NATIONAL RADIO ASTRONOMY OBSERVATORY
GREEN BANK, WEST VIRGINIA

ELECTRONICS DIVISION INTERNAL REPORT No. 143

ERROR IN RECEIVER NOISE TEMPERATURE MEASUREMENT USING HOT AND COLD LOADS DUE TO EFFECT OF SOURCE REFLECTION COEFFICIENT ON GAIN

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MAY 1974

NUMBER OF COPIES: 150

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ABSTRACT

From generalized scattering parameters for a two-port network, equations are developed for the maximum possible effect of source impedance variations on the Y factor noise temperature measurement. A technique is suggested for measuring the magnitude of the reverse power flow, which results in gain changes and thus Y factor errors when practical (non-identical) hot and cold loads are employed. Examples and curves are given for the case of a negative resistance amplifier with circulator matched to a positive real transmission line.

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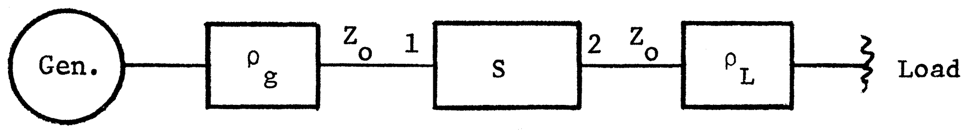
A. Gain Expression

Consider an arbitrary two-port network with generalized scattering parameters S_{11}, S_{22}, S_{12}, S_{21}.



Define:

S_{11} = input reflection coefficient.

S_{22} = output reflection coefficient.

S_{21} = forward voltage transfer 1 -> 2.

S_{12} = reverse voltage transfer 2 -> 1.

ρ_g = complex reflection coefficient at source (ρ_{g_C} => Cold load, ρ_{g_H} => Hot load).

ρ_L = complex reflection coefficient of load.

Kurakawa [1] and Bodway [2] have developed expresstions for the gain of such a network in terms of the scattering parameters and generator and load impedances. From Bodway [2] we obtain the expression for transducer gain:

$$1) \quad G_T = \frac{|S_{21}|^2 \left(1 - |r_1|^2\right) \left(1 - |r_2|^2\right)}{|1 - r_1 S_{11} - r_2 S_{22} + r_1 r_2 \Delta|^2} = \frac{\text{Power into Load}}{\text{Power Available from Generator}}$$

where: $$r_i = \frac{Z_i - Z_o}{Z_i + Z_o^*} \qquad \Delta = S_{11} S_{22} - S_{12} S_{21}$$

* Indicates the conjugate.

For the practical case where Z_o is real, such as a 50 ohm transmission line, r_i is the Voltage Reflection Coefficient, ρ_i (or Γ in some notations).

Then:

2) $$G_T = \frac{|S_{21}|^2 \left(1 - |\rho_g|^2\right) \left(1 - |\rho_L|^2\right)}{|1 - \rho_g S_{11} - \rho_L S_{22} + \rho_g \rho_L \Delta|^2}$$

If we assume the two-port to be matched to the transmission line (Z_o), then S_{11} = 0, S_{22} = 0, and the gain expression simplifies to:

3) $$G_T = \frac{|S_{21}|^2 \left(1 - |\rho_g|^2\right) \left(1 - |\rho_L|^2\right)}{|1 - \rho_g \rho_L S_{12} S_{21}|^2}$$

Gain dependence on $\left(\rho_L S_{12}\right)$ can be shown by letting $\rho_g = 1 e^{j\Theta_g}$, i.e., a sliding short on the input. In practice the gain would be measured by injecting a test signal into the input through a directional coupler placed between the sliding short and the input. We note that the term:

$$1 - |\rho_g|^2 = \frac{\text{Power Transmitted}}{\text{Incident Power}}$$

represents the power loss at the generator/transmission line interface. Obviously an open or short here $\left(\rho_g = 1 e^{j\Theta_g}\right)$ results in no transmitted power and G_T = 0. However, in our practical case the test signal is injected after the sliding short and thus the term $1 - |\rho_g|^2$ has no bearing on the observed gain, G.

