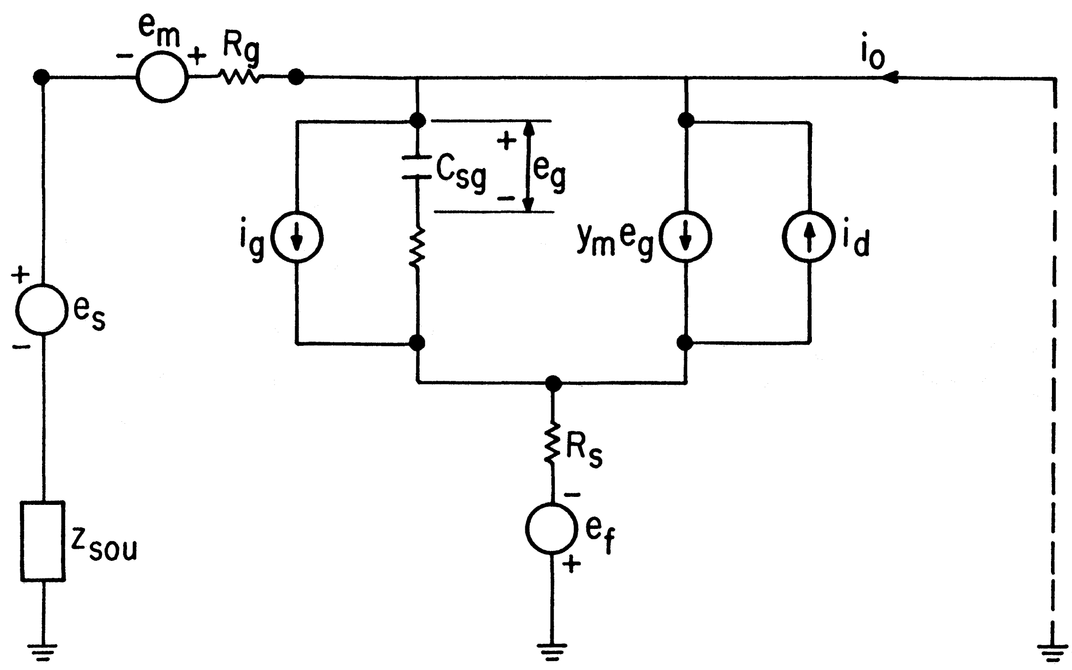

Figure three The equivalent circuit for the HEMT used in the noise analysis. Note that there are no feedback elements.

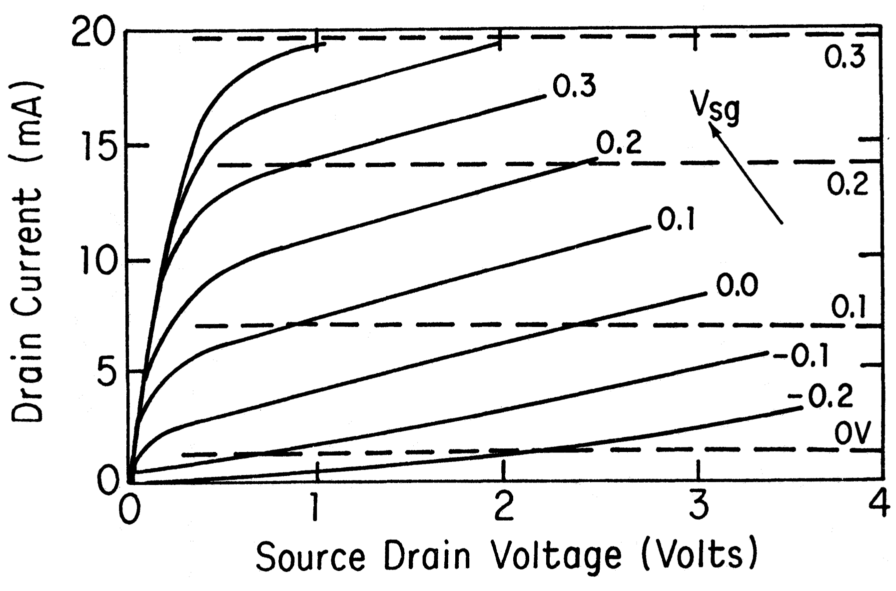

Figure four The measured (solid lines) and calculated (dashed lines) I-V characteristics for the HEMT of Table 1. The discrepancy at low currents is caused by the difference between the measured (-0.2V) and calculated (-0.05V) pinch off voltages , and also by neglecting the parasistic source and drain resistances and substrate conduction in the calculation.

theory and experiment will be limited to the noise temperature which is invariant under lossless transformations. In addition, the DC and small signal parameters predicted by the theory can be compared with measurements, noting that the measured transconductance is related to the calculated transconductance by

$$g_m^* = \frac{g_m}{1 + R_s g_m}$$

The theory has been applied to devices supplied to NRAO by Cornell University. The noise performance and the material parameters have already been reported by Camnitz et al [9]. The parameters of the device are given in Table 1. Figures 4, 5 and 6 show the I-V characteristics for the transistor, the transconductance and the minimum noise temperature. The agreement between the calculated and measured currents is fair, most of the discrepancy being due to a difference in the threshold voltage (measured -0.2 and calculated -0.05) . At larger forward bias, the measured and calculated drain currents are within 20% of each other.

In comparing the noise temperature, the measurements and calculations at the same drain current are used. The dissagreement at low currents is probably caused by the drop in the transconductance in the transistor, which does not occur in the model. The agreement is greatly improved by using measured transconductances, shown in the second curve . This clearly shows the importance of maintaining a high transconductace at low currents if low noise temperatures are to be achieved . Small non- uniformities in the thickness of the epi-layer have been shown to produce a large drop in the transconductance in the MESFET at low currents and a consequent increase in the noise temperature [11] . A similar effect could be occuring here, particularly if it is noted that the epi-layer is only 0.05 microns thick about 200 atomic layers . It is important then to maintain the uniformity of the epilayer in low noise devices.

