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THE NRAO VLBI MARK III PROCESSOR

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NOTE: THE WORK FOR THIS PROCESSOR WAS DONE IN FEBRUARY 1978.

CHAPTER 1: NRAO VLBI System

The NRAO VLBI system should be viewed in its entirety as a single radio astronomy "telescope". The individual components of this "telescope" include the network of radio telescopes, the record terminals, the processor, and the post-processor hardware and software. All of these components are essential to a system which is able to serve the astronomy community by making high quality maps of galactic and extra-galactic sources with angular resolution of 0"5 to 0"0005. It would be a mistake to consider the processor as the fundamental component in a VLBI system in which the rest is assumed to naturally follow. Without adequate planning for the nature of the observing programs, and the possible data reduction schemes, MKIII VLBI would be little more than a more sensitive MKII VLBI , producing calibrated amplitudes and phases for the expert and dedicated user. This would not justify the expense of building a processor.

Most users of the MKI and MKII VLBI systems now in use agree that getting final models or maps from their data is an arduous task. The effort involved and the almost black magic appearance to non-practitioners have prevented the full application of VLBI studies to many problems in astrophysics. It therefore is beneficial to develop data handling and analysis software in step with the hardware for the MKIII correlator. This note delineates a proposed data flow path for such a system.

What is needed in the NRAO system is a way to produce synthesis maps of radio sources with much the same ease and speed as the VLA will provide. For the case of spectral line observations precise amplitude and phase calibration can often be accomplished by using "calibration sources", which are separated from the desired "sources" not in angle in the sky, but in frequency. A few spectral line synthesis maps have been made with the present MKII system, and although the effort required to do this is great, more are being planned. Spectral line VLBI is in great need of a much faster and simple map making system which the NRAO VLBI could provide.

Continuum VLBI has been plagued primarily by problems of phase calibration introduced by independent frequency standards and by propagation effects in the earth's atmosphere and ionosphere. Many techniques are currently being investigated which would allow continuum VLBI studies to advance from amplitude model fitting techniques to synthesis mapping. There are two promising techniques to do this. First, in some cases a calibration source can be found which is close enough to the source to be mapped so that rapid switching allows calibration of the frequency standards and of the atmosphere and ionosphere. This allows phase-coherent observation and data reduction in the same manner as a conventional interferometer. Second, there has been much effort to use the phase information available around the three baselines of a three element interferometer (phase closure) to help map sources. The use of phase closure information has recently been demonstrated to allow one to produce synthesis maps without phase calibration of fairly complex source structures by using an iterative CLEAN scheme (Readhead and Wilkinson, 1978). This technique works best with a large number of telescopes, since the ratio of interferometer triangles (from which a closure phase relation can be obtained) to the number of baselines approaching unit, and the amount of missing phase information becomes small.

The goal of an NRAO VLBI system should be to allow a user to employ the techniques mentioned above to produce radio source maps in a fairly automated manner. This would require certain observing formats or modes around which the processor and post-processor data reduction can be organized to proceed rapidly to maps. There is no fundamental problem prohibiting a carefully integrated data acquisition, data processor <u>and</u> data reduction system from achieving this goal. Such a system need not exclude the possibility of specialized or unusual experiments, but would require extra data reduction procedures. It would, however, be a fatal mistake to attempt to make a VLBI system so flexible that it could handle any foreseeable form of observation. The benefits in terms of scientific output of making MKIII VLBI a "mapping machine" would far outweigh the difficulties it would introduce in a few special cases.

The VLBI system is outlined in block form in Figure 1. The data acquisition terminals have already been designed and built. These terminals are under the control of a minicomputer, with the observing program preloaded into a disk file. This guarantees identical observing procedures at all telescopes and facilitates observation by operators alone. Various logging data are gathered by the minicomputer and used later for data reduction and analysis. Observing programs could thus be formulated with an ease approaching the current use of the NRAO interferometer.

Data taken at each recording site includes system temperatures and antenna temperatures. Also, phase calibration information for each tape recorder track is necessary in order to combine the tracks later. Both a signal injection system and observations of strong sources are required. Atmospheric parameters such as temperature, pressure, and humidity are necessary for post-processing of geodetic data and would be useful for astronomical data as well.

The data disk would contain enough information to allow processing of the data in an automatic fashion. Names of telescopes and the sources being observed would be recorded and used later in the correlator to bring in the appropriate coordinates to do all the delay and fringe rate calculations. These features and a good data flag would eliminate all the hassles now associated with REDPREP. Once a set of clock parameters was found, a large experiment could be processed. The only intervention required would be to change tapes.

Most calibration information resides on the auxiliary data disk. Amplitudes and phases would be calibrated both at correlation time and in post-processing. System temperature and antenna gain effects, as well as some atmospheric phase perturbations would be corrected for during processing.

The processor would produce output tapes at a rate of one per hour to less than one per day. The data must then be processed in an off-line minicomputer. This computer should be an interactive, time sharing system with rapid I/O facilities, large ($\sim$100 Mbyte) disks, and graphics display. This computer will be responsible for editing data and calibrating amplitudes and phases.

In the continuum mode, it will fit fringes, allow the user to model data, and prepare data for closure phase mapping. For spectral line operation, it can transform autocorrelation data to the total power spectra needed for amplitude calibration, perform phase referencing, and fit fringes for fringe rate and/or synthesis mapping. At this point the data would be indistinguishable from VLA data and should be handled by VLA inversion/clean programs and NIPS.

CHAPTER 2: Description of a 3-station Processor

The basic configuration of a 3-station processor is shown in Figure 2-1. The processor reproduces data tapes recorded on any VLBI MKIII record terminal. The data is decoded, aligned in time, fringe rotated, and then correlated in a digital correlator. The correlation function is averaged for 0.156 to 10 seconds in the continuum mode, or is converted into a cross-spectrum by the FFT processor and then averaged up to 10 seconds in spectral mode. Data is written on a 9-track computer tape for further off-line processing.

