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CHARLOTTESVILLE, VIRGINIA

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MILLIMETER-WAVE MIXER TRANSMISSION LINE ANALYSIS

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## I. LOW PASS FILTER CHARACTERISTICS

The requirements of the low pass filter are that it reject the L.O. and signal across the entire band (70 to 120 GHz) and that it present a characteristic impedance of 50 to 70 ohms at the I.F. frequency. Also, it is important that the input impedance of the choke be small at the signal and L.O. frequencies since the choke appears in series with the diode at these frequencies. In the stop band, the choke is almost purely reactive and it is necessary that at midband it look like a short rather than an open circuit. At the low and high ends of the band, both the resistive and reactive components of the choke input impedance increase. It is desirable that  $\text{Re } Z_c < 1\Omega$  and  $\text{Im } Z_c < 10\Omega$  at the band edges to prevent excessive conversion loss and noise.

The low pass filter is composed of alternating sections of low and high impedance line. The section lengths are approximately  $\lambda/4$  long at the middle of the stop band ( $\lambda/2$  long at the center of the 2nd pass band). The cutoff frequency of the low pass characteristic is determined by the bandwidth and center frequency of the 2nd ( $\lambda/2$ ) passband. The design of this type of filter is covered in Matthai, Young, and Jones, Chapters 6 and 7<sup>1</sup>. For the 80-120 GHz mixer, this choke has been realized in quartz microstrip line using a six section design. The quartz is 12 mils. thick by 15 mils. wide by 125 mils. long. The low and high impedance line sections alternate in width between 14 and 2 mils. respectively. A relaxation analysis of quasi-TEM mode characteristic impedance and propagation velocity for this structure is given in the next section.

Two choke designs are presently extant and have been tested. The first, T2, is a design by Tony Kerr<sup>2</sup> having section lengths of 16 mils. for the low impedance (14 mils. wide) line and 17 mils. for the high impedance (2 mils. wide) line the difference in length being, presumably, to compensate for discontinuity capacitance. The second is a design by Jesse Davis, D1, having section lengths of 19 mils. and 22 mils. for the low and high impedance lines respectively. The D2 choke is composed of only five sections (3 low Z, 2 high Z) because of the increased section length. These two designs have been analyzed for input impedance (or alternately reflection coefficient  $\rho$ ) versus frequency using a program written for the 9830 Calculator. These results include the effect of discontinuity capacitance as calculated by Davis and by Farrar and Adams<sup>3</sup>. The impedances and propagation velocities used in the calculation are those

calculated for the case E=3 by S. Hansen using J. Greenberg's Fortran routine<sup>4</sup>.

The calculated response of choke T2 is plotted on the Smith chart in Figure 1.1. It is apparent that the design center is too high in frequency and that below 80 GHz (or possibly even 85 GHz) the mixer performance is expected to degrade due to excessive choke impedance. This is the choke currently in active use in the 80-120 GHz mixer.

Figure 1.2 shows the calculated response of choke D2. The design center has been lowered to about 106 GHz for this choke. The real part of the input impedance in the stop band is somewhat larger than for T2 due to the loss of one section and resulting lower Q. However, the input impedance is within reasonable bounds between 70 to 120 GHz and it is reasonable to assume that this choke will perform adequately over this frequency range if T2 is working well down to 80 GHz.

If a further improvement in choke performance is required to get full 70-120 GHz performance, the design center frequency can be lowered to 95 GHz and the sixth section can be reinserted resulting in longer overall choke length. Also, the high and low impedance ratios can be calculated to provide Tchebycheff response rather than just alternating equal impedances. Any redesign undertaken should rely heavily on low frequency modeling techniques as discussed in the third section to provide design normalization before fabrication and testing of the mm-wave quartz parts.

## II. CHARACTERISTIC IMPEDANCE OF SHIELDED MICROSTRIP

### 2.1 Propagating Mode and $Z_0$

The value of characteristic impedance  $Z_0$  of a transmission system is defined in terms of the mode of wave propagation assumed to exist on the line. A hybrid TE/TM mode is required to strictly represent the wave propagation in shielded microstrip (Fig. 2.1). The dispersion relations for hybrid modes on this structure have not been calculated in closed form<sup>5</sup>. In the majority of literature, the analysis of microstrip is carried out on the basis of an assumed TEM mode of propagation. This involves an approximation, the most critical feature of which for segmented-line filter design is the dispersion behavior, or frequency-wavelength characteristic. For a true TEM mode,

$$\lambda = 1/f\sqrt{\mu\epsilon} = 3 \times 10^8 / f\sqrt{K}$$

where  $K$  is the relative dielectric constant of the medium filling the region in which waves propagate. Due to the partial filling of the cross section with dielectric in microstrip, frequency-dependent departures from the foregoing relation occur. These departures tend to have a slow dependence on frequency. Figure 2.2 shows the dispersion behavior of a typical shielded microstrip. Also plotted for comparison is the dispersion behavior of the dominant TE mode in a quartz-filled rectangular K-band waveguide (W/G in Fig. 2,2). In the absence of dispersion relations in closed form for shielded microstrip, experimental design verification, as described in Part III of this report, is necessary.

Characteristic impedance  $Z_o$  can be defined for a two-conductor TEM system as:

$$Z_o = V/I$$

where traveling wave voltage and current are well-defined. For TE and TM mode hollow guide propagation,  $Z_o$  may be defined:

$$Z_o = \frac{V^2}{2P}$$

where  $V$  is a defined field integral and  $P$  is mode power propagated. The latter definition requires knowledge of the electromagnetic field distribution.

The electric vector satisfies:

$$\nabla^2 \underline{E} = \mu \epsilon \ddot{\underline{E}}$$

For propagating waves, in rectangular coordinates,

$$\nabla_t^2 E_i - \beta^2 E_i = \mu \epsilon \omega^2 E_i$$

for  $i = x, y, \text{ or } z$  component, where  $\beta$  is the propagation constant and  $\nabla_t^2$  is the transverse Laplacian. For TE and TM modes the wave solution satisfying boundary conditions is required to obtain the line parameters. This problem has been solved only in approximate form for hybrid modes in shielded microstrip<sup>6</sup>.