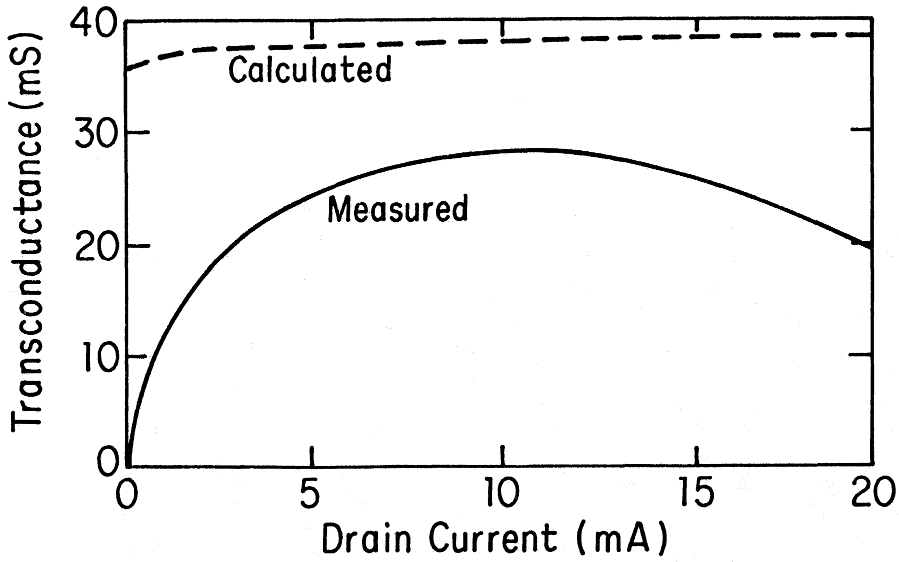

Figure five The measured and calculated transconductance for the HEMT described in the text. The measured g_m falls much more rapidly at low currents than that calculated, possibly due to small variations in the epilayer thickness. The reduction in g_m at high currents is probalbly caused by the formation of a parasistic MESFET in the structure [13].

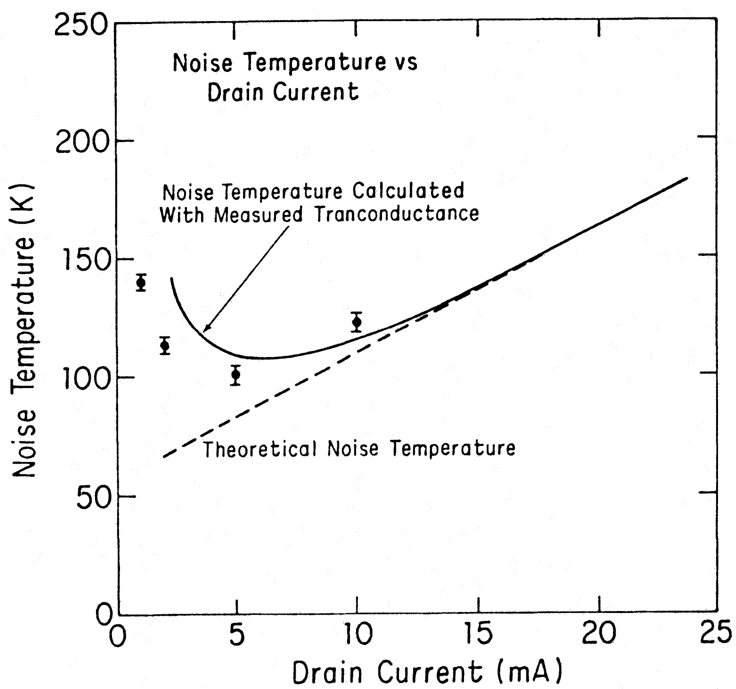

Figure six The calculated and measured noise temperatures for the HEMT. The curves were obtained using measured and calulated values of transconductance, thus indicating the importance of maintaining a large transconductance at low currents if a low noise temperature is to be obtained.

The agreement between the theory and experiment is fair. Thus, it seems justified to use the theory see find the dependance of the noise parameters on the various material parameters. The device whose parameters are given in Table 1 was used as the basis for this analysis .All the results are for a drain voltage of 2 volts and 5 mA drain current. Further restrictions were placed on the analysis to maintain a gate length to epilayer thickness ratio of greater than 3. Figures 7 and 8 show how the minimum noise temperature changes as the gate length and doped epilayer thickness were varied. The noise temperature is very nearly proportional to gate length in this model. Thus reducing the gate length from 0.4μm to 0.3μm will decrease the noise temperature by 25%. Some effects which occur in short gate length devices, such as velocity overshoot, may cause the rate of change of noise temperature with gate length to be different, but the trend is correct.

The noise temperature is roughly inversely proportional to the thickness of the doped epi-layer. Combining these two results show that a device with as low a ratio of gate length to epi-layer thickness as possible should produce the lowest noise, providing the transconductance can be maintained at the low operating current.

The variation of the noise temperature with doping density and with the thickness of the undoped epilayer was also explored. For a fixed drain current little variation of the noise temperature was seen as these parameters were altered over a range of approximatly 10 to 1.

Cryogenic Performance.

The noise properties of several devices have been measured at cryogenic temperatures. A general feature of the noise temperature at these temperatures is that it is light dependant, illuminating the transistor changes the noise temperature. If the device is left in darkness,

