WHY WOULD WE SEE 2-D TURBULENCE IN INTERSTELLAR GASES?

Anthony H. Minter (NRAO)

ABSTRACT

Neutral gas in the galaxy traced through the H I 21 cm line and the CO (J 1→0) line, as well as the ionized gas seen in Hα and radio recombination lines have power spectra of density, column density and velocity whose slopes are consistent with 2-Dimensional turbulence on large spatial scales ($> 0.01 - 1$ pc). We know, however, from ***in situ*** measurements that the turbulence in fluids on the Earth and in the solar wind is fully 3-Dimensional. We should also expect the observed turbulence in the interstellar medium to be 3-Dimensional.

A method had been devised to make "snapshot" models of the density and velocity fields of a turbulent gas. The desired power spectra (density and velocity) are the only inputs into the model. These models have been used to study how propagation effects and the various modes of observing can change the 3-Dimensional Kolmogorov-like turbulence input into the models into the observed 2-Dimensional turbulence. The following effects can make the observed turbulence appear 2-Dimensional: 1) if the turbulence is contained in a thin filament or slab; 2) if the medium has a high optical depth; and 3) if any method of observation or analysis is used which effectively limits the emission from the medium under study to a thin slab, for example, by analyzing an individual channel map. Straightforward analysis of data leads to misleading or incomplete results if these effects are not taken into account.

