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A DIRECTIONAL FILTER FOR LOCAL OSCILLATOR  
INJECTION IN A MILLIMETER-WAVE MIXER RADIOMETER

JESSE E. DAVIS

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ABSTRACT

The advent of mixer radiometers with large tuning ranges operating at a wavelength of three millimeters creates the need for a local oscillator injection scheme which will provide low signal power and local oscillator power losses over the entire operating range of the receiver. The purpose of this paper is to report on the development of a directional filter for use in an 80-120 GHz mixer radiometer. A unique use of the mode properties of a resonant ring filter allows a very wide tuning range (85-115 GHz LO) with little degradation in filter performance. The directional nature of the filter, together with the resonant characteristic, provides both low signal path loss (0.2 dB) and low local oscillator path loss (4 dB).

I. BACKGROUND

The proposed radiometer will tune continuously from 80 to 120 GHz (signal frequency) with an IF frequency of 4.75 GHz and an instantaneous bandwidth of 650 MHz. The first stage of the receiver is a broadband diode mixer with no RF tuning, allowing both the signal and image bands to be down converted to the IF. The entire front end and first IF amplifier will be cooled to 20° K to improve the noise performance. A method of supplying the local oscillator power to the mixer is required. The injection scheme must provide a low loss path for the signal and the image simultaneously and should provide a reasonable loss in the local oscillator path because of local oscillator power limitations. The injection scheme must cover the entire operating range of the receiver and, if tunable, must be adjustable remotely without opening the dewar.

The traditional solution to the problem of local oscillator injection is presented in Figure 1. The directional coupler is chosen to provide the least signal loss due to coupling, consistent with available local oscillator power. A typical coupler for use in a 100 GHz receiver will have a -10 dB coupling factor and a mainline resistive loss of about 0.5 dB. The total signal power loss for this coupler is about 1 dB (0.55 dB coupling loss + 0.5 dB resistive loss), representing a significant decrease in receiver sensitivity. The local oscillator power loss is about 10 dB. The broadband nature of the coupling allows receiver operation over essentially the full waveguide band with no tuning required. The directional coupler represents an inexpensive, broadband solution to the problem of local oscillator injection; however, a solution yielding a lower signal path and local oscillator loss would be desirable. There exist several other techniques for injecting the local oscillator power, each with its attendant disadvantages.

## II. RING FILTER

The proposed solution to the problem of local oscillator injection is presented in Figure 2. The injection filter is a 4-port device consisting of a traveling-wave resonant ring filter tuned to the local oscillator frequency and two directional couplers. The signal power is coupled from port 2 to port 3. The local oscillator power is coupled from port 1 to the resonant ring via one directional coupler and then to port 3 via the second directional coupler. The directional nature of the couplers prevents the flow of local oscillator power to ports 2 and 4, while the resonant ring prevents the coupling of signal power from port 2 to ports 1 and 4. Signal power loss is limited to resistive loss in the 2 to 3 path. There is virtually no signal loss due to coupling because of the resonant nature of the ring and the signal-local oscillator

frequency separation. Local oscillator power loss is limited to the resistive loss in the resonant ring. The resonant frequency of the ring filter is varied by altering the guide wavelength in the waveguide forming the ring. This directional filter represents a low loss solution to the problem of local oscillator injection.

The design of the traveling-wave filter is based largely on the work of Coale (1) and Tischer (2). The ring is formed by a circular section of rectangular waveguide ( $TE_{10}$ ) which is closed upon itself to form a ring. The waveguide ring is split along the center line of the broad wall of the waveguide. This allows the broad dimension, and hence the guide wavelength in the ring, to be varied for tuning. Power is coupled into and out of the ring through two narrow-wall multi-hole couplers. For a ring of broad dimension  $a$  and mean circumferential length,  $L$ , the resonant wavelength,  $\lambda$ , is given approximately by (1):

$$\lambda_{\text{res}} = \frac{1}{\sqrt{n^2/L^2 + 1/4a^2}}$$

where  $n$  represents the integer number of wavelengths around the ring at resonance. For a given guide width,  $a$ , there are an infinite number of resonances which will couple power from port 1 to port 3. The first few are shown in Figure 3. As we alter the guide width the "comb structure" representing the filter transmission modes moves along the frequency axis. One can operate the filter over any tuning range by increasing the ring waveguide width,  $a$ . There are three pertinent points to be noted.

The first point involves the restrictions which must be placed on the number of wavelengths,  $n$ , which one can fit around the ring. For a given guide wavelength there is a lower limit placed on the circumference of the ring, and

hence  $n$ , by the requirement that two directional couplers be realized along this circumference. The upper limit is placed on  $n$  by the minimum mode spacing requirements and loss considerations. The longer the guide, the greater the local oscillator power transmission loss and the smaller the mode spacing.

The second point involves the allowable range of the guide width,  $a$ . If the guide width is too narrow the guide wavelength becomes too long to be of use and the loss increases. As the guide width is increased to effect the tuning of the filter a gap appears between the ring halves. As the width of the gap increases, power is coupled through the gap, representing a loss, significantly lowering the cavity  $Q$  thereby increasing the transmission loss of the filter.

The third point involves the presence of more than one transmission frequency or mode. Multiple modes do not seriously affect the local oscillator to mixer coupling; however, if a mode were to fall in either the signal or image frequency band some power would be coupled from port 2 to port 4 representing a loss in the input path, decreasing the receiver sensitivity.

The practical filter is limited in tuning range by the finite range of the guide width,  $a$ , and the limitations placed on the mode number,  $n$ . As an example consider the following design, based on the receiver requirements listed above. Consideration of the above points, in particular the third point, leads to the following choices for the ring parameters:

$$\begin{aligned} n &= 16 \\ L &= 3'' \\ a &= .085'' \text{ to } .125'' \text{ (.040'' gap)} \end{aligned}$$

The tuning range of this filter will then be:

$$\begin{aligned} a = .085'' &= .126'' & f &= 93.7 \text{ GHz} \\ a = .125'' &= .150'' & f &= 78.6 \text{ GHz} \end{aligned}$$

This represents a fractional tuning range of less than 20%, far short of the

40% (80-120 GHz) required for receiver operation. The gap of .040" would represent a large power loss, considerably affecting the transmission loss of the filter. The gap in the ring must be kept small.

One solution to this problem is to note that any of the resonances may be used. It is not necessary to move one resonance, as was done in the example above, to achieve the tuning range. It is necessary only to move the  $n^{\text{th}}$  resonance to the frequency location of the  $n+1$  resonance. Since the entire comb moves, a large frequency range can be covered simply by choosing the proper mode and alternating the waveguide width to move that mode to the required frequency. In the example above, to move the  $n = 16$  mode to the frequency of the  $n = 17$  mode requires a change in guide width of .004 inches. This is a significant improvement in gap width. It is possible, using this technique, to construct a filter which has both low loss properties and a wide tuning range.

### III. DESIGN

The first step in the design is the selection of the ring circumferential length,  $L$ , and the range of the ring guide width,  $a$ . The ring length is chosen to give the desired mode spacing consistent with coupler reliability. Care must be taken to assure that neither the image nor the signal band falls at the frequency of an adjacent mode. It is desirable to keep the ring as short as possible, in order to minimize loss. The ring waveguide width range,  $\Delta a$ , is chosen to give the tuning required to completely cover the region between the modes. Since the mode spacing and tuning rate are non-linear, care must be taken to assure that each mode covers its required range. It is desirable to choose values for  $a$  which yield a guide wavelength in a lower loss region of the guide attenuation curve.

The second step is the design of the couplers. It is usual to make the couplers identical. The following equation relates the transmission loss through the filter to the voltage coupling coefficient,  $C$ , and the loaded  $Q$ ,  $Q_L$ , of the filter.

$$L \text{ (dB)} = 20 \log \frac{1 - (1 - C^2) C^{-\alpha L}}{C^2 e^{-\alpha L/2}}$$

$$Q_L \approx n\pi/C^2 10^{\text{Loss (dB)}/20}$$

where  $\alpha$  is the attenuation constant of the ring waveguide. The transmission loss of the filter is the more important of the two parameters in this case. Since we wish to minimize the local oscillator power loss the couplers will be designed accordingly. The accompanying decrease in the loaded  $Q$  is not a problem because of the high IF frequency. The coupler directivity requirement is determined from microwave circuit considerations for the case in question. The coupler is designed by any of the techniques presented in the literature.

#### IV. THE FILTER

The design of the filter proceeded as outlined above. To achieve a signal frequency range of 80-120 GHz the local oscillator must be tuned from about 85-115 GHz (LO above at the lower end and below at the upper end). A ring length of 3 inches was chosen to give the signal-LO-image relationship shown in Figure 4. Note that one mode has been allowed to fall between the LO (main mode) and the signal-image bands.

The couplers are chosen to each give a -10 dB coupling factor with a directivity in excess of 20 dB over the 85-115 GHz band, representing the best balance between loss and coupler reliability. The coupler design is based on



the designs given by Levy (3). A Chebyshev tapered multi-hole array with circular apertures is chosen. Machining considerations dictate the use of the less efficient circular apertures. The coupling array is located in the very thin (.005 inch) common wall of the ring guide and signal/LO guides. The couplers are arranged symmetrically about the circular ring. Both couplers are mounted in the same half of the ring guide to allow for the tuning motion. The pertinent coupler dimensions are shown in Figure 5. Calculated coupler response is shown in Figures 5a and 5b.

The filter is fabricated of tellurium-copper, chosen because of its favorable machining characteristics. The filter is constructed in three pieces to allow for machining. The structure is aligned with steel guide pins. The rather odd waveguide pattern is chosen both for convenient final mounting considerations and to allow for relatively easy machining of the guides. The pattern is machined using an end mill equipped with a rotary table. The completed filter is gold plated for corrosion protection. Mechanical drawings and photographs of the filter are present in Appendix A.

The tuning of the filter presents special problems because of the high tuning sensitivity (600 GHz/inch) and the fact that the tuning must be accomplished at 20° Kelvin. The high tuning sensitivity leads to the use of a differential screw (400:1). The position of the screw is measured by a differential transformer, the core being connected to the high speed center screw of the differential pair. This type of position sensor gives the high resolution needed to measure the ring gap accurately and will operate at 20° Kelvin. A gear box is provided to interface the screw with the motor shaft in a convenient manner. Ball bearings are used throughout to minimize friction since no lubricant can be used. Careful choice of materials to minimize problems due to differential expansion is of the utmost importance.

Experience with the type of tuning mechanism described above leads to a decision to simplify the arrangement. While the above arrangement works adequately it is painfully slow. While the springs remove most of the backlash in the screw, stiction in the screw leads to a jerky tuning motion, making the tuning of the filter difficult in practice. An unexpected problem occurs when the servo fails. Since the servo remains on at all times any failure in the electronics causes the motor to drive the screw past the mechanical limits causing considerable damage to the tuning mechanism. In practice this has happened often enough to warrant attention. A step toward remedying these problems has been undertaken. Mechanical drawings and photographs of the tuning mechanism are present in Appendix B.

A new tuning mechanism is illustrated in Appendix C. The motion necessary to tune the ring is provided by a cam driven by a gear box. All gears, as well as the cam yoke, are spring loaded to remove backlash. The mechanism provides the required tuning resolution without the use of screws. The cam provides a very smooth movement without the binding tendency inherent in the differential screw. Since the cam is cyclical there are no mechanical limits which can be exceeded, causing damage to the mechanism, as with the differential screw arrangement. The mechanism is designed to be directly interchangeable with the existing mechanism. Tests on the new unit are not complete at this time.

## V. ANALYSIS

A computer program to evaluate the performance of the filter is included in Appendix D. The program computes the amount of power coupled from the local oscillator port to the other three ports. An effort has been made to use the most appropriate model for the directional coupler and ring. The program is partitioned in the form of subroutines to provide for ease of implementation.

In addition to the main routines several utility routines have been included since they are not widely available. The usage should be clear from the listing. A sample problem is included. A description of each subroutine follows.

The subroutine CPLR calculates the complex reflection and coupling coefficients for two transmission lines coupled together periodically. The program utilizes a rigorous 4-port network analysis, rather than the usual weak coupling analysis, to yield accurate estimates of coupling and directivity. The analysis takes into account the dispersive nature of the transmission lines, transmission line losses, and the infinite number of waves being coupled back and forth by the coupler. The coupling coefficients of the apertures are based on Bethe's small hole theory as modified by Cohn to consider aperture resonance phenomena and the finite thickness of the wall containing the apertures. Aperture interaction due to evanescent modes is not considered. The true electrical length of the coupler is calculated to allow improved estimates of the resonant frequencies and mode spacings of the ring resonator. The analysis is based on articles by Riblat (4) and Levy (5). It should be noted that the use of this program is not limited to the coupler in question but may be used for any coupled transmission line problem by insertion of the appropriate expressions for the coupling reactances.

The conventions used are illustrated in Figure 6. Consider a wave of amplitude  $a_1$  incident on port 1. With all ports terminated in the characteristic impedance of the transmission line the waves emerging from the four ports are as follows:

$$\text{Port 1: } b_1 = R_0 a_1$$

$$\text{Port 2: } b_2 = KR a_1$$

$$\text{Port 3: } b_3 = KH a_1$$

$$\text{Port 4: } b_4 = T_{12} a_1$$

where  $R_0$ ,  $T_{12}$ ,  $KR$ , and  $KH$  are the complex reflection and coupling coefficients. Since the coupler is symmetrical and reciprocity holds, any port may be taken as the input with the same results. Therefore, the reflection coefficient,  $R_0$ , for all ports is the same; the thru arm forward coupling,  $T_{12}$ , is the same as the thru arm reverse coupling,  $T_{21}$ , etc. An example of the application of these quantities for the coupler as it is excited in the filter is presented in Figure 7. The figure is self-explanatory.

The subroutine FILTER calculates the power delivered to each of the other three ports with a wave of unit amplitude entering port 1. All ports are assumed terminated in their characteristic impedance. The port numbering conventions are presented in Figure 2. The analysis considers waves traveling in both directions around the ring and resistive losses in the ring. A brief description of the method is presented.

The analysis is carried out in terms of the wave cascading matrix  $[R]$ . If the amplitudes of the incident waves are  $a_1$  and  $a_2$  and the amplitudes of outgoing waves are  $b_1$  and  $b_2$  as illustrated in Figure 8, then the waves are related as follows:

$$\begin{bmatrix} a_1 \\ b_1 \end{bmatrix} = \begin{bmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{bmatrix} \begin{bmatrix} b_2 \\ a_2 \end{bmatrix}$$

The ring is divided into segments, each of which is characterized by its R-cascading matrix. Figure 9 illustrates the segmentation. The length of the coupler waveguides which are part of the ring is just the number of coupling holes times the hole spacing. The length of the waveguides which connect the two couplers to form the ring is half the balance necessary to make up the ring. Thus,

$$2 (\# \text{ holes} \times \text{hole spacing} + \text{length connecting guide}) = L.$$

The upper directional filter is characterized as a 2-port with accompanying R matrix. This is allowed if we restrict incident waves to ports 1 and 2. The R matrix for this portion of the ring is:

$$\begin{bmatrix} a_1 \\ b_1 \end{bmatrix} = \frac{1}{T_{12}} \begin{bmatrix} 1 & -R_0 \\ R_0 & T_{12}^2 - R_0^2 \end{bmatrix} \begin{bmatrix} b_2 \\ a_2 \end{bmatrix}$$

where the quantities are defined as above.

The two sections of connecting waveguide are represented by:

$$\begin{bmatrix} a_1 \\ b_1 \end{bmatrix} = \begin{bmatrix} e^{\Gamma L} & 0 \\ 0 & e^{-\Gamma L} \end{bmatrix} \begin{bmatrix} b_2 \\ a_2 \end{bmatrix}$$

where  $\Gamma = \alpha + j\beta$  is the complex propagation constant.