The processor will be able to:

a) Expand up to 10 stations for the VLB Array in an efficient and inexpensive manner.

b) Process spectroscopic data simultaneously for all baselines for all 10 stations.

c) Operate at twice the present standard rate of 4 Mb/sec. allowing for future tape recorder development.

Figure 2-2 shows the number of station modules and baseline modules and power requirements as a function of number of stations. For a 3-station processor one has as many station modules as baseline modules. For more than 3-stations one has more baseline modules than station modules. We have therefore put as much circuitry as possible into the station module to save not only circuitry but also power. Our 10-station processor with a total of 20,160 lags will dissipate only 6K watts. For comparison we show in Figure 2-3 that a Haystack processor for 10-stations* with only 10,080 lags would require >30K watts.

The system divides into three parts:

1) The playback recorder
2) The correlator
3) The on-line computer system

1) The playback recorder:

The recorder contains a Honeywell Model 96 tape drive with a 28-track reproduce head, preamplifiers and phase equalizers for 28 tracks and 4 Mbits/sec. and recorder motion control logic. There is one recorder needed for each station.

*Haystack is designed only for up to 8-station operation.

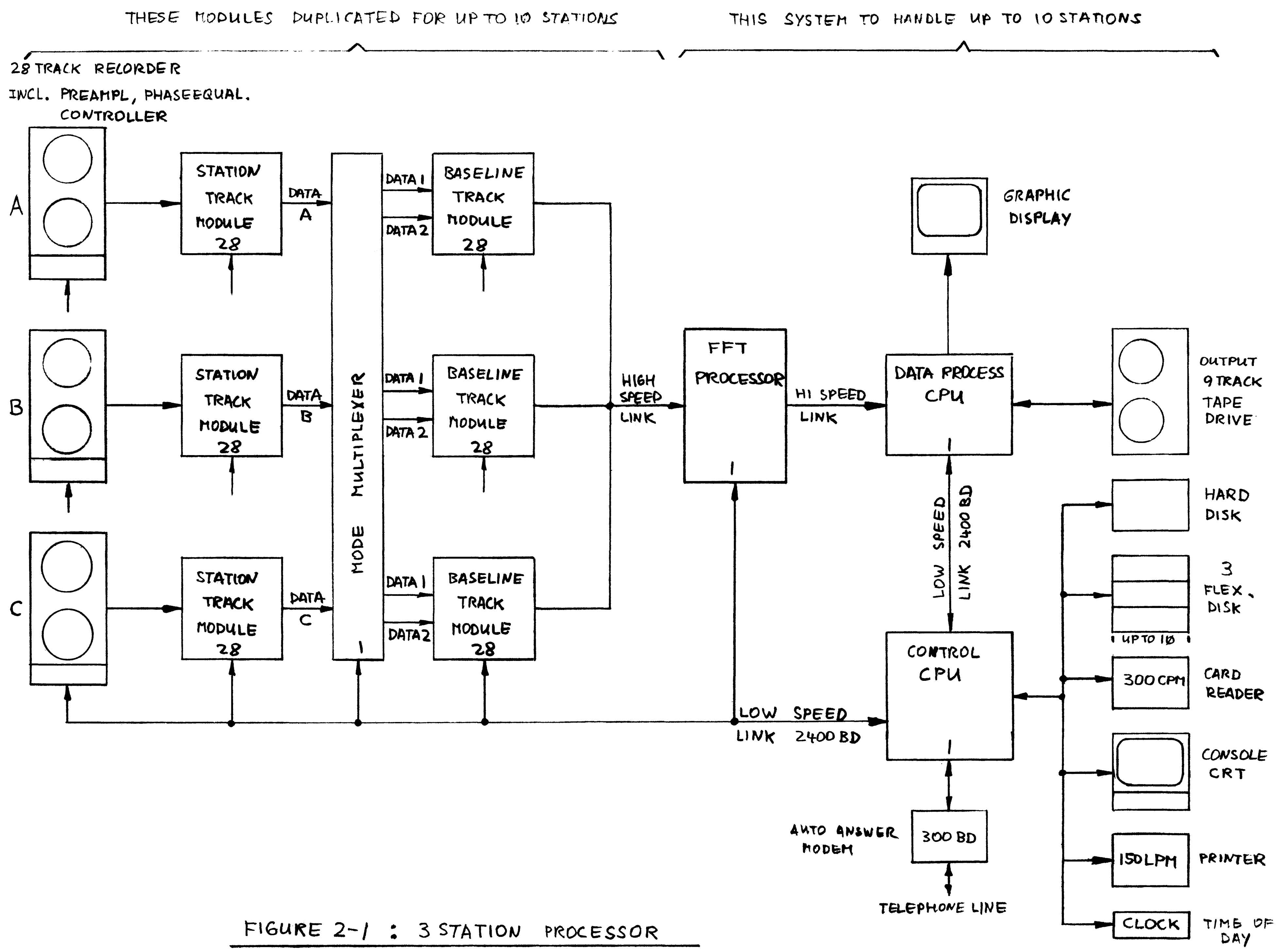


FIGURE 2-1 : 3 STATION PROCESSOR

# OF STATIONS	# OF BASELINES	STATION MODULES #	STATION MODULES POWER/KW	BASELINE MODULES #	BASELINE MODULES POWER/KW	BASELINE MODULES # OF CROSSLAGS	TOTAL DISSAP. FOR CORRELATOR KW
2	1	56	0.3	28	0.1	448	0.4
3	3	84	0.4	84	0.3	1344	0.7
4	6	112	0.6	168	0.6	2688	1.2
5	10	140	0.7	280	1.0	4480	1.7
6	15	168	0.8	420	1.5	6720	2.3
7	21	196	1.0	588	2.0	9408	3.0
8	28	224	1.1	784	2.7	12544	3.8
9	36	252	1.3	1008	3.5	16128	4.8
10	45	280	1.4	1260	4.4	20160	5.8

STATION MODULE: BIT SYNCH, DECODER, BUFFER, FRINGE RATE SYNTH, AUTOCORR., NORMALIZING COUNTER

BASELINE MODULE: FRINGE ROTATOR, CROSS CORRELATOR, NORMALIZING COUNTER

FIGURE 2-2: NRAO MK III CORRELATOR.