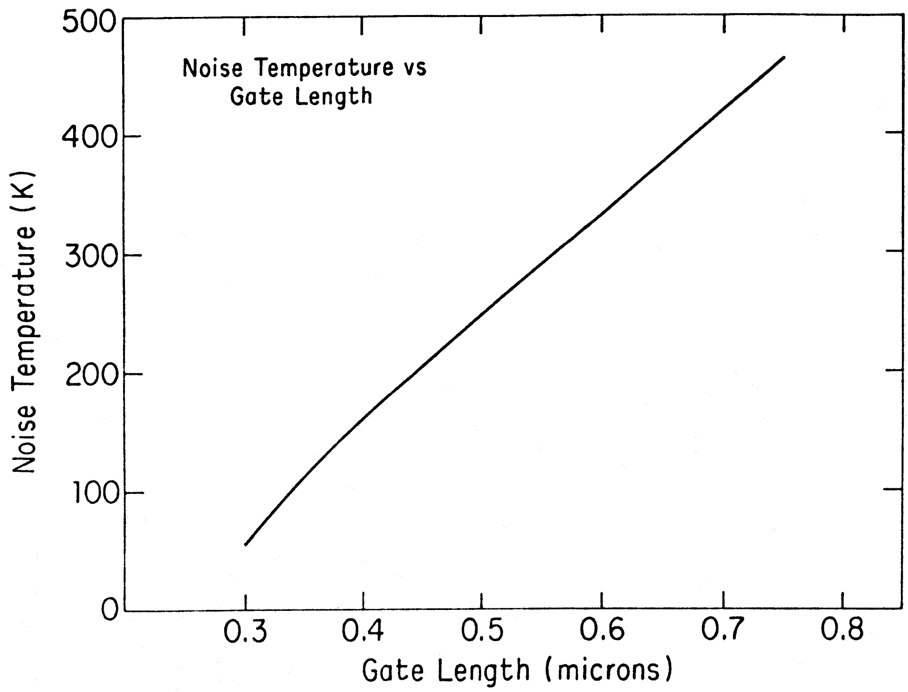

Figure seven The calculated variation of noise temperature with gate length for the device of Table 1. The model restricts the analysis to ratio's of gate length to epithickness greater than 3 .

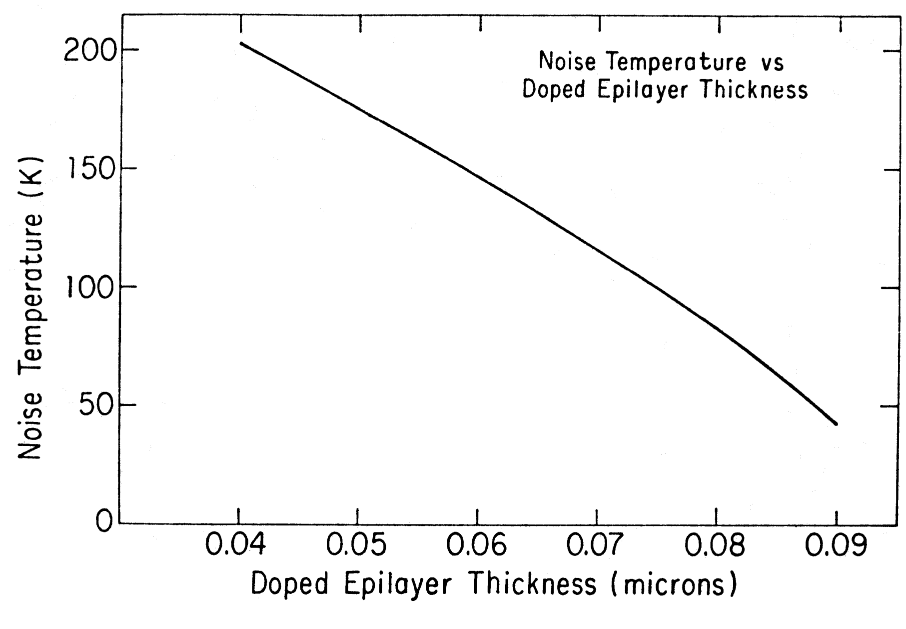

Figure eight The calculated variation of noise temperature with doped epithickness for the device of Table 1. The analysis is restricted to ratios of gate length to epithickness greater than 3 where the model is valid. The reduction in the noise temperature is primarily caused by a decrease in the gate to source capacitance.

the noise temperature usually increases to several times its initial value after a period of many hours. Illuminating the transistor returns the noise to its inital value after a period of a few seconds. The mechanism responsible for this noise is probably associated with deep level traps in the GaAlAs . A generation-recombination process, as the electrons are captured and subsequently re-emitted could cause noise, and the long time scale is caused by the high activation energy of the traps in relation to the thermal energy of the electrons. Illuminating the transistor would release the electrons from the traps as they absorbed energy from the incident photons. This noise source is not included in the model, explaining why the noise temperature and its current dependance are different from those measured.

Figure 9 shows the noise temperature at different ambient temperatures for a TRW HEMT operated at 24GHz. It should be noted that the true minimum noise temperature was not measured, only a single source impedance being presented to the transistor, but the temperature dependance of T_{min} should be similar. The qualitative agreement between theory and experiment is fair, both being linearly proportional to temperature. The fit could be improved by allowing the hot electron noise coefficient, δ , to be a function of temperature ; an effect which has been observed in MESFET's [12], where values ranging from 1.2 to 8 have been observed. An alternative explanation for the rapid decrease in the noise temperature is that the mobility of the GaAs is increasing as the temperature decreases. This has also been observed experimentaly and is the more likely of the two explanations. Figure 10 compares the theoretical noise temperature, calculated using a linear variation of mobility with temperature and a saturation velocity proportional to mobility, and the measured noise temperatures for this transistor The intercept at 0 K shows the contribution to the total noise from the hot electrons, as thermal noise has

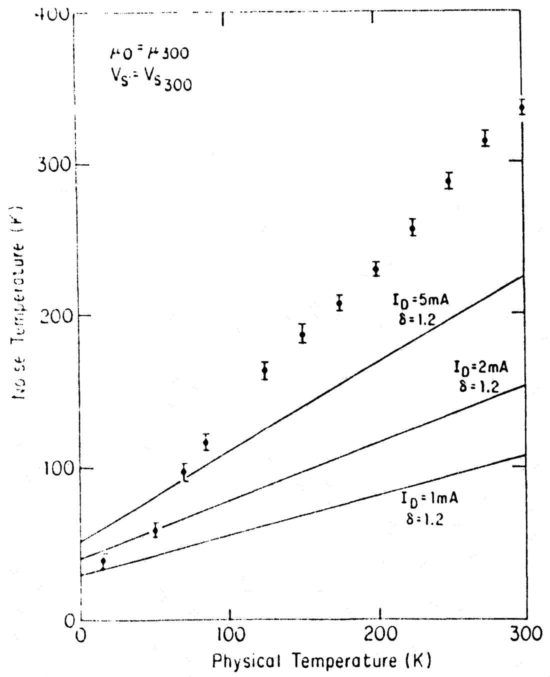

Figure nine The noise temperature , calculated for constant mobility, is compared with measured data. Better agreement would be obtained if the "hot electron noise " coefficient δ were a function of temperature.