These three elements of the ring are now cascaded to yield a representation of the upper half of the ring as illustrated in Figure 10. The R matrix for the entire structure is:

$$\begin{bmatrix} a_1 \\ b_1 \end{bmatrix} = \frac{1}{T_{12}} \begin{bmatrix} e^{\Gamma L} & 0 \\ 0 & e^{-\Gamma L} \end{bmatrix} \begin{bmatrix} 1 & -R_0 \\ R_0 & T_{12}^2 - R_0^2 \end{bmatrix} \times$$

$$\times \begin{bmatrix} e^{\Gamma L} & 0 \\ 0 & e^{-\Gamma L} \end{bmatrix} \begin{bmatrix} b_2 \\ a_2 \end{bmatrix}$$

The upper portion of the ring is interfaced with the lower portion using the conventions illustrated in Figure 11. With the coupler parameters as defined above the relations among the various waves are:

$$\begin{aligned}
 b_1 &= a_1 R_0 + a_3 KH + a_4 KR \\
 b_2 &= a_1 T_{12} + a_3 KR + a_4 KH \\
 b_3 &= a_1 KH + a_3 R_0 + a_4 T_{12} \\
 b_4 &= a_1 KR + a_3 T_{12} + a_4 R_0
 \end{aligned}$$

We now define the matrix:

$$\begin{bmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{bmatrix} = \frac{1}{T_{12}} \begin{bmatrix} e^{\Gamma L} & 0 \\ 0 & e^{-\Gamma L} \end{bmatrix} \begin{bmatrix} 1 & -R_0 \\ R_0 & T_{12}^2 - R_0^2 \end{bmatrix} \begin{bmatrix} e^{\Gamma L} & 0 \\ 0 & e^{-\Gamma L} \end{bmatrix} .$$

Then solving the following for  $a_3$  and  $a_4$

$$\begin{bmatrix} a_3 \\ b_3 \end{bmatrix} = \begin{bmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{bmatrix} \begin{bmatrix} b_4 \\ a_4 \end{bmatrix}$$

yields

$$\begin{aligned}
 a_3 &= (R_{21} - R_{22} R_{11}/R_{12}) b_4 + \frac{R_{22}}{R_{12}} b_3 \\
 a_4 &= \frac{b_3 - R_{11} b_4}{R_{12}}
 \end{aligned}$$

which when substituted in the set of four simultaneous equations above yields in matrix form:

$$\begin{bmatrix} -a_1 K R \\ -a_1 K H \\ -a_1 T_{12} \\ -a_1 R_0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & (R_0/R_{12} + R_{22} T_{12}/R_{12}) \\ 0 & 0 & (R_0 R_{22}/R_{12} + T_{12}/R_{12} - 1) \\ 0 & -1 & \left( \frac{K R R_{22}}{R_{12}} + \frac{K H}{R_{12}} \right) \\ -1 & 0 & \left( \frac{K H R_{22}}{R_{12}} + \frac{K R}{R_{12}} \right) \end{bmatrix} \begin{bmatrix} \left\{ T_{12} \left( R_{21} - \frac{R_{22}}{R_{12}} R_{11} \right) - \frac{R_0 R_{11}}{R_{12}} \right\} - 1 \\ \left[ R_0 \left( R_{21} - \frac{R_{22}}{R_{12}} R_{11} \right) - \frac{T_{12} R_{11}}{R_{12}} \right] \\ \left[ K R \left( R_{21} - \frac{R_{22}}{R_{12}} R_{11} \right) - \frac{K H R_{11}}{R_{12}} \right] \\ \left[ K H \left( R_{21} - \frac{R_{22}}{R_{12}} R_{11} \right) - \frac{R_{11}}{R_{12}} K R \right] \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix}$$

This represents a set of four simultaneous equations in four unknowns. If we call the 4 x 4 matrix BFM, then

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix} = [BFM^{-1}] \begin{bmatrix} -a_1 K R \\ -a_1 K H \\ -a_1 T_{12} \\ -a_1 R_0 \end{bmatrix}$$

We now have an expression for the four waves necessary to completely characterize the filter. Waves  $b_3$  and  $b_4$  characterize the output side of the filter. Waves  $b_1$  and  $b_2$  characterize the input side of the filter. Output waves  $b_5$  and  $b_6$  can be found as follows: Waves  $b_3$  and  $b_4$  represent the waves at the exits of the input coupler. These waves are transformed around the ring to the input of the output coupler and then:

$$b_5 = a_4' K H + a_3' K R$$

$$b_6 = a_4' K R + a_3' K H$$

where the primed quantities are the waves after transformation around the ring as described above. Waves  $b_1$  and  $b_2$  are the outward flowing waves in the input coupler main line.

The quantities used in the above analysis are complex; consequently the programs are written to handle complex expressions. Several mathematical functions such as the complex hyperbolic sine and a routine for the inversion of a complex matrix were not available on the system used by myself. The subroutines were written and are included. The methods used are straightforward. The matrix inversion is accomplished using the standard Gauss-Jordan pivoted method, with thanks to IBM. These routines are general and may be useful elsewhere. Plotting routines have not been included as they are very system dependent.

## VI. RESULTS

The measured response of the filter at room temperature is presented in Figures 12 and 13. No measurements of the antenna port to mixer port loss is included due to the difficulty of measuring these small quantities accurately in this frequency range. The signal loss is about .3 dB. There is a slight improvement in local oscillator transmission loss when the filter is cooled to 20° Kelvin. The filter response agrees fairly well with the design. The discrepancies are thought to be due largely to inaccuracies in the coupler design and the uncertainty in the high frequency value of the skin depth. The overall response of the filter is considered to be very good, representing a significant improvement over other injection schemes.

The choice of the mode spacing together with the mode characteristics of the filter yield a filter without spurious modes. This property allows for ease of tuning. Mechanical limits prevent the filter from being tuned more than one mode spacing. This assures that at any operating frequency the filter need be tuned only to the closest mode without fear of reduced filter performance due to mode selection errors. In practice one need not be concerned with which mode he is operating on.



This injection filter represents an improved solution to the problem of local oscillator injection in millimeter wave radiometers. The filter gives excellent electrical and mechanical performance and can be implemented with moderate efforts, as compared to other schemes. In addition to use an an LO injection filter, this type of filter has many other uses. Filters of this type are currently in use in receivers of the NRAO at Kitt Peak, Arizona.

#### ACKNOWLEDGMENTS

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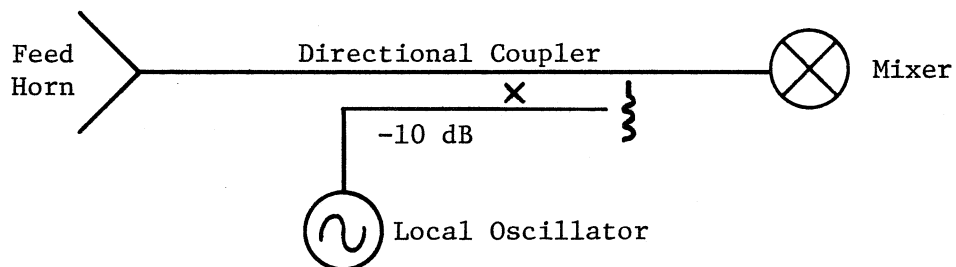


Figure 1: Directional coupler used in local oscillator injection scheme.

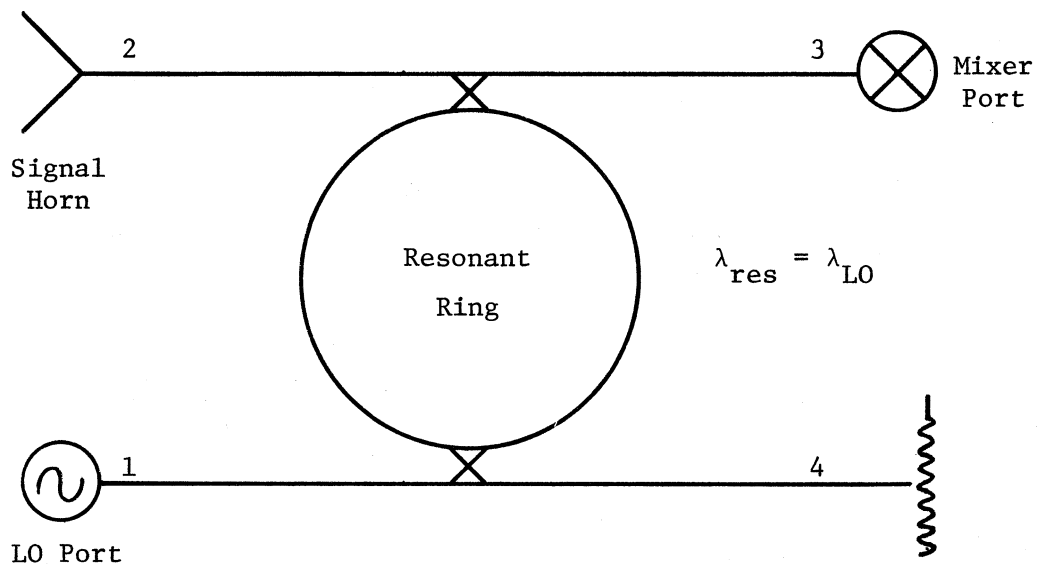
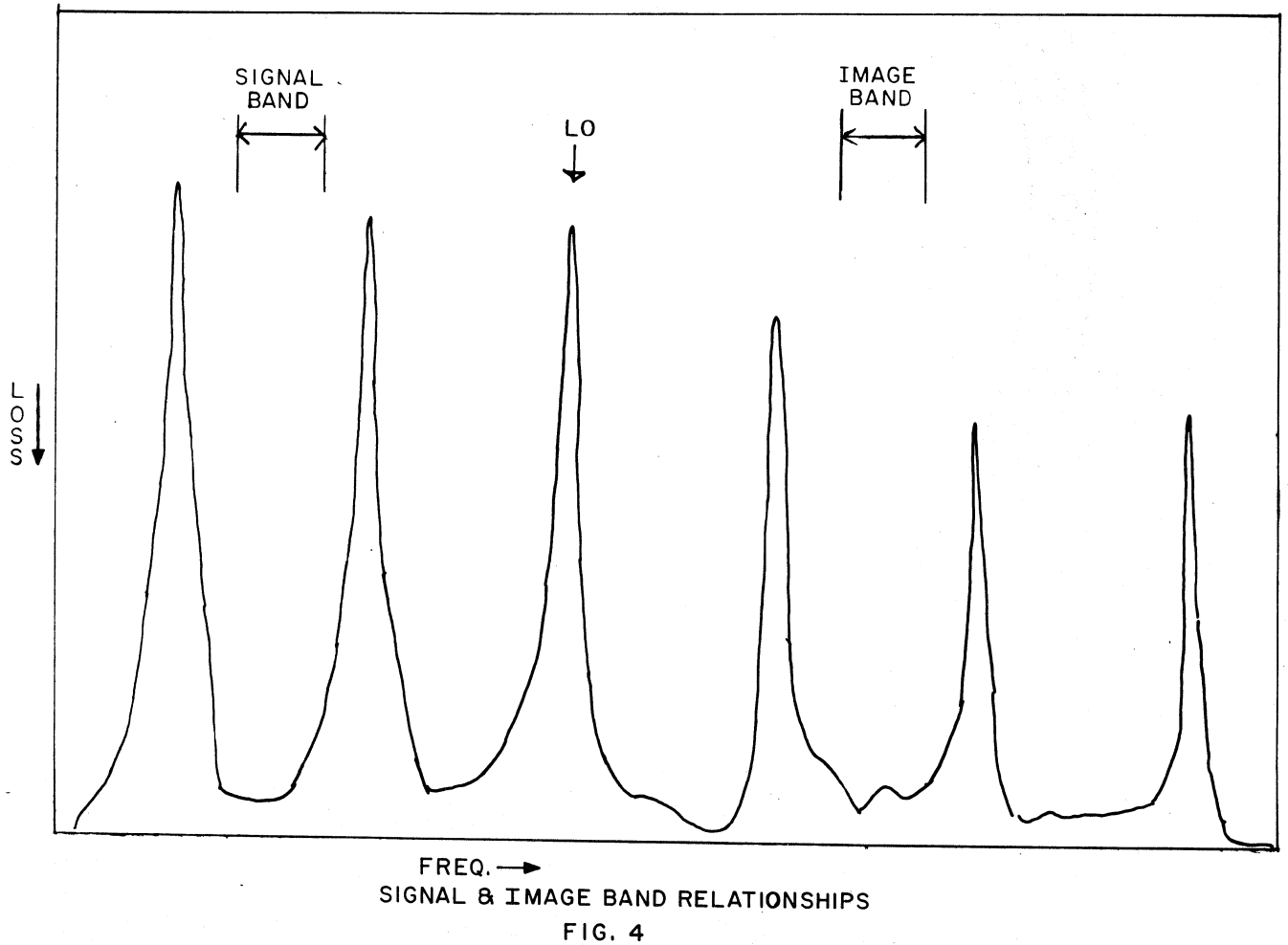
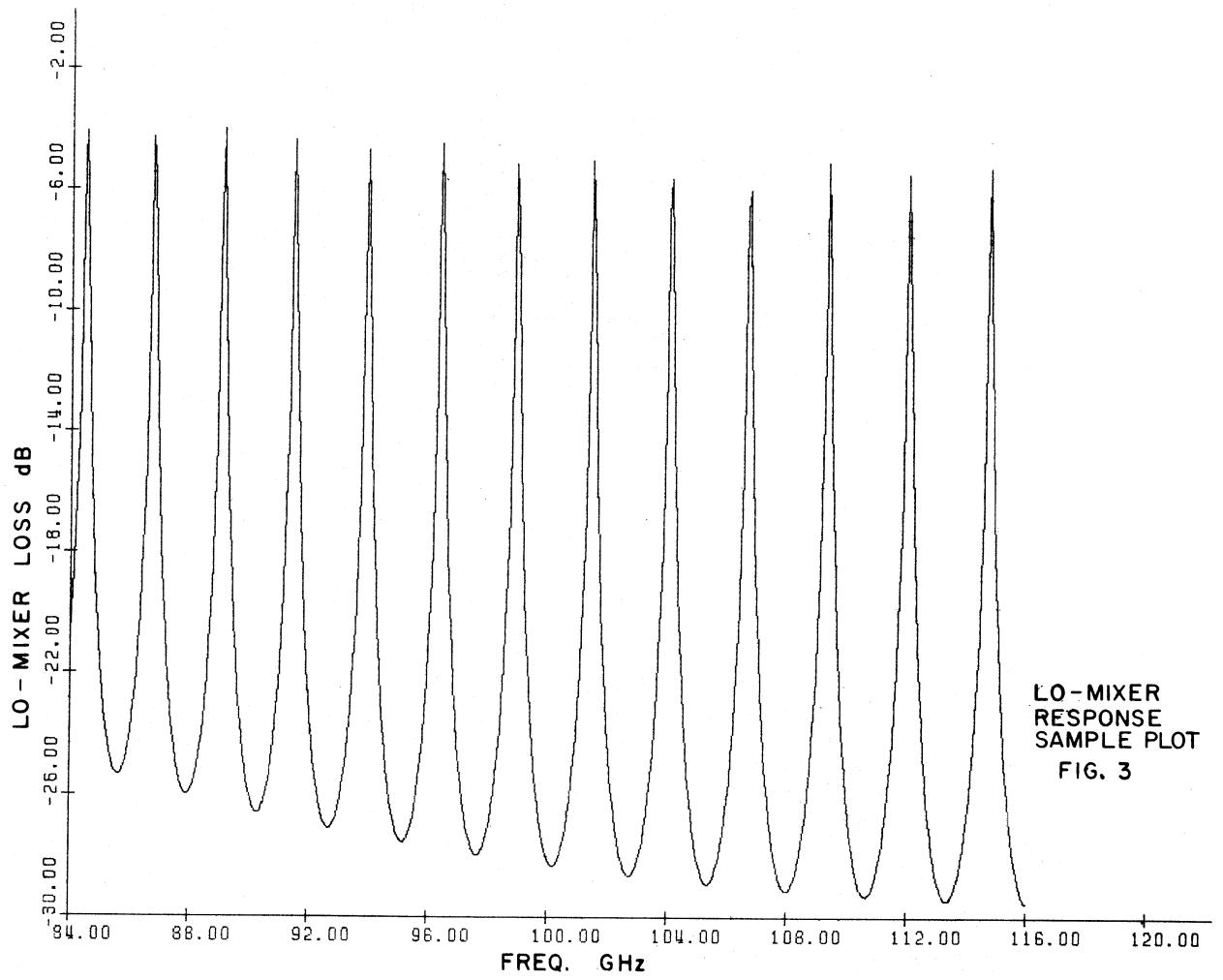


Figure 2: Resonant ring filter used for local oscillator injection.