# OF STATIONS	# OF BASELINES	# OF MODULES	POWER /kW	# OF CROSS LAGS
2	1	28	0.8	224
3	3	84	2.2	672
4	6	168	4.4	1344
5	10	280	7.3	2240
6	15	420	11	3360
7	21	588	16	4704
8	28	784	21	6272
9	36	1008	26	8064
10	45	1260	32	10080

PER BASELINE MODULE : 2DECODERS , BUFFER, FROTATOR, AUTO CORR., CROSS CORR, TCCOUNTER

FIGURE 2-3 : HAYSTACK MK III PROCESSOR

The recorder is completely compatible with all MKIII record terminals. All data will be played back at 4 Mb/sec. independent of record speed. Data recorded at reduced speed (e.g. reduced bandwidth) will be played faster than real time, therefore processing time will be considerably faster than real time. This is a large advantage over the MKII system.

2) The correlator:

The correlator is divided into station modules, a multiplexer and baseline modules. See Figure 2-1.

a) Station modules:

The station module contains all circuitry unique to a station including an autocorrelator. See Figure 2-4.

The largest part of the fringe rotator, the fringe rate synthesizer, is also included in this module. For a detailed description of the fringe rotator see Chapter 5 and Appendices A, B and C. There will be 1 station module per track for each station, or a total of 84 modules for a 3-station processor.

b) Multiplexer:

The multiplexer has to multiplex three lines per station module, or a total of 252 lines for a 3-station processor. There is a large number of different configurations possible and we will design this multiplexer for complete flexibility. However, we will initially implement only a limited number of modes. When desired, one will be able to implement others as well fairly easily. Initially implemented modes for a 3-station processor will be:

(1) Playback all tracks for 3-station continuum observations with 16 correlator lags per baseline track.
(2) Playback 4-tracks for 2-station spectral line observations with 512 correlator lags per baseline track.