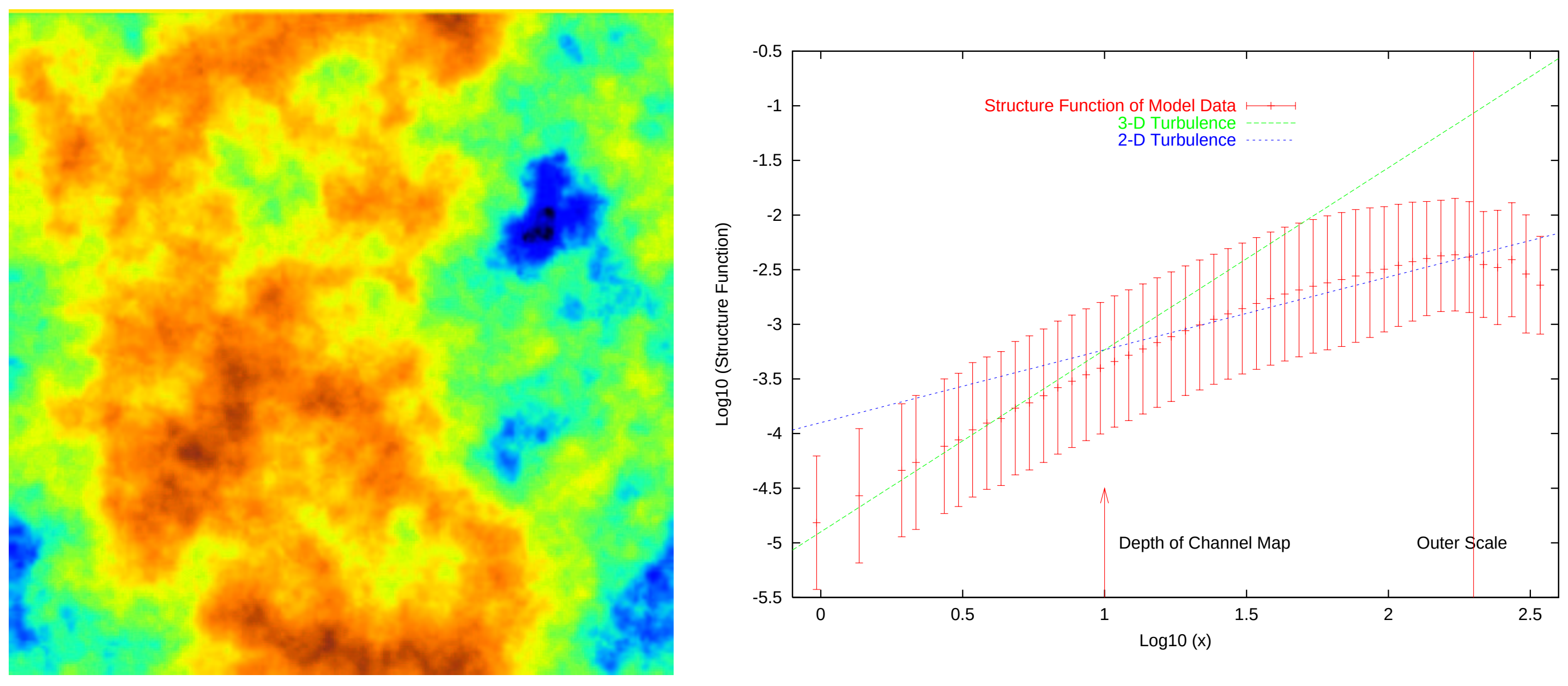

Figure 1. Image of "observable" flux from a gas whose emission is constrained to a single channel map and its corresponding structure function. The channel map has an equivalent width of 10 pixels. The outer scale of the turbulence is 200 pixels and is shown by the solid line on the left. It is easily seen that there is a transition from 3-D turbulence to 2-D turbulence on the size scale of ∼ 10 pixels.

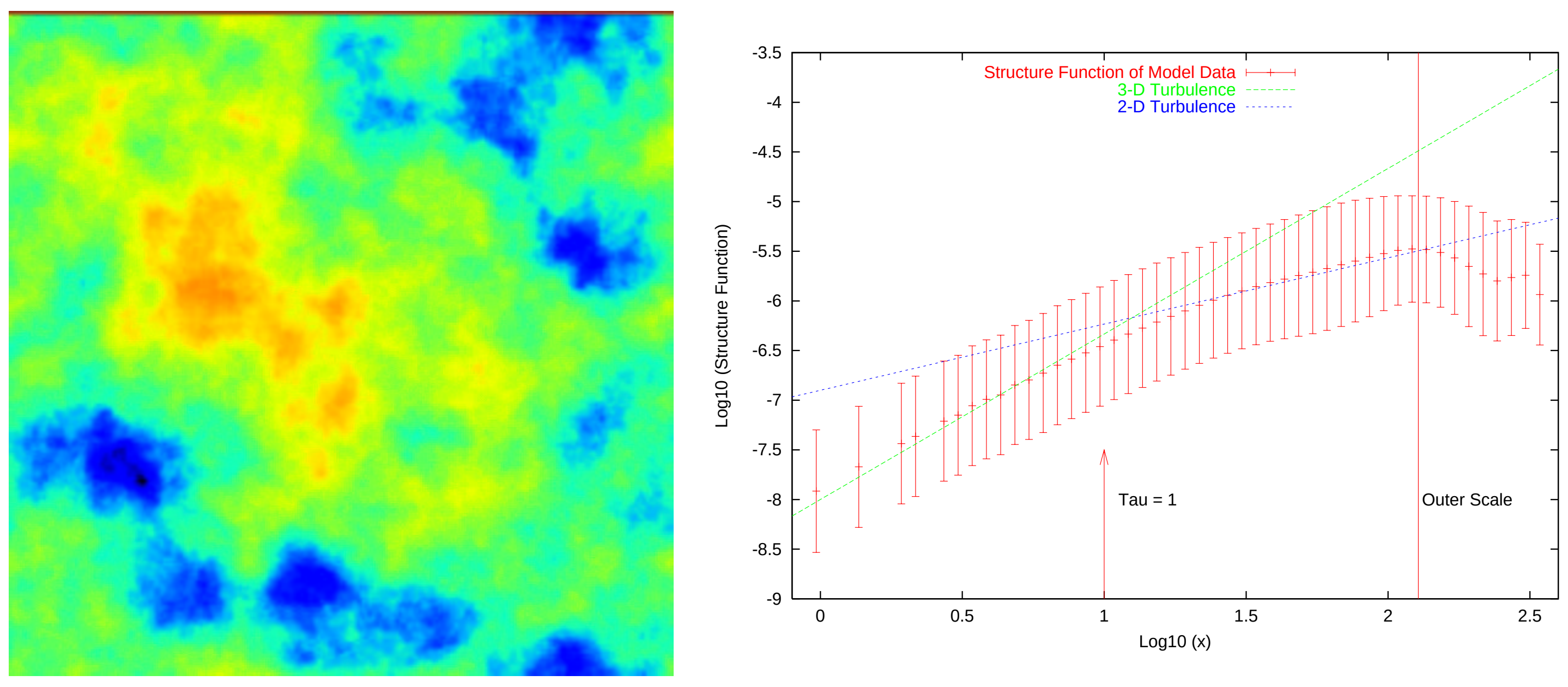

Figure 2. Image of "observable" flux from a gas which is optically thick and its corresponding structure function. The outer scale of the turbulence is 128 pixels and is shown by the solid line on the left. The optical depth at ten pixels is $\tau \sim 1$. It is easily seen that there is a transition from 3-D turbulence to 2-D turbulence around an optical depth of $\tau \sim 1$.