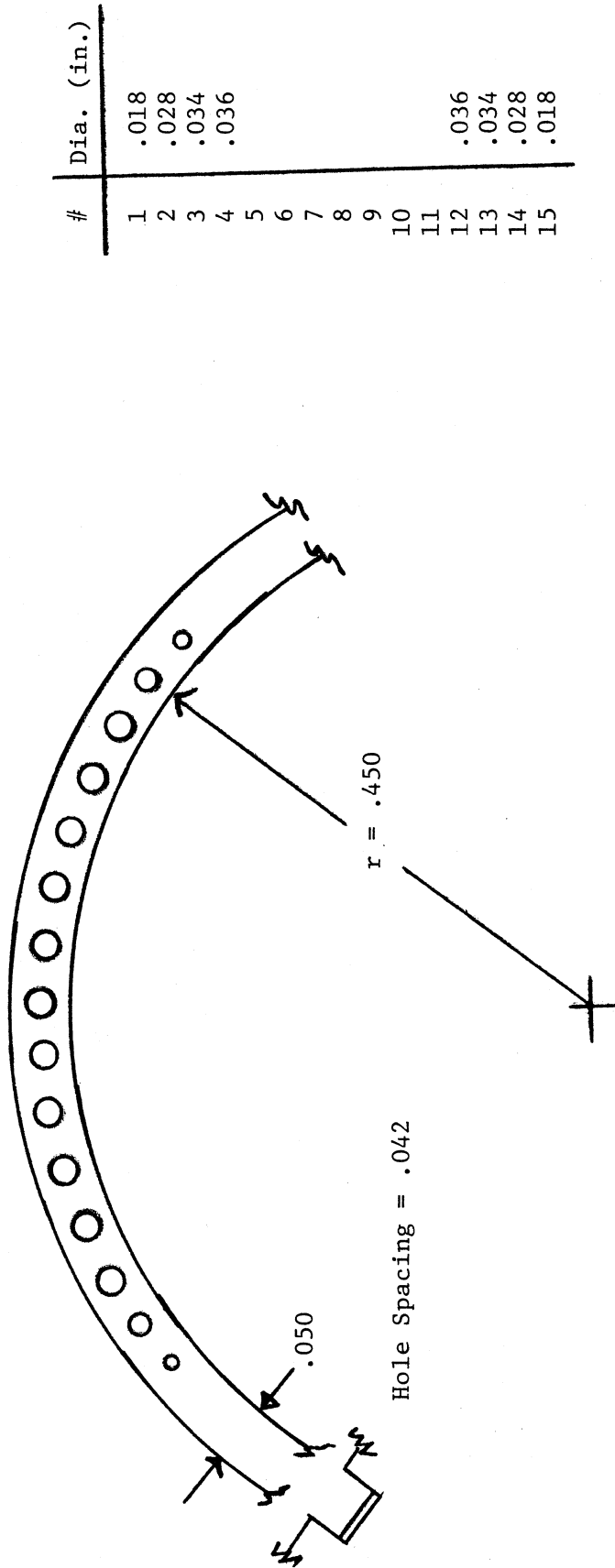
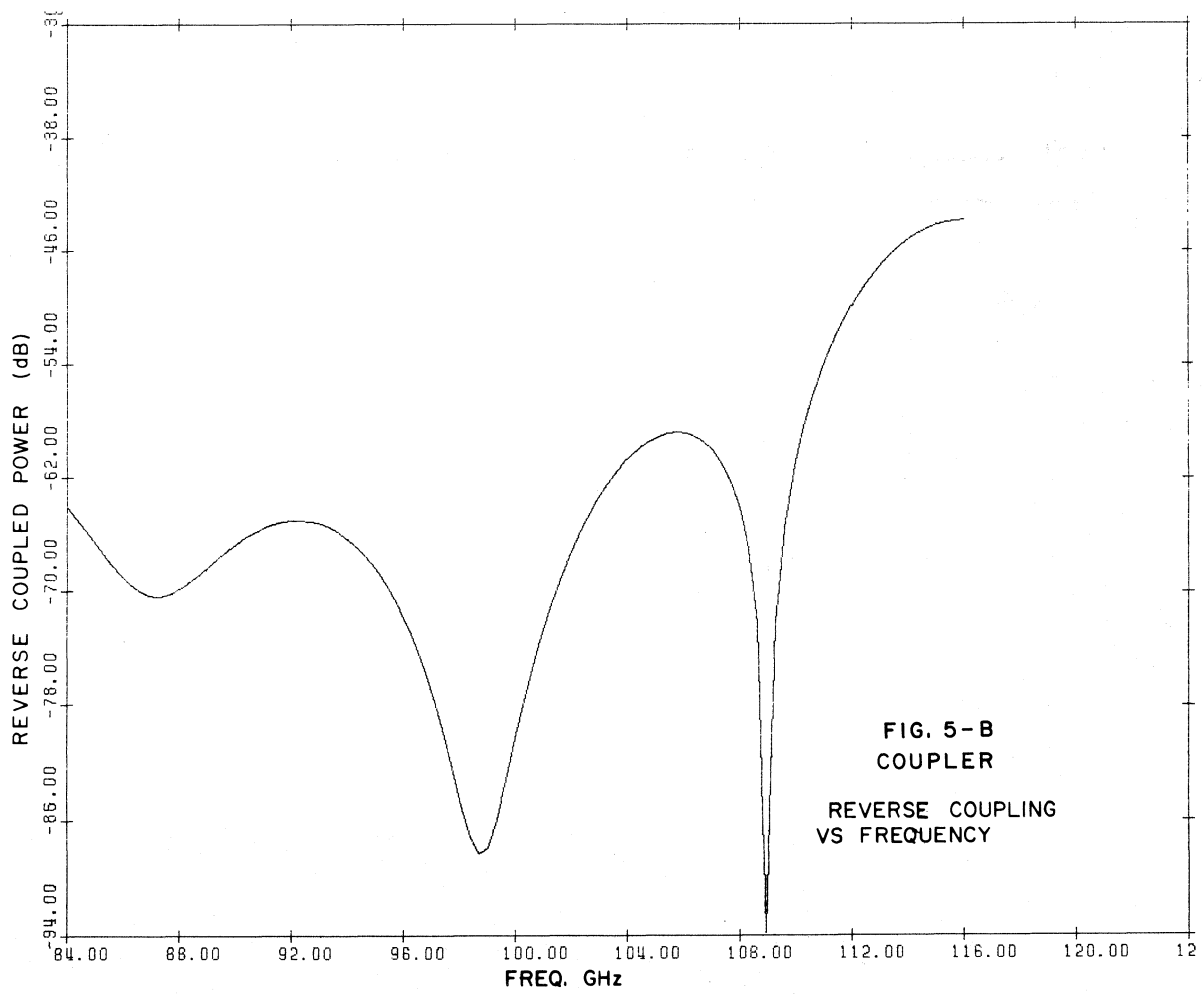
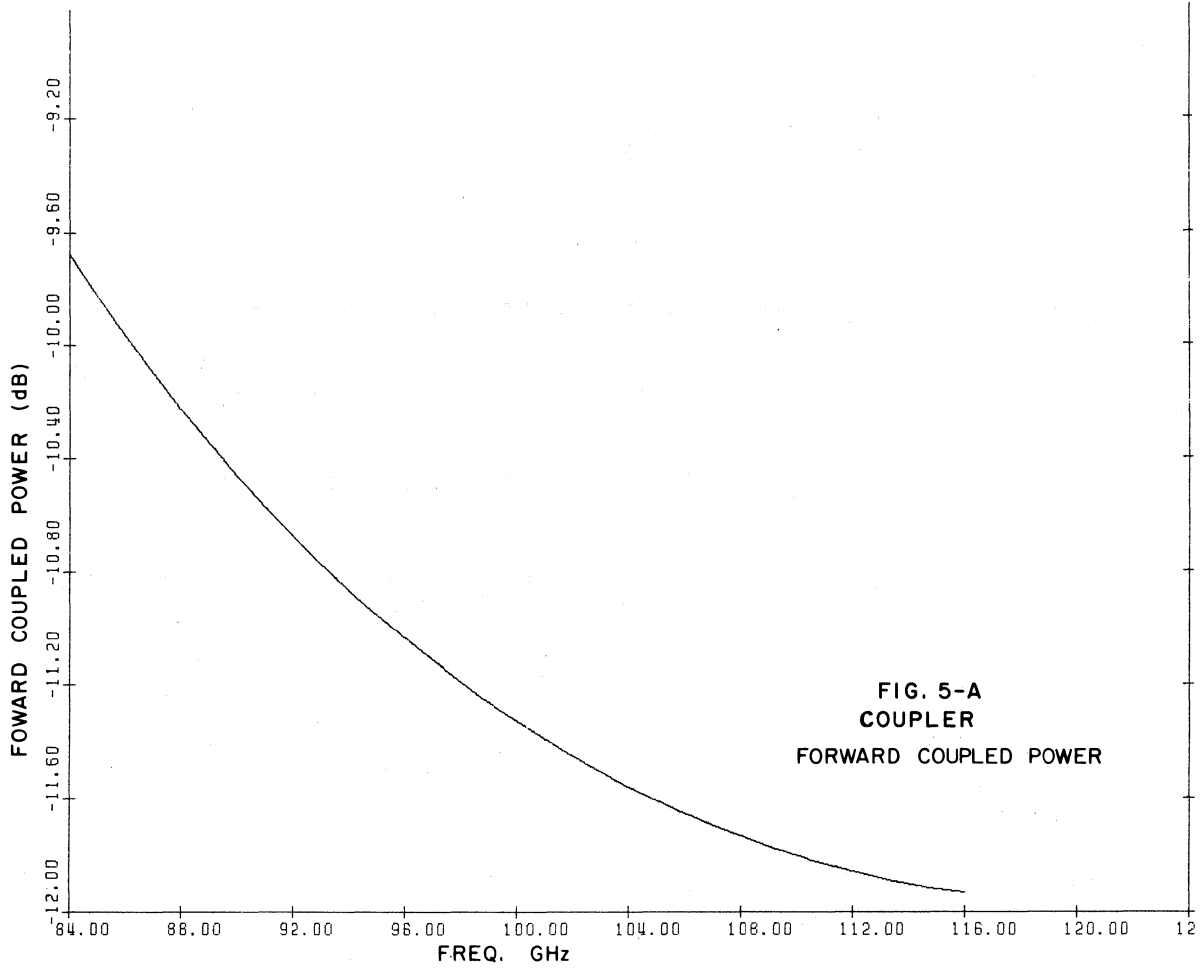


Figure 5: Directional coupler dimensions for 10 dB coupling, 85-115 GHz.



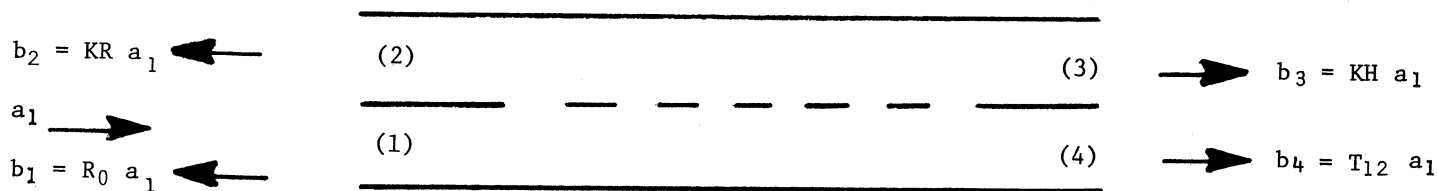
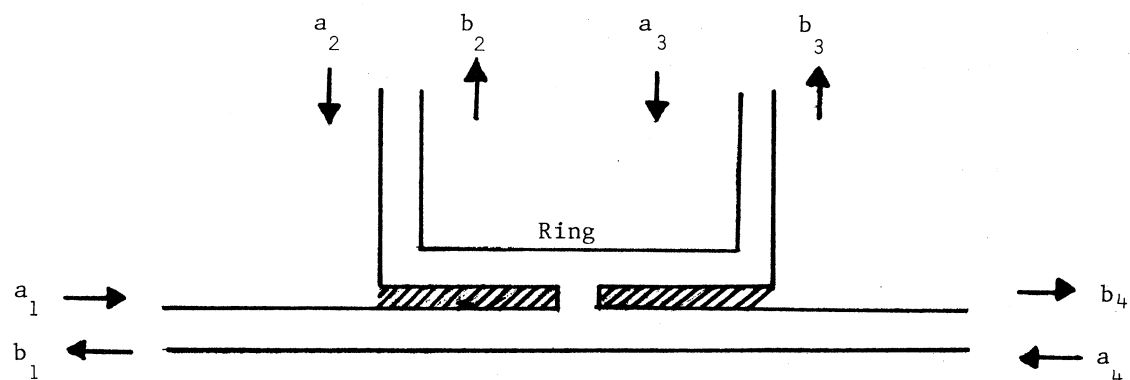


Figure 6: Coupler wave conventions.



$$b_1 = a_1 R_0 + a_2 KR + a_3 KH + a_4 T_{12}$$

$$b_2 = a_1 KR + a_2 R_0 + a_3 T_{12} + a_4 KH$$

$$b_3 = a_1 KH + a_2 T_{12} + a_3 R_0 + a_4 KR$$

$$b_4 = a_1 T_{12} + a_2 KH + a_3 KR + a_4 R_0$$

$R_0$  = Reflection coefficient.

$KR$  = Reverse coupling.

$KH$  = Forward coupling.

$T_{12}$  = Thru arm coupling.

Figure 7: Power flow in coupler.

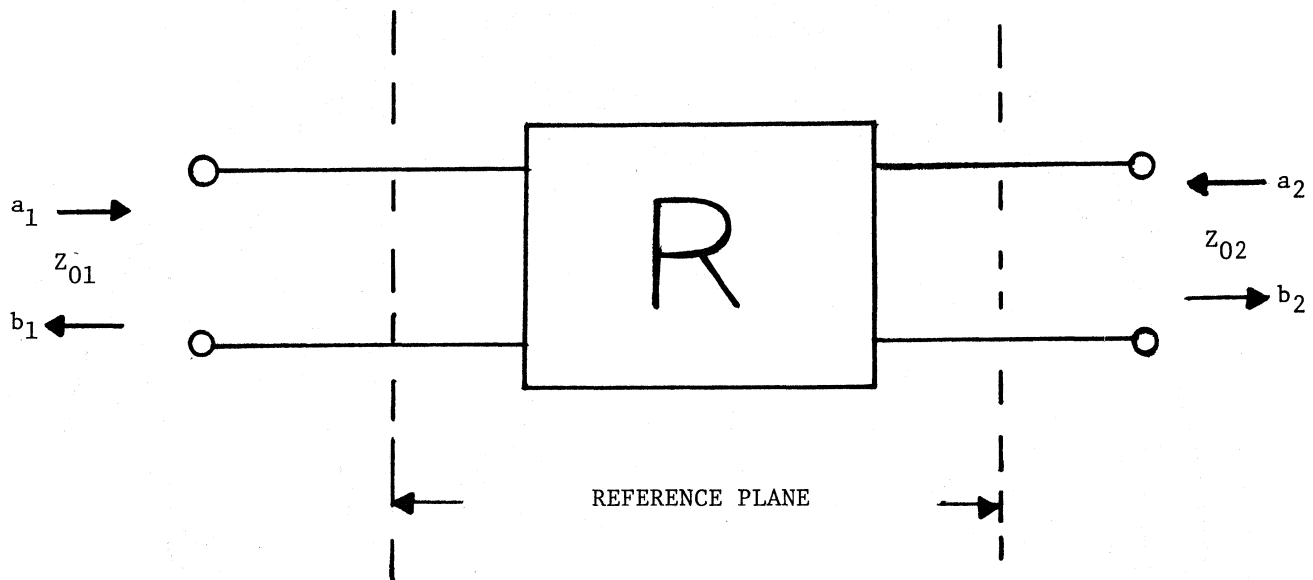
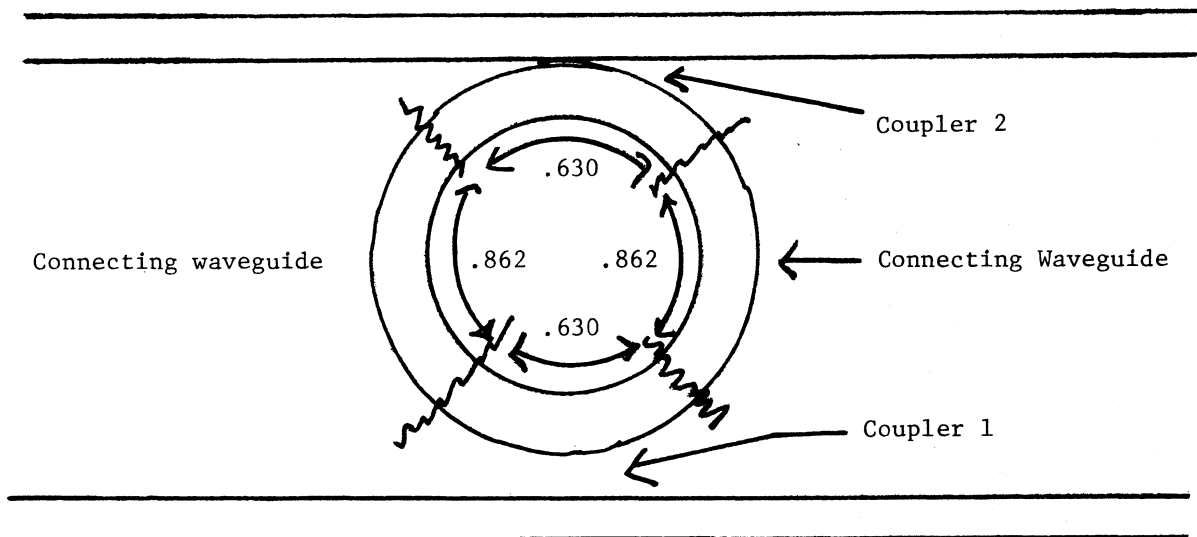


Figure 8. Wave Cascading Matrix Conventions.



Waveguide lengths are as in example.

$$.630 = 15 \text{ (holes)} \times .042 \text{ (hole) spacing}$$

Figure 9: Segmentation of filter.

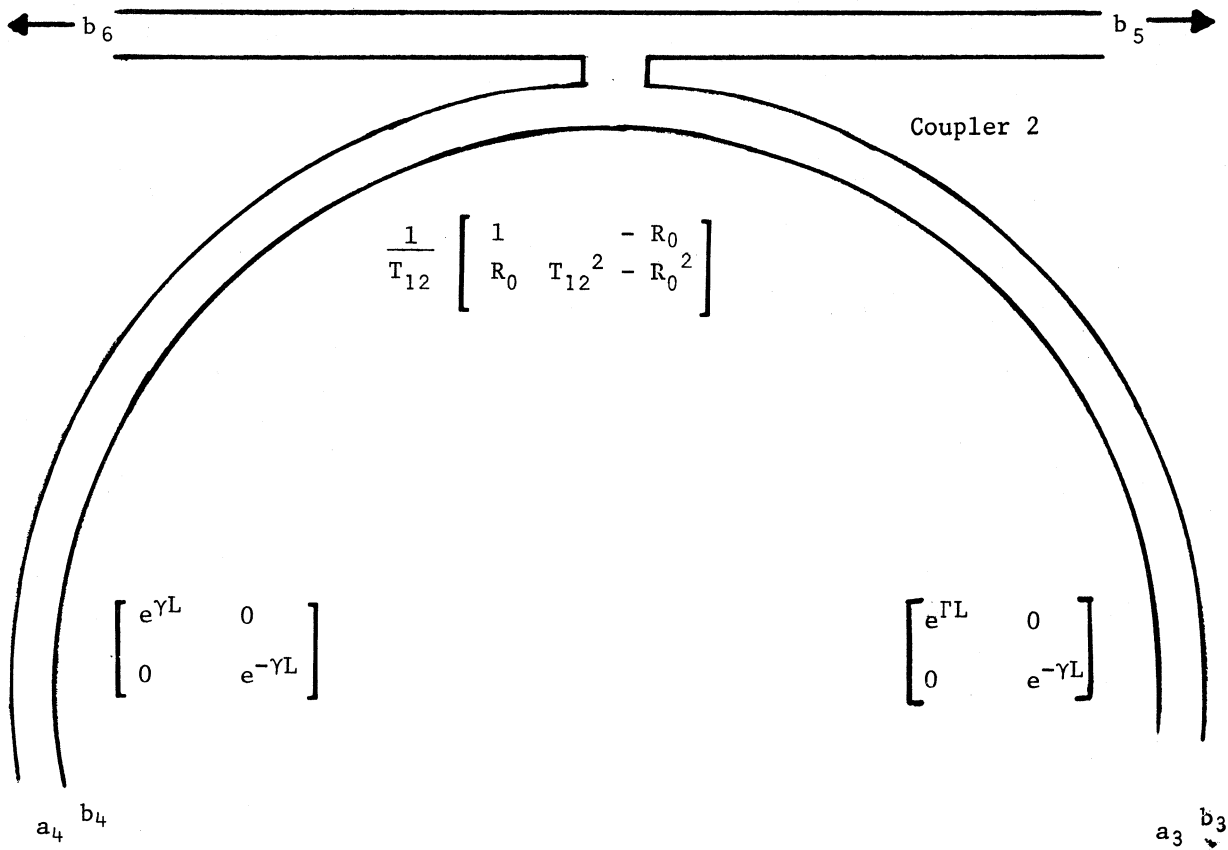


Figure 10: Representation of upper ring and coupler.