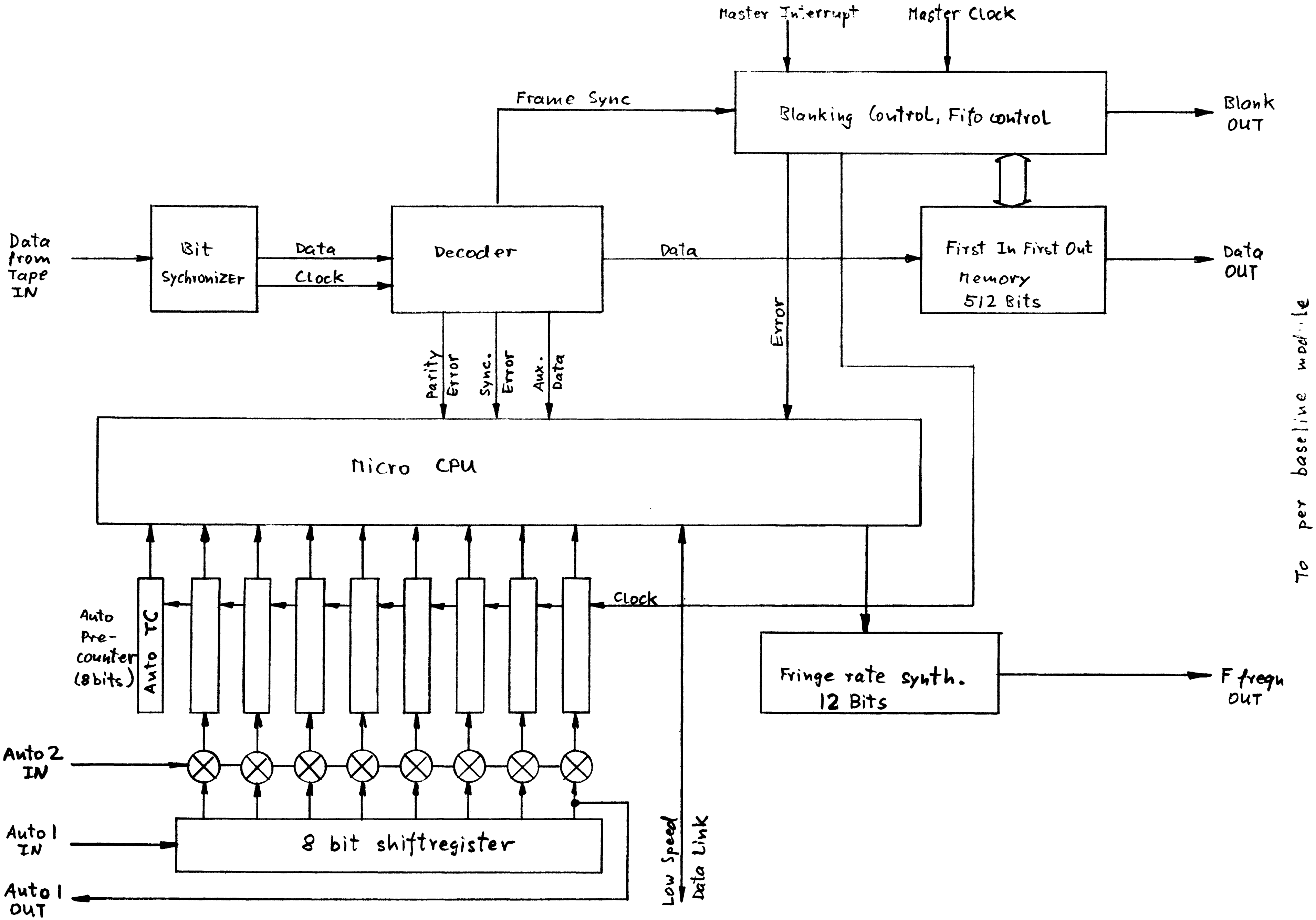


FIGURE 2-4: PER STATION - TRACK MODULE

(3) Playback 2-tracks for 2-station spectral line observations with 1024 correlator lags.

(4) Playback 1-track for 3-station spectral line observations with 512 correlator lags.

(5) Playback 1-track for 2-station spectral line observations with 2,048 lags.

c) Baseline modules:

The baseline module contains the cross correlator for 16 lags and the fringe rotator (see Figure 2-5). For correlator details see also Chapter 6 and for fringe rotator details see Chapter 5 and Appendices A, B, and C. There will be one module per baseline track and 28 modules per baseline. Each set of 28 modules for one baseline is followed by an optimal 28 track combiner.

For modes described earlier we need 128 per baseline modules and 3 combiners.

3) The computer system:

The computer system is divided into three parts: FFT processor, data process CPU, and control CPU. The system is designed so that it will be able to handle a full 10-station correlator (see Figure 2-1).

The FFT processor accepts data from the correlator, performs a fast transform and fractional bit shift correction in spectral line mode and passes the cross-spectrum on to the data process CPU. For continuum observation the FFT processor can be used to increase the SNR by transforming the correlation into a spectrum, doing the fractional-bit-shift correction, filtering the unwanted sideband and then transforming the spectrum back into its correlation function. (It will also be possible to pass raw correlator data onto the the data process CPU. In this case an approximate fractional bit shift correction is accomplished by incrementing the fringe rotator by 90° each time when delay is updated by 1 bit.)

The FFT processor will have to be a rather fast hardware array processor capable of transforming 1024 complex channels in 6 msec.

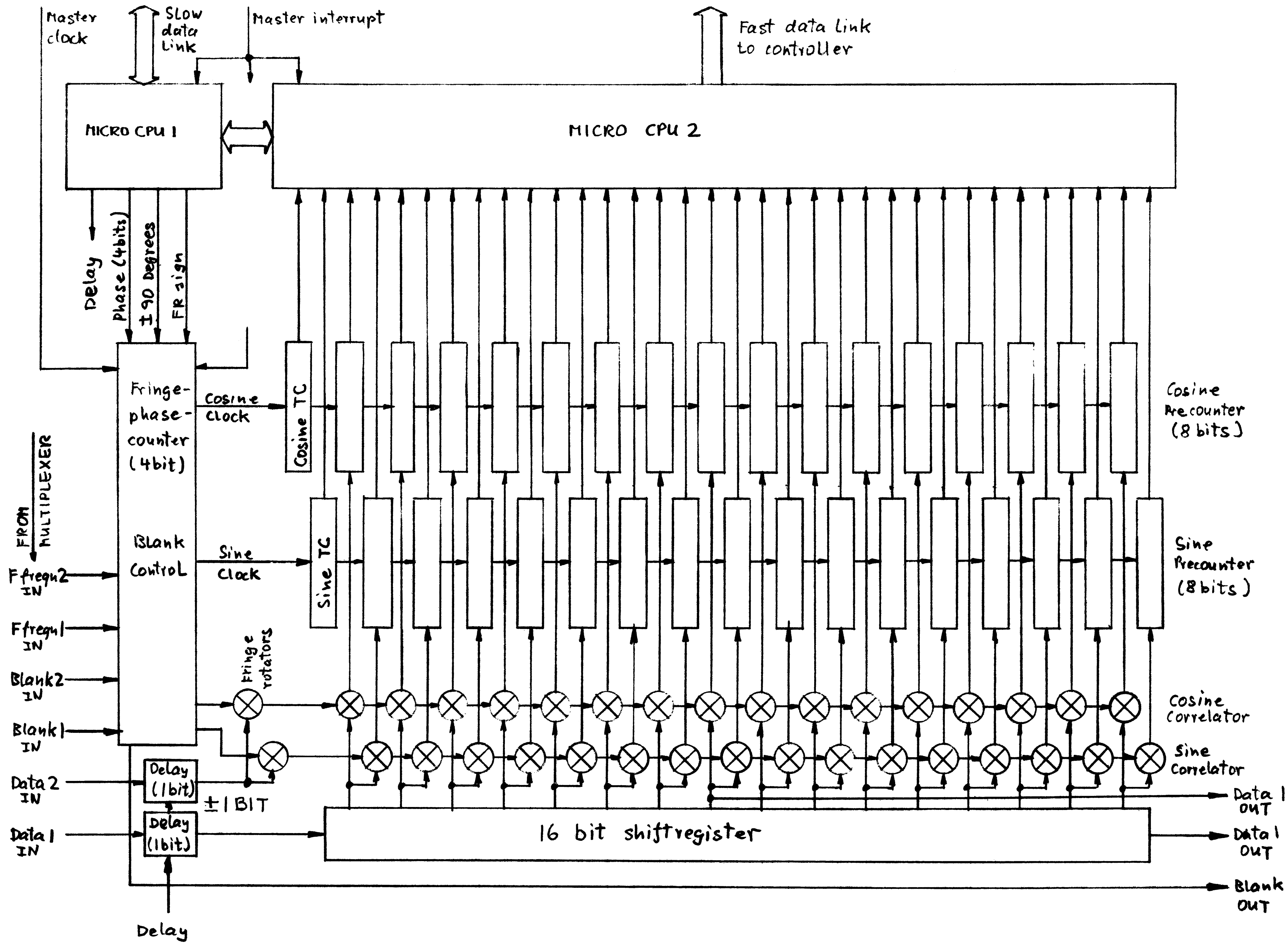


FIGURE 2-5: PER BASELINE-TRACK MODULE

The data process CPU is a general purpose minicomputer that will:

(1) block data for efficient tape usage

(2) integrate data up to 10 seconds

(3) display "first look" fringes or spectra

(4) perform hardware diagnostic

There will be no operating system on this CPU. The program will be developed and assembled in the control CPU and then simply loaded into the data process CPU and executed.

The control CPU is also a general purpose minicomputer, but it is set up to be quite flexible and will perform a number of different tasks. It will operate under a real time operating system with a number of peripherals. It will provide capabilities for program development, communication with the user, communication with the operator and controlling operation of the processor. Controlling operation of the processor is divided into:

(1) Preparation to process data (same as PREPTAPE)

(2) Setting up the hardware parameters for each scan

(3) Controlling and updating the processor during run

(4) Monitor performance of processor

(5) Write log for each run

There are two parameters to be calculated for updating the processor during run. These are delay and fringe rotator. The calculations are described in Chapters 4 and 5.

CHAPTER 3: Cost and Manpower Estimates

Cost of 3-Station (3 baselines) Processor

3 tape recorders incl. heads, electronics	$ 22K ea.	$ 66K
Correlator, incl. 84 per station modules, multiplexer, 128 per baseline modules,		55K
Computer system		175K
	TOTAL	$296K

Cost of 6-Station (15 baselines) Processor

6 tape recorders incl. heads, electronics		$132K
Correlator		121K
Computer		180K
	TOTAL	$433K

Cost of 10-Station (45 baselines) Processor

10 tape recorders incl. heads, electronics		$220K
Correlator incl. 280 per station modules, multiplexer 1260 per baseline module 45 combiners		192K
Computer System		193K
	TOTAL	$615K

Cost of 4-Station (6 baselines) Processor

4 tape recorders incl. heads, electronics		$ 88K
Correlator incl. 112 station modules, multiplexer 256 baseline modules		82K
Computer System		180K
	TOTAL	$350K

Manpower Required for Construction of a 3-Station Processor

Scientific Advisor	2 man-years
Electronics Engineer	4 man-years
Electronics Technician	4 man-years
Programmer	4 man-years
	14 man-years

Manpower Required for Construction of a 4-Station Processor

Scientific Advisor	2 man-years
Electronics Engineer	4 man-years
Electronics Technician	4-1/2 man-years
Programmer	4-1/2 man-years
	15 man-years

Manpower Required for Construction of a 10-Station Processor

Scientific Advisor	2 man-years
Electronics Engineer	5 man-years
Electronics Technician	6 man-years
Programmer	5 man-years
	18 man-years

Manpower Required to Operate 3-Station Processor

1 Scientific Advisor	1/2 time
1 Electronics Engineer	1/2 time
1 Maintenance Technician	Full time
3 Operators	Full time
1 Clerk	Full time

With enough manpower on the project we expect to have a 3-station procesor operational within 2 years.

CHAPTER 4: Calculation of Fringe Rate and Time Delay

A station's coordinates may be specified by its geocentric latitude, longitude and radius ϕ, λ, R or its x, y, z components in a left-handed coordinate system. x and y are in the equatorial plane and x is along longitude 0°. z is parallel to the earth's axis (North is positive).

$$x = R \cos \phi \cos \lambda \qquad \text{(West longitude is positive } \lambda\text{)}$$

$$y = R \cos \phi \sin \lambda$$

$$z = R \sin \phi$$

Consider the vector from station 1 to station 2.

So

$$b_x = x_2 - x_1$$

$$b_y = y_2 - y_1$$

$$b_z = z_2 - z_1$$

and the baseline hour angle with respect to Greenwich is

$$GBHA = \tan^{-1}\left(\frac{b_y}{b_x}\right)$$

The sidereal time at midnight is the sidereal time at Greenwich at

0000 U.T. $\equiv$ STM

if r is the ratio of sidereal rate to atomic time rate

($\approx$1.0027378)

then,

$$IHA = STM + UT*r - \alpha - GBHA - \pi/2$$

at a given UT.

(Note that for a given UT, $\Delta IHA = - \Delta\alpha$ since everything else is a constant for a given time.)

The time delay from station 1 to 2 is

$$\tau = \frac{b_1}{c} \sin \delta + \frac{b_2}{c} \cos \delta \sin (IHA)$$

and fringe rate is

$$f = - \Omega B_2 \cos \delta \cos (IHA)$$

where

$$b_1 = b_z$$

$$b_2 = \sqrt{b_x^2 + b_y^2} \qquad B_2 = \frac{b_2}{\lambda}$$ or, equatorial component, in wavelengths

$$\Omega = \text{rotation rate of earth} \approx 7.2722 \times 10^{-5} \frac{\text{radians}}{\text{sec.}}$$

These may be derived by noting that

$$\tau = \frac{\vec{b} \cdot \hat{k}}{c}$$ where $\hat{k}$ is unit vector in direction of source and $\vec{b}$ is the baseline vector

$$\hat{k} = \left[\cos \delta \cos GHA, \cos \delta \sin GHA, \sin \delta\right]$$

where GHA is the hour angle of the source wrt Greenwich.

