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A PROTOTYPE DIGITAL CROSS-CORRELATOR FOR THE NRAO INTERFEROMETER

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I. Abstract

A prototype digital cross-correlator is considered, which is capable of giving extremely accurate phase information on one component of the Fourier transformation of a source brightness distribution.

II. Theoretical Considerations

Weinreb [1] has suggested a method of cross-correlating gaussian noise voltages, which we shall consider for the correlation interferometer. If x(t) is a sample function of a gaussian random process with zero mean, and y(t) is the function formed by infinitely clipping x(t), we have

$$y(t) = A \quad \text{when} \quad x(t) > 0$$
$$y(t) = 0 \quad \text{when} \quad x(t) = 0$$
$$y(t) = -A \quad \text{when} \quad x(t) < 0$$

where A is a constant. For infinite clipping we can ignore y(t) = 0. According to Van Vleck the normalized (to mean square) autocorrelation functions of x(t) and y(t) are related by

$$\rho_x(\tau) = \sin\left[\frac{\pi}{2}\rho_y(\tau)\right]$$

This is also true in our case for cross-correlation functions.

In order to cross-correlate the two noise voltages, each noise voltage is filtered (in "rectangular" filters from zero to B), infinitely clipped, sampled at 2B, and corresponding samples multiplied and summed (integrated). Since each sample can only have values +1 and -1, the products can only have these values. In fact, due to the nature of the digital equipment we let -1 be represented by zero, which will give $\rho_x(\tau) = \frac{1}{2}$ for zero correlation. We will consider this aspect later. In order that corresponding samples are in fact multiplied, sampling pulses have to reach each sampler simultaneously.

The interferometer system with a digital cross-correlator is shown in Figure I. Considering the signals arriving from the IF amplifiers of an interferometer (from zero frequency upward), the correlated component varies cosinusoidally as a point source traverses the interferometer fringes on the sky. For the correlation interferometer in the region of zero delay (i.e., neglecting fringe amplitude modulation [2]) we have

$$\rho_x(\tau) = \cos 2\pi \frac{\tau}{T}$$

where T = fringe period

Also we have

$$\rho_x(\tau) = \sin \frac{\pi}{2} \rho_y(\tau)$$

from $\rho_x = 1$ to $\rho_x = 0$. Hence

$$\cos 2 \frac{\tau}{T} = \sin \pm [\frac{\pi}{2} \rho_y(\tau)]$$

$$= \cos \pm [\frac{\pi}{2} \rho_y(\tau) - \frac{\pi}{2}]$$

from $-\frac{\pi}{2}$ to $+\frac{\pi}{2}$. Thus we have

$$\rho_y(\tau) = \frac{4\tau}{T} - 1 \quad \text{from } \tau = 0 \text{ to } \tau = \frac{T}{4}$$

$$\rho_y(\tau) = \frac{4\tau}{T} + 1 \quad \text{from } \tau = \frac{T}{4} \text{ to } \tau = 0$$

This is a consequence of the fact that the Van Vleck equation is only concerned with magnitudes. Hence we should have the record shown in Figure II. This form of response is easily checked experimentally by varying τ in small steps.

Remembering that $\rho_y(\tau)$ is the observed correlation function, a shift in zero level and dynamic range of ρ_y does not affect the form of ρ_y. Hence, for a digital system where +1 gives 1 and -1 gives 0, ρ_y is as shown in Figure III provided that the interferometer signals are completely correlated at $\tau = 0$. This condition is not fulfilled when the system noise temperature, T_s, exceeds zero. Let us consider signal-to-noise ratio in more detail.

For parametric amplifiers $T_s = 150\ °K$. Thus the signal-to-noise ratio is given by

$$\frac{S}{N} = \frac{T_A}{150\sqrt{B\Theta}}$$

where T_A = source contribution to antenna temperature,

B = bandpass (rectangular, from zero to B),

and Θ = pre-integration time.