Thus:

4) $$G = \frac{|S_{21}|^2 \left(1 - |\rho_L|^2\right)}{|1 - 1 e^{j\Theta_g} \rho_L S_{12} S_{21}|^2} = \text{Test Gain}$$

for the test situation.

Then letting Θ_g range over 2π, by adjusting the sliding short:

$$5)\qquad \frac{G_{max}}{G_{min}} = \frac{\left(1 + |\rho_L S_{12} S_{21}|\right)^2}{\left(1 - |\rho_L S_{12} S_{21}|\right)^2}$$

From such a test one can determine the value of the term $|\rho_L S_{12} S_{21}|$, which is a measure of the reverse power flow in a two-port terminated by ρ_L.

B. Parametric or Tunnel Diode Amplifiers

For the special case of a negative resistance amplifier with a circulator matched to input and output transmission lines (Z_o real):

$$|S_{21}| = a = \text{forward transducer voltage gain}$$

$$|\rho_L S_{12}| = \frac{1}{\text{Log}^{-1}\ (I/20)}$$

where I = isolation of circulator in dB.

Then from equation (5):

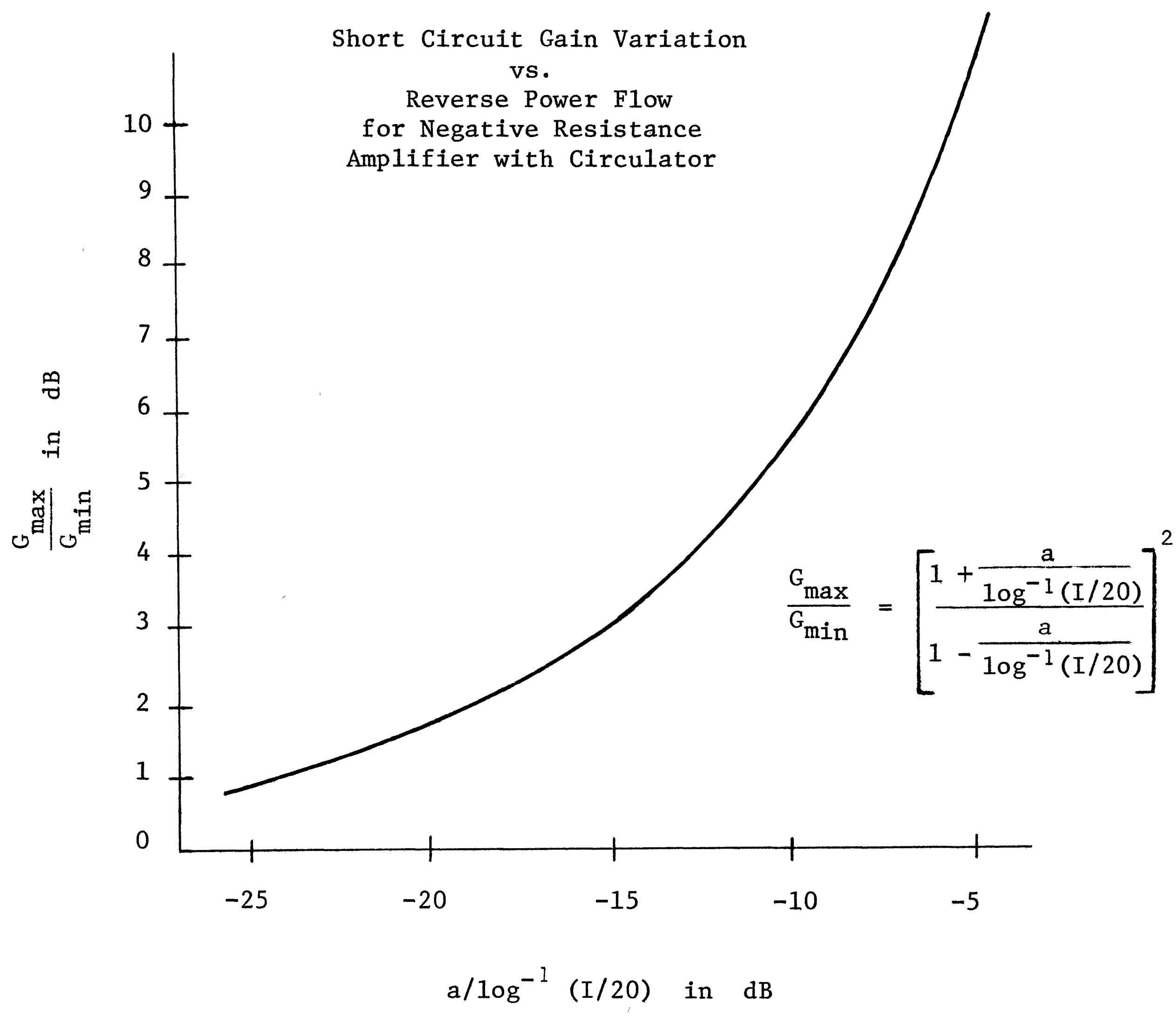
$$6)\qquad \frac{G_{max}}{G_{min}} = \left[\frac{1 + \dfrac{a}{\text{Log}^{-1}\ (I/20)}}{1 - \dfrac{a}{\text{Log}^{-1}\ (I/20)}}\right]^2$$