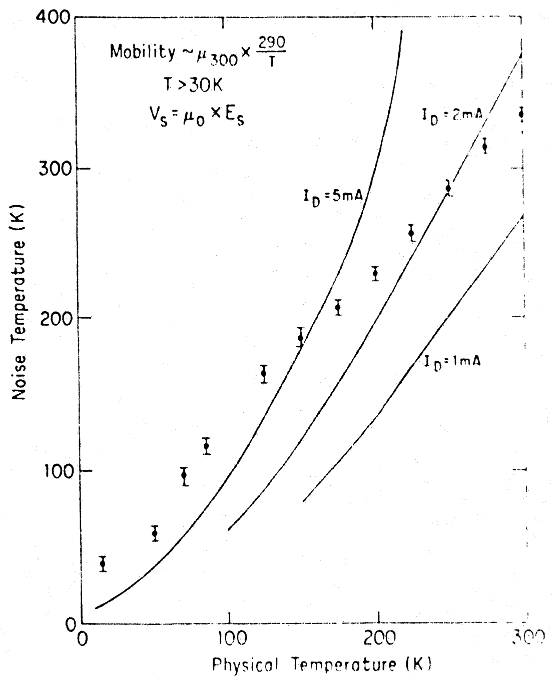

Figure ten The noise temperature, calculated with a temperature dependant mobility is shown. Whilst the agreement between calculated and measured data is better than that shown in figure nine, the agreement between the calculated and measured I-V characteristics is poor, indicating the importance of some other mechanism to controll the current flowing at low temperatures.

been eliminated by cooling. The three curves plotted for three different currents shows that best cryogenic performance will be attained for transistors which maintain a high transconductance at low currents.

Conclusions.

The simple model of a HEMT presented here has been shown to be in reasonable agreement with experiment. The primary noise mechanism at room temperature is thermal noise, high field diffusion noise important in the MESFET can be ignored. Simple simulations have shown that it is important to be able to operate the device at low currents and with a high transconductance to obtain the lowest noise temperature. Non-uniformities in the epilayer thickness have been suggested as the reason for the fall in transconductance at low currents, so by growing more uniform layers a better device should result. It may be possible in future to use this simple model to design a low noise HEMT, or at least to guide the designer in which parameters are to be tightly controlled.

3.0 Instructions on how to use NOISE.

The program was written for interactive use, but can be used on batch systems. As far as possible, standard FORTRAN 77 was used, but anyone using the program on a machine other than a VAX may have problems.

The program is split into 3 parts;

1) entry of the material parameters and the device dimensions
2) defining what type of analysis is required
3) writing out the results

In future, it will be possible to plot the results, but the code to perform this will be written in SGS, a device independant graphical language implimented on the STARLINK network. This system is a high level graphical language based on the GKS graphics system and will probably be availlable to Universities and Astronomical Institutes in the near future.

When the program is run, the following prompt will appear

NOISE >

When this is visible, the program is requesting the user to enter a command to tell the program what to do. The first thing to do is to enter the material parameters so the program knows what device to analyse.

(3.1) Data Entry.

To enter the data into the program, first type *DATA* and the prompt

$$DATA >$$

will appear on the screen. Then enter the values needed. The same format of command is used to enter all the parameters. It is as follows

$$PARAMETER \quad = \quad VALUE$$

The parameter, =, and value are separated by one or more spaces. More then one parameters may be entered on each line. The values can be any real or integer format eg.

1.2

10

1.5*E*17

would be valid values. Either uppercase or lower case letters may be used as they are converted to uppercase internally. For some parameters, there are two possible key words, either will do and they can be abbreviated until they are no longer unique. An error message will be produced if this occurs, but the program will still run in interactive systems. It is FATAL in a batch enviroment.

The parameters needed are as follows:

Doping Density   *DOPING_DENSITY*   *ND*

Dielectric constant   The relative dielectric constant for the material

DIELECTRIC_CONSTANT   *EPS2*

Schottky Barrier Potential   *SHOTTKY_VOLTAGE*   *PHI*   Volts

Mobility *MOBILITY*

Saturation Field The electric field at which velocity saturation occurs

SATURATION_FIELD *ESAT* V/m

Delta Ec *DELEC* V

Total Epi thickness *EPI_THICKNESS* *D*2 microns

Undoped Epi Thickness *UNDOPED_EPI* *E*2 microns

Gate Length *GATE_LENGTH* *L* microns

Gate Width *GATE_WIDTH* *Z* microns

Drain Resistance *DRAIN_RESISTANCE* *RD* ohms

Gate Resistance *GATE_RESISTANCE* *RG* ohms

Source Resistance *SOURCE_RESISTANCE* *RS* ohms

Intrinsic Resistance *RI* ohms

When all of the above parameters have been set, type *ALL* *SET* (two words), and control will return to the main program. If you want to change any value during a session, type DATA again and change the values as required. It is not nescessary to re-enter all of the values.

A simple way to do the above, particulaly if many runs are to be carried out on the same device, is to enter the data into a file of type .DAT. Then when you want to enter the data, type *OBEY* filename when the prompt appears. This saves having to type all the values each time, and single values can be easily changed when the program is running.

3.2 Types of Analysis.

Having entered the data, some form of analysis is then required. Again 3 basic types are possible;

1) DC parameters (default)

2) Small signal parameters (type *SMALL_SIGNAL* or an abreviation)

3) Noise parameters (type *NOISE*)

The required analysis will then be performed. To set the frequency for the analysis the command *FREQUENCY* (freq in Hz default 22GHz) is used.

For example, the I-V characteristics of the device are needed. This requires the gate and drain voltages to be 'swept'. To do this, fix either of them by typing *VSG* (or *VSD* if the drain voltage is to remain constant) followed by the value. Then type the other parameter as follows

VSD start stop no of steps

where ***start*** is the start voltage , ***stop*** is the stop voltage and ***no of steps*** is the number of increments between the start and stop voltages

This type of swept analysis may be carried out for other parameters, but in these cases fix the bias voltages first. Possible analyses are

TEMPERATURE	change in ambient temperature
DOPING_DENSITY ND	change in doping density
MOBILITY	change in mobility
GATE_LENGTH L	change in gate length
EPI_THICKNESS D2	change in the epi-thickness
UNDOPED_EPI E2	change in the undoped epi-thickness

To perform the analysis, type *ANALYSE* ,(it will abreviate to *A*).

