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MEASUREMENT OF OUTPUT CHARACTERISTICS OF A RADIOMETER

By

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The characteristics of a radiometer system are quite well known as far as the effects of thermal noise in the system are concerned. The necessary formulas for treating the sensitivity of a total power receiver (under idealized conditions) were given by Rice (1945), and the sensitivity of the switched radiometer has been discussed by Goldstein (1956), among others. The effect of instabilities in the system gain upon the overall sensitivity, however, has not been thoroughly discussed. The switching scheme was invented by Dicke (1946) to overcome the effects of gain instabilities in a total power receiver. Strum (1958) mentioned the loss in sensitivity which gain instabilities would introduce and depicted the gain as being a random function of time and having a certain probability distribution function.

In order to get some idea as to the effect instabilities in the system gain may have upon a radiometer output characteristics, it was felt that an analysis of a radiometer's output noise was of some interest, and this note gives the results from such an analysis.

The output noise from a switched radiometer of the type described by Orhaug and Waltman (1962) was recorded by using a digital output system. The system noise temperature was 1200 °K (at 1400 mc) including the terminating load at the input to the switch. Two different runs were taken, each run occupying 3 hours, and the output time constant was 2 seconds while the gate time of the digital readout was 5 seconds. The system time constant is thus determined by the gate time and is a finite memory type integrator. In the first run the input to the receiver was balanced while in the second run an unbalance signal of 10 °K was inserted through a directional coupler. In the first case the effects of gain instabilities should be absent, according to the theory of the switched radiometer, and the only output fluctuations should be noise fluctuations and eventually zero drift. In the second case, however, there should be additional fluctuations caused by gain instabilities, and the output fluctuation term should be proportional to the unbalance signal (in our case 10 °K).

Our first task is to analyze the output fluctuations for different effective integration time. We know that if only thermal noise is present in the system then the rms-value of the output fluctuations is given in terms of equivalent input noise temperature variations.

$$\Delta T = C\frac{T_{tot}}{\sqrt{B\tau}} \tag{1}$$

where T_{tot} is the total system temperature, B is the rf-bandwidth, and τ the effective integration time. C is a constant dependent upon the actual system parameters (approximately equal to unity). The rms-fluctuations were then computed (by the use of a Bendix G-20 computer). The next time two consecutive output readouts were averaged (thereby doubling the effective integration time) and the rms-value was computed. The third time three consecutive readouts were averaged, giving the effective integration time three times the initial integration time. The result is then a series of computed rms-values as a function of integration time, $\Delta T_c(n\tau_o)$, where τ_o is the initial integration time (= 5 seconds). From Equation (1) we have

$$\Delta T\sqrt{n\tau_o} = C\frac{T_{tot}}{\sqrt{B}} \tag{2}$$

which means that a plot of $\Delta T_c\sqrt{n\tau_o}$ against n is a constant, dependent only upon the system parameters, if only the effect of thermal noise is present.

If, on the other hand, we assume that the gain instabilities can be approximated with a gaussian random process and if the power spectrum associated with the instabilities is concentrated to low frequencies (i.e., to frequencies below b_1), then we have the approximate situation as depicted in Figure 1.