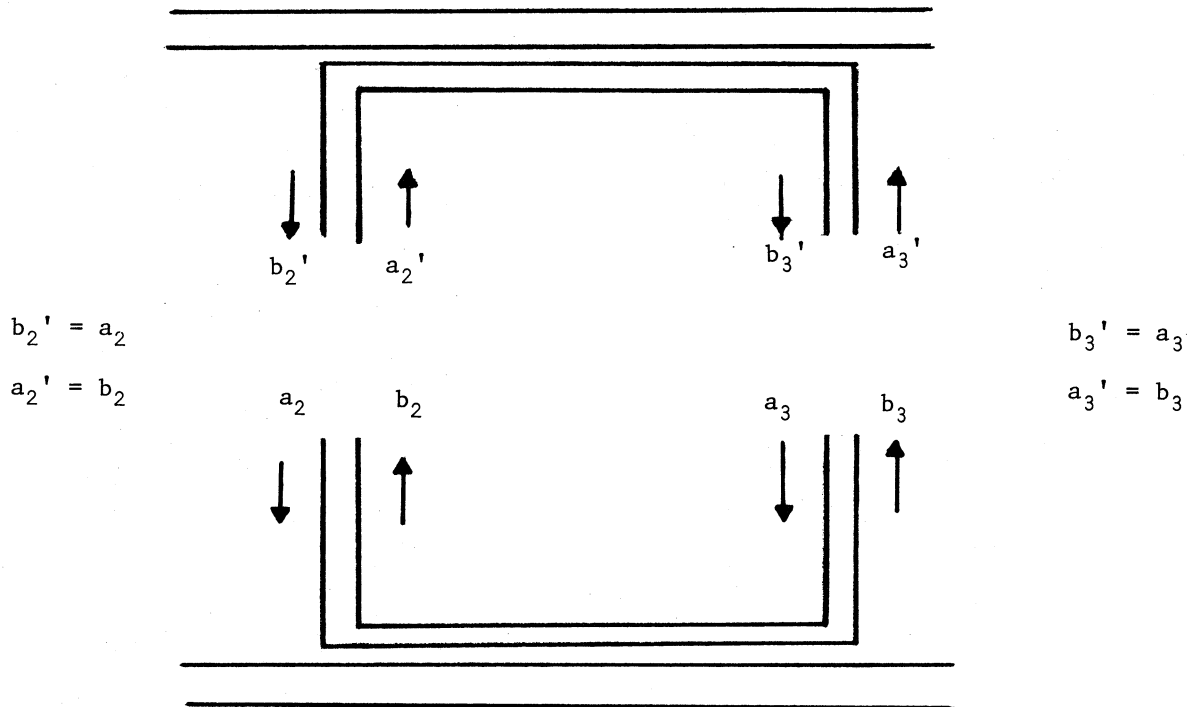
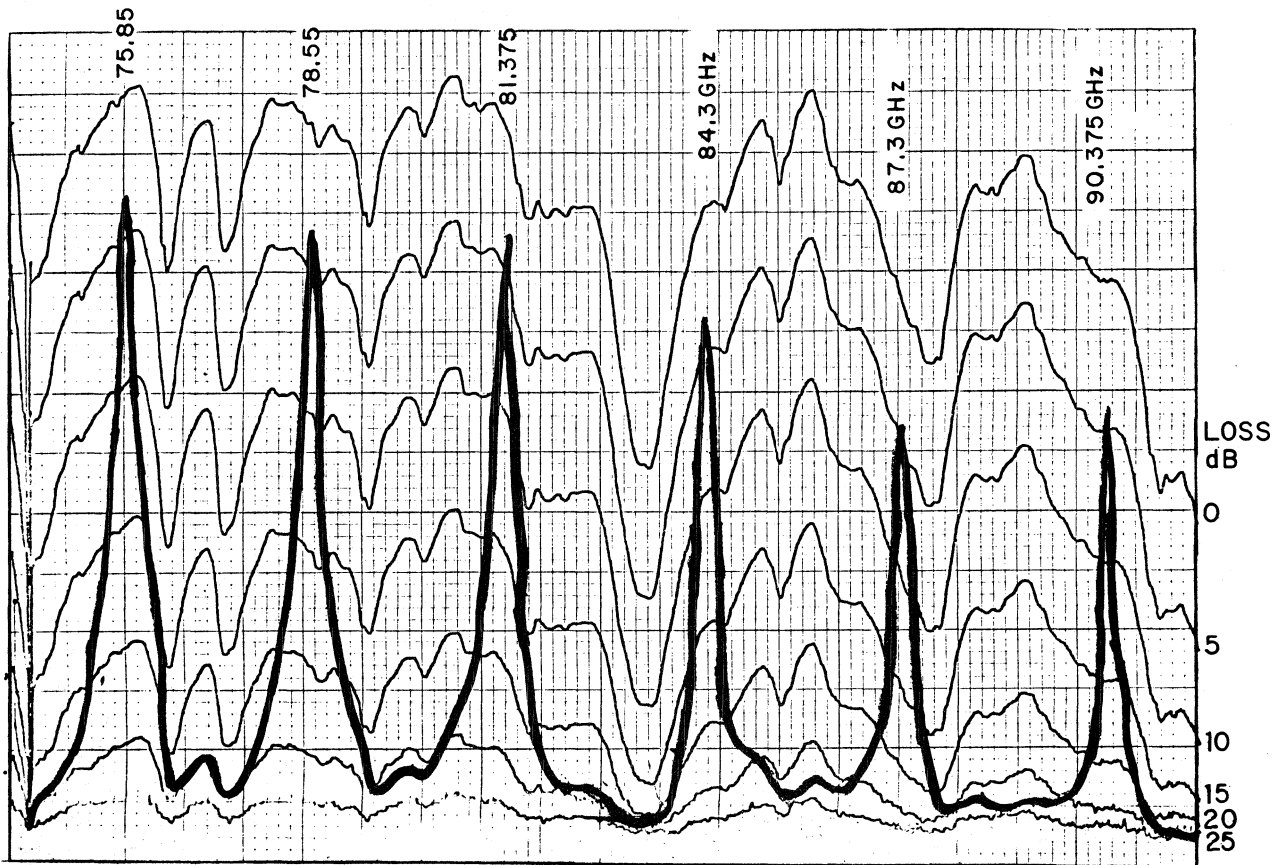
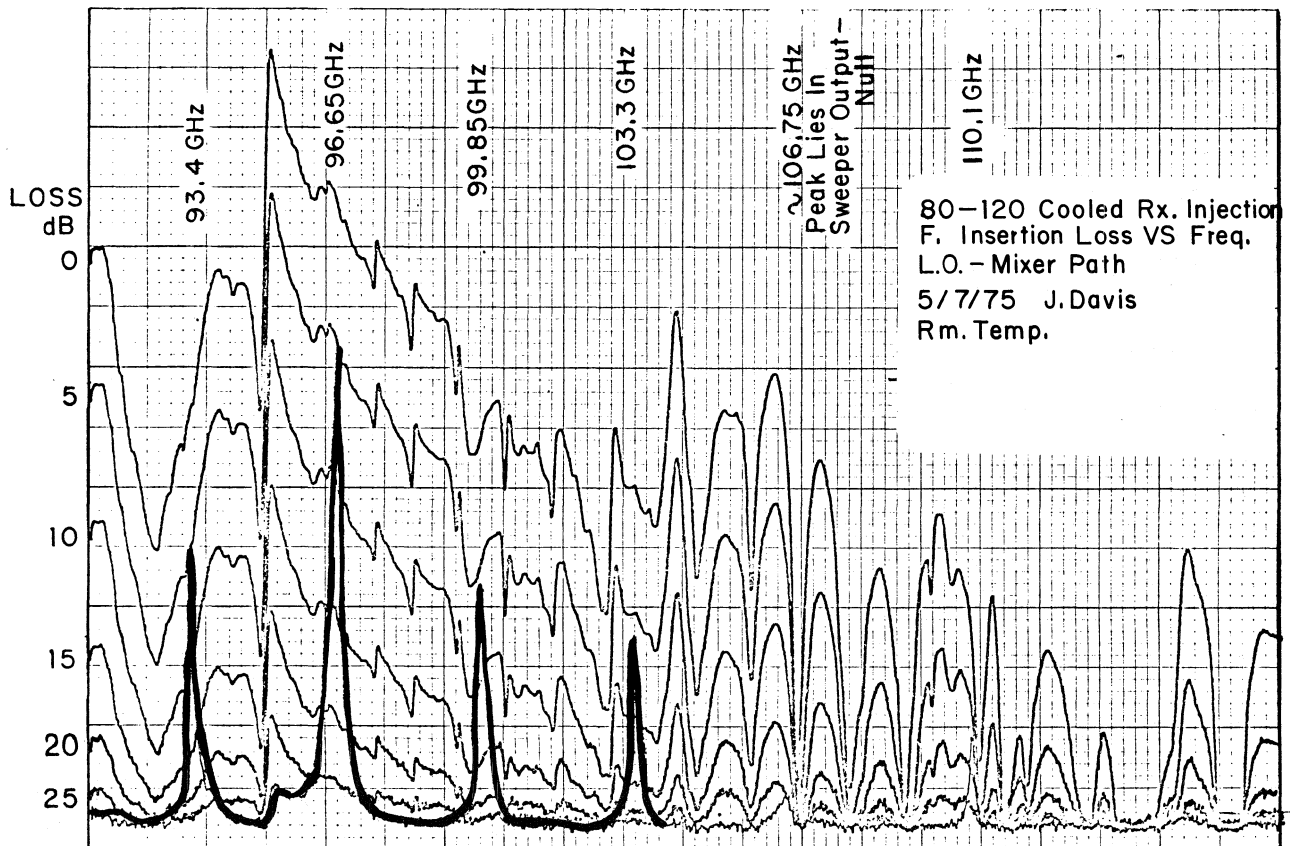


Figure 11: Wave conventions for ring connections.

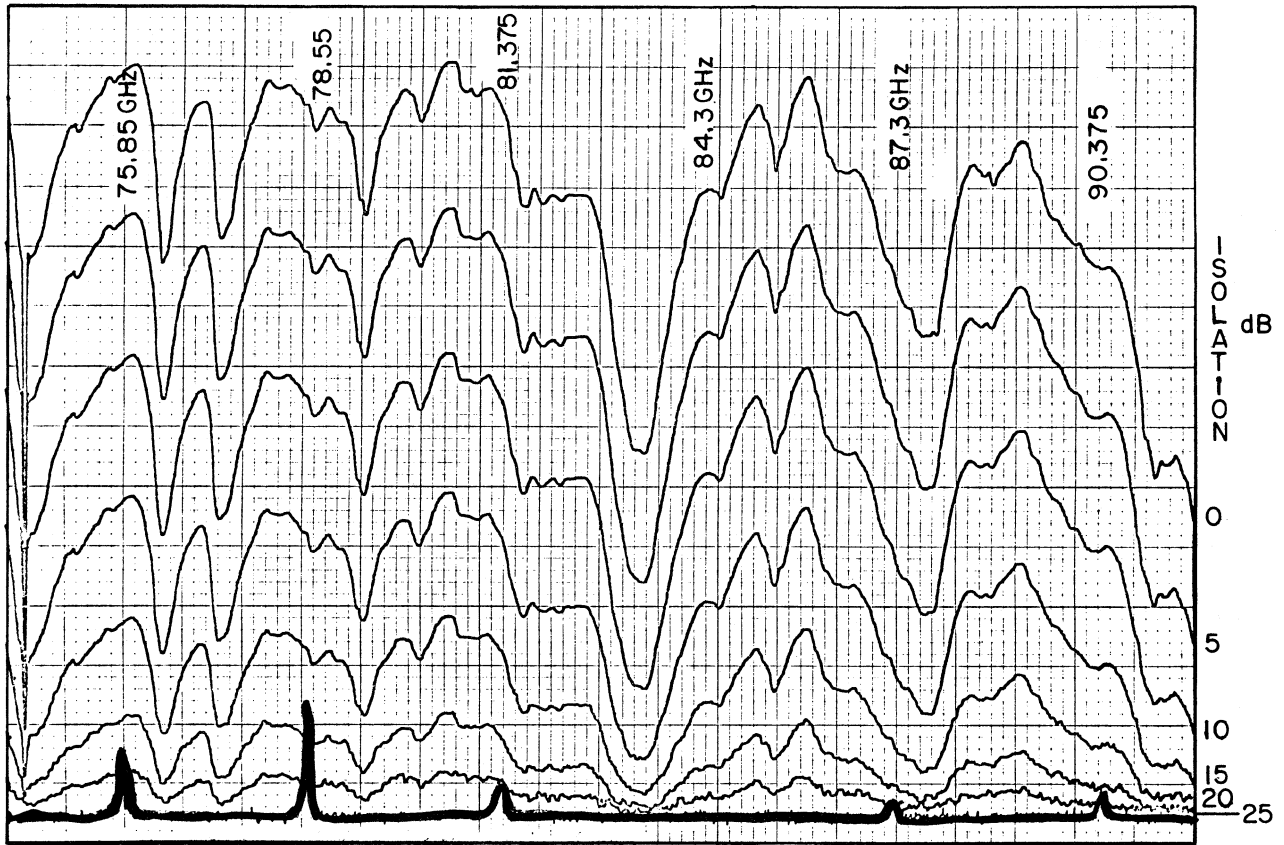




ROOM TEMP. MAY 7, 1975 80-120 COOLED RX. INJECTION FILTER  
 J. DAVIS INSERTION LOSS VS FREQUENCY  
 L.O. TO MIXER PATH OTHER PORTS TERMINATED  
 FIG. 12-A



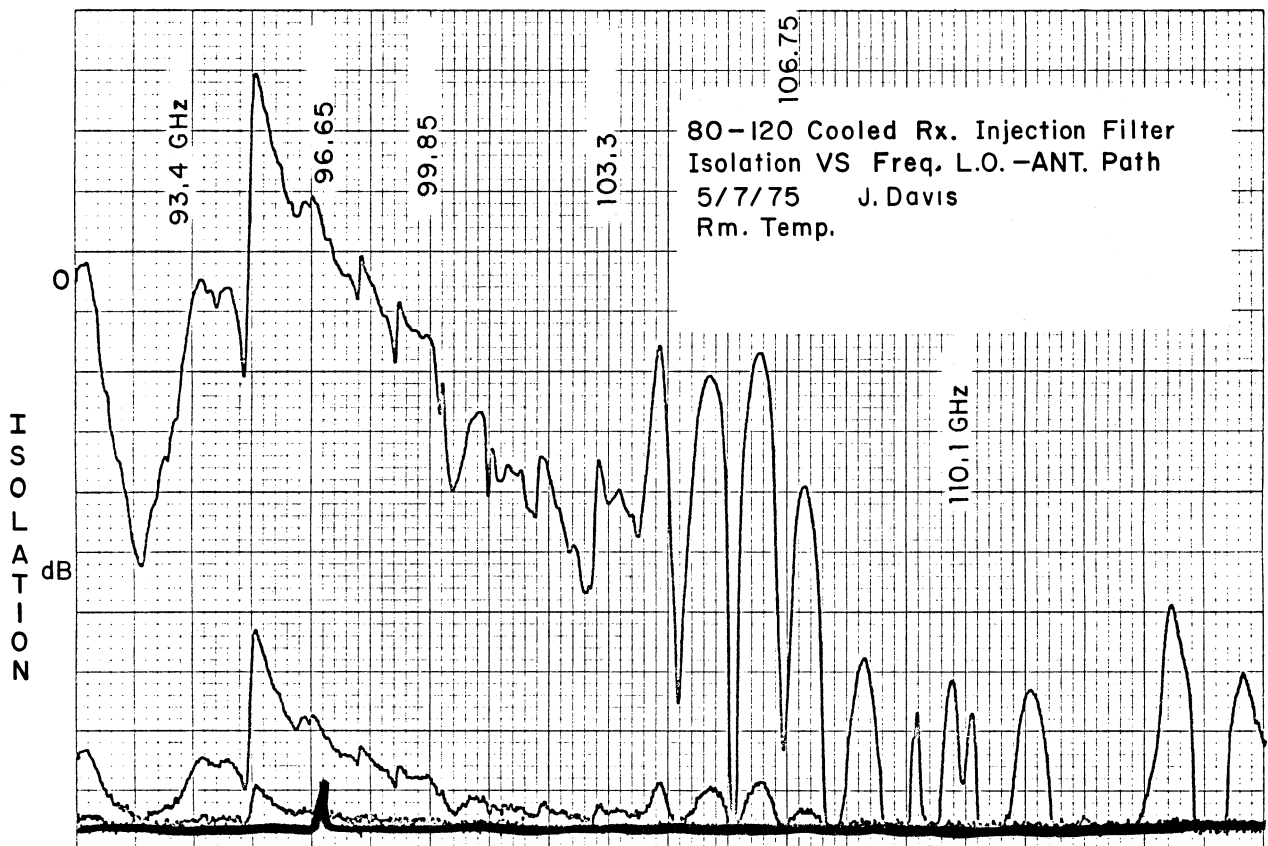
$f \rightarrow$   
 FIG. 12-B



80-120 GHz Cooled Rx. Injection Filter  
L.O. - ANT. Path Isolation VS Freq.