for a sliding short on the input. By measuring $\frac{G_{max}}{G_{min}}$, one can then determine the value $\frac{a}{\text{Log}^{-1}\ (I/20)}$ from equation (6). This expression is shown in the curve below. We point out that:

$$a = \text{Log}^{-1} \frac{G_t}{20}$$

where G_t = transducer power gain in dB under matched conditions, i.e., $S_{11} = S_{22} = \rho_g = \rho_L = 0$.

Note that $S_{12} = 1$ for an amplifier employing a 3-port circulator while $S_{12} < 1$ for a 4- or more port circulator. Isolation of 30 dB for a 3-port circulator requires a load VSWR of 1.06 $\left(|\rho| = \frac{\text{VSWR} - 1}{\text{VSWR} + 1} \right)$.



For an amplifier with 30 dB isolation [$\log^{-1}$ (I/20) = .032] and 15 dB matched gain (a = 5.62), then:

$$a/\log^{-1}\left(\frac{I}{20}\right) = -15 \text{ dB}$$

and the gain can vary as much as 3 dB due to changes in reflection coefficient seen by the input.

C. Noise Temperature Measurement Errors

Bounds on noise temperature error can be established by determining the effects of maximum and minimum gain on the Y factor.

$$Y_{max} = \frac{T_R + T_H}{T_R + T_C} \cdot \left(\frac{G_{max_H}}{G_{min_C}}\right)$$

7)

$$Y_{min} = \frac{T_R + T_H}{T_R + T_C} \cdot \left(\frac{G_{min_H}}{G_{max_C}}\right)$$

where T_R = true receiver noise temperature.

T_H = hot load temperature.

T_C = cold load temperature.

G_{max_H} $\left(G_{min_H}\right)$ = maximum (minimum) gain when hot load connected.

G_{max_C} $\left(G_{min_C}\right)$ = maximum (minimum) gain when cold load connected.

Hence, $T_{R_{max}}$ and $T_{R_{min}}$ will be measured as:

8)

$$T_{R_{min}} = \frac{T_H - Y_{max} T_C}{Y_{max} - 1}$$

$$T_{R_{max}} = \frac{T_H - Y_{min} T_C}{Y_{min} - 1}$$

where from equation (3):