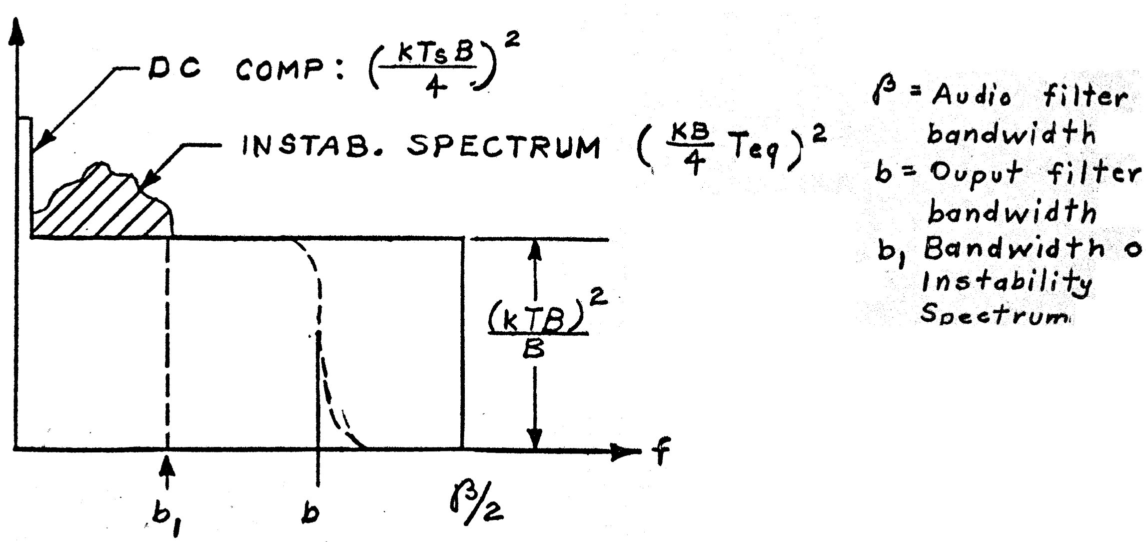

Fig. 1 -- Approximate power spectrum when signal (T_s) and gain instabilities are present. Situation depicted for output of phase detector. Square law detector assumed.

We have assumed an input signal T_s °K and also assume that the gain instabilities in the system have a mean square value of $\overline{(\Delta G/G)^2}$. If a switched receiver with a gain modulator is used, then the equivalent rms input noise temperature variation is, for a square law detector (Orhaug and Waltman (1962))

$$T_{eq} = (T_C + T_R)\left[\frac{T_A + T_R}{T_C + T_R} - K\right]\sqrt{\overline{\left(\frac{\Delta G}{G}\right)^2}} \tag{3}$$

where T_A is the antenna temperature, T_C is the temperature of the comparison channel, and K is the gain modulation factor ($K = G_C/G_A$). For K = 1 (as in our case) we have

$$T_{eq} = T_{unb}\sqrt{\overline{\left(\frac{\Delta G}{G}\right)^2}} \tag{4}$$

where

$$T_{unb} = T_A - T_C = 10\ °K$$

The signal to noise ratio is now

$$\frac{s}{n} = \frac{\left(\frac{kT_s B}{4}\right)^2}{\left(\frac{kT_{eq} B}{4}\right)^2 + \frac{(kTB)^2}{B} \cdot b}$$

provided that the output filter bandwidth is larger than the largest frequency of the instability spectrum ($b > b_1$). Setting $s/n = 1$ gives in the usual way the rms output noise fluctuations

$$T_s = \Delta T = \left[T^2_{eq} + \left(\frac{CT}{\sqrt{B\tau}}\right)^2\right]^{1/2} \tag{5}$$

where

$$C \mp 4\sqrt{C_1}$$

and

$$C_1 = b\tau$$

We notice that $C_1 = 1/2$ for an integration of the finite memory type ($b = \frac{1}{2\tau}$) (as we have actually used by taking output readings with a 5-second gate time). Writing $\tau = n\tau_o$ we get

$$\frac{\Delta T\sqrt{n\tau_o}}{\dfrac{CT}{\sqrt{B}}} = \left\{1 + \frac{T^2_{eq}\, n\tau_o}{\left(\dfrac{CT}{\sqrt{B}}\right)^2}\right\}^{1/2}$$

when n = 1 then $\dfrac{CT}{\sqrt{B\tau_o}} >> T_{eq}$ if τ_o is small enough, and therefore

$$(\Delta T)_N\sqrt{n\tau_o} \simeq \sqrt{1 + \left(\frac{T_{eq}}{\Delta T_1}\right) n\tau_o}$$