WHY NOT 3-D TURBULENCE

- *in situ* measurements of turbulence (Earth, Solar Wind) are 3-D.
- Small scale n_e variations in the ISM are 3-D.
- Troposphere exhibits change from 3-D to 2-D for $l >$ few km.
- Could physical conditions or observation techniques → 2-D.
- Can test this hypothesis with models.

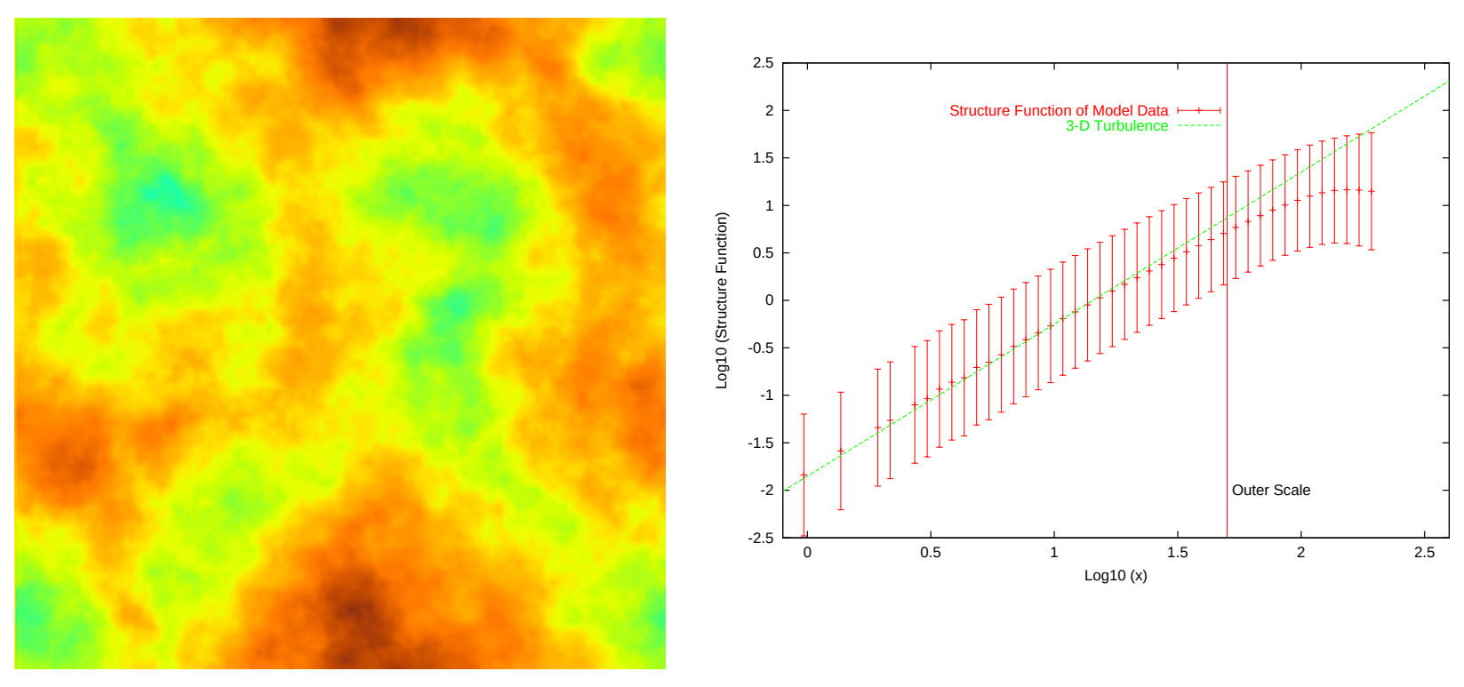

Figure 3. Image of "observable" flux from a gas which the full data cube is sampled and its corresponding structure function. The outer scale of the turbulence is 1000 pixels. It is easily seen that 3-D turbulence is observed on all relevant size scales. The outer scale for the turbulence is 50 pixels and is shown by the solid vertical line.