Notice that $\hat{k}$ is continuously changing.

The phase of the source is

$$\phi = \frac{\vec{b} \cdot \hat{k}}{\lambda}$$

The fringe rate is the derivative of the phase (i.e. rate of change of phase, expressed in cycles per second)

$$f = \frac{1}{2\pi} \frac{\partial \phi}{\partial t}$$

By much trigonometric manipulation, the delay and fringe rate equations can be gotten from these vector expressions.

$$\frac{\partial \hat{k}}{\partial t} = \vec{\Omega} \times \hat{k},$$ where $\vec{\Omega}$ is spin vector of the earth.

if $f = -\Omega B_2 \cos\delta \cos(IHA)$

then $\frac{\partial f}{\partial t} = \Omega B_2 \cos\delta \sin(IHA) \frac{\partial (IHA)}{\partial t}$

but $\frac{\partial IHA}{\partial t}$ is just rotation rate of the earth

so $\left|\frac{\partial f}{\partial t}\right| = \Omega^2 B_2 \cos\delta \sin(IHA) \leqslant \Omega^2 B_2$

The delay and fringe rate expressions are not quite right, because they don't allow for the fact that the earth rotates in the interval between the arrival of the radio waves at station 1 and 2. A proper treatment of this effect is to difference the delays and fringe rates for each station.

$$\tau = \frac{\hat{k}\cdot\vec{b}}{c} = \frac{\hat{k}_2\cdot\vec{r}_2}{c} - \frac{\hat{k}_1\cdot\vec{r}_1}{c}$$

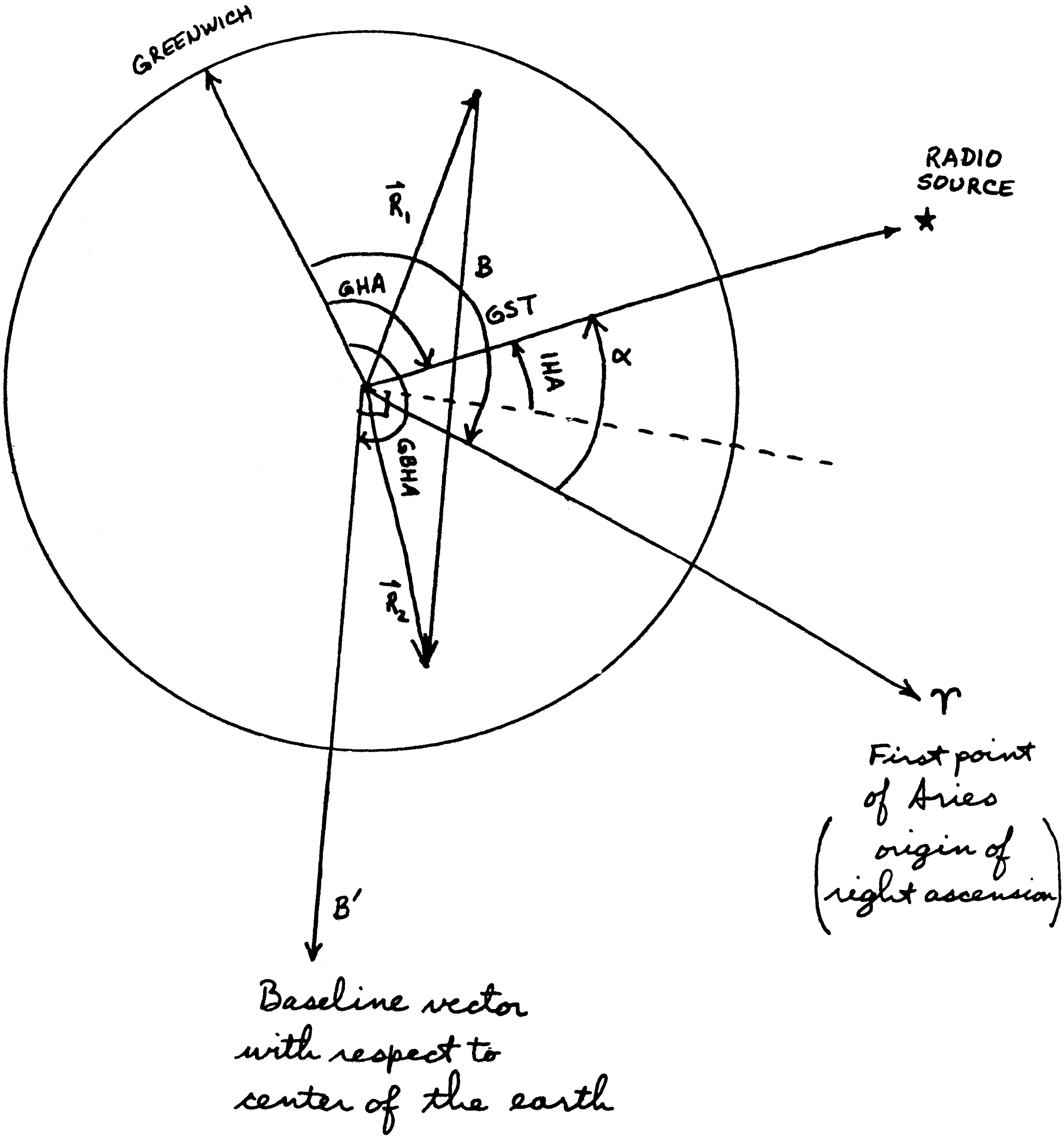
where $\hat{k}_i$ is the correct value of $\hat{k}$ at the arrival time at station i.

$$\text{Roughly } \hat{k}_2 = \hat{k}_1 + \frac{\partial \hat{k}_1}{\partial t}\tau$$

and fringe rate

$$f = \frac{1}{2\pi\lambda}\frac{\partial}{\partial t}\left(\hat{k}_2\cdot\vec{r}_2 - \hat{k}_1\cdot\vec{r}_1\right)$$

It's easy to calculate the appropriate values of $\hat{k}_i$ and then difference things in the correlator.