9)

$$Y_{max} = \left(\frac{T_R + T_H}{T_R + T_C}\right) \left(\frac{1 + \left|\rho_{g_C} \rho_L S_{12} S_{21}\right|}{1 - \left|\rho_{g_H} \rho_L S_{12} S_{21}\right|}\right)^2 \cdot \left(\frac{1 - \left|\rho_{g_H}\right|^2}{1 - \left|\rho_{g_C}\right|^2}\right)$$

$$Y_{min} = \left(\frac{T_R + T_C}{T_R + T_C}\right) \left(\frac{1 - \left|\rho_{g_C} \rho_L S_{12} S_{21}\right|}{1 + \left|\rho_{g_H} \rho_L S_{12} S_{21}\right|}\right)^2 \cdot \left(\frac{1 - \left|\rho_{g_H}\right|^2}{1 - \left|\rho_{g_C}\right|^2}\right)$$

Again for a negative resistance amplifier with a circulator matched to Z_o:

$$|S_{21}| = a$$

$$|\rho_L S_{12}| = \frac{1}{\text{Log}^{-1} (I/20)}$$

$$|\rho_g| = \frac{\text{VSWR}_g - 1}{\text{VSWR}_g + 1}$$

And:

$$Y_{max} = \left(\frac{T_R + T_H}{T_R + T_C}\right) \left(\frac{1 + \left|\rho_{g_C}\right| \dfrac{a}{\mathrm{Log}^{-1}\ (I/20)}}{1 - \left|\rho_{g_H}\right| \dfrac{a}{\mathrm{Log}^{-1}\ (I/20)}}\right)^2 \left(\frac{1 - \left|\rho_{g_H}\right|^2}{1 - \left|\rho_{g_C}\right|^2}\right)$$

10)

$$Y_{min} = \left(\frac{T_R + T_H}{T_R + T_C}\right) \left(\frac{1 - \left|\rho_{g_C}\right| \dfrac{a}{\mathrm{Log}^{-1}\ (I/20)}}{1 + \left|\rho_{g_H}\right| \dfrac{a}{\mathrm{Log}^{-1}\ (I/20)}}\right)^2 \left(\frac{1 - \left|\rho_{g_H}\right|^2}{1 - \left|\rho_{g_C}\right|^2}\right)$$

D. Examples:

1. A single stage ambient parametric amplifier operating at 6090 MHz has the following parameters in a radiometer system:

$T_R = 260\ \mathrm{K} = T_{R\ true}$ (includes second stage noise)

$a = 7.07$ (17 dB)

When tested with a sliding short on the input,

$$\frac{G_{max}}{G_{min}} = 15.3\ \mathrm{dB}$$

which indicates I = 20 dB from equation 6.

We measure the noise temperature of this receiver with hot/cold loads having the following parameters:

$T_H = 293$ K $\mathrm{VSWR}_{g_H} = 1.06$

$T_C = 85$ K $\mathrm{VSWR}_{g_C} = 1.03$ (includes coax/WG adapter).

Then:

$$\frac{a}{\mathrm{Log}^{-1}\ (I/20)} = -3\ \mathrm{dB} = 0.707 \text{ from equation (6).}$$

$$\left|\rho_{g_H}\right| = .0291$$

$$\left|\rho_{g_C}\right| = .0148$$

And:

$$Y_{max} = \left(\frac{260 + 293}{260 + 85}\right) \left(\frac{1 + (.0148)\ (.707)}{1 - (.0291)\ (.707)}\right)^2 \left(\frac{1 - (.0291)^2}{1 - (.0148)^2}\right) = 1.705$$

$$Y_{min} = \left(\frac{260 + 293}{260 + 85}\right) \left(\frac{1 - (.0148)\ (.707)}{1 + (.0291)\ (.707)}\right)^2 \left(\frac{1 - (.0291)^2}{1 - (.0148)^2}\right) = 1.506$$

Then:

$$T_{R_{min}} = \frac{293 - (1.705)\ (85)}{1.705 - 1} = 210.0 \text{ K } (-19\% \text{ error})$$

$$T_{R_{max}} = \frac{293 - (1.506)\ (85)}{1.506 - 1} = 326.1 \text{ K } (+25\% \text{ error})$$

It is shown that very large errors can result even with hot/cold loads having reasonably good VSWR's.