5/7/75 J. Davis  
Rm. Temp.

FIG. 13-A



80-120 Cooled Rx. Injection Filter  
Isolation VS Freq. L.O. - ANT. Path  
5/7/75 J. Davis  
Rm. Temp.

$f \rightarrow$

FIG. 13-B

APPENDIX A

MECHANICAL DRAWINGS:

INJECTION FILTER

APPENDIX A

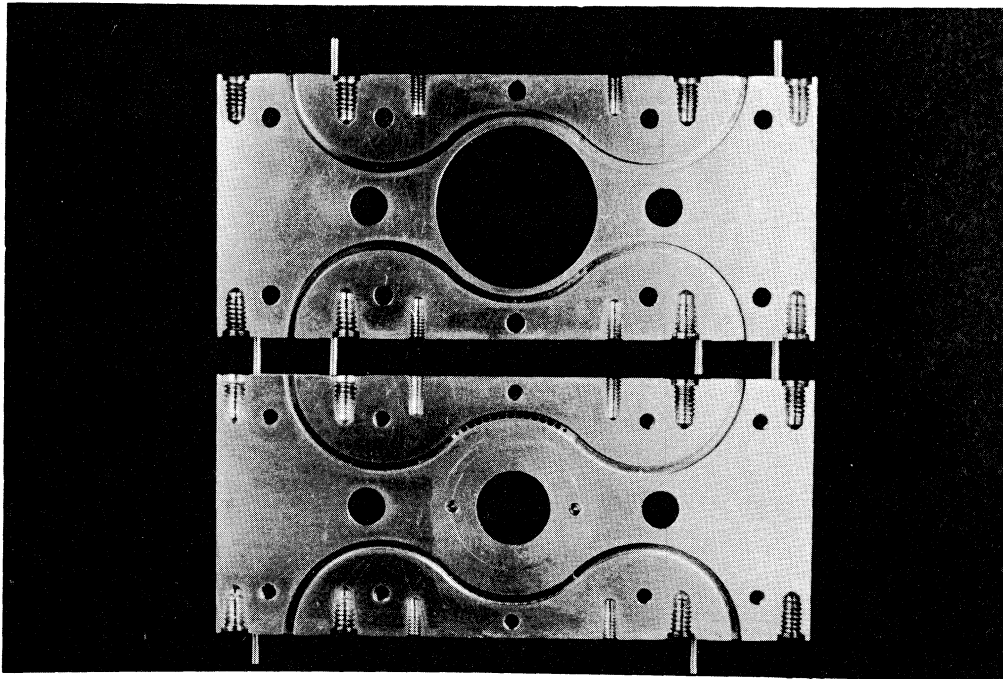


Photo 1: Injection Filter, Internal View.

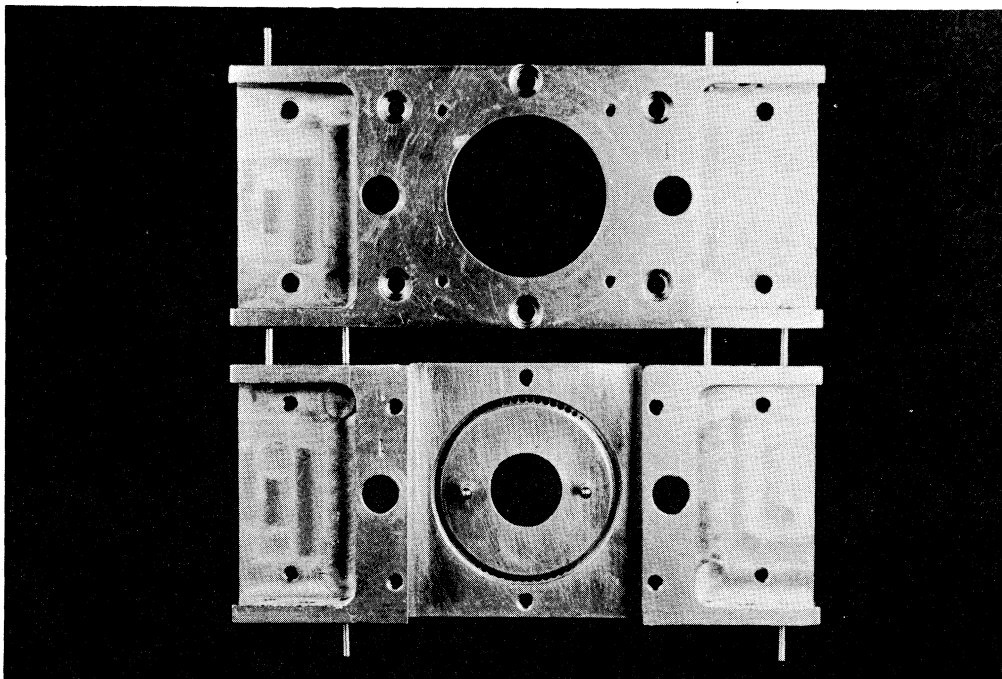


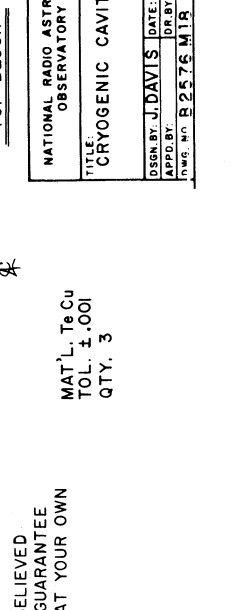
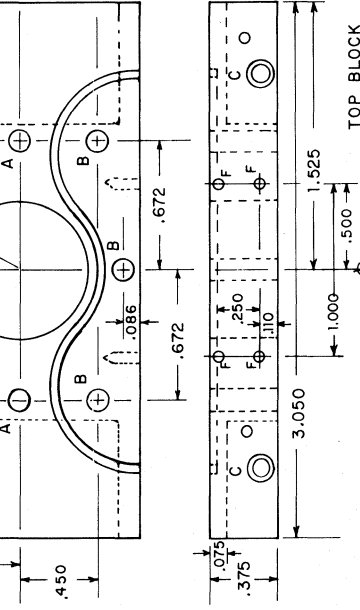
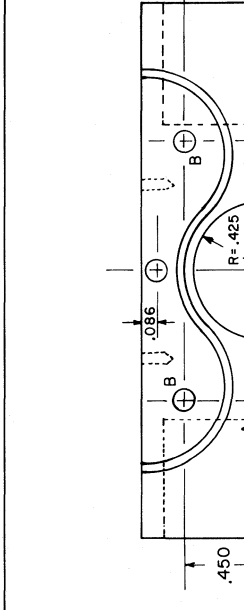
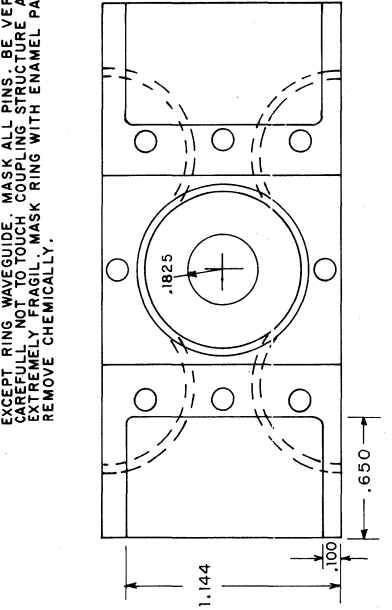
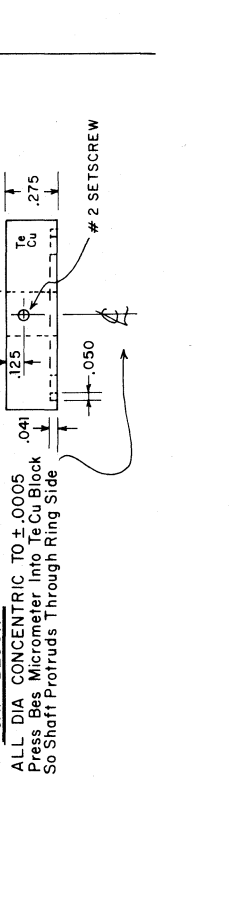
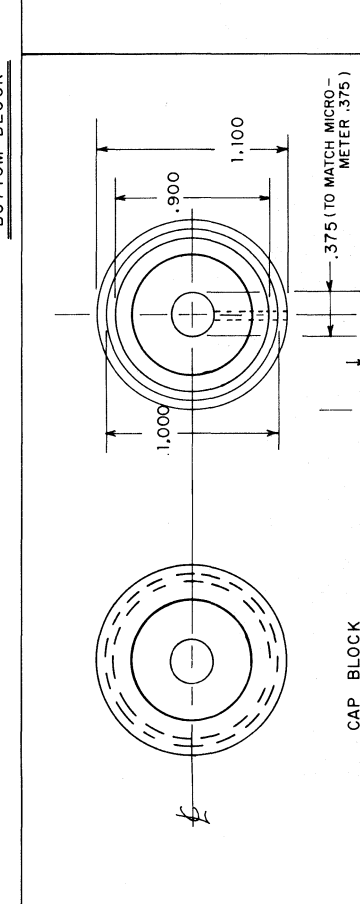
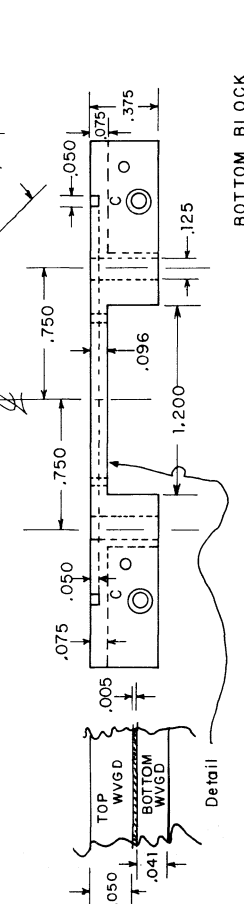
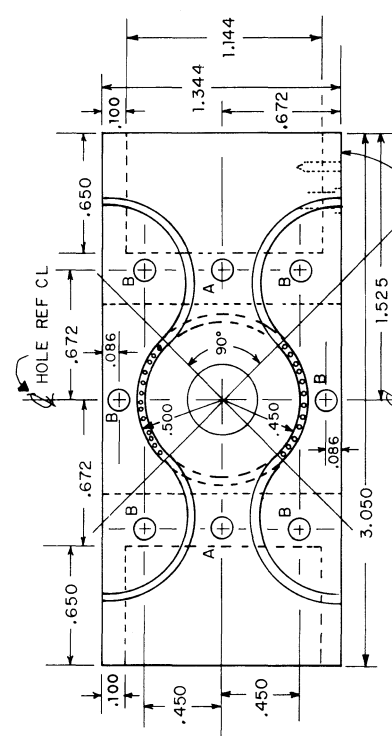
Photo 2: Injection Filter, External View.

**COUPLING HOLES**  
 Spaced Center To Center .042 On WVDG Centerline  
 HOLE DIAMETERS PLACE CENTER HOLE ON CENTERLINE  
 DO NOT BREAK THRU BETWEEN HOLES.  
 SEE ATTACHED SHEET FOR HOLE SIZES

**NOTES:**  
 A: 1/8" ST. ALIGN PIN 2PLCS.  
 TO WVDG TO ±.001  
 B: #2 CLEAR IN TOP HALF THREADED IN BOTTOM 6PLCS.  
 C: STANDARD WVDG FLANGE 4PLCS.  
 D: ALL WAVEGUIDES INNER RADIUS .450; OUTER RADIUS .500  
 CENTERS OF CURVATURE LOCATED AT BLOCK CENTER &  
 .672 X AND Y MOVEMENT FROM CENTER.  
 E: #2-.56 X .150 DEEP 4 EACH SIDE (ON BOTH SIDES, 8 TOTAL)  
 TO PROTECT FROM CORROSION, PLATE ALL SURFACES  
 EXCEPT RING WAVEGUIDE MASK ALL PINS. BE VERY  
 CAREFULLY NOT TO TOUCH COUPLING STRUCTURE AS IT IS  
 EXTREMELY FRAGILE. MASK RING WITH ENAMEL PAINT.  
 REMOVE CHEMICALLY.

**COUPLING HOLES**  
 Spaced Center To Center .042 On WVDG Centerline  
 HOLE DIAMETERS PLACE CENTER HOLE ON CENTERLINE  
 DO NOT BREAK THRU BETWEEN HOLES.  
 SEE ATTACHED SHEET FOR HOLE SIZES

5.06° BETWEEN CENTERS ON ROLAY TABLE  
 5° 3' 36"



**WARNING**  
 THIS DRAWING IS BELIEVED  
 ACCURATE. I DO NOT GUARANTEE  
 THIS. CONSTRUCT IT AT YOUR OWN  
 RISK.

MAT. L. TeCu  
 TOL. ±.001  
 QTY. 3

NATIONAL RADIO ASTRONOMY OBSERVATORY
TITLE CRYOGENIC CAVITY
DATE: 5/25/73
DR. BY: J. DAVIS
APP. BY: [Signature]
FIG. NO. B 2576 M1A

## APPENDIX B

MECHANICAL DRAWINGS:

DIFFERENTIAL SCREW AND GEAR BOX

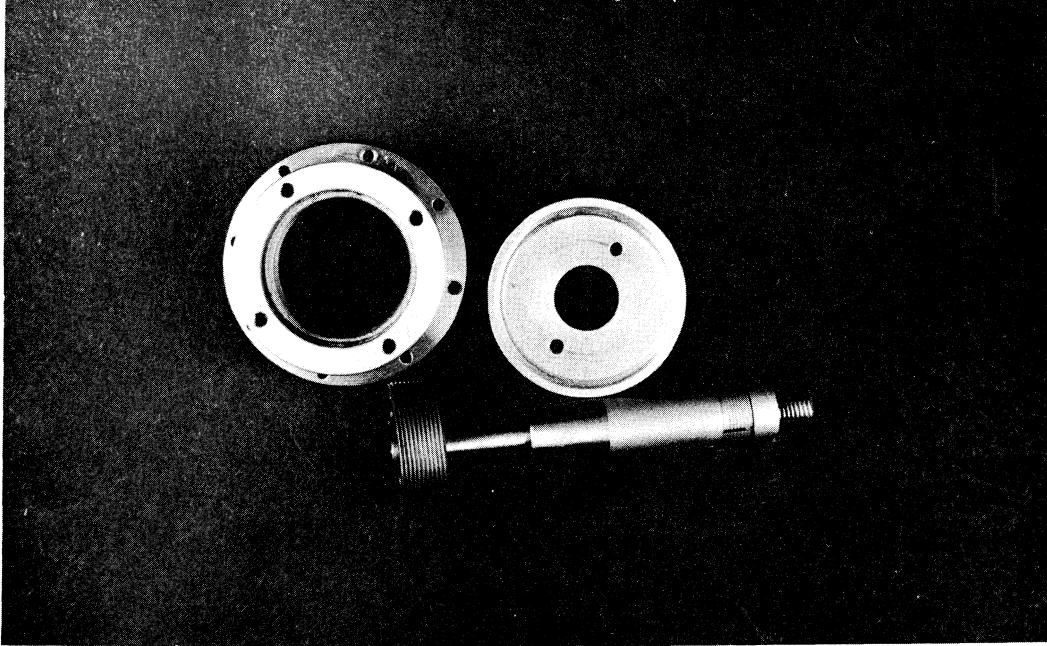


Photo 1: Differential Screw and Cap Block.