For the TEM mode,  $\beta^2 = \mu \epsilon \omega^2$ , and the electric field satisfies, in rectangular coordinates:

$$\nabla^2 \underline{E}_t = 0$$

Moreover, the transverse E field distribution in the TEM mode is identical with the electrostatic field for the same geometry, and may be derived from  $\underline{E} = -\nabla V$ . Hence,

$$\nabla^2 V = 0 \quad (\text{TEM mode})$$

## 2.2 Calculation of Characteristic Impedance

With the TEM wave mode assumed, the standard formulae of TEM wave propagation are available. The characteristic impedance is:

$$Z_o = \sqrt{\frac{L}{C}} = \frac{1}{v_p C} \quad (1)$$

where  $v_p$  = phase velocity and  $C$  = capacitance/meter of line. For the microstrip line, eliminating  $L$  from (1) yields:

$$Z_o = \frac{1}{c \sqrt{C_d C_a}} \quad (2)$$

where  $C_d$  is line capacitance with dielectric substrate present,  $C_a$  is capacitance with air replacing the dielectric and  $c = 3 \times 10^8$  m/sec. The line capacitance is given by:

$$C = Q$$

where  $Q$  is the charge per conductor with an assumed 1 volt potential difference. In terms of electric flux vector  $D$ ,

$$C = \iint D_n da = \epsilon_o \iint K E_n da = \epsilon_o \iint K \frac{dV}{dn} da \quad (4)$$

where the integral is taken over any cylindrical surface between conductors, and  $V$  is the electrostatic potential. In the TEM assumption,  $V$  satisfies Laplace's equation:

$$\nabla^2 V = 0$$

The potential distribution  $V$  in the shielded microstrip may be calculated numerically by means of a relaxation procedure. The potential is defined on a square network of field points (Fig. 2.3). The finite-difference form of

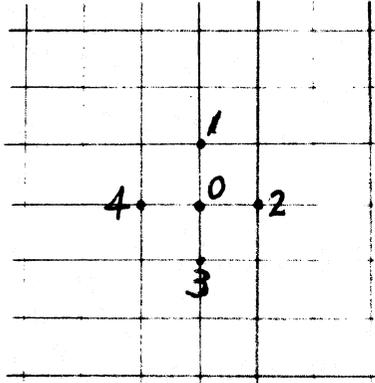


FIG. 2.3a

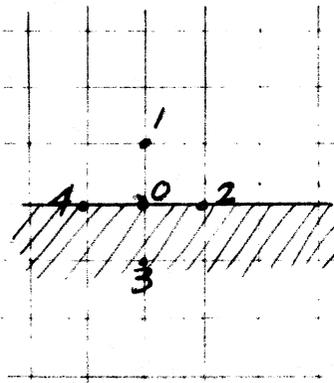


FIG 2.3b

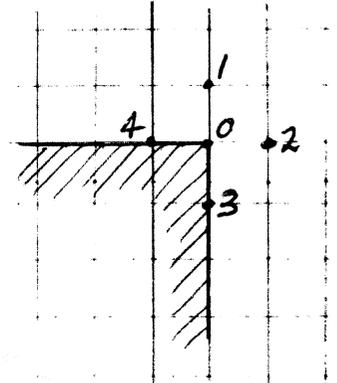


FIG 2.3c

Eq. (5) for a free field point as in Fig. 2.3a is:

$$V_1 + V_2 + V_3 + V_4 - 4V_0 = 0 \quad (6a)$$

At a dielectric-air interface (Fig. 2.3b) the form is:

$$V_2 + V_4 + 2(KV_3 + V_1)/(K + 1) - 4V_0 = 0 \quad (6b)$$

and at an external corner of the dielectric (7) as in Fig. 2.3c):

$$2(V_1 + V_2) + (K + 1)(V_3 + V_4) - 2(K + 3)V_0 = 0 \quad (6c)$$

In the relaxation calculation, initial values of potential  $V$  are assigned to every field point. On the outer boundary  $V = 0$ , and on the strip conductor  $V = 1$ . Equations (6) are then applied respectively to every field point until a near-equilibrium potential distribution is obtained. A measure of the nearness to equilibrium is given by the magnitude of the residue at each point:

$$\begin{aligned} R_a &= V_1 + V_2 + V_4 - 4V_0 \\ R_b &= V_2 + V_4 + 2(KV_3 + V_1)/(K + 1) - 4V_0 \\ R_c &= 2(V_1 + V_2) + (K + 1)(V_3 + V_4) - 2(K + 3)V_0 \end{aligned} \quad (6d)$$

for the respective cases of Figs. 2.3a to 2.3c.

The approach to equilibrium can be accelerated by applying an over-relaxation factor  $x$ . On the  $n^{\text{th}}$  iteration the  $(n+1)^{\text{st}}$  estimate of the potential is taken as:

$$V^{n+1} = (1 - x) V^n + x \left( \sum_j^4 V_j / 4 \right) \quad (7)$$

where the summation in the last term is the appropriate calculation for  $V_0$  from Eq. (6a) to (6c). The overrelaxation process is convergent for  $1 < x < 2$ . The optimum  $x$  is problem-dependent and was found by trial to be in the range 1.6-1.75 for this case. Figure 2.4 shows the effect of various values of  $x$  on the approach of the calculated capacitance to a final value.

### 2.3 Computer Program for $Z_0$

A computer program was written in BASIC for the relaxation calculation of  $Z_0$  of the shielded microstrip transmission line. The program consists of:

- 1) Fitting line cross section to a square mesh of field points.
- 2) Assignment of initial values of potential.
- 3) Relaxation of potentials, with quartz dielectric.
- 4) Calculation of capacitance  $C_d$  with quartz.
- 5) Assignment of dielectric constant  $K = 1$  to quartz region and repetition of steps (3) and (4) for  $C_a$  with air.
- 6) Calculation of  $Z_0$  from capacitances.

Because of the symmetry of the line cross section, a half cross section was calculated and the resulting capacitance doubled. This requires that all field points on the center line be relaxed with potentials assumed to be symmetric about the center.

In setting up the network, field points are identified by rows  $R$  and columns  $C$ , e.g.: potential  $V(R, C)$ . The first row and column are numbered 1 (not zero), as in Fig. 2.5. An arbitrary number  $N+1$  of columns is assigned, corresponding to the width of the half shield-box. (Typical values:  $N=12$  to  $N=20$ .) Based on this number, all edges and corners of the box are then placed on network points of the square network. Use of too small a number of field points introduces excessive "quantization error" in fitting the box to the net; use of too many increases the computation time (as the square of the number of columns). With  $N=20$  each relaxation cycle requires approximately 2-1/2 minutes of Hewlett-Packard 9830 computing time.

The program was written with sufficient flexibility to permit a wide variety of transmission line geometries. This involves use of a number of IF statements, to eliminate ambiguity in the assignment and relaxation of potentials on points between the quartz substrate and metal shield. The quartz may be brought into contact with the metal at the bottom. As a practical measure, a minimum air gap of at least one row of field points is maintained at sides and top of quartz.