3.3 Writing Out the Results.

Having analyses the device, you want to print out the results you are interested in. The command

WRITE_OUT

enables you to select which of the parameters are to be printed. note that you have to have first decided to calculate them using *SMALL* or *NOISE*. The parameters which can be printed are as follows:

DRAIN_CURRENT *ID*

SOURCE_VOLTAGE *VSD*

GATE_VOLTAGE *VSG*

TEMPERATURE

DOPING_DENSITY *ND*

MOBILITY

EPI_THICKNESS *D2*

UNDOPED_EPI *E2*

TRANSCONDUCTANCE *GM*

GATE_CAPACITANCE *CSG*

DRAIN_RESISTANCE *RD*

(Note that this is the intrinsic drain resistance

and NOT the resistance between the channel and the drain contact)

SOURCE_RESISTANCE *RS*

GATE_RESISTANCE *RG*

NOISE_TEMP *TN*

NOISE_IMPEDANCE *ZOPT*

GN The noise conductance

CORRELATION_IMPEDANCE *ZC*

*P*1

*R*1 The basic noise parameters

*C*1

KC

KR Pucels derived noise parameters

KG

The names given above may be shortened until they are no longer unique.

3.4 An example of how to run the program.

Let us suppose that the device data is in a file called HEMT.DAT, and the noise temperature as a function of drain current at 10Ghz and 3 volts drain voltage is needed.

The following commands will achieve this.

NOISE >data

DATA >obey hemt

DATA >all set

NOISE >frequency 10E9

NOISE >vsd 3

NOISE >vsg 0 0.5 6

NOISE >small

NOISE >noise

NOISE >analyse

NOISE >write_out id tn

Finally to leave the program and return controll to the monitor ,type *EXIT*.

Appendix A : The Calculation of the Drain Noise Voltage.

The spectral components of drain noise voltage fluctuations are to be calculated. The noise voltage at the end of region I, caused by a noise fluctation at a single point in the channel is determined. As this fluctuation is assumed to be caused by thermal noise, its magnitude can easily be calculated, if the equivalent resistance of the channel is known. The noise voltage at the end of region I is then found by adding up all the contributions from all points in the channel. Finally, the mean square magnitude of the drain noise voltage is obtained by calculating how the noise voltage at the end of region I is enhanced by region II.

From the definition of w, we have

$$w = \frac{V_{\mathrm{sg}} - V_{\mathrm{off}} - V}{V_{\mathrm{off}}} \tag{A1}$$

Purturbing this equation gives,

$$-\Delta w V_{\mathrm{off}} = \Delta V \tag{A2}$$

where V is the channel voltage at a distance x from the source. The drain current I_{d} is given by,

$$I_{\mathrm{d}} = Z\mu_0 V_{\mathrm{off}} \frac{\epsilon_2}{(d + \Delta d)} w \frac{dV}{dx}$$

If I_{d} is held constant,this equation can be written as,

$$I_{\mathrm{d}} = \mathrm{const}(w + \Delta w) \left(\frac{dV}{dx} + \frac{d\Delta V}{dx} \right)$$

which to first order implies that,

$$\Delta w \frac{dV}{dx} + w \frac{d\Delta V}{dx} = 0$$

or,

$$\frac{d\Delta V}{\Delta w} = -\frac{dV}{w}$$

Using equation (A2), this becomes,

$$\frac{d\Delta V}{\Delta V} = -\frac{dw}{w}$$

Integrating this expression between x and L_1 gives,

$$\int_{\Delta V_x}^{\Delta V_{L_1}} \frac{d\Delta V}{\Delta V} = -\int_w^p \frac{dw}{w}$$

So that,

$$\ln \Delta V_{L_1} - \ln \Delta V_x = \ln w - \ln p$$

and so,

$$\Delta V_{L_1} = \frac{w}{p} \Delta V_x \qquad \text{(A3)}$$

ΔV_x, is assumed to be caused by thermal noise in this instance, so that its value may be determined from the following equation. The equivalent resistance, R, of the channel is equal to,

$$R = \frac{\Delta x}{Z \mu_0 Q(x)}$$

so that the mean square noise voltage[1] caused by thermal noise is,

$$\overline{\Delta V_x^2} = 4kT_e\Delta f\frac{\Delta x}{Q(x).\mathrm{const}}$$
$$= -4kT_e\Delta f\frac{V_{\mathrm{off}}\,dw}{I_{\mathrm{d}}}$$

Introducing this value into the square of equation (A3) gives,

$$\overline{\Delta V_{L_1}^2} = -4kT_e\Delta f\frac{V_{\mathrm{off}}}{I_{\mathrm{d}}}\frac{w^2}{p^2}\,dw \qquad \text{(A4)}$$

The electron temperature, T_e, is assumed to be given by the following expression due to Baechtold [5],

$$T_e = T_a + T_0\delta\left(\frac{E}{E_s}\right)^3$$

or,

$$T_e = T_0\left[t + \delta\left(\frac{E}{E_s}\right)^3\right]$$

where, T_0 = 290K, $t = \frac{T_a}{T_0}$, and, T_a, is the ambient temperature. The electric field in the channel can be derived simply as follows. The drain current, I_{d}, is equal to,

$$I_{\mathrm{d}} = Z\mu_0\frac{\epsilon_2}{(d+\Delta d)}wV_{\mathrm{off}}\frac{dV}{dx} \quad \text{, and also,} \quad Z\frac{\epsilon_2}{(d+\Delta d)}v_sV_{\mathrm{off}}p$$