For $B = 500$ Kc and $\Theta = 2$ sec (or $B = 10$ Mc and $\Theta = 0.1$ sec)

$$\frac{S}{N} = 6.67\ T_A$$

Let the system noise contribution reduce the amplitude of $\rho_y(\tau)$ from that in Figure III (for zero system noise) to that in Figure IV. Here we have values of $\rho_y(\tau)$ between R and 1 - R, where $\frac{1}{2} < R < 1$. Hence

$$(\rho_y)_{max} = 1 - R$$

$$\frac{S}{N} = \frac{(\rho_y)_{max} - \frac{1}{2}}{1 - (\rho_y)_{max}} = \frac{\frac{1}{2} - R}{R}$$

Hence for parametric amplifiers

$$\frac{\frac{1}{2} - R}{R} = 6.67\ T_A$$

$$R = \frac{1}{13.3\ T_A + 2}$$

Several important facts emerge from the above considerations:

(a) Fringe phase is determined by the zero crossover of the sawtooth waveform (Figure IV). The accuracy of this crossover depends strongly on the stability of the clipping zero;

(b) Since the signal has been clipped, amplitude information can only be obtained by comparing the correlated (interferometer) component with the uncorrelated (system noise) component;

(c) If the sampling pulses arrive at the same relative times prior to sampling, the digital samples are time independent provided they are kept in the correct order.

A possible method of obtaining amplitude information is to sum the analog signals, and to compare the fringe amplitude with the detected system noise (DC). The record of the analog sum is shown in Figure V(a). In Figure V(b) the increase in average (DC) level in the region of the fringes is due to a finite source, so that the source is partially resolved at the interferometer spacing considered. This is equivalent to a temporary increase in system noise, and must be taken into account when determining fringe amplitudes.

III. A Prototype Digital Cross-Correlator

Since it is desirable to test such a system before considering it as an alternative to analog multiplication, a system is outlined which utilizes components already required for the digital autocorrelation radiometer being built at NRAO. The simplicity of the system suggested here is such that tests could be completed before the components are required for the autocorrelation interferometer. Since we are only interested in the performance of the system IF's, we may test the system by passing (Gaussian) noise through two noisy amplifiers, and then filters. The filters are essentially rectangular from 0 to 500 Kc. The filtered noise is clipped and sampled, the samples being multiplied and summed in a digital counter. The system is shown in Figure VI, and its variation of output with τ is shown in Figure VII. It is interesting to note that no RF fringes occur, which is to be expected when no RF noise has been provided. The system in Figure VIII will give fringes, since we are now cross-correlating RF noise as in the interferometer. The reduction in frequency is to facilitate the construction of such a system. Since the problems of path stability will not arise here, and since the RF is considerably lower than for the NRAO interferometer, a 0.1 μs delay adjustment in the RF section of the circuit will give a "scan" of three fringes, which is quite adequate for testing the system. Furthermore, for f_{LO} = 25 Mc and Δf = 500 Kc we have very little dispersion over three fringes, provided that the center fringe in the zero-delay fringe (which is easily arranged in this case). Hence the single-sideband system designed for the auto-correlation radiometer could be used.

References

[1] "A Digital Spectral Analysis Technique and Its Applications to Radio Astronomy", S. Weinreb. PhD Thesis, MIT (1963).

[2] "Design Considerations for the NRAO Interferometer", N. J. Keen. Electronics Division Internal Report No. 14 (August 1963).