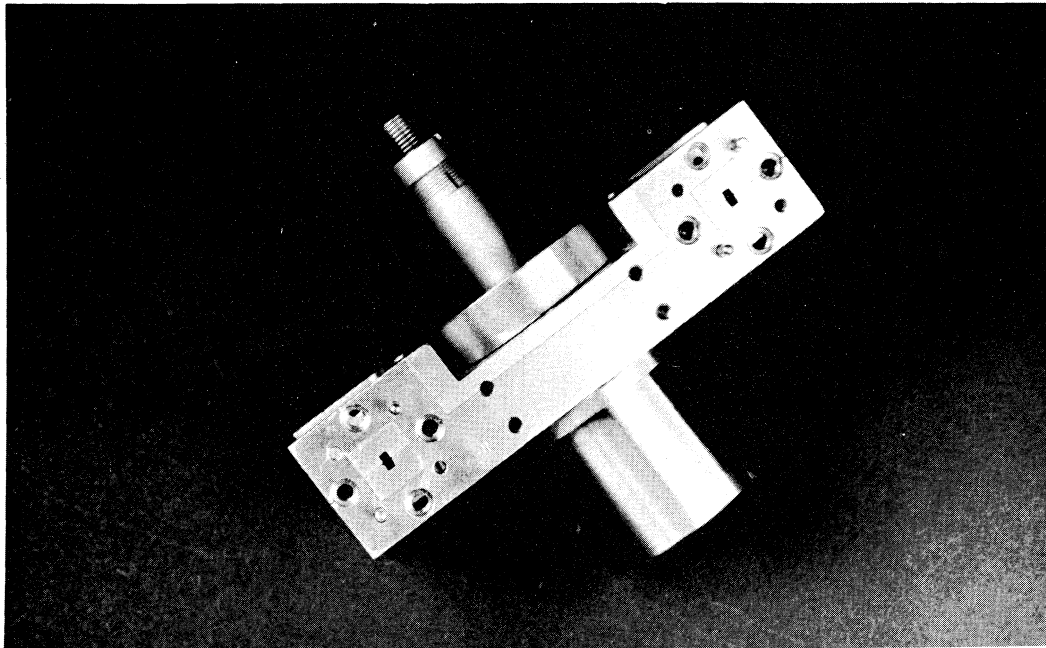
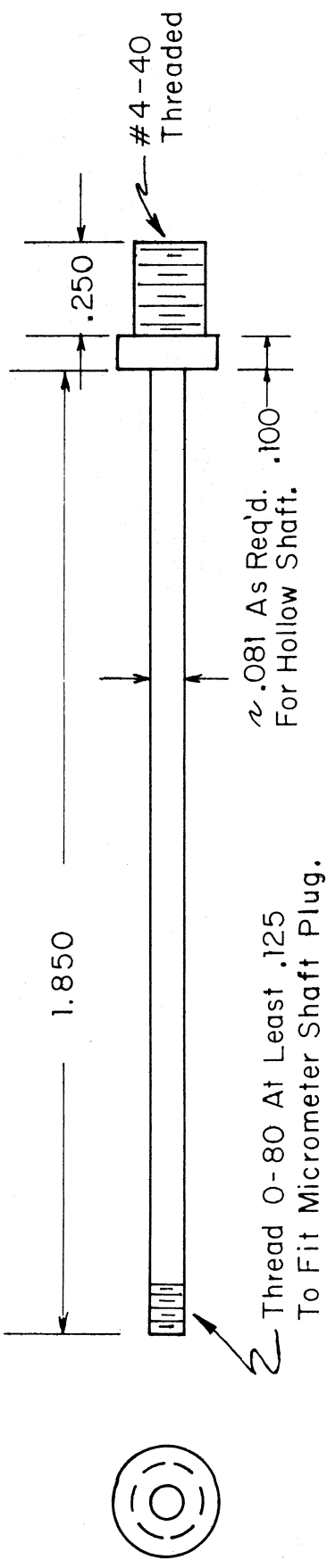
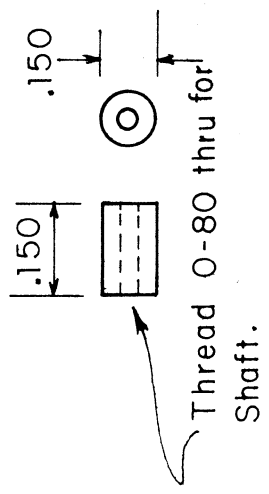


Photo 2: Cavity with Differential Screw and Cap Block Attached.



Make from AL.    SENSOR ROD    2 REQ'D.



Make from Brass  
Press into micrometer shaft.

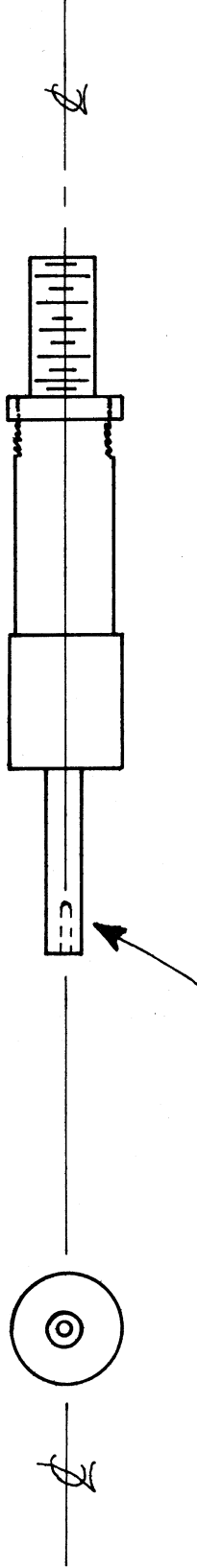
THREADED PLUG    2 REQ'D.

<b>NATIONAL RADIO ASTRONOMY OBSERVATORY</b>	
TITLE: CRYOGENIC CAVITY 80-120 GHz SENSOR BITS & PIECES	
DSGN. BY: J. DAVIS	DATE: 7-30-73
APPD. BY:	DR. BY:
DWG. NO. A 2576 M 25-1	



Turn Down First .300 of barrel  
until concentric with shaft  $\pm .0005$

Make cap block to match - OR - turn  
down sufficiently to press brass sleeve  
on barrel and then turn brass sleeve to  
 $\downarrow .375$  dia.  $\pm .0005$



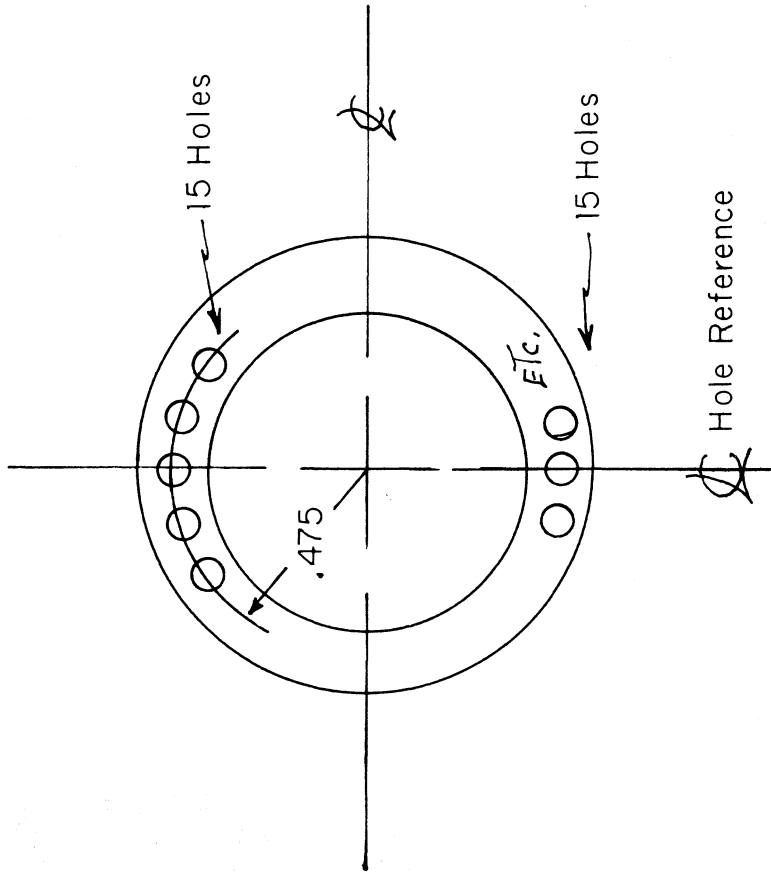
Soften micrometer shaft

Tap and thread end #0-80 x .100 DP to  
receive sensor rod. concentric  $\pm .001$

Centerline is defined to be centerline of rotation of .195 dia. micrometer shaft.  
All rotating parts of differential screw refer to this  $\phi$ .

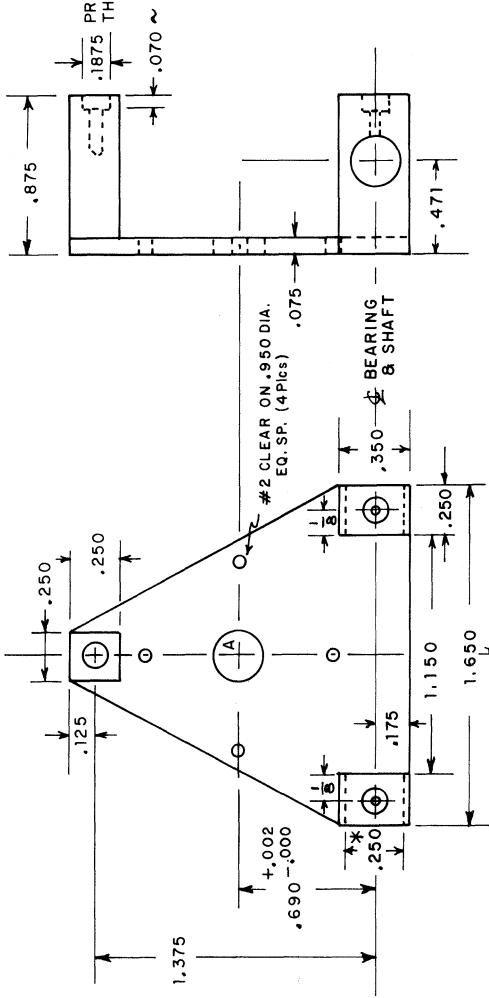
NATIONAL RADIO ASTRONOMY OBSERVATORY	
TITLE: Modifications to Micrometer for 80-120 Cooled Injection Filter	
DSGN. BY: J. DAVIS	DATE: 7/75
APPD. BY:	DR. BY:
DWG. NO. A2576 M25-2	

Coupling holes: Set up Rotary table so the Zero Degree - 180° line is colinear with Reference  $\phi$  (See dwg. no. B2576M18 Hole Ref.  $\phi$  ) then place holes on .475 radius about Block  $\phi$ 's according to following table.  
 Rotate block 180° and Repeat

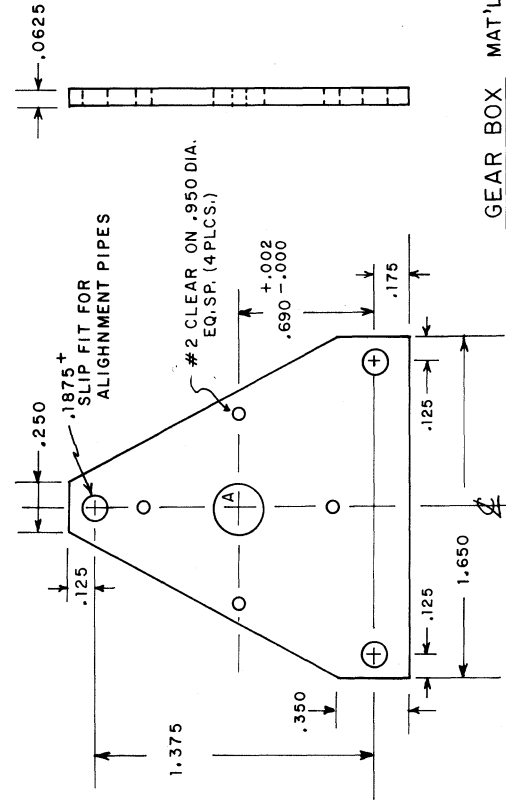


ANGLE	DIAMETER
146° 55' 0"	.018
151 59 0	.028
159 3 0	.034
162 7 0	.036
167 11 0	.036
172 15 0	.036
177 19 0	.036
182 23 0	.036
187 27 0	.036
192 31 0	.036
197 35 0	.036
202 39 0	.036
207 43 0	.034
212 47 0	.028
217 51 0	.018

NATIONAL RADIO ASTRONOMY OBSERVATORY	
TITLE: COUPLER DETAIL	
DSGN. BY: J. DAVIS	DATE: 7/75
APPD. BY:	DR. BY:
DWG. NO. A2576 M 25-3	

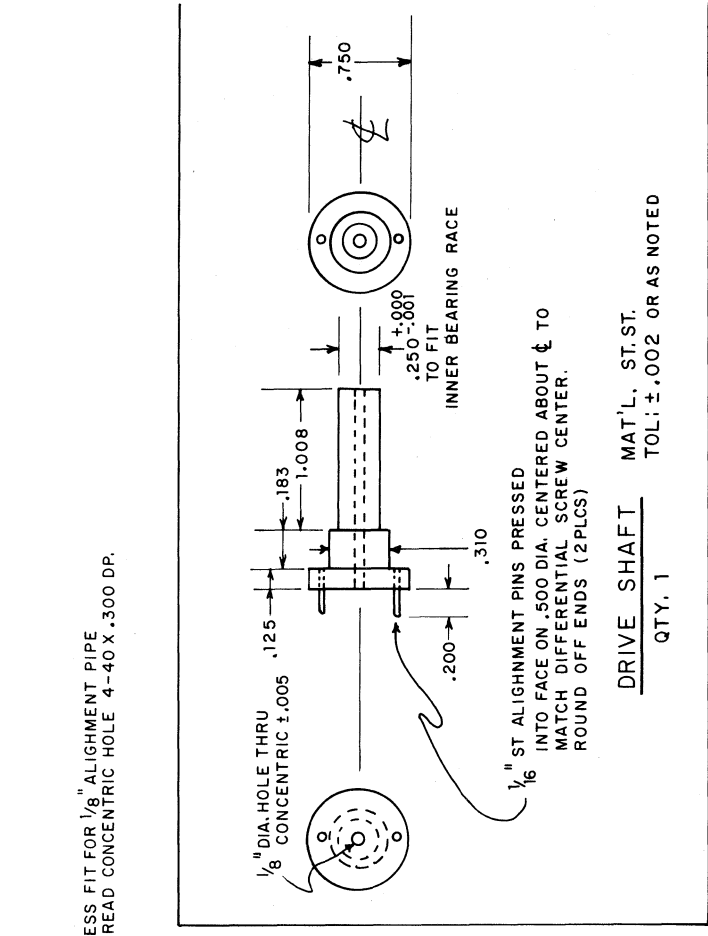


\* PRESS FIT FOR BEARING



NOTE A: BORE TO .3750  
PRESS FIT FOR E2-8 BALL BEARINGS.

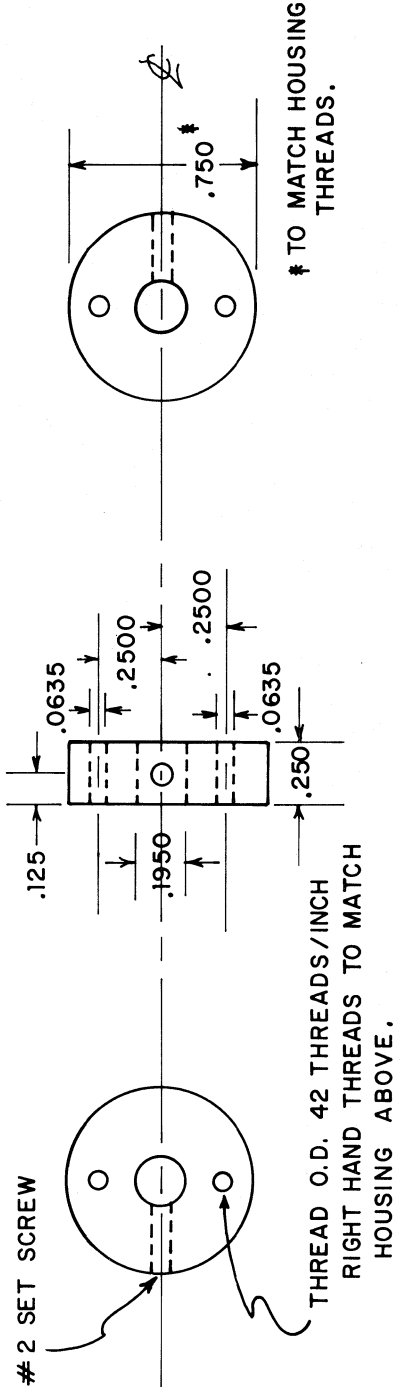
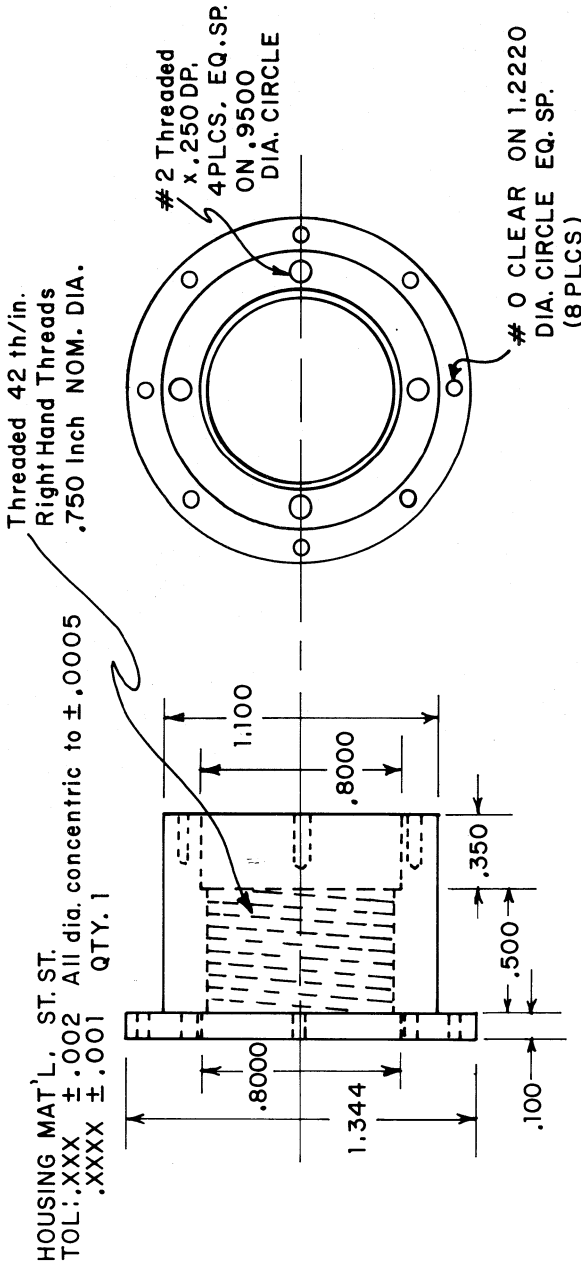
GEAR BOX MAT'L. AL  
QTY. 3 TOL: ±.002  
OR AS NOTED