Identifying, $\frac{dV}{dx}$, as the channel field, then,

$$E = \frac{pv_s}{w\mu_0} = \frac{p}{w}E_s$$

[1]the product of V_{off} and dw is always negative

The mean square noise voltage can then be calculated by integrating equation (A4) over the range,

$$0 < x < L_1$$

or,

$$s < w < p$$

so,

$$\overline{|v_{\mathrm{d}}^2|} = -4kT_0\Delta f \frac{V_{\mathrm{off}}}{I_{\mathrm{d}}} \cosh^2\left(\frac{\pi L_2}{2\,(d+\Delta d)}\right) \int_s^p \frac{w^2}{p^2}\left[t + \delta\left(\frac{p^3}{w^3}\right)\right] dw \qquad \text{(A5)}$$

where the $\cosh^2$ term represents the enhancement of the mean square noise voltage, $\Delta\overline{|v_{\mathrm{d}}^2|}$, by region II, (Pucel et al [2]). So, the final value for the voltage fluctuation at the drain end of the channel caused by thermal noise in region I can be written as,

$$\overline{|v_{\mathrm{d}}^2|} = -4kT_0\Delta f \frac{V_{\mathrm{off}}}{I_{\mathrm{d}}} \cosh^2\left(\frac{\pi L_2}{2\,(d+\Delta d)}\right) \left\{\frac{t}{3p^2}(p^3 - s^3) + \delta p \ln\left(\frac{p}{s}\right)\right\} \qquad \text{(A6)}$$

or with,

$$\begin{aligned} P_0 &= \frac{t}{3p^2}(p^3 - s^3) \quad \text{and,} \\ P_\delta &= \delta p \ln\left(\frac{p}{s}\right) \end{aligned} \qquad \text{(A7)}$$

$$\overline{|v_{\mathrm{d}}^2|} = -4kT_0\Delta f \frac{V_{\mathrm{off}}}{I_{\mathrm{d}}} \cosh^2\left(\frac{\pi L_2}{2\,(d+\Delta d)}\right) \{P_0 + P_\delta\} \qquad \text{(A8)}$$

Appendix B : The calculation of the induced gate noise and the correlation coefficient between the gate and drain circuit noise..

The analysis will procede in several stages to calculate the spectral components of the noise current fluctuations. The fluctuating charge on the gate caused by a noise fluctuation at one point in the channel will be determined. The total fluctating charge induced by disturbances along the channel will then be obtained by integrating along the channel. Then, the spectral components of the charge fluctuations will be determined and subsequently multiplied by the angular frequency to obtain the spectral components of the current fluctuation. The correlation coefficient between the gate and drain circuit noise will be derived in a similar manner.

From the DC results,

$$Q(x) = \frac{\epsilon_2}{(d+\Delta d)}(V_{\mathrm{sg}} - V(x) - V_{\mathrm{off}}) = \frac{\epsilon_2}{(d+\Delta d)} V_{\mathrm{off}} w$$

so the total charge in the channel in a length dx, at a position x is equal to,

$$Q(x)\,dx = \frac{\epsilon_2}{(d+\Delta d)} V_{\mathrm{off}} w dx$$

Consequently, a fluctuation in the voltage, J, at the point, x_0, will produce a changing charge,

$$\Delta q dx = \frac{\epsilon_2}{(d+\Delta d)} V_{\mathrm{off}} \Delta w dx \tag{B1}$$

where Δw is the change in the reduced potential w at x caused by the fluctuation J at x_0 . To procede further we require the value of Δw for a noise fluctuation.

The Drain current $I_{\rm d}$ is given by the following equation,

$$I_{\rm d} = Aw\frac{dV}{dx}$$

where,

$$A = \mu_0 Z \frac{\epsilon_2}{(d + \Delta d)} V_{\rm off}$$

Purturbing this equation yields,

$$I_{\rm d} + \Delta i_{\rm d} = A\left[(w + \Delta w)\frac{d}{dx}(V + \Delta V)\right]$$

Taking only the first order terms in variation gives,

$$\Delta i_{\rm d} = A\left[w\frac{d\Delta V}{dx} + \Delta w\frac{dV}{dx}\right]$$

Now,

$$\frac{d}{dx}(w\Delta V) \quad = \quad \frac{dw}{dx}\Delta V + w\frac{d\Delta V}{dx} \quad = \quad \Delta w\frac{dV}{dx} + w\frac{d\Delta V}{dx}$$

Subsituting this result gives a value for $\Delta i_{\rm d}$,

$$\Delta i_{\rm d} = A\frac{d}{dx}(w\Delta V)$$

Pucel et al [2] derived a similar equation for the GaAs FET. Applying their boundary conditions to the above equation gives,

$$\begin{aligned} \Delta i_{\rm d}\frac{x}{A} &= w\Delta V \qquad x < x < x_0 \\ \Delta i_{\rm d}\frac{x - \gamma L_1}{A} &= w\Delta V \qquad x_0 < x < L_1 \end{aligned} \tag{B2}$$

Suppose that there is a discontinuty in the solution to these equations only at x_0, caused by the noise fluctuation at this point. The constant, γ, can then be determined because the size of the discontinuity is equal to $-J$.

From equation A3, The fluctuation Δv_d at the drain may be written as,

$$\Delta v_\mathrm{d} = \frac{w}{p} \cosh\left(\frac{\pi L_2}{2\,(d+\Delta d)}\right) J$$

Under short circuit conditions, the noise voltage is transformed into a noise current,

$$\Delta i_\mathrm{d} = \frac{\Delta v_\mathrm{d}}{r_\mathrm{d}} = \frac{w}{r_\mathrm{d} p} \cosh\left(\frac{\pi L_2}{2\,(d+\Delta d)}\right) J$$

so,

$$\Delta i_\mathrm{d} \frac{(d+\Delta d)}{\epsilon_2} = w \Delta v_\mathrm{d}$$

Substituting for Δi_d gives a value for γ,

$$\gamma = \frac{Z \epsilon_2 \mu_0 r_\mathrm{d} p V_\mathrm{off}}{L_1\,(d+\Delta d) \cosh\left(\frac{\pi L_2}{2(d+\Delta d)}\right)} \tag{B3}$$

We now have the information required to calculate the charge induced on the gate by a noise fluctuation at the point x_0 .