where $(\Delta T)_N = 1$ for n = 1, and $\Delta T_1 = \dfrac{CT}{\sqrt{B}}$ = noise fluctuations for 1 second integration time. For no gain instability, $T_{eq} = 0$, and therefore $(\Delta T)_N\sqrt{n\tau_o} = 1$ independent of integration time. It should be emphasized that we have arrived at this expression for the output fluctuations by assuming that the instability power spectrum is concentrated to frequencies below b_1, and that b_1 always is smaller than the output filtering bandwidth b. If, on the other hand, the instability power spectrum consists of a white continuous spectrum in addition to a low frequency part just discussed, the only difference will be that the noise fluctuations would be larger; the "white" part of the instability power spectrum would have the same effect as an increase in receiver noise temperature. Since no absolute calibration was applied in the observations reported here, there is no way to distinguish between the instability power spectrum according to Figure 1 and the same power spectrum in addition to a white spectrum. Figure 2 shows values of $(\Delta T)_c\sqrt{n\tau_o}$ plotted as a function of $n\tau_o$. The normalized rms output fluctuations are here called $(\Delta T)_c$. We should notice that if N_o is the number of output readouts used for the computation of $(\Delta T)_c$ for n = 1, then $N = N_o/n$ is the number of effective output readouts for arbitrary n. The standard deviation of the computed $(\Delta T)_c$-values is now $(\Delta T)_{TH}(2N)^{-1/2}$, where $(\Delta T)_{TH}$ is the theoretical rms-value, and the uncertainty of the computation therefore increases as $\sqrt{n}$. In Figure 2 is also shown the standard deviation where $(\Delta T)_{TH}$ is taken to be unity. The graph shows that the experimental $(\Delta T)_c$-values hardly significantly deviate from unity. There is a slight trend for smaller $(\Delta T)_c$-values for increasing n, for n larger than approximately 15. This point will be discussed later.

The output rms-fluctuations for the case with an input unbalance signal of 10 °K are shown in Figure 3 together with the curve $\left(1 + \left(\dfrac{T_{eq}}{\Delta T_1}\right)^2 n\tau_o\right)^{1/2}$. The value $\left(\dfrac{T_{eq}}{\Delta T_1}\right)^2 \tau_o = \alpha\tau_o = \dfrac{1}{70}$ is fitting the observational results best. The observed rms-values are seen to agree extremely well with the simple picture recently outlined.

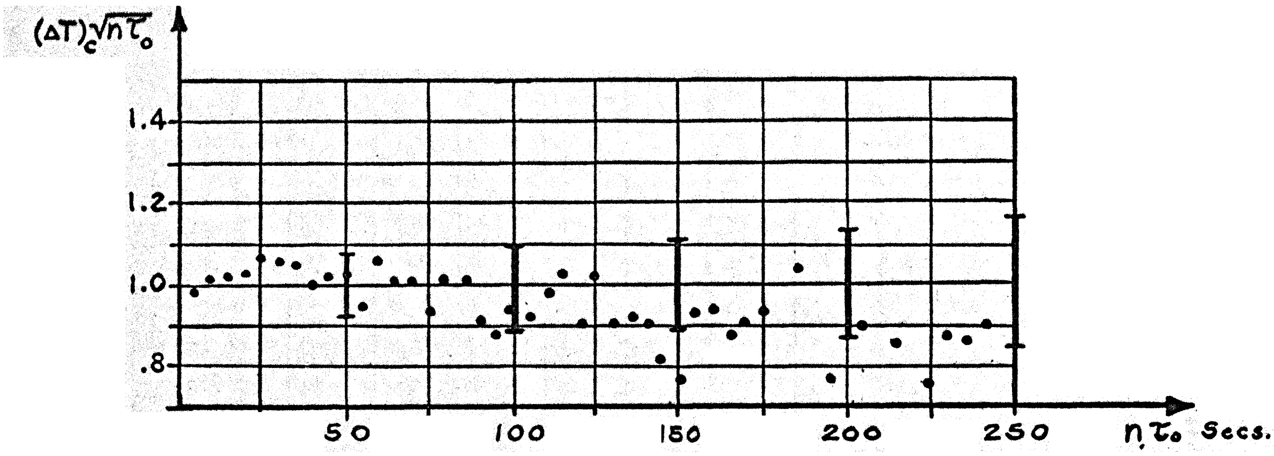

Fig. 2 -- Experimental rms output fluctuations as a function of effective integration time $n\tau_o$ for a switched radiometer having balanced input.