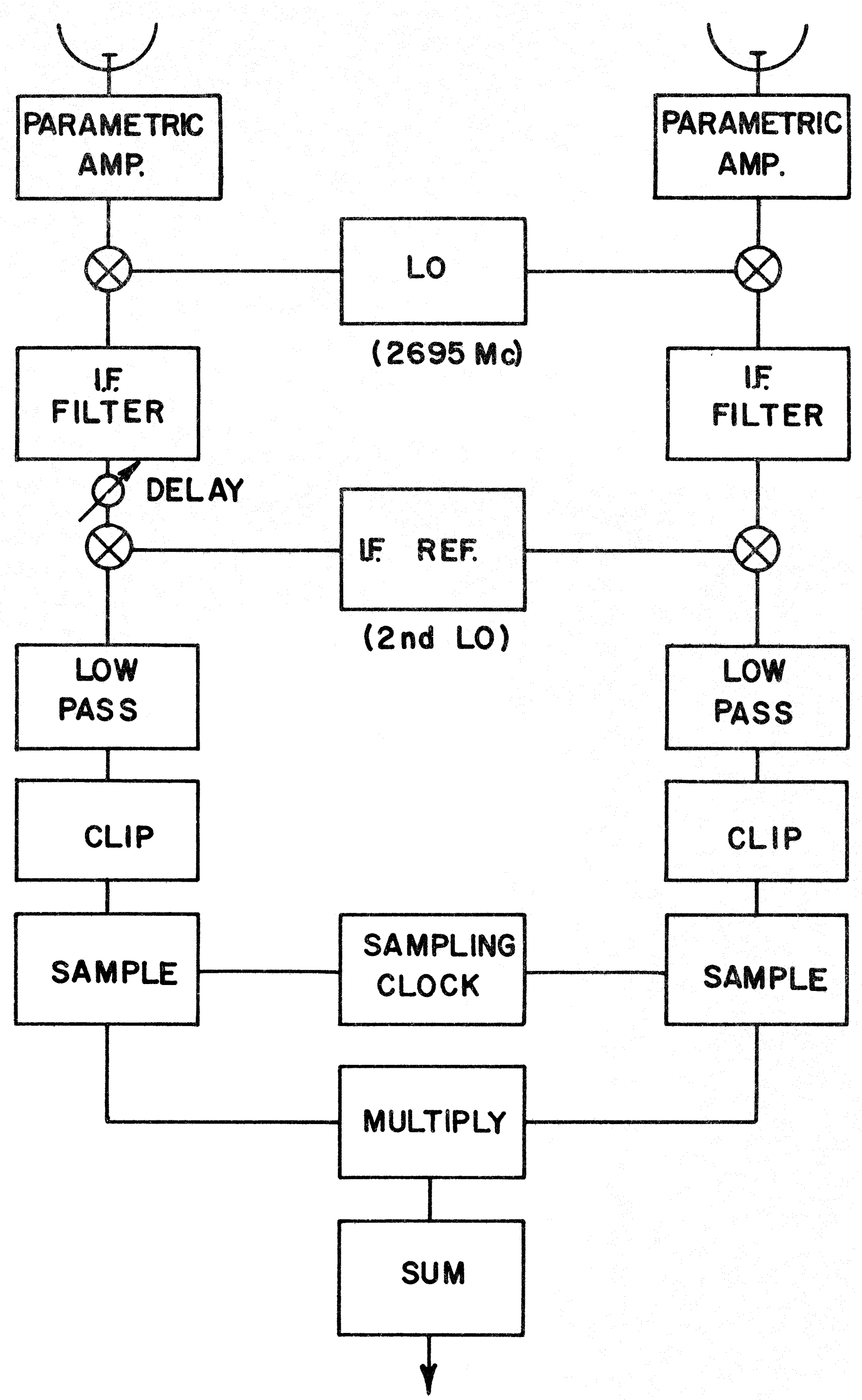


FIGURE I-INTERFEROMETER WITH DIGITAL CROSS CORRELATION

FIGURE II

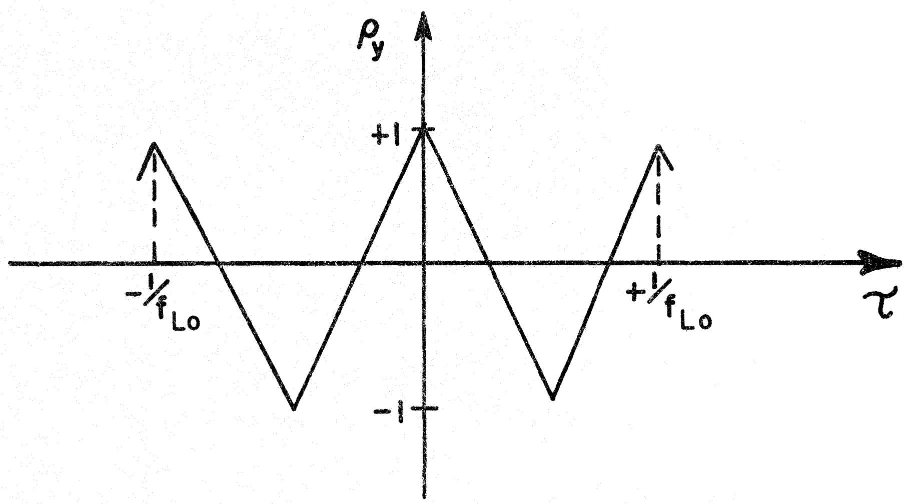
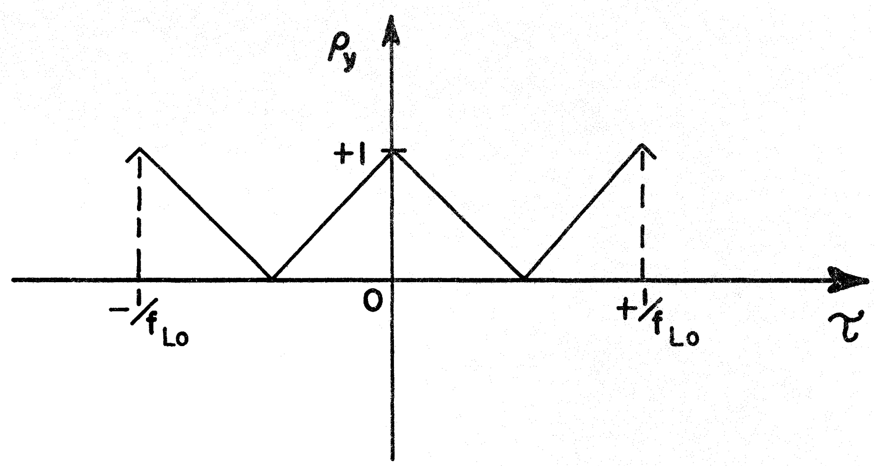