1/16" ST ALIGNMENT PINS PRESSED INTO FACE ON .500 DIA. CENTERED ABOUT  $\phi$  TO MATCH DIFFERENTIAL SCREW CENTER. ROUND OFF ENDS (2PLCS)

DRIVE SHAFT MAT'L. ST. ST.  
QTY. 1 TOL: ±.002 OR AS NOTED

NATIONAL RADIO ASTRONOMY OBSERVATORY	
TITLE: GEAR BOX & DRIVE SHAFT	
DSGN. BY: J. DAVIS	DATE: 9/11/73
APPD. BY:	DR. BY:
DWG. NO. B2576 M19	



KEEP PLUG AND HOUSING AXIS CONCENTRIC TO WITHIN ±.001 OR ±.002 IF POSSIBLE  
 THREADS SHOULD BE A SMOOTH EASY TURNING FIT  
 AXIAL SLOP < .001

DIFFERENTIAL SCREW CENTER QTY. 1  
 MAT'L. BRASS  
 TOL.: .XXX ±.002 ALL DIA. CONCENTRIC  
 .XXXX ±.001 TO ±.0005

NATIONAL RADIO ASTRONOMY OBSERVATORY	
TITLE: SCREW DRIVE & CAP BLOCK	
DSGN. BY: J. DAVIS	DATE: 5/31/73
APPD. BY:	DR. BY:
DWG. NO. B2576 M20	

APPENDIX C

MECHANICAL DRAWINGS:

PRECISION MOVER

APPENDIX C

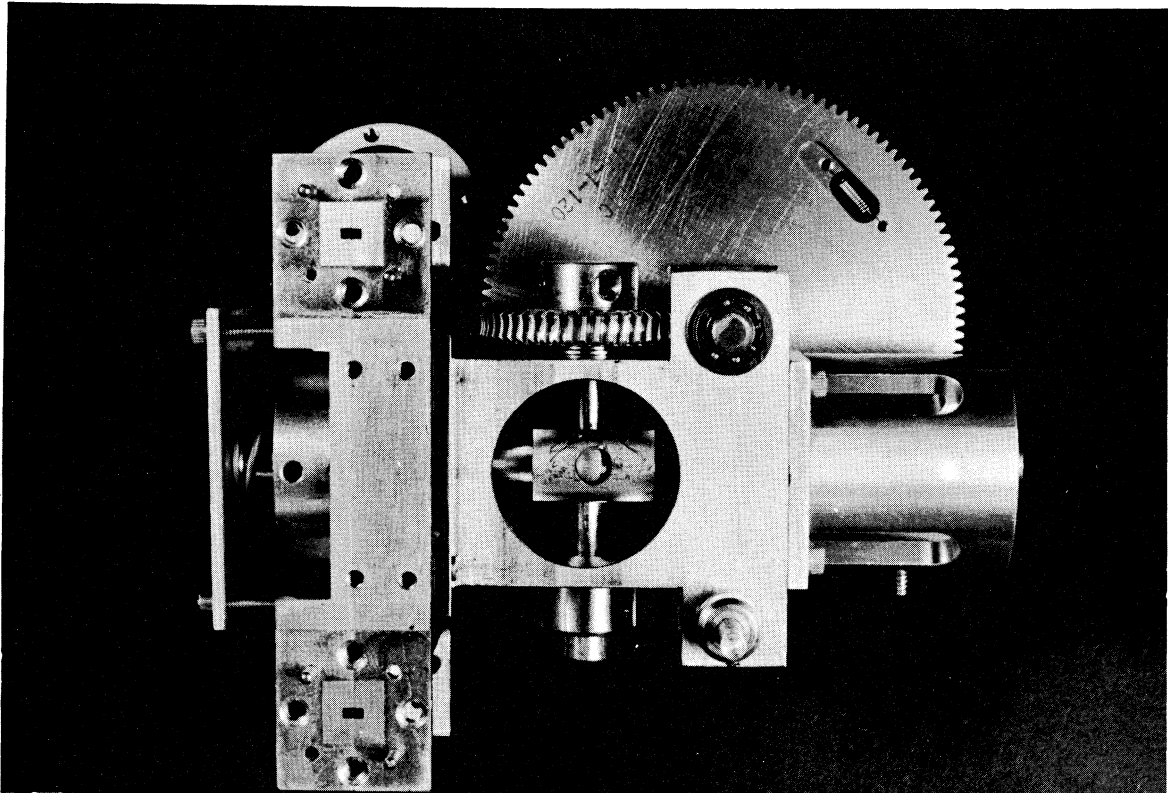
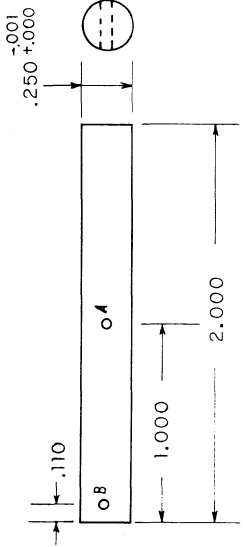
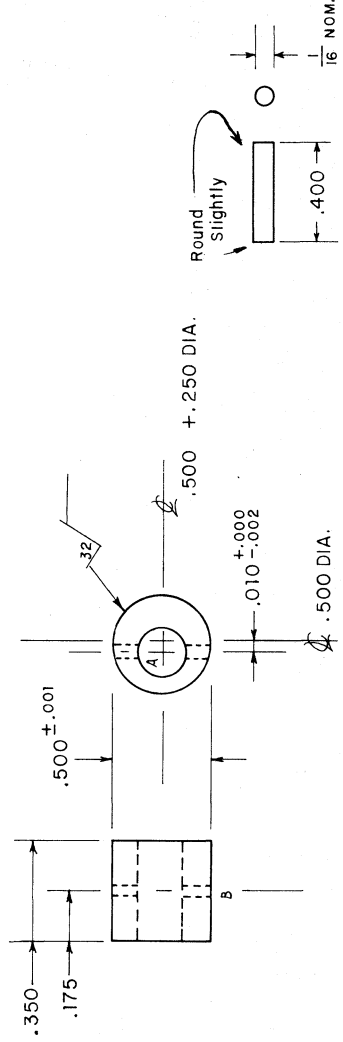


Photo 1: Cavity with Precision Mover Attached.



NOTES  
 A. Slip Fit For 1/16" ST ST PIN ON Dia. To Match CAM.  
 B. Slip Fit For 1/16" ST ST PIN To Match Worm Wheel.

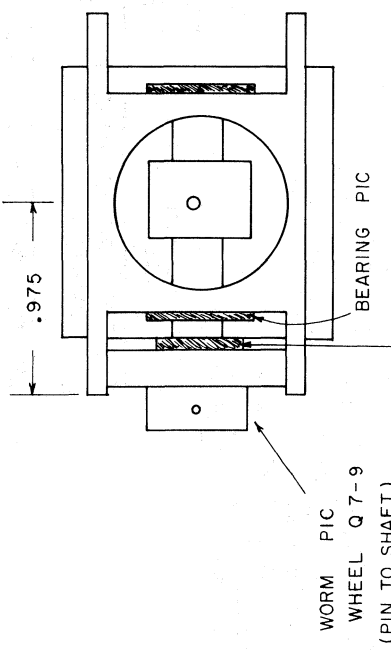
SHAFT MAT. ST. ST.  
 TOL. ±.005



A: Bore .250 Dia. Hole To Be Slip Fit Over .250 Dia. Shaft, Above. Center of Bore is offset by .010 from center of .500 Dia. to form cam. inside surface RMS 32μ INCH OR Better.  
 B: Hole for 1/16" PIN to match shaft above. To be slip fit one side and snug fit other side holes located on diameter of .250 Hole.

NOTE This cam is to give about .016 mils. of travel for yoke rod when shaft is rotated.

CAM MAT. ST. ST.  
 TOL. ±.005



SHIM (Adjust Shim Thickness To Provide For Smooth Running When Engaged With Worm About (.025" Thick) To Run On Inner Race Only.

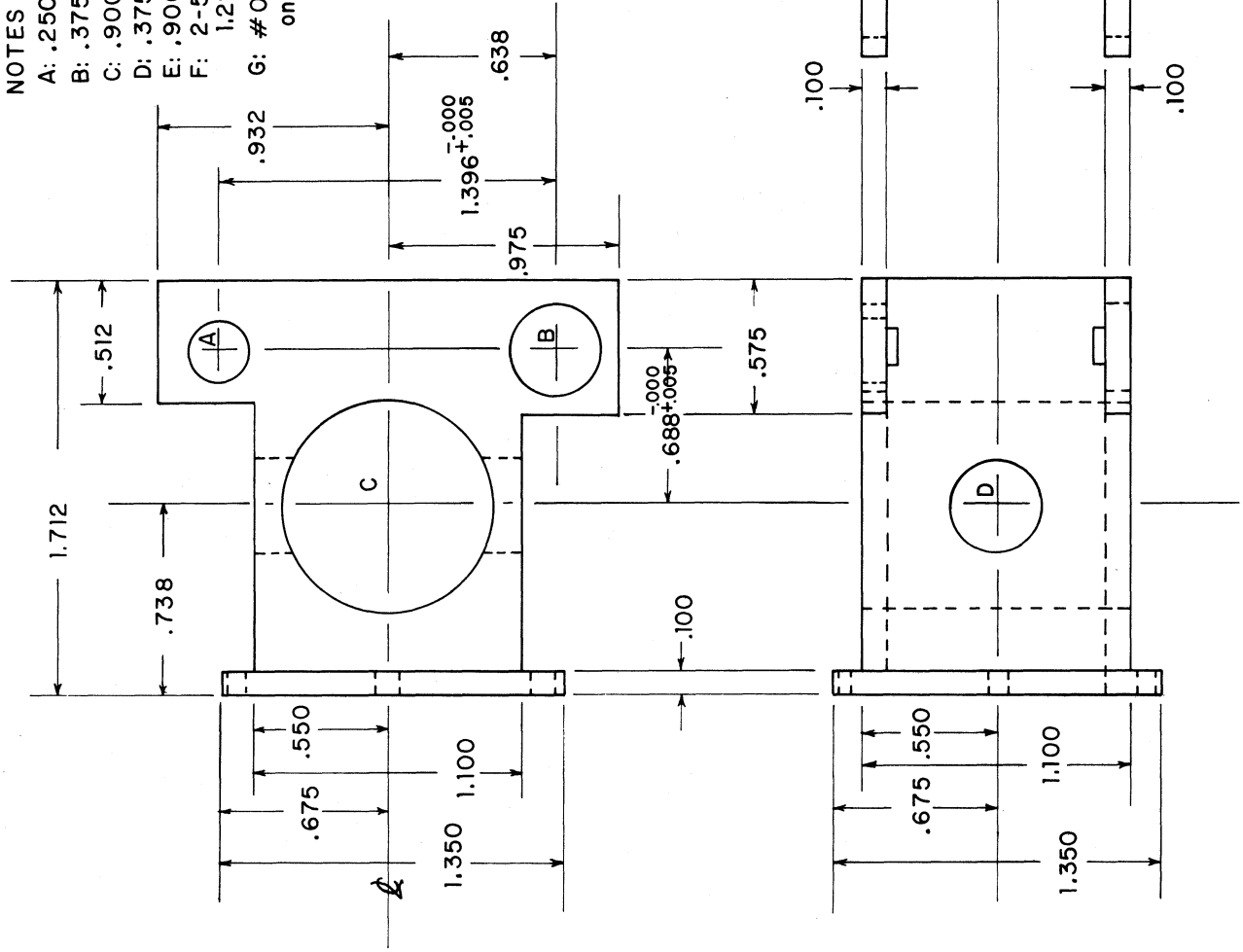
NATIONAL RADIO ASTRONOMY OBSERVATORY	
TITLE: SHAFT / CAM / SHIMS / & ASSY.	
DSGN. BY: J. DAVIS	DATE:
APPD. BY:	DR. BY:
DWG. NO.	

**NOTES**

- A: .250 Dia. to be snug fit for PIC E4-5 Bearing (2 plcs)
- B: .375 Dia. to be snug fit for PIC E2-2 Bearing (2 plcs)
- C: .900 Dia. Thru.
- D: .375 Dia. to be snug fit for PIC E2-8 Bearing (2 plcs)
- E: .900 Dia. Thru.
- F: 2-56 M.S. x .250 min. thread depth eq. spaced on 1.225 Dia. B.C. centered on  $\pm .005$  (4 plcs)
- G: #0 Clear Thru eq. spaced on 1.225 Dia. B.C. centered on  $\pm .005$  (4 plcs)

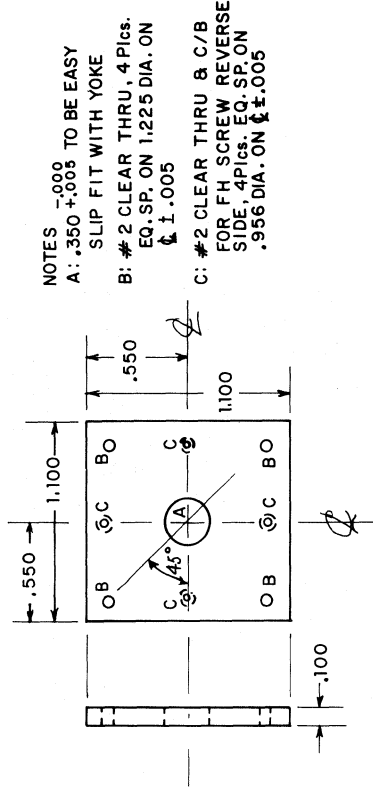
- PIC GEARS G62-14
- " " P16-1-120
- " WORM Q8-2
- " " WHEEL Q7-9
- " SHAFT SPACERS B6-8 & B4-8
- " COLLARS C1-1 & C1-2

MAT'L. ST. ST.  
TOL:  $\pm .005$  or as noted



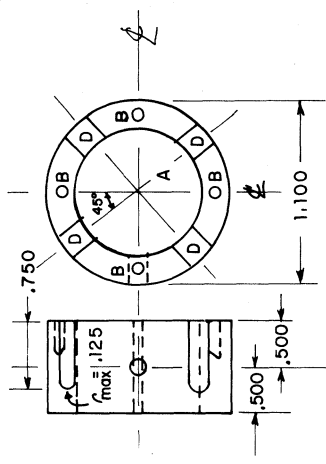
NATIONAL RADIO ASTRONOMY OBSERVATORY	
TITLE: PRECISION MOVER BODY BLOCK	
DSGN. BY: J. DAVIS	DATE:
APPD. BY:	DR. BY:
DWG. NO.	





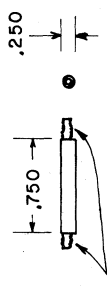
NOTES  
 -0.005  
 A: .350 +.005 TO BE EASY  
 SLIP FIT WITH YOKE  
 B: #2 CLEAR THRU, 4 PICS.  
 EQ. SP. ON 1.225 DIA. ON  
 $\phi \pm .005$   
 C: #2 CLEAR THRU & C/B  
 FOR FH SCREW REVERSE  
 SIDE, 4 PICS. EQ. SP. ON  
 .956 DIA. ON  $\phi \pm .005$

END PLATE  
 MAT'L. AL.  
 TOL:  $\pm .005$   
 QTY. 1



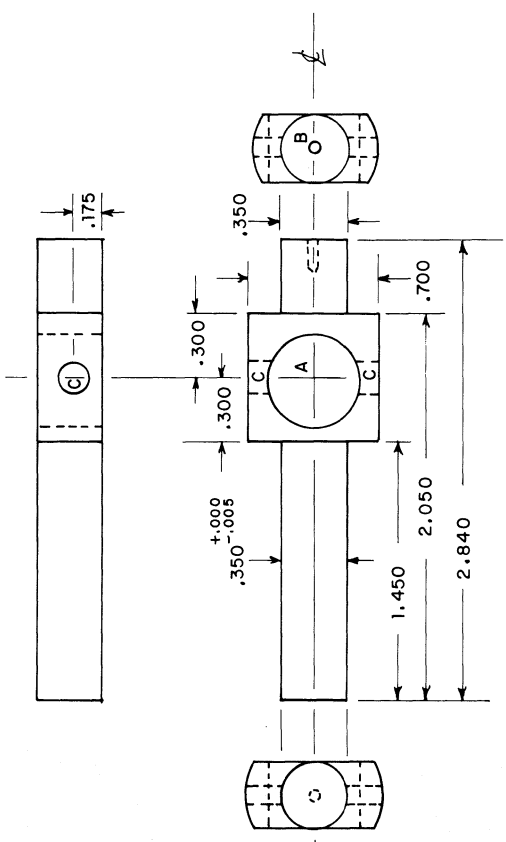
NOTES:  
 A: .820  $\pm .010$  DIA. THRU  
 B: #2-56 x .200 MIN. THREAD DEPTH  
 4 PICS. EQ. SP. ON .956 DIA. ON  $\phi \pm .005$   
 D: .250  $\pm .005$  WIDE SLOT x .750 DEEP  
 4 PICS. EQ. SP. AROUND PART.  
 C: #4-40 THRU ON DIA. THRU  $\phi$ .