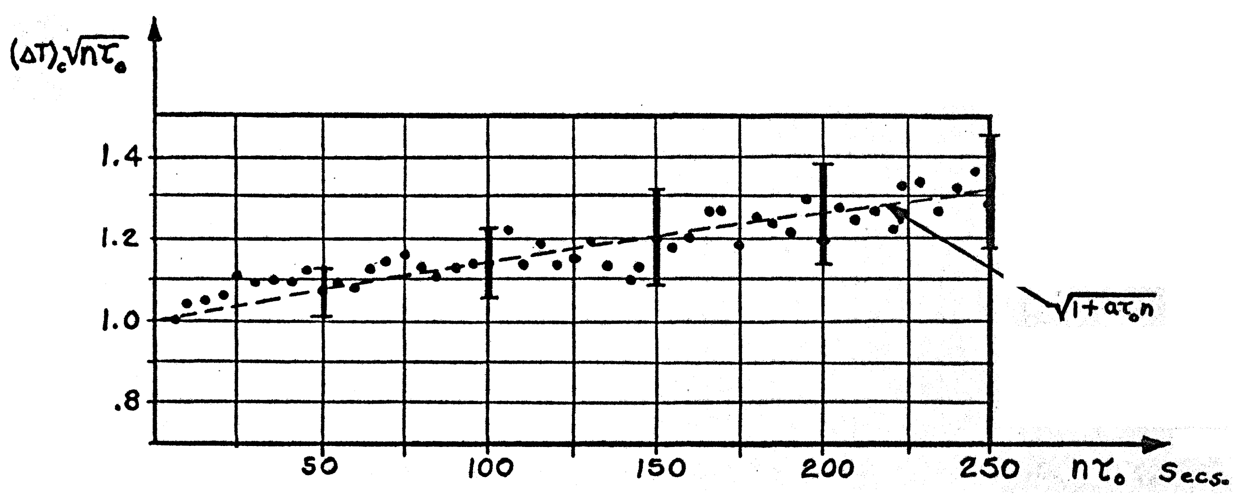

Fig. 3 -- Experimental rms output fluctuations as a function of effective integration time $n\tau_o$ for a switched radiometer having an input unbalance of 10 °K.

By means of the 10 °K input signal the rms-fluctuations for n = 1 are found to be 0.45 °K. This means that $T_{\sim\sim} = 0.042$ °K, or

$$\left[\overline{\left(\frac{\Delta G}{G}\right)^2}\right]^{1/2} \simeq 0.4\%$$

This value of the instability agrees with that found when using the radiometer as a total power radiometer.

For very large effective integration time when $b < b_1$, it can easily be shown that

$$\Delta T\sqrt{n\tau_o} = \left[T^2_{eq}\frac{C_1}{b_1} + \left(\frac{CT}{\sqrt{B}}\right)^2\right]^{1/2}$$

provided that the instability spectrum is flat between 0 and b_1 c/s. The normalized expression is

$$(\Delta T)_N\sqrt{n\tau_o} = \left[1 + \frac{T^2_{eq}C_1B}{(CT)^2b_1}\right]^{1/2} = \sqrt{1+\beta}$$

For very long integration times, the experimental rms-value should therefore be parallel with the n-axis, and the transition should occur for a n-value approximately given by

$$n \approx \frac{1}{b_1\tau_o}$$

In Figure 3 there is no sign to a transition to a horizontal curve for the experimental points, and we therefore conclude that the main power in the instability spectrum is situated below approximately $b = \frac{1}{n_{max}\tau_o} = \frac{1}{50 \times 5} = 4 \times 10^{-3}$ c/s = 0.24 c/min.