From equation B1,

$$\Delta q\, dx = \frac{\epsilon_2}{(d+\Delta d)} V_\mathrm{off} \Delta w_x\, dx$$

and so,

$$\Delta q = \int_0^{x_0} \frac{\epsilon_2}{(d+\Delta d)} V_\mathrm{off} \Delta w\, dx + \int_{x_0}^{L_1} \frac{\epsilon_2}{(d+\Delta d)} V_\mathrm{off} \Delta w\, dx$$

Now from equation's B2 and A2,

$$\Delta q = -\int_0^{x_0} \frac{\epsilon_2}{(d+\Delta d)} \frac{\Delta i_{\mathrm{d}} x}{V_{\mathrm{off}} A w}\, dx - \int_{x_0}^{L_1} \frac{\epsilon_2}{(d+\Delta d)} \frac{\Delta i_{\mathrm{d}} x}{V_{\mathrm{off}} A w}\, dx + \int_{x_0}^{L_1} \frac{\Delta i_{\mathrm{d}} \gamma L_1}{V_{\mathrm{off}} A w}\, dx$$

To solve this equation we require dx and x in terms of the reduced potentials dw and w. These can be obtained from the equation for the drain current.

$$I_{\mathrm{d}} = Aw \frac{dV}{dx}$$

so,

$$dx = A \frac{w}{I_{\mathrm{d}}}\, dV$$

which as, $V_{\mathrm{off}} dw = -dV$, gives,

$$dx = -AV_{\mathrm{off}} \frac{w}{I_{\mathrm{d}}}\, dw$$

To determine x, this equation can be integrated in the range, 0 to x, which corresponds to the range, s to w, in the transformed variable. So,

$$\int_0^x dx = -\int_s^w AV_{\mathrm{off}} \frac{w}{I_{\mathrm{d}}}\, dw$$

$$x = \frac{AV_{\mathrm{off}}}{2I_{\mathrm{d}}} \left(s^2 - w^2\right)$$

Substituting these values and changing the limits of integration,

$$\begin{aligned} \Delta q = &\int_s^w \frac{\epsilon_2}{(d+\Delta d)} \frac{\Delta i_{\mathrm{d}}}{AV_{\mathrm{off}} w} \frac{AV_{\mathrm{off}}(s^2 - w^2)}{2I_{\mathrm{d}}} \frac{AV_{\mathrm{off}} w}{I_{\mathrm{d}}}\, dw \\ &+ \int_w^p \frac{\epsilon_2}{(d+\Delta d)} \frac{\Delta i_{\mathrm{d}}}{2I_{\mathrm{d}}^2} AV_{\mathrm{off}} (s^2 - w^2)\, dw \\ &- \int_w^{L_1} \frac{\epsilon_2}{(d+\Delta d)} A \frac{\Delta i_{\mathrm{d}}}{I_{\mathrm{d}}} \gamma L_1\, dw \end{aligned}$$

Collecting similar terms,

$$\Delta q = \frac{\epsilon_2}{(d+\Delta d)} A \frac{V_{\rm off}^2}{I_{\rm d}^2} \Delta i_{\rm d} \int_s^p \frac{s^2 - w^2}{2}\, dw \quad - V_{\rm off} A \frac{\Delta i_{\rm d}}{I_{\rm d}} \frac{\epsilon_2}{(d+\Delta d)} \gamma L_1 \int_w^p dw$$

Performing the integration,

$$\Delta q = \frac{\epsilon_2}{(d+\Delta d)} \frac{\Delta i_{\rm d}}{I_{\rm d}} V_{\rm off} \left\{ \frac{A V_{\rm off}}{2 I_{\rm d}} \left[s^2 (p-s) - \frac{(p^3 - s^3)}{3} \right] - \gamma L_1 (p - w) \right\}$$

and as,

$$I_{\rm d} = \frac{\epsilon_2 Z \mu_0 V_{\rm off}^2 (s^2 - p^2)}{2\,(d+\Delta d)\, L_1}$$

so,

$$\Delta q = \frac{\epsilon_2 \Delta i_{\rm d} L_1 V_{\rm off}}{(d+\Delta d)\, I_{\rm d}} \left\{ \frac{1}{(s^2 - p^2)} \left[(s^2 (p-s)) - \frac{(p^3 - s^3)}{3} \right] - \gamma (p - w) \right\}$$

Let,

$$\kappa = \frac{1}{(s^2 - p^2)} \left[s^2 (p-s) - \frac{(p^3 - s^3)}{3} \right]$$

so,

$$\Delta q = \frac{\epsilon_2}{(d+\Delta d)} \frac{\Delta i_{\rm d}}{I_{\rm d}} V_{\rm off} L_1 (\kappa - \gamma (p - w))$$

The charge stored in region II of the channel (the velocity saturated region) has to be added to the quantity of charge already calculated. This charge is given by,

$$\Delta q' = \Delta i_{\rm d} \frac{L_2}{v_s}$$

which as, $I_{\mathrm{d}} = \frac{\epsilon_2}{(d+\Delta d)} v_s p V_{\mathrm{off}}$, per unit gate width, is,

$$\Delta q' = \Delta i_{\mathrm{d}} L_2 \frac{\epsilon_2}{(d+\Delta d)} \frac{p}{I_{\mathrm{d}}} V_{\mathrm{off}}$$

so adding this to the charge stored in region I gives the total charge stored in the channel, Δq_r, which equals,

$$\Delta q_r = \frac{\epsilon_2}{(d+\Delta d)} L_1 \frac{\Delta i_{\mathrm{d}}}{I_{\mathrm{d}}} V_{\mathrm{off}} \left[\kappa - \gamma(p-w) - \frac{L_2}{L_1} p \right] \tag{B4}$$

The mean square charge is thus,

$$\overline{|\Delta q_r^2|} = C \Delta \overline{|i_{\mathrm{d}}^2|} \left[\kappa' - \gamma(p-w) \right]^2$$

where,

$$C = \left[\frac{\epsilon_2 L_1 V_{\mathrm{off}}}{(d+\Delta d) I_{\mathrm{d}}} \right]^2$$

and,

$$\kappa' = \kappa + \frac{L_2}{L_1}$$

From equation A5, the mean square noise voltage on the drain is,

$$\Delta \overline{|v_{\mathrm{d}}^2|} = \frac{4k T_e(x_0) \Delta f}{I_{\mathrm{d}}} V_{\mathrm{off}} \cosh^2 \left(\frac{\pi L_2}{2(d+\Delta d)} \right) \frac{w^2}{p^2} \, dw$$