2. Using the above example, we can note the effect of increasing the isolation (I) from 20 to 30 dB.

$$\frac{a}{\text{Log}^{-1}\ (I/20)} = -13 \text{ dB} = .224$$

and

$$T_{R_{min}} = 243.1 \text{ K } (-6.5\% \text{ error})$$

$$T_{R_{max}} = 279.4 \text{ K } (+7.5\% \text{ error})$$

We can express this effect in another form by combining equations (8) and (10) to yield:

$$11)\quad \frac{T_{R_{max}}}{T_{R_{min}}} = \frac{(T)\;(L)\left\{T_H\left(\dfrac{1+\left|\rho_{g_C}\right|\dfrac{a}{\mathrm{Log}^{-1}\ I/20}}{1-\left|\rho_{g_H}\right|\dfrac{a}{\mathrm{Log}^{-1}\ I/20}}\right)^2 + T_C\left(\dfrac{1-\left|\rho_{g_C}\right|\dfrac{a}{\mathrm{Log}^{-1}\ I/20}}{1+\left|\rho_{g_H}\right|\dfrac{a}{\mathrm{Log}^{-1}\ I/20}}\right)^2\left[1-(T)\;(L)\left(\dfrac{1+\left|\rho_{g_C}\right|\dfrac{a}{\mathrm{Log}^{-1}\ I/20}}{1-\left|\rho_{g_H}\right|\dfrac{a}{\mathrm{Log}^{-1}\ I/20}}\right)^2\right]\right\}-T_H}{(T)\;(L)\left\{T_H\left(\dfrac{1-\left|\rho_{g_C}\right|\dfrac{a}{\mathrm{Log}^{-1}\ I/20}}{1+\left|\rho_{g_H}\right|\dfrac{a}{\mathrm{Log}^{-1}\ I/20}}\right)^2 + T_C\left(\dfrac{1+\left|\rho_{g_C}\right|\dfrac{a}{\mathrm{Log}^{-1}\ I/20}}{1-\left|\rho_{g_H}\right|\dfrac{a}{\mathrm{Log}^{-1}\ I/20}}\right)^2\left[1-(T)\;(L)\left(\dfrac{1-\left|\rho_{g_C}\right|\dfrac{a}{\mathrm{Log}^{-1}\ I/20}}{1+\left|\rho_{g_H}\right|\dfrac{a}{\mathrm{Log}^{-1}\ I/20}}\right)^2\right]\right\}-T_H}$$

where:

$$(T) = \left(\frac{T_R + T_H}{T_R + T_C}\right)$$

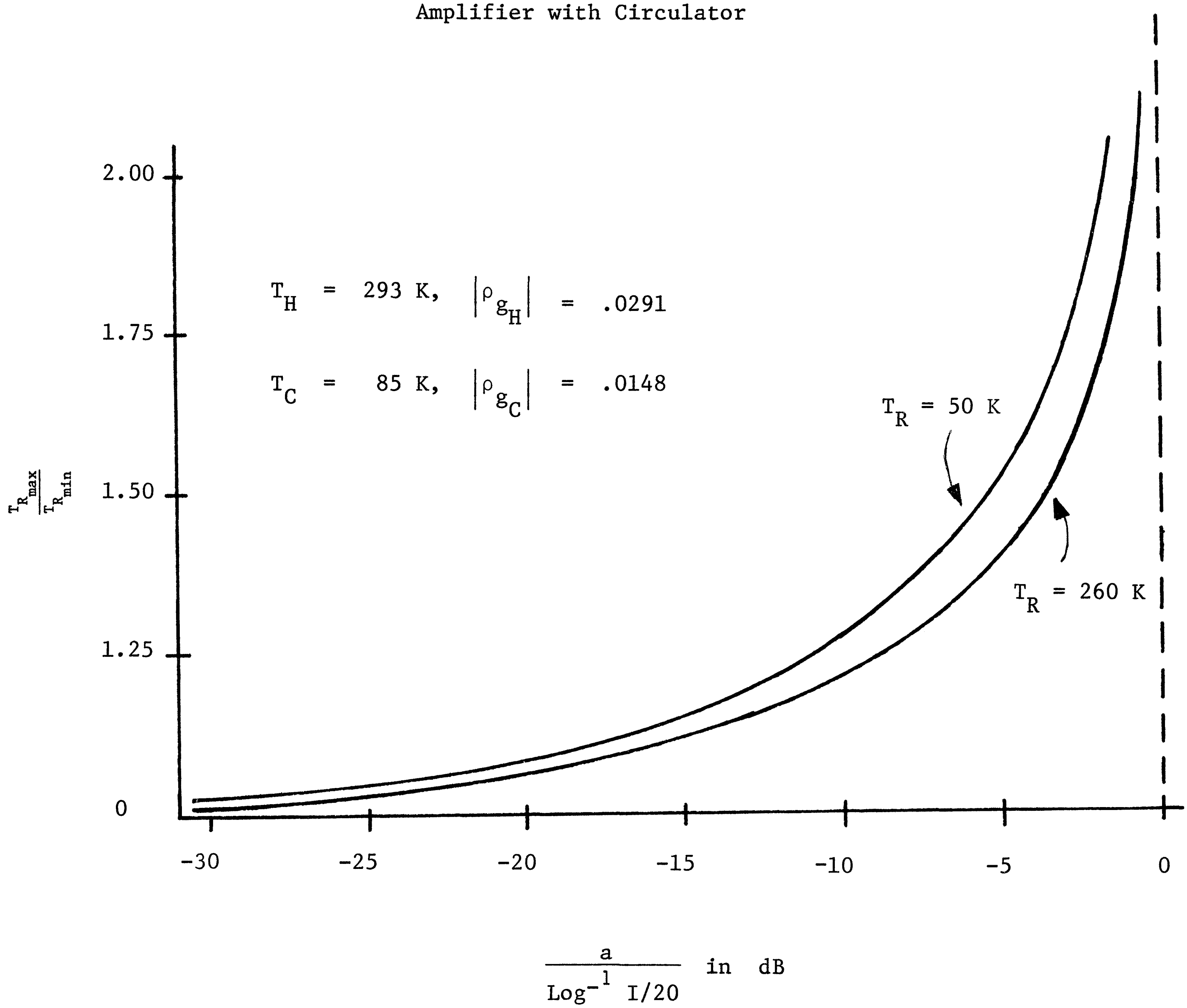
$$(L) = \left(\frac{1 - \left|\rho_{g_H}\right|^2}{1 - \left|\rho_{g_C}\right|^2}\right)$$

The maximum noise temperature error is plotted below for a given set of hot/cold loads and an expected receiver noise temperature. Obviously, the measurement error due to gain change is reduced for increasing isolation.

Noise Temperature Measurement Error

vs.

Reverse Power Flow
for Negative Resistance
Amplifier with Circulator



E. Conclusions

We have shown that gain changes induced by source impedance variations can have a significant contribution to noise temperature measurement error when using hot/cold loads, particularly for a negative resistance amplifier. The equations for expressing this error are general enough to be valid for any practical two-port network (Z_o = real and network reflection coefficients S_{11}, S_{22} matched to zero.)

Further, we have suggested a method for measuring the magnitude of the reverse power flow when this term is not known. This value can then be used to determine the possible noise temperature measurement error introduced by hot/cold load reflection coefficients.

We should point out that the noise temperature is affected by the source impedance in an additional way. Power flowing out of the network due to internally generated noise (termination on circulator for instance) has temperature T_{eff}. This will reappear at the input as ρ_g T_{eff} and increase the system noise temperature. For good hot/cold loads ρ_g is small, and this contribution is minimal. However, it cannot be ignored when the network is connected to an antenna or feed horn, where ρ_g is no longer small.

References

(1) K. Kurakawa, "Power Waves and the Scattering Matrix," IEEE Trans. Microwave Theory Tech., Vol. MTT-13, pp. 194-202, March 1965.

(2) G. Bodway, "Two-Port Power Flow Analysis using Generalized Scattering Parameters," Microwave J., Vol. 10, pp. 61-69, May 1967.