Another way of representing the result in Figure 2 is to make a plot of $(\Delta T)_c$ as a function of $n\tau_o$, and this is done in Figure 3. Using logarithmic scales, Equation (1) is represented by parallel straight lines, and three such curves are shown for three different (normalized) system temperatures.

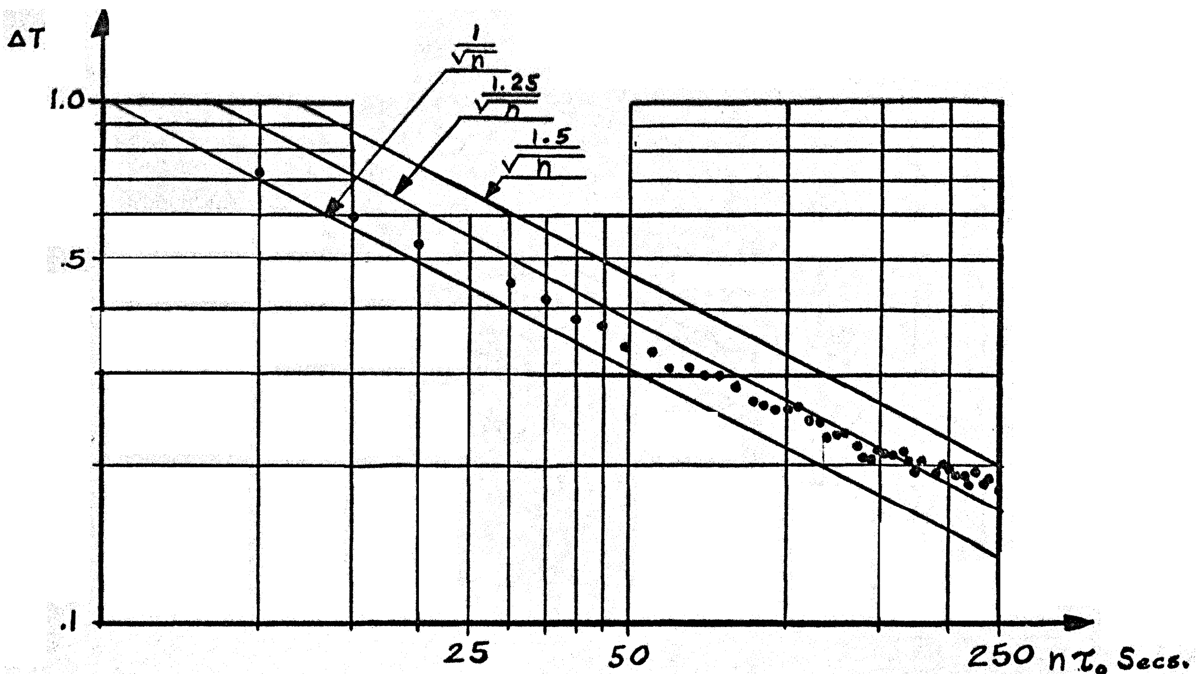

Fig. 4 -- Normalized rms output fluctuations as a function of normalized integration time for different normalized noise temperatures.

We can again see the feature of the output fluctuations when an unbalance is present. The output fluctuations do not decrease according to (integration time)$^{-1/2}$. For longer integration time, the output fluctuations decrease with a rate which is less than that expected from thermal noise alone in the system. We can describe the situation by saying that the system acts like it has, for longer integration time, a larger noise temperature than actually is the case. From Figure 4 we can see that an increase of time constant from 5 seconds to 250 seconds increases the "equivalent" noise temperature with approximately 35 percent.