Equation (4) for capacitance becomes a sum of finite differences of  $V$  over a path chosen in air at top and sides, and in quartz at the bottom of the surface of integration:

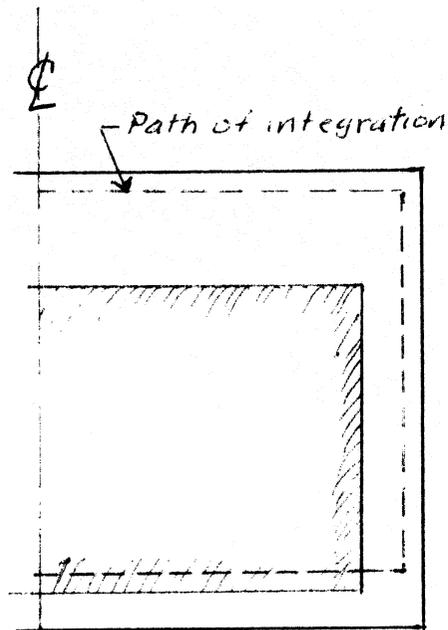


FIG. 2.6.

This eliminates the question of appropriate  $K$  value as the air gap under the quartz is closed to zero.

Typical output values of characteristic impedance as a function of strip conductor width  $w$ , with shield box dimensions: width 18 mils, height 19 mils; quartz width 15 mils, height 12 mils, and air gap over quartz 3 mils. are given in Table 2.1.

TABLE 2.1

Z <sub>0</sub> OF SHIELDED MICROSTRIP	
STRIP WIDTH W	Z <sub>0</sub>
2 mils.	97.6 ohms
5	66.4
7	55.5
10	44.5
12	39.0
14	34.9

The sum of squares of residues (Eqs. 6d) of all the field points within the area of the quartz block was arbitrarily chosen as a measure of the nearness of approach of the potential distribution to its final form. Setting a limit of: (sum of squares) =  $5 \times 10^{-5}$  for typical lattice sizes of 500-750 points restricts error in calculated Z<sub>0</sub> to order of tenths of an ohm. Reduction of the lattice size reduces the computing time radically. For example, calculation of Z<sub>0</sub> for a typical transmission line cross section with a lattice of 903 points yielded a value of Z<sub>0</sub> = 44.5 ohms, and required approximately 16 hours. When the lattice size was reduced to 253 points, the program yielded Z<sub>0</sub> = 44.4 ohms in about 3-1/2 hours. This non-proportional reduction in computing time is a result of the fact that the efficiency of the overrelaxation factor  $\alpha$  depends markedly on the lattice size<sup>8</sup>. The lattice size is easily reduced by editing the program, at line 250. (Program reproduced as Figs. 2.6 to 2.8.)

### III. MEASUREMENTS ON SCALED MODEL OF MIXER LINE AND SCALED CHOKE

An 18:1 scale model of choke T2 was built by J. Greenberg<sup>4</sup> but never tested. This structure has been tested at C band to obtain scaled performance in the frequency range 70 to 120 GHz. Also tested was a scale model of 15 mil. wide microstrip on 14 mil. thick by 15 mil. wide quartz to obtain an electrical length from which the scaled propagation velocity may be obtained.

The electrical length measurements were made between 4.5 and 7.0 GHz which gives a scaled frequency range of 81 to 126 GHz. The stripline geometry is similar to that described in the previous section except that the scaled quartz thickness is 14 rather than 12 mils. The spacing between the quartz and bottom of the channel was about  $E = 1$  mil. scaled which gives a top spacing of  $G = 4$  mil. which is approximately equivalent to a bottom spacing of  $E = 3$  with 12 mil. thick quartz. The effective velocity  $V_e$  was obtained by dividing the physical length of the quartz microstrip by the measured electrical length. The electrical length was measured by inserting an equivalent length of reference line on the network analyzer and carefully correcting for the electrical length of the connectors and enclosure in which the microstrip was mounted. Table 3.1 shows the measured effective propagation velocity  $V_e = V/C$ .

TABLE 3.1

#### EFFECTIVE PROPAGATION VELOCITY VERSUS FREQUENCY

$f$ (GHz)	$f_s$ (GHz)	$V_e = V/C$
4.5	81	0.751
5.0	90	0.749
5.5	99	0.747
6.0	108	0.743
6.5	117	0.737
7.0	126	0.732

These data show a slight velocity dispersion for the microstrip which is expected from a hybrid mode analysis of shielded microstrip at these frequencies<sup>9</sup>. Corresponding calculated effective velocity from TEM mode analysis (non-dispersive) is 0.78 to 0.80 depending on whether 12 mil. or 14 mil. quartz thickness is used respectively. Therefore the amount of dispersion that needs to be considered in

designing chokes at these frequencies appears to be in the range of 5 to 10%.

Reflection and transmission measurements have been made of the scaled choke between 4 and 8 GHz. These data give some idea of the insertion loss of the choke and the input impedance versus frequency respectively. The  $S_{21}$  data show that the insertion loss begins to rise above -20 dB at 105 GHz for the original T2 design shown in Figure 3.1. An attempt to improve the performance of the model choke by reducing the high impedance line width to 1 mil. equivalent was successful in reducing the -20 dB insertion loss point to 95 GHz. A comparison with calculated  $S_{12}$  data also shown in Figure 3.1 indicates that there is a disagreement in frequency between calculated and scaled measured data of about 5 GHz or 5%. However, addition of discontinuity capacitance will increase this discrepancy to about 10%. Also, use of the measured propagation velocity instead of the calculated TEM velocity is also in the direction of increasing the discrepancy between calculated and scale measured data. Some careful work to resolve these discrepancies would have a high payoff in understanding and implementing choke design.

The reflection measurements on the scaled choke are presented in Smith chart form in Figure 4. These measurements were taken after the high Z linewidths were reduced to 1 mil. equivalent, and are not directly comparable with the calculated curve of Figure 1 which was for 2 mil. linewidths. The section lengths are equivalent, however, and the frequency versus reactance should be comparable except that there is possibly some error in the rotation performed to get the data of Figure 3.2. When allowance is made for these possible errors, there is quite good agreement between Figures 1.1 and 3.2. The measured data show a reactance versus frequency characteristic about 2 GHz lower in frequency at the high end which is consistent with the decrease of propagation velocity measured.

Taken together, the measurements on the scaled microstrip line and choke indicate that this is a highly useful technique for design checking and verification. It appears desirable to construct a scale model of choke D1 to verify its performance between 70 and 120 GHz before proceeding with any further new choke designs.