Substituting for, $\overline{|i_{\mathrm{d}}^2|}$ gives,

$$\Delta \overline{|q_{\mathrm{r}}^2|} = \frac{C}{r_{\mathrm{d}}^2} \cosh^2 \left(\frac{\pi L_2}{2(d+\Delta d)} \right) \left[\kappa' - \gamma(p-w) \right]^2 \left[4kT_0 \Delta f \frac{V_{\mathrm{off}}}{I_{\mathrm{d}}} \left\{ t + \delta \frac{p^3}{w^3} \right\} \frac{w^2}{p^2} \right] dw$$

Integrating with respect to w gives,

$$\overline{|q_r^2|} = D \int_s^p \left[\kappa' - \gamma p + \gamma w\right]^2 \left[t + \delta\left(\frac{p^3}{w^3}\right)\right] \frac{w^2}{p^2}\, dw$$

where,

$$D = -4kT_0 \frac{\Delta f}{I_d} \frac{V_{\text{off}}}{r_d^2} \cosh^2\left(\frac{\pi L_2}{2\,(d + \Delta d)}\right) C$$

Collecting terms in delta and t yields,

$$\begin{aligned}\overline{|q_r^2|} = D \quad & \int_s^p \left[\frac{w^2}{p^2}\left[\kappa'^2 + \gamma^2 p^2 - 2\kappa'\gamma p\right] + \frac{2w^3}{p^2}\left[\gamma^2 p - \kappa'\gamma\right] + \frac{w^4\gamma^2}{p^2}\right] dw \\ + D\delta \quad & \int_s^p \left[\frac{p}{w}\left[\kappa'^2 + \gamma^2 p^2 - 2\kappa'\gamma p\right] + 2p\left[\gamma^2 p - \kappa'\gamma\right] + pw\gamma^2\right] dw\end{aligned}$$

Let R_0 equal the first integral and R_δ the second. These correspond to the noise induced by normal thermal noise and the 'hot electrons' . They have the following values,

$$R_0 = \frac{(p^3 - s^3)}{3p^2}\left[\kappa'^2 + \gamma^2 p^2 - 2\kappa'\gamma p\right] + \frac{2(p^4 - s^4)}{4p^2}\left[\gamma^2 p - \kappa'\gamma\right] + \frac{(p^5 - s^5)}{5p^2}\gamma^2 \tag{B5a}$$

$$R_\delta = \delta\left\{p \ln\left(\frac{p}{s}\right)\left[\kappa'^2 + \gamma^2 p^2 - 2\kappa\gamma' p\right] + \frac{p}{2}(p - s)\left[\gamma^2 p - \kappa'\gamma\right] + \frac{(p^2 - s^2)}{2} p\gamma^2\right\} \tag{B5b}$$

Having calculated the fluctuating charge on the gate, the noise current flowing, $\overline{|i_g^2|}$, is simply,

$$\overline{|i_g^2|} = \omega^2 D\ (R_0 + R_\delta) \tag{B6}$$

The correlation coefficient between the drain circuit noise and the gate cicuit noise will now be calculated. It has been shown that, $i_{\mathrm{d}} = \frac{v_{\mathrm{d}}}{r_{\mathrm{d}}}$, which under short circuit conditions is equal to,

$$i_{\mathrm{d}} = \frac{w}{p}\cosh\left(\frac{\pi L_2}{2\,(d+\Delta d)}\right)\frac{1}{r_{\mathrm{d}}}v_{\mathrm{d}}$$

Let,

$$K = \omega\frac{\epsilon_2 V_{\mathrm{off}} L_1}{(d+\Delta d)\, I_{\mathrm{d}} r_{\mathrm{d}}^2 p^2}\cosh^2\left(\frac{\pi L_2}{2\,(d+\Delta d)}\right)$$

so,

$$\Delta i_{\mathrm{d}}\Delta i_{\mathrm{g}}^* = -jK(\kappa' - \gamma(p-w))w^2\,\overline{|v_{\mathrm{d}}^2|}$$

Now,

$$\overline{|v_{\mathrm{d}}^2|} = \frac{-4kT_0\Delta f}{I_{\mathrm{d}}}V_{\mathrm{off}}\frac{w^2}{p^2}\,dw$$

so let,

$$K' = -K\,\frac{4kT\Delta f}{I_{\mathrm{d}}}V_{\mathrm{off}}$$

so that,

$$\Delta i_{\mathrm{d}}\Delta i_{\mathrm{g}}^* = jK'(\kappa' - \gamma(p-w))w^2\left[1+\delta\frac{p^3}{w^3}\right]\frac{w^2}{p^2}\,dw$$

Performing the integration gives,

$$\overline{i_{\mathrm{d}} i_{\mathrm{g}}^*} = jK'\left[\int_s^p (\kappa' - \gamma(p-w))w^2\frac{w^2}{p^2}\,dw \quad + \quad \delta\int_s^p (\kappa' - \gamma(p-w))w^2\frac{p^2 w^3}{w^2 p^2}\,dw\right]$$

or,

$$\overline{i_{\rm d} i_{\rm g}^*} = jK' \left[\int_s^p (\kappa' - \gamma(p-w)) \frac{w^4}{p^2} \, dw \quad + \quad \delta \int_s^p (\kappa' - \gamma(p-w)) \, pw \, dw \right]$$

Let the first integral equal, S_0, and the second S_δ . So,

$$S_0 = \frac{1}{p^2} \left[\frac{(\kappa' - \gamma p)(p^5 - s^5)}{5} + \frac{\gamma(p^6 - s^6)}{6} \right] \tag{B7a}$$

and,

$$S_\delta = \frac{\delta}{p^2} \left[\frac{(\kappa' - \gamma p)(p^2 - s^2)}{2} + \frac{\gamma(p^3 - s^3)}{3} \right] \tag{B7b}$$

The normalised correlation coefficient is defined as,

$$C = \frac{\overline{i_{\rm d} i_{\rm g}^*}}{\sqrt{|\overline{i_{\rm d}^2}| |\overline{i_{\rm g}^2}|}}$$

Substituting the appropriate values and cancelling terms gives,

$$C = j \frac{\frac{1}{p^2} \left(S_0 + S_\delta \right)}{\sqrt{(P_0 + P_\delta) \, (R_0 + R_\delta)}} \tag{B8}$$

The noise voltages and correlation coefficient between them have now been calculated and can be applied to determine the noise temperature of a HEMT.

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