A Fourier analysis was also made for the output data for the two different conditions described before. The computer program was made according to a paper by Goertzel (1960), where the coefficients in the expansion

$$f_M = \frac{1}{2}\alpha_o + \sum_{p=1}^{N} \left[\alpha_p \cos\frac{2\pi Mp}{2N+1} + \beta_p \sin\frac{2\pi Mp}{2N+1} \right]$$

are computed.

f_M is here the values of the output function f(t) at the points

$$t_M = \frac{2\pi M}{2N+1} \quad (M = 0, 1 \ldots 2N)$$

The lowest frequency in this series expansion is $(2N + 1)\tau_o^{-1}$. Because of the finite memory space of the computer used, it was necessary to divide the 3-hour run into 4 blocks, each block containing approximately 425 data points or 2125 seconds. This means that the lowest frequency in our expansion is 0.028 c/min., or approximately one-tenth of the upper frequency limit for the instability spectrum. Figures 5 and 6 show the results of the Fourier analysis. Figure 5 is the result of the receiver run during balanced condition, and we notice that for lower frequencies the spectrum has a slope toward the p-axis. The rms-fluctuations for balanced conditions followed the simple theory for $\tau < 15 \times 5 = 75$ seconds, and this corresponds to an equivalent noise bandwidth of

$$b = \frac{1}{2n\tau_o} = \frac{1}{2 \times 15 \times 5}$$

and the p-value corresponding to this frequency is

$$p = \frac{N}{n} = 28$$

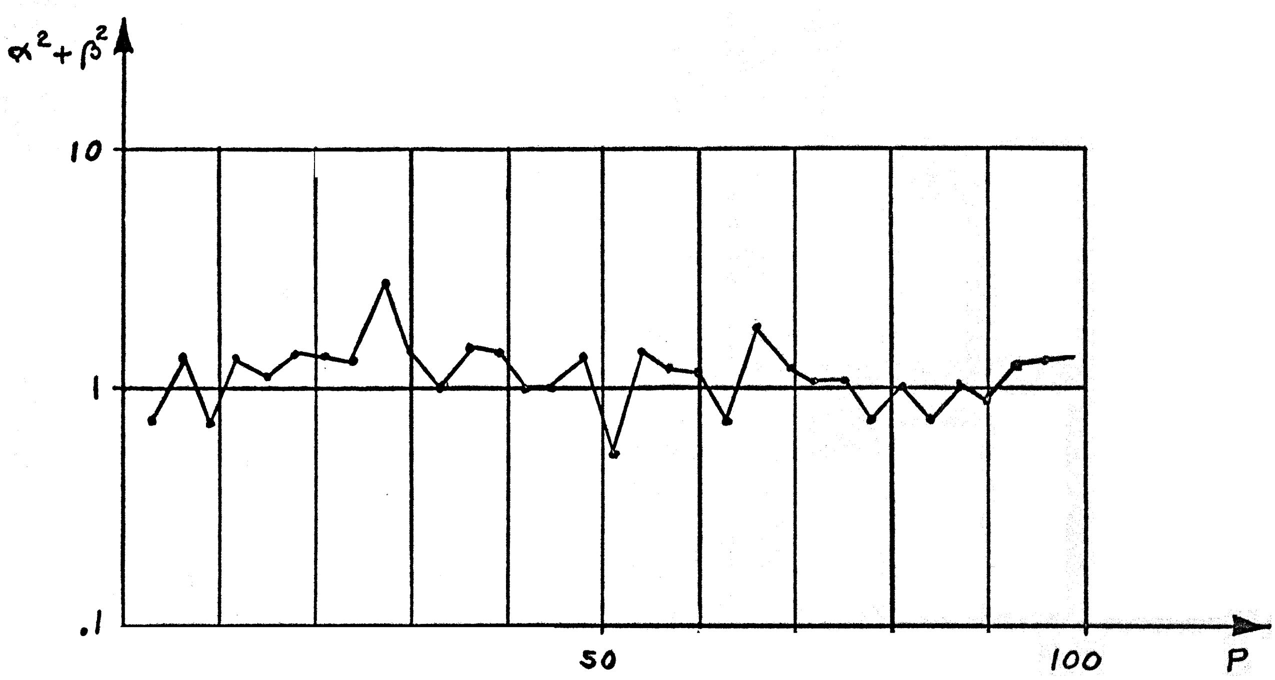

Fig. 5 -- Power spectrum for balanced condition.