α = right ascension of source

GST = Hour angle of ♈ with respect to Greenwich ≡ Greenwich sidereal time

IHA = interferometer hour angle of radio source (measured wrt equatorial component. Equals zero when perpendicular to baseline.)

Figure 4-1.

CHAPTER 5: The Fringe Rotator

We calculate here the requirements for the hardware fringe rotator using a numerical example of f_o = 44 GHz (λ = 7 mm) for SiO maser line.

The velocity at each station is:

$$v = \hat{k} \cdot \vec{\Omega} \times \vec{r}$$

where $\vec{\Omega}$ is spin vector of the earth = $[0, 0, 7.27 \cdot 10^{-5}]$

then $v = R\Omega \cos\delta \cos\phi \sin(GHA - \lambda)$

Worst case: $V_{max} = R\Omega = 465$ m/sec.

The Doppler shift is $\Delta f = f_o \left(1 \pm \sqrt{\frac{c - V}{c + V}}\right)$

$$\text{or } \Delta f = f_o \left(\frac{V}{c} + \frac{V^2}{c^2} + \frac{V^3}{c^3} + \ldots\right) = \text{fringe rate}$$

for 44 GHz: $\Delta f = f_o \frac{V}{c} = 68{,}200$ Hz

and $f_o \frac{V^2}{c^2} = 0.106$ Hz

and $f_o \frac{V^3}{c^3} = 0.164 \cdot 10^{-6}$ Hz

If we neglect any terms beyond second order the error in fringe rate will be $<0.16 \cdot 10^{-6}$ Hz. If we update the fringe rotator every 10 seconds we obtain a phase error of $<14.4 \cdot 10^{-6}$ lobes. We therefore calculate the fringe rate as:

$$\Delta f = f_o \left(\frac{V}{c} + \frac{V^2}{c}\right)$$

to an accuracy of $50 \cdot 10^{-6}$ Hz so that the phase error stays $<5 \times 10^{-4}$ lobes (0.2 degrees) for a 10 second update rate. Since the hardware fringe rotator will have a maximum possible fringe rate of 250 kHz the calculation needs to have a precision of 32 bits.

We also need to calculate the fringe phase of the rotator. The phase is

$$\phi < \frac{r}{\lambda} = \frac{6370\ 000\ \text{m}}{0.007\ \ \text{m}} = 9.1 \times 10^{8}$$

for $\lambda = 1$ mm: $\phi < 6.3 \cdot 10^{6}$

for a phase precision of 0.001 lobes we need to calculate ϕ to at least 43 bit accuracy. The phase that goes to the hardware rotator is

$$\phi' = \phi - n\ 2\pi$$

where n is an integer so that $0 < \phi' < 2\pi$

Since we want a phase precision of 0.001 lobes we need a phase register of 10 bits in the fringe rotator. The most significant 4 bits of these 10 bits are located in the per baseline module. We need to calculate the difference between these 4 bits for each baseline before we send it to the rotator. The least significant 6 bits are sent to the per station module.

We now need to calculate how the fringe rate changes with time. The fringe rate derivatives vary like the time delay derivatives, in gaining a factor of Ω for each derivative:

$$\dot{f}\ \alpha\ f_o\ \Omega < 5\ \text{Hz/sec.}$$

$$\ddot{f}\ \alpha\ f_o\ \Omega^2 < 3.6 \times 10^{-4}\ \text{Hz/sec}^2$$

$$\dddot{f}\ \alpha\ f_o\ \Omega^3 < 2.6 \times 10^{-8}\ \text{Hz/sec}^3$$

for a phase error of less than 0.001 lobe for a 10 second update rate only $\dot{f}$ and $\ddot{f}$ need to be calculated. The hardware fringe rotator will have two fringe rate accelerator registers, one for $\dot{f}$ and one for $\ddot{f}$. The accelerator registers will need to have an accuracy of 16 bits and 4 bits respectively.

The hardware rotator is thus specified:

Parameter period = 10 seconds

Fringe rate register = 32 bits

Maximum fringe rate = 250 kHz

1st fringe rate accelerator = 16 bits

2nd fringe rate accelerator = 4 bits

Phase register = 10 bits total

Phase register in p. sta. mod. = 6 bits

Phase register in p. base. mod. = 4 bits

Accuracy of phase = 0.001 lobe = 0.3 degrees (for 10 sec integration)

CHAPTER 6: Correlator Integration Time

Since the processor will always operate at a fixed data rate of 4 M bits/second (8 Mb/sec. for future developments) an observation which has been recorded at reduced bandwidth and reduced tape speed will be processed faster than real time. See Figure 6-1. This means that the update rate for delay in bits/second is independent of record bandwidth and is only a function of the interferometer geometry.

We calculate the update rate:

$$= \frac{4\omega_{\oplus} R_{\oplus} BW}{C}$$

Worst case condition:

$$V = 465 \text{ m/sec.}, \qquad BW = 2 \text{ MHz}$$

we get: update rate = 12.4 bits/sec.

or 1 bit every 80 msec.

One needs to make the minimum integration time so that the delay does not change more than one bit. Fractional bit shift correction may then be applied on the spectrum without restricting the field of view of the interferometer. This worst case condition is unrealistic and we have chosen the minimum integration time as 156.25 msec.