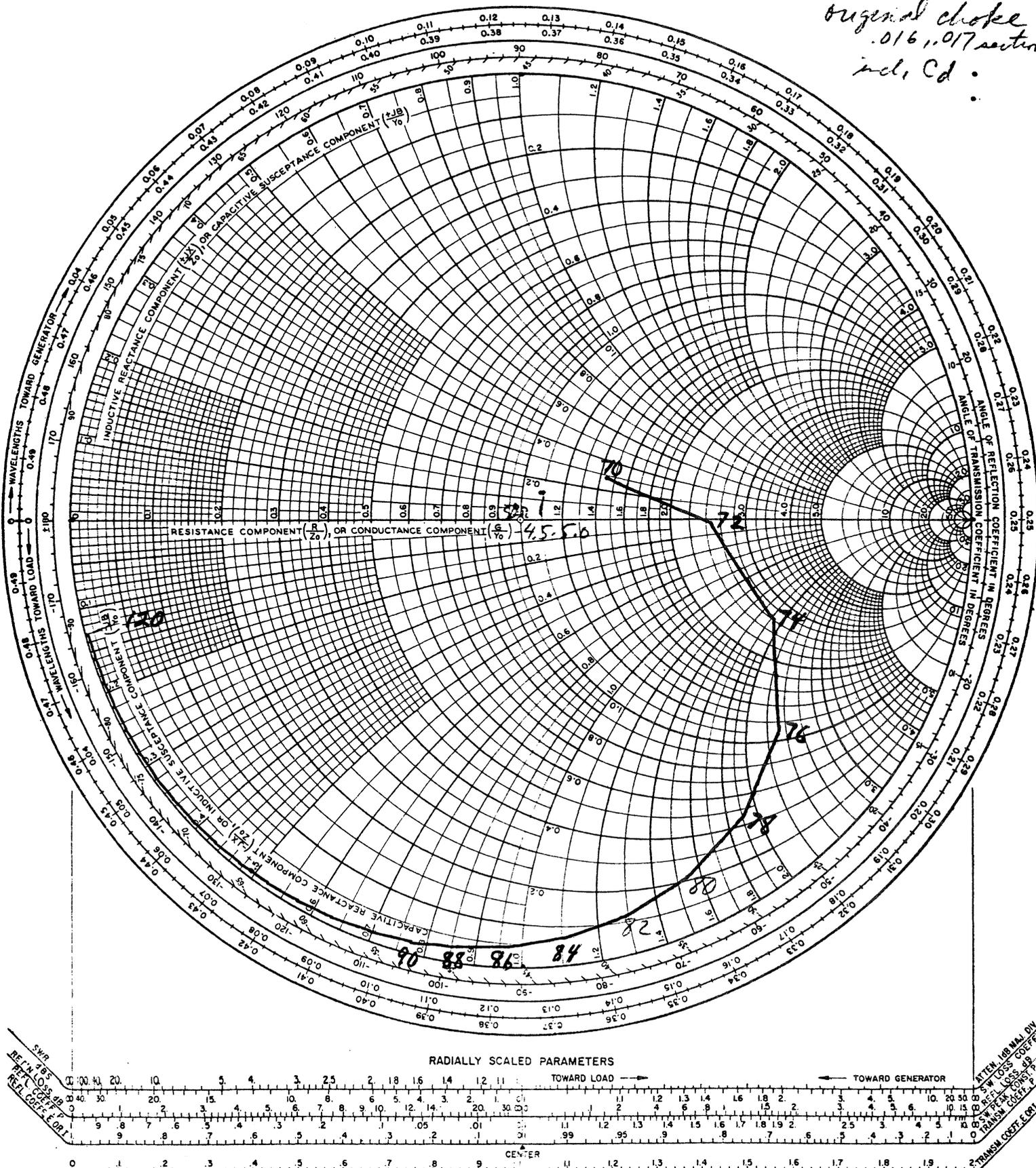
## REFERENCES

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- <sup>4</sup>J. Greenberg, "Stripline Impedance Determination", NRAO Electronics Division Internal Report
- <sup>5</sup>R. Mittra, T. Itoh, "Analysis of Microstrip Transmission Lines", in Advances in Microwaves, Vol. 8, Academic Press, New York (1974).
- <sup>6</sup>J. S. Hornsby, A. Gopinath, IEEE Trans. MTT-17, 684 (1969).
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- <sup>8</sup>W. S. Metcalf, "Characteristic Impedance of Rectangular Transmission Lines", Proc. IEE, 112, 2033 (1963).
- <sup>9</sup>R. Mittra, T. Itoh, "A New Technique for the Analysis of the Dispersion Characteristics of Microstrip Lines", IEEE Trans. MTT-19, 47 (1971).

NAME	TITLE <b>Choke T2 Performance</b>	DWG. NO. <i>Fig 1.1</i>
SMITH CHART FORM 82-BSPR(9-66)	KAY ELECTRIC COMPANY, PINE BROOK, N.J., © 1966. PRINTED IN U.S.A.	DATE

IMPEDANCE OR ADMITTANCE COORDINATES

*DRD 7/12/76*  
*original choke*  
*.016, .017 section*  
*incl. Cd.*

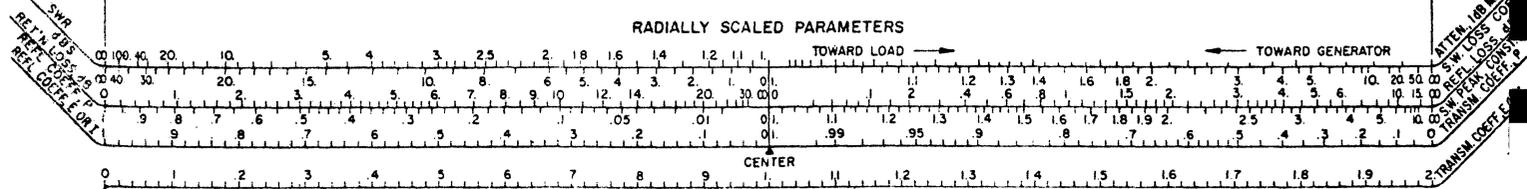
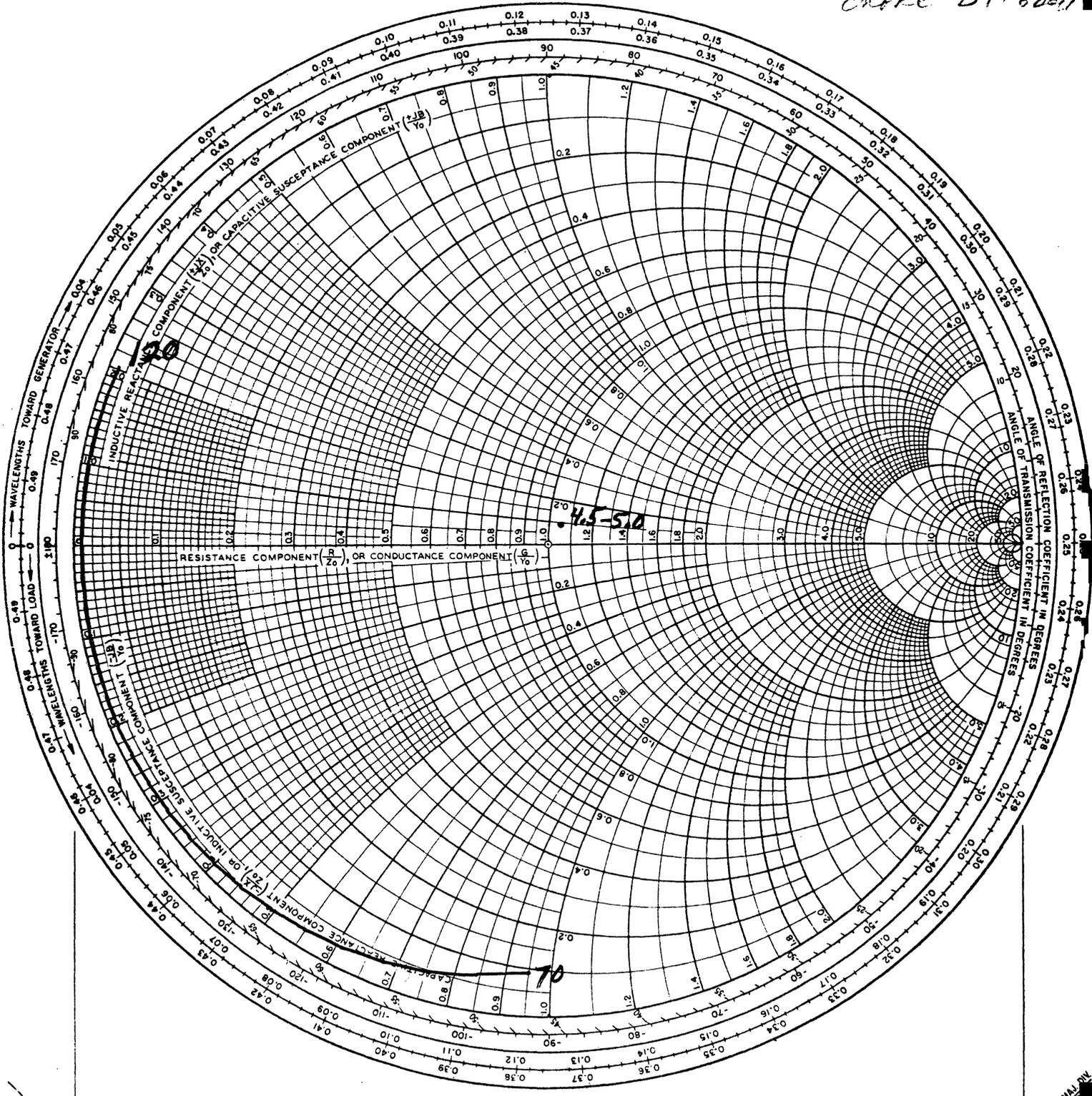