FIGURE III



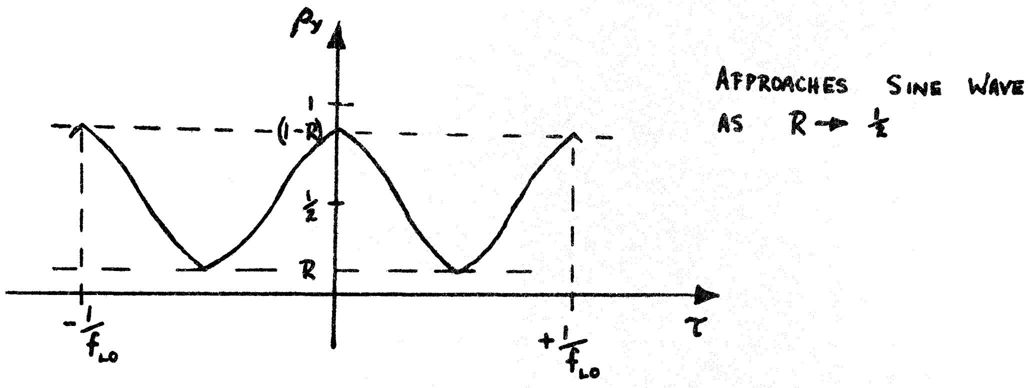


FIGURE IV

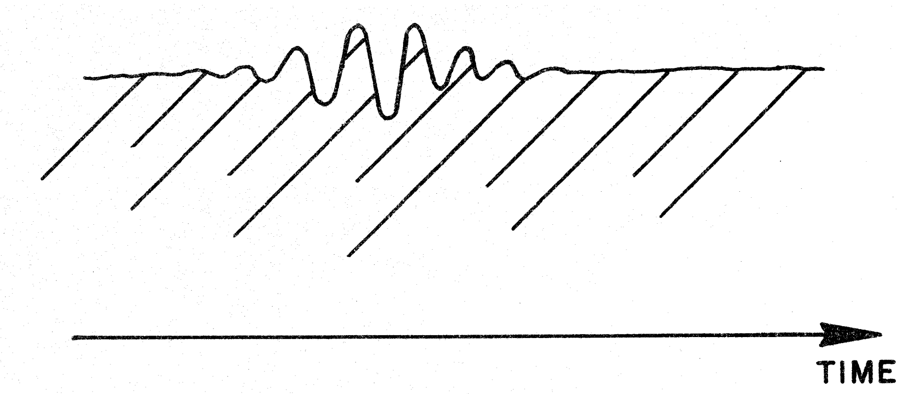


FIGURE V (a)