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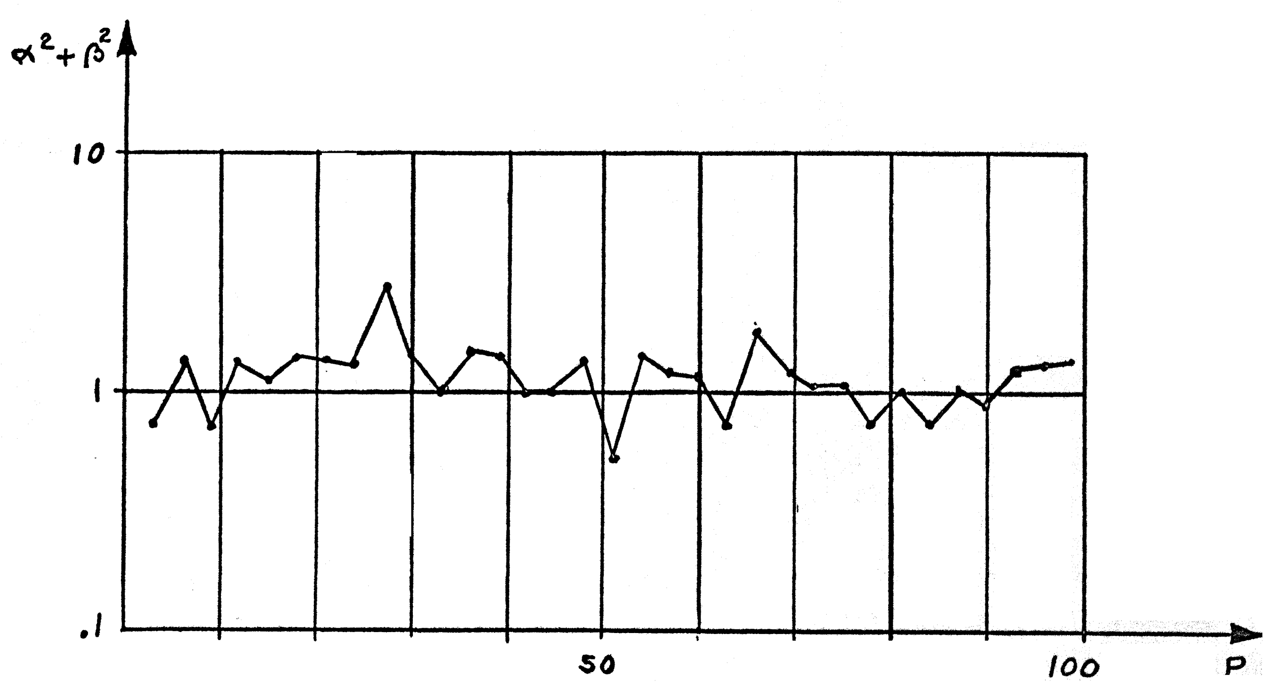

Fig. 5 -- Power spectrum for balanced condition.

This agrees very well with the p-value of the maximum in the power spectrum as shown in Figure 5. Also, the shape of the power spectrum at lower frequencies would cause the $(\Delta T)_c\sqrt{\tau}$-values to decrease with τ. We therefore conclude that the rms-fluctuations and the power spectrum agree well with each other's performance. The power spectrum (Figure 6) associated with the unbalanced condition is more flat than the previous one, but there is a slight increase for the lowest frequencies. his is probably the indication of the instability power spectrum.