**FIG. 1.1**

NAME	TITLE <i>Choke D1 Performance</i>	DWG. NO. <i>Fig 12</i>
SMITH CHART FORM 82-BSPR (9-66)	KAY ELECTRIC COMPANY, PINE BROOK, N. J., © 1966. PRINTED IN U.S.A.	DATE

IMPEDANCE OR ADMITTANCE COORDINATES

*DRD 7/12/76*  
*choke D1/600ft*



**FIG. 12**

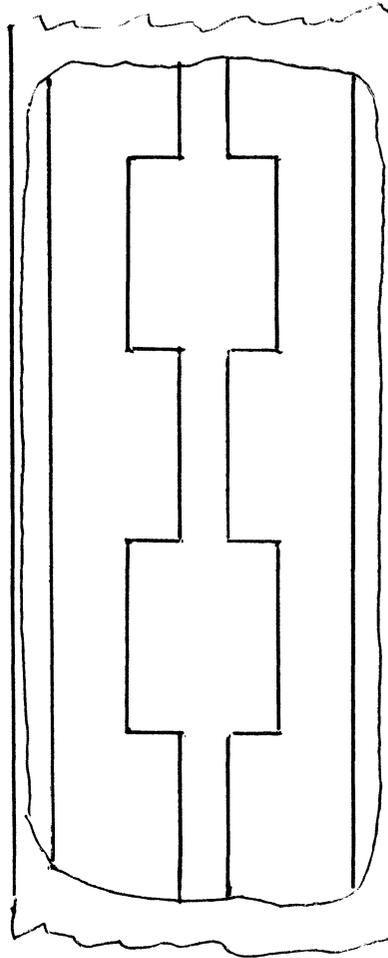
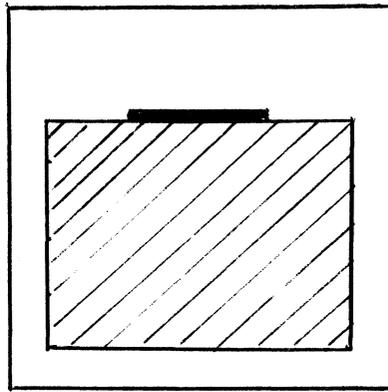


FIG. 2.1

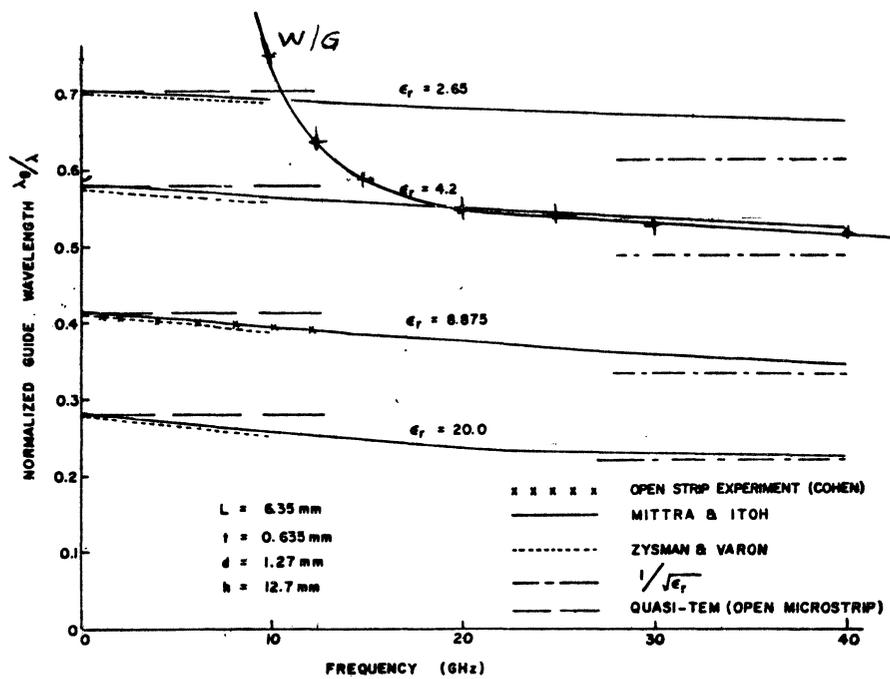


FIG. 2.2

See Figure 2.3, page 5.