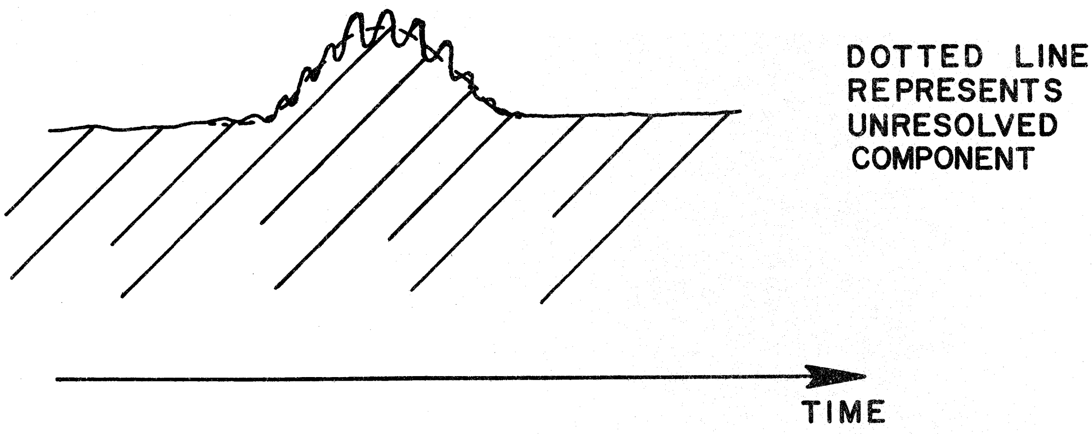


FIGURE V (b)