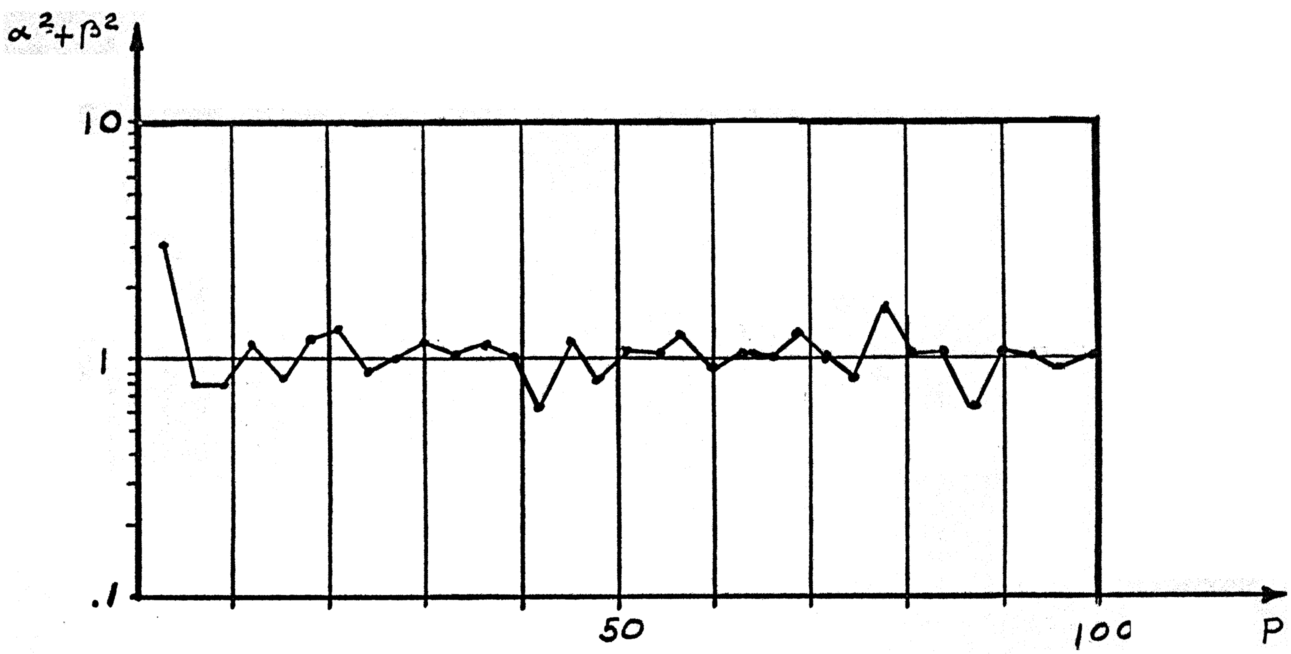

Fig. 6 -- Power spectrum for unbalanced condition.

Let us next investigate the expected density of the instability power spectrum. If we assume that the power is uniformly distributed between o and b_1 c/s, then the density in ratio to the density in the spectrum of the thermal noise is

$$\frac{\sigma_{inst}}{\sigma_{noise}} = \left(\frac{T_{eq}}{T}\right)^2 \frac{B}{4b_1} = \overline{\left(\frac{\Delta G}{G}\right)^2}\left(\frac{T_{unb}}{T}\right)^2 \frac{B}{4b_1}$$

Assuming B = 6 mc, $\overline{\left(\frac{\Delta G}{G}\right)^2} = 10^{-4}$, T = 1200 °K, T_{unb} = 10 °K, and $b_1 = 4.10^{-3}$ c/s, we get

$$\frac{\sigma_{inst}}{\sigma_{noise}} \simeq 1,6$$

From Figure 6 we find that the ratio between the density of the instability power spectrum and the thermal noise spectrum is approximately 2, but the instability spectrum is probably increasing quite rapidly towards lower frequencies. The magnitude of the power density of the instability power spectrum therefore agrees with the order of magnitude to be expected from the other information we have obtained.

The results presented here show that even in a switched radiometer the results of the instabilities may play an important role. The analysis of the output fluctuations for various effective integration time shows that the result is in agreement with a model assuming the gain instabilities to be a gaussian random process whose power spectrum is concentrated to low frequencies. An upper limit of the highest frequency of the power spectrum is found to be 0.24 c/min., and this result is in agreement with the Fourier analysis made of the radiometer output fluctuations.

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