FIGURE 2.3

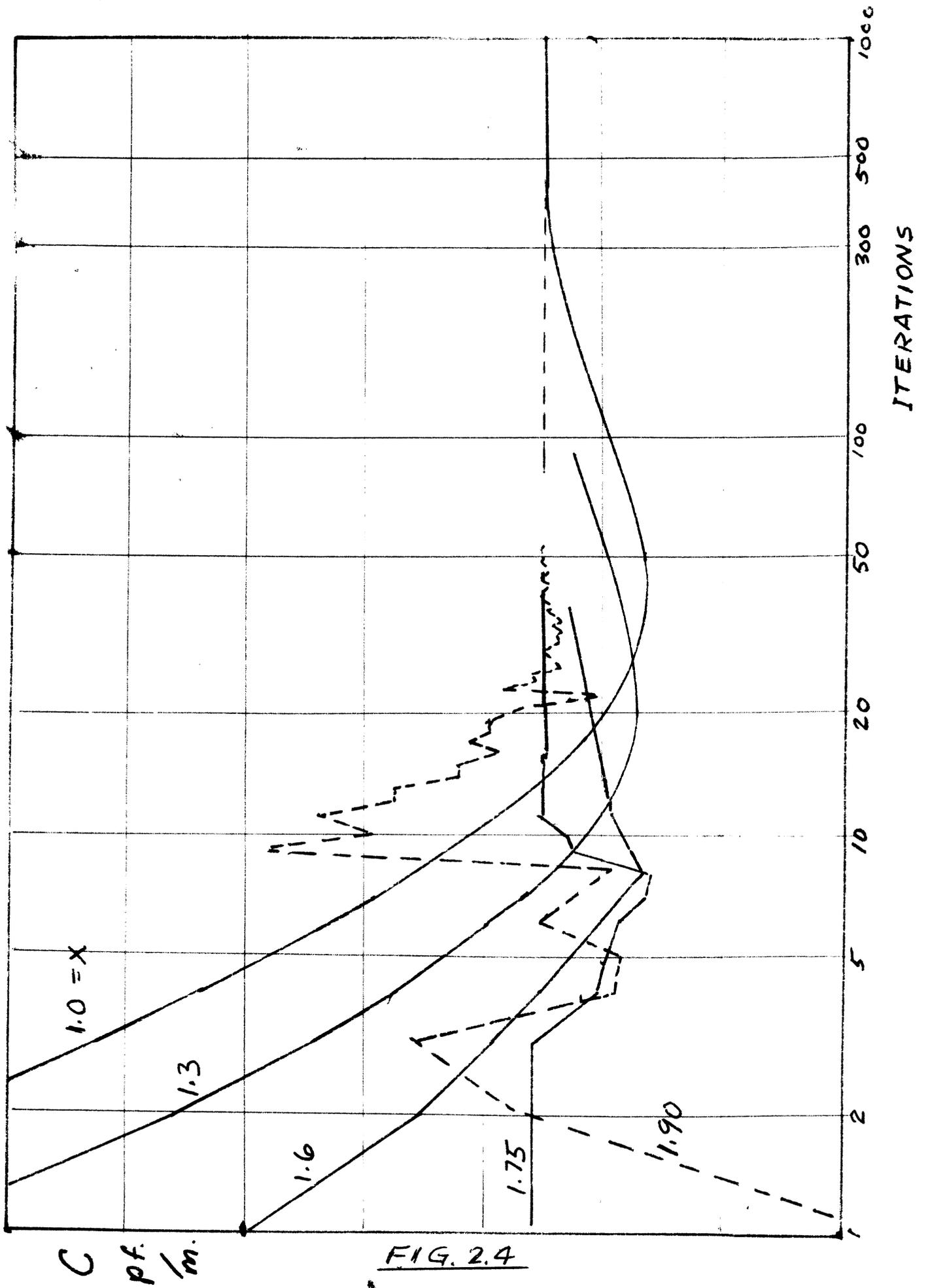


FIG. 2.4

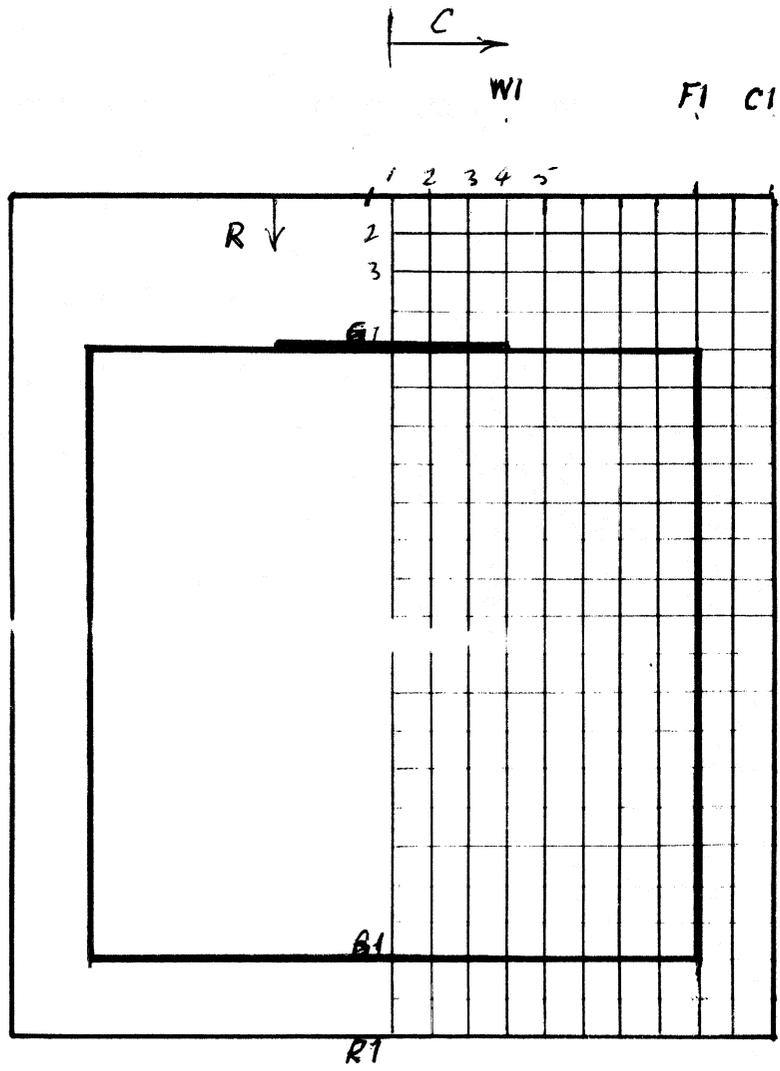


FIG. 2.5

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10 DISP "CAPACITY:BOXED STRIP W/CHPS
20 WAIT 3000
30 X=1.7
40 M1=M2=0
50 PRINT
60 PRINT " OVERRELAXATION FACTOR X="X"."
70 PRINT
80 DISP " DIMENSIONS IN MILS"
90 WAIT 2000
100 DIM V$(70,35)
110 DISP " ENTER BOX WIDTH & HT.:"
120 INPUT D,A
130 DISP "QUARTZ WPTH.&HT.:"
140 INPUT F,B
150 K=3.8
160 DISP "CONDOC.STRP. WPTH,& GAP OVER":
170 INPUT W,G
180 PRINT " BOX SIZE: WIDTH="D"MILS, HEIGHT="A"MILS."
190 PRINT
200 PRINT " QUARTZ WPTH="F"MILS, HT.="B"MILS, DIEL. CONST.="K"."
210 PRINT
220 PRINT " AIR GAP OVER QUARTZ="G"MILS, CONDOC. STRIP WIDTH="W"MILS.
230 PRINT
240 REM BEGIN ASSIGNING NET POINTS
250 N=16
260 C1=N+1
270 R1=1+INT(2*A*N/D)
280 W1=1+INT(W*N/D)
290 F1=1+INT(F*N/D)
300 IF (C1-F1)>1 THEN 320
310 F1=C1-2
320 G1=1+INT(2*G*N/D)
330 IF G1 >= 3 THEN 350
340 G1=3
350 B1=1+INT(2*(B+G)*N/D)
360 IF (R1-B1)>2 THEN 380
370 B1=R1
380 REM BEGIN ASSIGNING INITIAL POTENTIAL VALUES
390 PRINT "C1="C1;"R1="R1;
400 PRINT "F1="F1;"B1="B1;
410 PRINT "W1="W1;"G1="G1
420 PRINT
430 FOR C=1 TO C1
440 V(1,C)=0
450 NEXT C
460 FOR R=2 TO R1-1
470 V(R,C1)=0
480 NEXT R
490 FOR C=1 TO C1
500 V(R1,C)=0
510 NEXT C
520 FOR C=1 TO W1
530 V(G1,C)=1
540 NEXT C
550 FOR R=2 TO (G1-1)
560 FOR C=1 TO (C1-1)
570 V(R,C)=0.3
580 NEXT C
590 NEXT R

```

FIG.2.6

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600 FOR R=G1 TO B1-1
610 FOR C=(F1+1) TO (C1-1)
620 V(R,C)=0.1
630 NEXT C
640 NEXT R
650 IF B1=R1 THEN 750
660 FOR C=(F1+1) TO (C1-1)
670 V(B1,C)=0.1
680 NEXT C
690 IF B1 >= (R1-1) THEN 750
700 FOR R=(B1+1) TO (R1-1)
710 FOR C=1 TO (C1-1)
720 V(R,C)=0.1
730 NEXT C
740 NEXT R
750 FOR C=(W1+1) TO (F1-1)
760 V(G1,C)=0.5
770 NEXT C
780 FOR R=(G1+1) TO B1
790 FOR C=1 TO (F1-1)
800 V(R,C)=0.3
810 NEXT C
820 NEXT R
830 FOR R=G1 TO B1
840 V(R,F1)=0.2
850 NEXT R
860 IF B1<R1 THEN 900
870 FOR C=1 TO C1
880 V(R1,C)=0
890 NEXT C
900 REM BEGIN RELAXING POTENTIAL NETWORK
910 FOR R=2 TO (G1-1)
920 S1=(V(R+1,1)+V(R-1,1)+2*V(R,2))/4
930 V(R,1)=(1-X)*V(R,1)+X*S1
940 NEXT R
950 IF B1 >= (R1-1) THEN 1000
960 FOR R=(B1+1) TO (R1-1)