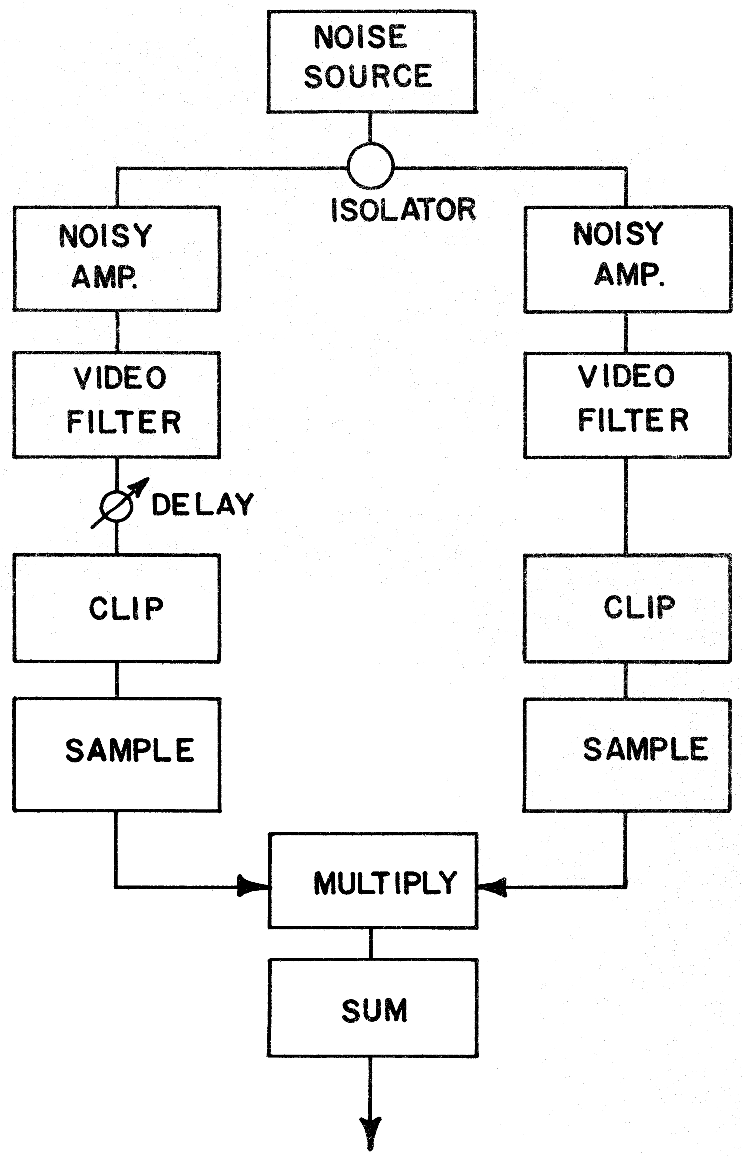


FIGURE VI

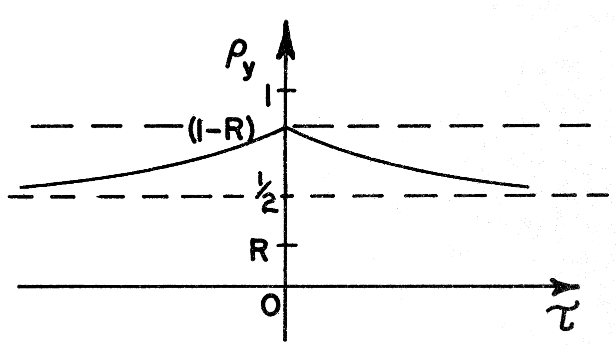


FIGURE VII

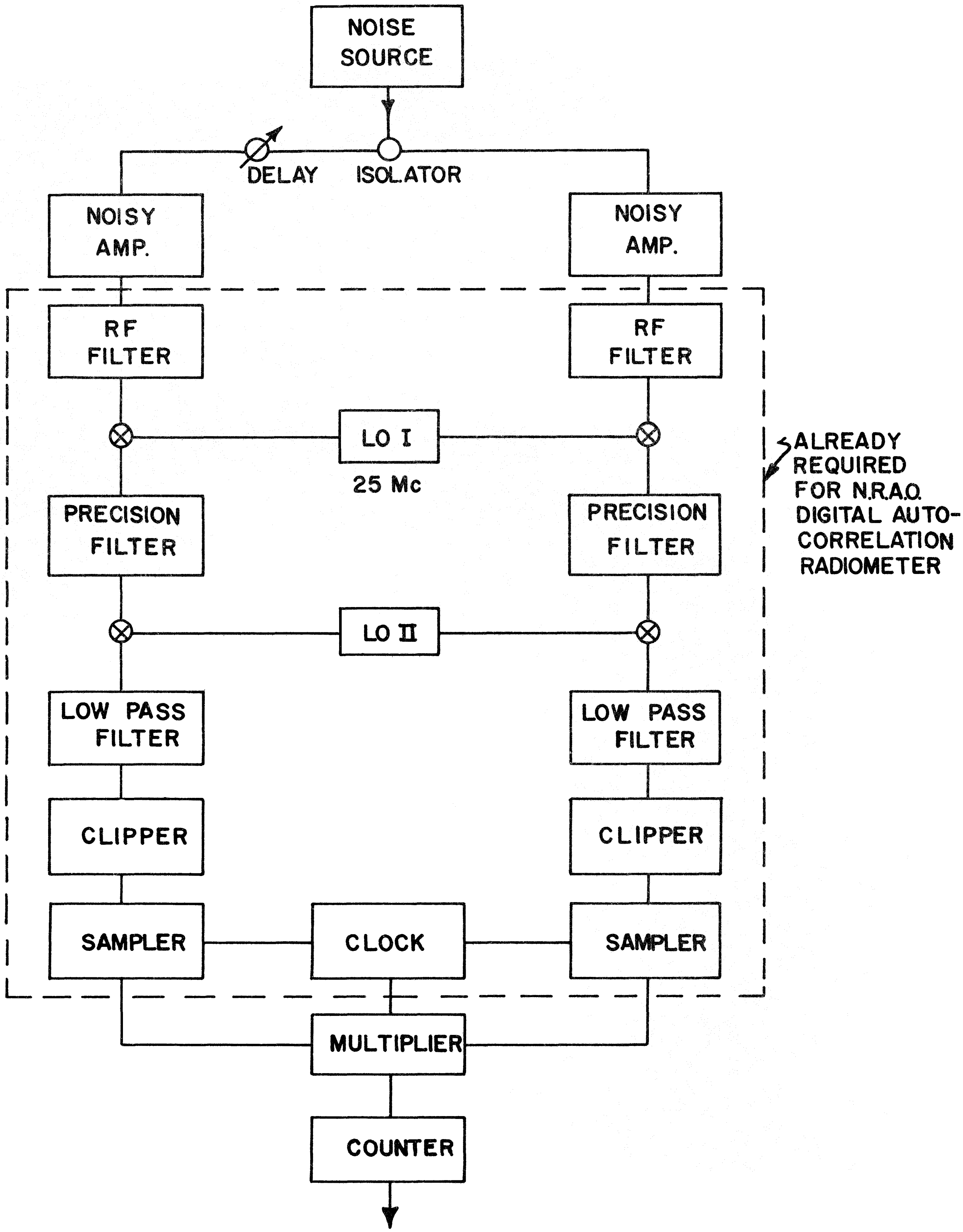

NOISE SOURCE
DELAY
ISOLATOR
NOISY AMP.
NOISY AMP.
RF FILTER
RF FILTER
LO I
25 Mc
ALREADY REQUIRED FOR N.R.A.O. DIGITAL AUTO-CORRELATION RADIOMETER
PRECISION FILTER
PRECISION FILTER
LO II
LOW PASS FILTER
LOW PASS FILTER
CLIPPER
CLIPPER
SAMPLER
CLOCK
SAMPLER
MULTIPLIER
COUNTER

FIGURE VIII