A maximum integration time of 10 sec. was chosen to be compatible with VLA integration time. Under program control one may thus select any correlator integration time between 156.25 msec. and 10 sec. in increments of 156.25 msec. Note that an experiment recorded at 62.5 kHz and processed at 8 Mb/sec. is integrated for 10 sec. real time.

The correlator counters are divided into an 8 bit precounter and a 19 bit counter. The precounter will not be read out, however, the precounter will be rounded before the counter is read. Of the 19 counter bits only 16 bits are read out to the computer whereby the data is left-justified to avoid overflows or loss of accuracy.

RECORD BW/MHz	SPEED-UP FACTOR	MINIMUM REAL TIME INTEGRATION TIME - SEC	MAXIMUM REAL TIME INTEGRATION TIME - SEC
4	1/2 (1)	0.078125 (0.15625)	5 (10)
2	1 (2)	0.15625 (0.3125)	10 (20)
1	2 (4)	0.3125 (0.625)	20 (40)
0.5	4 (8)	0.625 (1.25)	40 (80)
0.25	8 (16)	1.25 (2.5)	80 (160)
0.125	16 (32)	2.5 (5)	160 (320)
0.0625	32 (64)	5 (10)	320 (640)

PLAYBACK RATE : 4 Mbit/sec (8 Mbit/sec)

MINIMUM HARDWARE INTEGRATION : 156.25 msec

MAXIMUM HARDWARE INTEGRATION : 10 sec

FIGURE 6-1 : REAL TIME INTEGRATION TIME

For τ = 156.25 msec. and data rate of 4 Mb/sec. (8 Mb/sec.) we get:

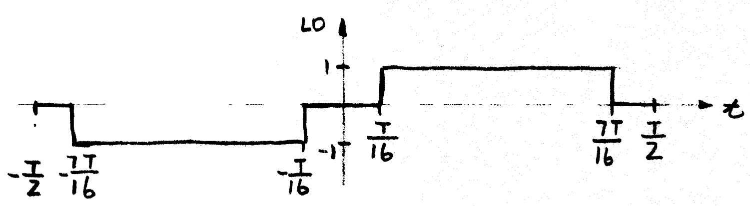
Noise RMS = 783 bits (1107 bits)

With an 8 bit precounter the quantization is 256 bits. This is 1/3 (1/4) of the RMS noise and will not affect S/N significantly.

APPENDIX A

3-Level Fringe Rotator

The rotator approximates its analog equivalent, the mixer, by quantizing the LO signal into three discrete levels:



$$f(t) = 0 \qquad -\frac{T}{2} < -\frac{7T}{16}$$

$$f(t) = -1 \qquad -\frac{7T}{16} < -\frac{T}{16}$$

$$f(t) = 0 \qquad -\frac{T}{16} < +\frac{T}{16}$$

$$f(t) = 1 \qquad +\frac{T}{16} < +\frac{7T}{16}$$

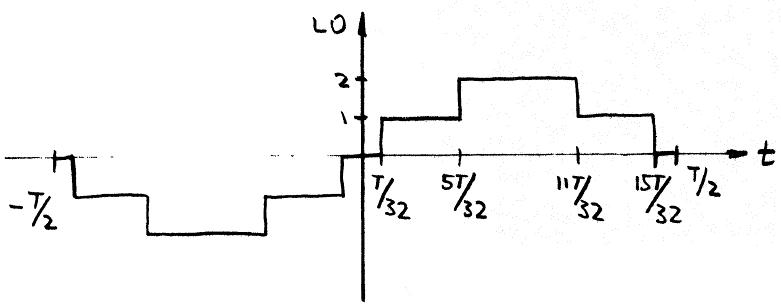
$$f(t) = 0 \qquad +\frac{7T}{16} < +\frac{T}{2}$$

This LO signal with period T has harmonic frequencies which, when mixed with input data which need to be assumed as broadband noise, will produce unwanted mixing products and reduce the S/N ratio by 7.4%

APPENDIX B

5-Level Fringe Rotator

The rotator approximates its analog equivalent, the mixer, by quantizing the LO signal into five discrete levels:



This gives a 3.8% less in SNR.

APPENDIX C

Quantization of Fringe Frequency:

Since there is a fringe frequency synthesizer in each per station module and the per baseline module takes the difference between two fringe frequencies we introduce a small loss in S/N ratio.

Assume a signal vector $S = A^{i\phi}$ and a fringe frequency vector $S^1 = A^{i\phi^1}$ which are mixed and result in a difference vector of $S^* = A\, e^{i(\phi-\phi^1)}$ then the probability of S^* is:

$$\langle S^*\rangle = \frac{\int_{-\infty}^{\infty} S^*\, P(\phi-\phi^1)\, d(\phi-\phi^1)}{\int_{-\infty}^{\infty} P\,(\phi-\phi^1)\, d(\phi-\phi^1)}$$

$$\text{whereby } P(\phi^1) = \begin{cases} 1 \text{ for } -\phi_M < (\phi-\phi^1) < \phi_M \\ 0 \text{ elsewhere} \end{cases}$$

and ϕ_M is peak deviation

then:

$$\langle S^*\rangle = \frac{A}{2\phi_M} \int_{-\phi_M}^{\phi_M} e^{i(\phi-\phi^1)}\, d(\phi-\phi^1)$$

Thus, $\dfrac{\langle S^*\rangle}{A} = \dfrac{\sin \phi_M}{\phi_M}$

The power is then

$$\frac{\langle S^*\rangle^2}{A^2} = \left(\frac{\sin \phi_M}{\phi_M}\right)^2$$

and the loss in S/N:

$$\text{LOSS} = 1 - \frac{\langle S* \rangle^2}{A^2} = 1 - {\frac{\sin \phi_M}{\phi_M}}^{2}$$

With $\phi_M = 11.25°$ for our design we obtain a 1.3% loss.