970 S2=(V(R+1,1)+V(R-1,1)+2*V(R,2))/4
980 V(R,1)=(1-X)*V(R,1)+X*S2
990 NEXT R
1000 FOR R=2 TO (G1-1)
1010 FOR C=2 TO (C1-1)
1020 S3=(V(R+1,C)+V(R-1,C)+V(R,C-1)+V(R,C+1))/4
1030 V(R,C)=(1-X)*V(R,C)+X*S3
1040 NEXT C
1050 NEXT R
1060 FOR R=G1 TO (B1-1)
1070 FOR C=(F1+1) TO (C1-1)
1080 S4=(V(R+1,C)+V(R-1,C)+V(R,C-1)+V(R,C+1))/4
1090 V(R,C)=(1-X)*V(R,C)+X*S4
1100 NEXT C
1110 NEXT R
1120 IF B1=R1 THEN 1240
1130 FOR C=(F1+1) TO (C1-1)
1140 Z4=(V(B1+1,C)+V(B1-1,C)+V(B1,C-1)+V(B1,C+1))/4
1150 V(B1,C)=(1-X)*V(B1,C)+X*Z4
1160 NEXT C
1170 IF B1 >= (R1-1) THEN 1240
1180 FOR R=(B1+1) TO (R1-1)
1190 FOR C=2 TO (C1-1)
1200 S5=(V(R+1,C)+V(R-1,C)+V(R,C-1)+V(R,C+1))/4
1210 V(R,C)=(1-X)*V(R,C)+X*S5

```

FIG. 2.7

```

1220 NEXT C
1230 NEXT R
1240 FOR R=(G1+1) TO (B1-1)
1250 S6=(V[R+1,1]+V[R-1,1]+2*V[R,2])/4
1260 V[R,1]=(1-X)*V[R,1]+X*S6
1270 NEXT R
1280 FOR R=(G1+1) TO (B1-1)
1290 FOR C=2 TO (F1-1)
1300 S7=(V[R+1,C]+V[R-1,C]+V[R,C-1]+V[R,C+1])/4
1310 V[R,C]=(1-X)*V[R,C]+X*S7
1320 NEXT C
1330 NEXT R
1340 FOR C=(M1+1) TO (F1-1)
1350 Z7=(V[G1,C-1]+V[G1,C+1])/4+(K*V[G1+1,C]+V[G1-1,C])/(2*K+2)
1360 V[G1,C]=(1-X)*V[G1,C]+X*Z7
1370 NEXT C
1380 FOR R=(G1+1) TO (B1-1)
1390 S8=(V[R-1,F1]+V[R+1,F1])/4+(K*V[R,F1-1]+V[R,F1+1])/(2*K+2)
1400 V[R,F1]=(1-X)*V[R,F1]+X*S8
1410 NEXT R
1420 IF B1=R1 THEN 1490
1430 FOR C=2 TO (F1-1)
1440 S9=(V[B1,C-1]+V[B1,C+1])/4+(K*V[B1-1,C]+V[B1+1,C])/(2*K+2)
1450 V[B1,C]=(1-X)*V[B1,C]+X*S9
1460 NEXT C
1470 T1=V[B1,2]/2+(K*V[B1-1,1]+V[B1+1,1])/(2*K+2)
1480 V[B1,1]=(1-X)*V[B1,1]+X*T1
1490 T3=(V[G1,F1+1]+V[G1-1,F1]+(V[G1,F1-1]+V[G1+1,F1])*(K+1)/2)/(K+3)
1500 V[G1,F1]=(1-X)*V[G1,F1]+X*T3
1510 IF B1=R1 THEN 1540
1520 T2=(V[B1,F1+1]+V[B1+1,F1]+(V[B1,F1-1]+V[B-1,F1])*(K+1)/2)/(K+3)
1530 V[B1,F1]=(1-X)*V[B1,F1]+X*T2
1540 REM END OF ONE RELAXATION CYCLE
1550 IF K=1 THEN 1600
1560 M2=M2+1
1570 DISP " M2="M2
1580 WAIT 2000
1590 IF K>1 THEN 1630
1600 M1=M1+1
1610 DISP " M1="M1
1620 WAIT 2000
1630 U=0
1640 FOR R=(G1+1) TO (B1-1)
1650 FOR C=2 TO (F1-1)
1660 U=U+(V[R-1,C]+V[R+1,C]+V[R,C-1]+V[R,C+1]-4*V[R,C])^2
1670 NEXT C
1680 NEXT R
1690 IF (ABS U)<0.0005 THEN 1750
1700 IF K=1 THEN 1730
1710 IF (M2/10)=INT(M2/10) THEN 1750
1720 GOTO 910
1730 IF (M1/10)=INT(M1/10) THEN 1750
1740 GOTO 910