SENSOR MTG.  
 MAT'L. AL, QTY. 1  
 TOL:  $\pm .005$



THREAD #6-40 BOTH ENDS  
 MIN. THREAD LENGTH = .200 MAX. .250

SENSOR ROD MAT'L. BRASS OR AL.  
 QTY. 1 TOL:  $\pm .005$



NOTES  
 A: .562  $\pm .005$  DIA. THRU; INSIDE SURFACE TO BE  $32\mu$  IN. RMS OR BETTER,  
 AXIS OF HOLE TO BE  $\perp$  TO AXIS OF .350 DIA. TO WITHIN  $\pm .5$  DEG.  
 B: THREADED 6-40 NF X .250 MIN. THREAD DEPTH,  
 SURFACE OF .350 DIA. TO BE FINISHED TO RMS  $32\mu$  IN. OR BETTER.  
 C: .187 DIA. THRU.

YOKE MAT'L. ST. ST. QTY. 1 TOL:  $\pm .005$   
 OR AS NOTED

NATIONAL RADIO ASTRONOMY OBSERVATORY	
TITLE: YOKE, END PLATE SENSOR MTG. & SENSOR ROD	
DESIGN BY: J. DAVIS	DATE: OCT 11, 76
APP'D BY:	DR. BT.
DWG. NO.	

APPENDIX D

COMPUTER PROGRAMS

```

FORTRAN IV          V01C-03A   MON 18-APR-77 12:54:24          PAGE 001
  C PROGRAM DIRCOUP(INPUT,OUTPUT,HAFWAY,TAPE5=HAFWAY)
0001  DIMENSION FWD(503),REV(503),R(103),T(103),K1(103),K2(103),F(503)
0002  DIMENSION D(20)
0003  COMMON A,B,D,X,TH,LEN,N,ALPHA,K
0004  REAL K1,K2,K,LEN
0005  COMPLEX RO,T12,KR,KH
0006  READ 98,N
0007  READ 100, LEN
0008  READ 100, A
0009  READ 100, B
0010  READ 100,X
0011  READ 100,TH
0012  DO 99 I=1,N
0013    99 READ 100,D(I)
0014    ALPHA = .014576
0015    K = .862
0016  100 FORMAT (F6.4)
0017    98 FORMAT (I2)
0018  DO 200 I = 1,501
0019    F(I) = I
0020    F(I) = 85.E9+10.E7*(F(I)-1.)
0021    CALL FILTER(F(I),FWD(I),REV(I))
0022  200 F(I) = F(I)/1.E9
0023    CALL SIMPLOT(F,FWD,501,0,'FREQ  GHZ','FWD  DB',5,0)
0024    CALL SIMPLOT(F,REV,501,0,'FREQ  GHZ','REV  DB',5,0)
0025    CALL ENDPLOT
0026  END

```

C\*DECK,FILT

```

0001     SUBROUTINE FILTER(F,FWD,REV)
      C
      C
      C     COMPUTES FOWARD (MIXER) AND REVERSE (ANTENNA) COUPLED POWER FOR
      C     RESONANT RING INJECTION FILTER VS FREQUENCY.
      C     F = FREQUENCY IN GHZ
      C     FWD = FOWARD COUPLED POWER (DB)
      C     REV = REVERSE COUPLED POWER (DB)
      C
      C     ***NOTE - COMMON STATEMENT MUST APPEAR IN CALLING ROUTINE
      C
      C
      C
      C
0002     COMMON A,B,D,X,TH,LEN,N,ALPHA,K
0003     DIMENSION D(20)
0004     DIMENSION T(2,2),R(2,2),TRM(2,2),TRIP(2,2),BFM(4,4),L(4),M(4)
0005     REAL LEN,LAM,LAMG,K
0006     COMPLEX RO,T12,T,R,TRM,TRIP,BFM,R11,R12,R21,R22, L,M,R5,R6
0007     COMPLEX KR,KH,DET
0008     CALL CPLR(F,RO,T12,KR,KH)
0009     LAM = 1.18E10/F
0010     LAMG = LAM/SQRT(1.-LAM**2/4./A**2)
0011     BETA = 6.283185/LAMG
0012     T(1,1) = CEXP(K*CMPLX(ALPHA,BETA))
0013     T(2,2) = 1./T(1,1)
0014     T(1,2) = 0.
0015     T(2,1) = T(1,2)
0016     R(1,1) = 1./T12
0017     R(1,2) = -RO/T12
0018     R(2,1) = RO/T12
0019     R(2,2) = (T12*T12-RO*RO)/T12
0020     CALL CMPY(R,T,TRM,2,2,2)
0021     CALL CMPY(T,TRM,TRIP,2,2,2)
0022     R11 = TRIP(1,2)
0023     R12 = TRIP(1,1)
0024     R21 = TRIP(2,2)
0025     R22 = TRIP(2,1)
0026     DO 101 I1= 1,4
0027     DO 101 J = 1,4
0028     101 BFM(I1,J)= (0.,0.)
0029     BFM(3,2) = (-1.,0.)
0030     BFM(4,1) = (-1.,0.)
0031     BFM(1,3) = RO/R12+R22*T12/R12
0032     BFM(1,4) = T12*(R21-R22*R11/R12)-RO*R11/R12-(1.,0.)
0033     BFM(2,3) = RO*R22/R12+T12/R12-(1.,0.)
0034     BFM(2,4) = RO*(R21-R22*R11/R12)-T12*R11/R12
0035     BFM(3,3) = KR*R22/R12-KH/R12
0036     BFM(3,4) = KR*(R21-R22*R11/R12)-KH*R11/R12
0037     BFM(4,3) = KH*R22/R12+KR/R12
0038     BFM(4,4) = KH*(R21-R22*R11/R12)-R11*KR/R12
0039     CALL CMINV(BFM,4,DET,L,M)
0040     L(1) = -KR

```

```
FORTRAN IV      V01C-03A   MON 18-APR-77 12:54:27
0041      L(2) = -KH
0042      L(3) = -T12
0043      L(4) = -RO
0044      CALL CMPY(BFM,L,M,4,4,1)
0045      M(3) = M(3)/T(1,1)
0046      M(4) = M(4)/T(1,1)
0047      R5 = M(4)*KH+M(3)*KR
0048      R6 = M(4)*KR+M(3)*KH
0049      REV = 10.*ALOG10(CABS(R5)**2)
0050      FWD = 10.*ALOG10(CABS(R6)**2)
0051      RETURN
0052      END
```

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C\*DECK,CCOSH

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C

COMPLEX HYPERBOLIC COSINE  
X = COMPLEX ARGUMENT  
\*\*\*NOTE - CCOSH MUST APPEAR IN COMPLEX STATEMENT IN CALLING PRGM

```
0001      COMPLEX FUNCTION CCOSH(X)
0002      COMPLEX X,A,B
0003      A = CEXP(X)
0004      B=1./A
0005      CCOSH =(A+B)/(2.,0.)
0006      RETURN
0007      END
```

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C*DECK,CPLR
0001 SUBROUTINE CPLR(F,RO,T12,KR,KH)
C
C
C F = FREQUENCY OF COUPLER EVALUATION
C RO = REFLECTION COEFFICIENT AT ANY PORT OF COUPLER (OTHER PORTS
C TERMINATED IN THE CHARACTERISTIC IMPEDANCE OF THE LINE)
C T12 = THROUGH ARM TRANSFER COEFFICIENT
C KR = REVERSE COUPLING COEFFICIENT
C KH = FOWARD COUPLING COEFFICIENT
C NOTE - RO, T12, KR, KH ARE COMPLEX
C A = WAVEGUIDE WIDTH
C B = WAVEGUIDE HEIGHT
C D = DIAMETER OF COUPLING HOLE
C X = DISTANCE FROM EDGE OF GUIDE TO CENTER OF COUPLING HOLE
C TH = THICKNESS OF WALL CONTAINING COUPLING HOLE
C LEN = DISTANCE BETWEEN COUPLING HOLES
C N = NUMBER OF COUPLING HOLES PER COUPLER
C ALPHA = ATTENUATION CONSTANT OF WAVEGUIDE (NEPERS)
C
C
C
C
0002 COMMON A,B,D,X,TH,LEN,N,ALPHA,A2
0003 COMPLEX T12,RO,KR,KH,ML,GAMMA,CSINH,CCOSH
0004 COMPLEX M1,M2,M3,MT,GAMMAO,GAMMAE,TO,TE,M,ME,MO
0005 DIMENSION D(20),M1(2,2),M2(2,2),M(2,2),MT(2,2)
0006 DIMENSION ME(2,2),MO(2,2),BZ(20),XX(20),BY(20),ML(2,2)
0007 REAL LAM,LAMG,LEN,K1,K2
0008 LAM = 1.18E10/F
0009 LAMG = LAM/(SQRT(1.-LAM**2/4./A**2))
0010 DO 102 K = 1,N
0011 FOE = 1.18E10/1.305/D(K)
0012 FOH = 1.18E10/1.705/D(K)
0013 A2 = (.065*D(K)+TH)/TH
0014 K1 = TAN(3.14159*F/2./FOH)/3.14159/F*2.*FOH
1*EXP(-6.28319*TH*A2/1.705/D(K)*SQRT(1.-F**2/FOH**2))
0015 K2 = TAN(3.14159*F/2./FOE)/3.14159/F*2.*FOE
1*EXP(-6.28319*TH*A2/1.305/D(K)*SQRT(1.-F**2/FOE**2))
0016 BY(K)=12.56637*LAMG*D(K)**3*(SIN(3.14159*X/A))**2/A/B/LAM**2/12.
0017 XX(K)=12.56637*D(K)**3*(SIN(3.14159*X/A))**2/A/B/LAMG/6.
0018 BZ(K)=-3.14159*LAMG*D(K)**3*(COS(3.14159*X/A))**2/A**3/B/6.
0019 BY(K) = BY(K)*K2
0020 XX(K) = XX(K)*K1
0021 102 BZ(K) = BZ(K)*K1
0022 M1(1,1) = (1.,0.)
0023 M1(1,2) = (0.,0.)
0024 M1(2,1) = (0.,0.)
0025 M1(2,2) = (1.,0.)
0026 M2(1,1) = (1.,0.)
0027 M2(1,2) = (0.,0.)
0028 M2(2,1) = (0.,0.)
0029 M2(2,2) = (1.,0.)
0030 GAMMA = CMPLX(ALPHA,6.283185/LAMG)

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0031 ML(1,1) = CCOSH(GAMMA*LEN/2.)
0032 ML(1,2) = CSINH(GAMMA*LEN/2.)
0033 ML(2,1) = ML(1,2)
0034 ML(2,2) = ML(1,1)
0035 DO 103 K = 1,N
0036 ME(1,1) = (1.,0.)
0037 ME(1,2) = CMPLX(0.,2.*XX(K))
0038 ME(2,1) = (0.,0.)
0039 ME(2,2) = (1.,0.)
0040 MO(1,1) = (1.,0.)
0041 MO(1,2) = (0.,0.)
0042 MO(2,1) = CMPLX(0.,2.*(BZ(K)+BY(K)))
0043 MO(2,2) = (1.,0.)
0044 CALL CMPY(ML,ME,M,2,2,2)
0045 CALL CMPY(M,ML,ME,2,2,2)
0046 CALL CMPY(ML,MO,MT,2,2,2)
0047 CALL CMPY(MT,ML,MO,2,2,2)
0048 CALL CMPY(M1,ME,M,2,2,2)
0049 CALL CMPY(M2,MO,MT,2,2,2)
0050 DO 104 I1 = 1,2
0051 DO 105 I2 = 1,2
0052 M1(I1,I2) = M(I1,I2)
0053 105 M2(I1,I2) = MT(I1,I2)
0054 104 CONTINUE
0055 103 CONTINUE
0056 DO 106 I1 = 1,2
0057 DO 107 I2 = 1,2
0058 ME(I1,I2) = M(I1,I2)
0059 107 MO(I1,I2) = MT(I1,I2)
0060 106 CONTINUE
0061 GAMMAO = (MO(1,1)-MO(2,2)+MO(1,2)-MO(2,1))
1/(MO(1,1)+MO(1,2)+MO(2,1)+MO(2,2))
0062 GAMMAE = (ME(1,1)-ME(2,2)+ME(1,2)-ME(2,1))
1/(ME(1,1)+ME(1,2)+ME(2,1)+ME(2,2))
0063 TO = (2.,0.)/(MO(1,1)+MO(1,2)+MO(2,1)+MO(2,2))
0064 TE = (2.,0.)/(ME(1,1)+ME(1,2)+ME(2,1)+ME(2,2))
0065 KH = (TE-TO)/(2.,0.)
0066 KR = (GAMMAE-GAMMAO)/(2.,0.)
0067 RO = (GAMMAE+GAMMAO)/(2.,0.)
0068 T12 = (TO+TE)/(2.,0.)
0069 RETURN
0070 END

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```
C*DECK,CMPY
0001      SUBROUTINE CMPY(A,B,C,I,J,K)
C          MULTIPLIES COMPLEX MATRICES      A*B = C
C          I = NUMBER OF ROWS IN A
C          J = NUMBER OF COLUMNS IN A
C          K = NUMBER OF COLUMNS IN B
0002      COMPLEX A,B,C
0003      DIMENSION A(I,J),B(J,K),C(I,K)
0004      DO 101 L=1,I
0005          DO 100 M = 1,K
0006      100 C(L,M) = (0.,0.)
0007      101 CONTINUE
0008          DO 104 L = 1,I
0009          DO 103 M = 1,K
0010          DO 102 N = 1,J
0011      102 C(L,M) = C(L,M)+A(L,N)*B(N,M)
0012      103 CONTINUE
0013      104 CONTINUE
0014          RETURN
0015      END
```

```

C*DECK,CMINV
0001      SUBROUTINE CMINV(A,N,D,L,M)
C
C
C      INVERTS A COMPLEX MATRIX
C      A = N X N COMPLEX MATRIX TO BE INVERTED.  MATRIX A IS WRITTEN
C      OVER WITH THE INVERSE OF A.
C      N = ORDER OF MATRIX A
C      D = DETERMINANT OF MATRIX A (RETURNED.  MAY BE TESTED FOR
C      SINGULARITY.)
C      L = WORK MATRIX OF LENGTH N
C      M = WORK MATRIX OF LENGTH N
C      ***ALL QUANTITIES EXCEPT N ARE COMPLEX, N IS INTEGER***
C
C
C
0002      DIMENSION A(1),L(1),M(1)
0003      COMPLEX A,BIGA,HOLD,D
0004      D = (1.,0.)
0005      NK = -N
0006      DO 80 K = 1,N
0007      NK = NK + N
0008      L(K) = K
0009      M(K) = K
0010      KK = NK + K
0011      BIGA = A(KK)
0012      DO 20 J = K,N
0013      IZ = N*(J-1)
0014      DO 20 I = K,N
0015      IJ = IZ + I
0016      10 IF(CABS(BIGA)-CABS(A(IJ))) 15,20,20
0017      15 BIGA = A(IJ)
0018      L(K) = I
0019      M(K) = J
0020      20 CONTINUE
0021      J = L(K)
0022      IF(J-K) 35,35,25
0023      25 KI = K-N
0024      DO 30 I = 1,N
0025      KI = KI+N
0026      HOLD = -A(KI)
0027      JI = KI-K+J
0028      A(KI) = A(JI)
0029      30 A(JI) = HOLD
0030      35 I = M(K)
0031      IF(I-K) 45,45,38
0032      38 JP = N*(I-1)
0033      DO 40 J = 1,N
0034      JK = NK + J
0035      JI = JP + J
0036      HOLD = -A(JK)
0037      A(JK) = A(JI)
0038      40 A(JI) = HOLD

```

```
0039 45 IF(CABS(BIGA)) 48,46,48
0040 46 D = (0.,0.)
0041 RETURN
0042 48 DO 55 I = 1,N
0043 IF(I-K)50,55,50
0044 50 IK = NK + I
0045 A(IK) = A(IK)/(-BIGA)
0046 55 CONTINUE
0047 DO 65 I = 1,N
0048 IK = NK + I
0049 HOLD = A(IK)
0050 IJ = I-N
0051 DO 65 J = 1,N
0052 IJ = IJ + N
0053 IF (I-K) 60,65,60
0054 60 IF(J-K) 62,65,62
0055 62 KJ = IJ-I+K
0056 A(IJ) = HOLD*A(KJ)+A(IJ)
0057 65 CONTINUE
0058 KJ = K-N
0059 DO 75 J = 1,N
0060 KJ = KJ+N
0061 IF(J-K) 70,75,70
0062 70 A(KJ)=A(KJ)/BIGA
0063 75 CONTINUE
0064 D = D*BIGA
0065 A(KK) = (1.,0.)/BIGA
0066 80 CONTINUE
0067 K = N
0068 100 K = K-1
0069 IF (K) 150,150,105
0070 105 I = L(K)
0071 IF(I-K) 120,120,108
0072 108 JQ = N*(K-1)
0073 JR = N*(I-1)
0074 DO 110 J = 1,N
0075 JK = JQ+J
0076 HOLD = A(JK)
0077 JI = JR+J
0078 A(JK) = -A(JI)
0079 110 A(JI) = HOLD
0080 120 J = M(K)
0081 IF(J-K) 100,100,125
0082 125 KI = K-N
0083 DO 130 I = 1,N
0084 KI = KI + N
0085 HOLD = A(KI)
0086 JI = KI-K+J
0087 A(KI) = -A(JI)
0088 130 A(JI) = HOLD
0089 GO TO 100
0090 150 RETURN
0091 END
```