```

```

1750 REM CALCULATE CAPACITANCE
1760 Q1=0
1770 Q1=V[2,1]/2
1780 FOR C=2 TO (C1-1)
1790 Q1=Q1+V[2,C]
1800 NEXT C
1810 FOR R=2 TO (B1-2)
1820 Q1=Q1+V[R,C1-1]
1830 NEXT R
1840 Q1=Q1+K*(V[B1-2,1]-V[B1-1,1])
1850 FOR C=2 TO (F1-1)
1860 Q1=Q1+K*(V[B1-2,C]-V[B1-1,C])
1870 NEXT C
1880 Q1=Q1+(V[B1-2,F1]-V[B1-1,F1])*(K+1)/2
1890 FOR C=(F1+1) TO (C1-1)
1900 Q1=Q1+V[B1-2,C]-V[B1-1,C]
1910 NEXT C
1920 C9=2*8.85*Q1
1930 IF K=1 THEN 1990
1940 WRITE (15,2010)"CAP.(QTZ)="+C9"PF./METER AFTER"M2" CYCLES. RESIDUE:"U
1950 C2=C9
1960 IF ABSU >= 0.0005 THEN 910
1970 LET K=1
1980 GOTO 910
1990 WRITE (15,2010)"CAP.(AIR)="+C9"PF./METER AFTER"M1" CYCLES. RESIDUE:"U
2000 C0=C9
2010 FORMAT F8.3,F4.0,F8.5
2020 IF ABSU >= 0.0005 THEN 910
2030 Z0=3333/(SQR(C0*C2))
2040 PRINT
2050 PRINT "CHARACTERISTIC IMPEDANCE IS Z0="Z0" OHMS"
2060 END

```

DPD 7/9/76

• original 2 calc model choke (Thromberg)  
△ all high  $\frac{1}{2}$  sections cut to limit (eg. 1.5")  
○ calculated  $\frac{1}{2}$  for original chokes (no limit)

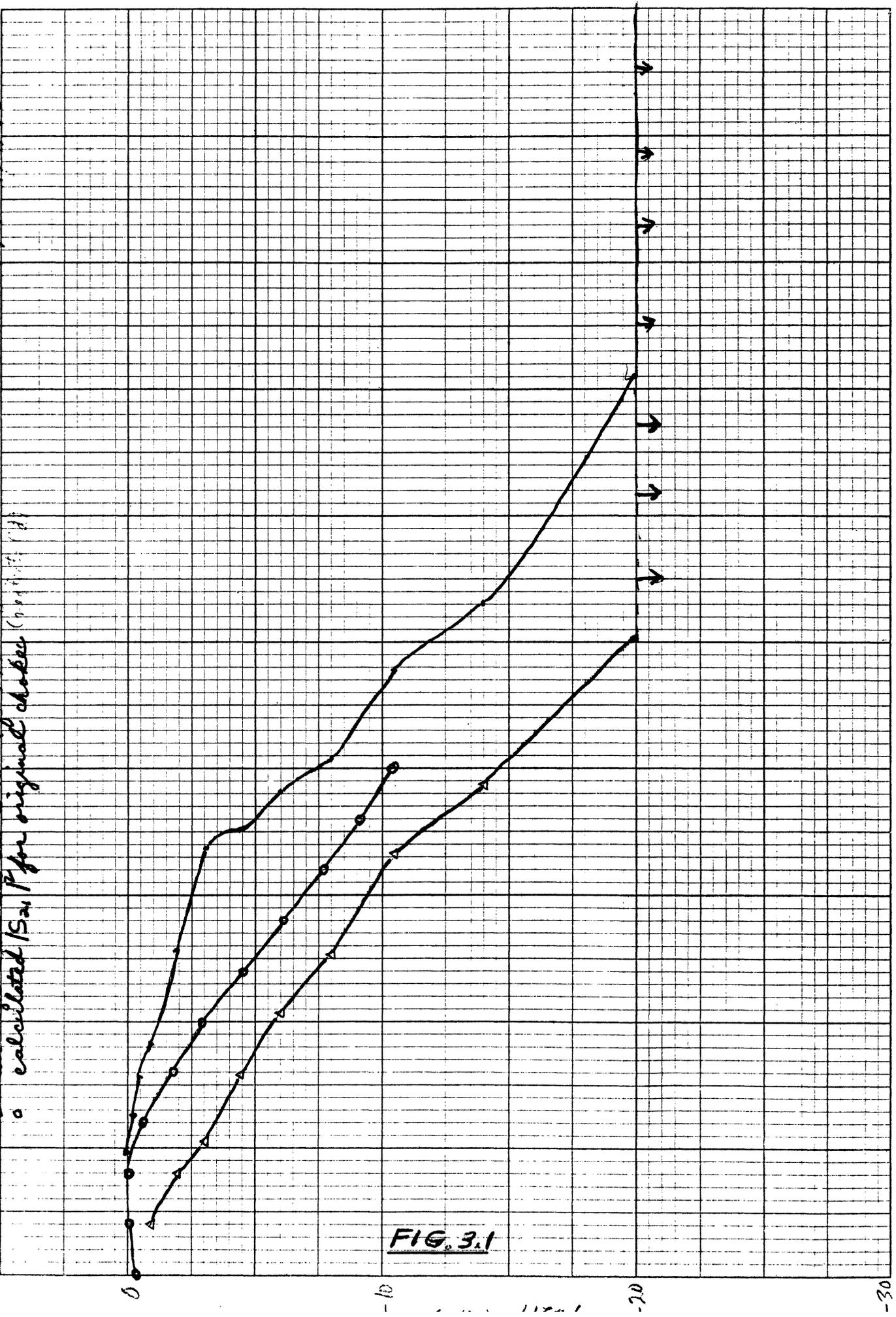


FIG. 3.1