OBSERVATIONS OF 2-D TURBULENCE IN THE GALAXY

H I 21 cm observations with interferometers

- Measure column density power spectrum directly.
- $\alpha \sim 2.8 \pm .2$ (Kolmogorov turbulence has $\alpha = 3.67$).
- $0.01\ pc < l < 300\ pc$.

CO (J 1 → 0) single dish maps

- Measure column density power spectrum through structure function.
- $\alpha \sim 2.8 \pm 0.2$.
- $0.01\ pc < l <\sim 10\ pc$.

Diffuse ionized gas observed in Hα

- Intensity proportional to EM= $\int n_e^2 dl$.
- Measure column density power spectrum through EM structure function.
- $\alpha \sim 2.7 \pm 0.1$.
- $5\ pc < l < 100\ pc$.
- 3-D turbulence for $l < 10^{15}\ cm$.

Velocity centroid fluctuations are consistent with 2-D velocity fluctuations on the same scales for all observations.

W 43

- Observed in Hydrogen radio recombination lines with VLA.
- In single channel map observe 2-D turbulence.
- Observe 2-D turbulence when image integrated over whole line.
- Opacity is not a factor in Hydrogen radio recombination lines.
- Conclusion: Turbulence contained in thin filaments ≈ 0.1 pc in size which is consistent with measured thickness of observed filaments.

TURBULENCE MODEL

- Snapshot (in time) model.
- Define power spectrum (slope, level, inner and outer scale).
- Use power spectra of density and velocity.
- Let $\Delta n \sim n/3$ and $\Delta v \sim v/3$.
- 512^3 grid.
- Use $\sqrt{\text{power spectrum}}$ and random phase to determine $\mathcal{N}$ and $\mathcal{V}$.
- FFT $\mathcal{N} \rightarrow n$ and $\mathcal{V} \rightarrow v$.
- Integrate along z-axis including "radiative transfer" to reproduce observations.

RESULTS

What can cause the observed 2-D turbulence?

- Emission from thin sheet or filaments.
- Opacity.
- Channel maps limiting emission to be from thin slices of space.

Incomplete or misleading results can be obtained if one does not take into account any effect which changes the observed turbulent characteristics from 3-D to 2-D.

Theory

In this section we will assume that the observed intensity is related to the column density of the neutral gas integrated to some depth z_o into the medium. The value of z_o can be a finite number for spectral line observations assuming that the galactic rotation curve can be converted into a distance, ignoring velocity fluctuations. It can be made finite by the opacity of the medium or it could be finite due to the neutral gas being contained in slabs or filaments with a size of z_o. We can write the column density as $N(x,y) = \int_0^{z_o} n(x,y,z)w(z)dz$ where $w(z)$ is a weighting function. The density $n(x,y,z)$ can be broken up into a constant term and a fluctuating term such that $n(x,y,z) = n_o + \delta n(x,y,z)$ and $\langle \delta n(x,y,z) \rangle = 0$ where $\langle \rangle$ signifies the ensemble average over all space. We will assume that the fluctuations are isotropic and homogeneous. We will assume that the gas contains Kolmogorov-like turbulence as is observed for neutral fluids on Earth. The power spectrum of the density fluctuations can thus be written as $PS_n(\vec{k}) = C_n^2 \left(k^2 + k_o^2\right)^{-\alpha/2}$ for $k < k_i$ and $PS_n(\vec{k}) = 0$ for $k > k_i$ where C_n^2 indicates the strength of the turbulence, $k^2 = k + x^2 + k_y^2 + k_z^2$, $k_o = \frac{2\pi}{l_o}$, $k_i = \frac{2\pi}{l_i}$, with l_o being the largest size scale of the turbulent density fluctuations and l_i is the smallest size scale of the turbulent fluctuations (*i.e.* the turbulent dissipative scale). For Kolmogorov-like turbulence, the power spectrum index is $\alpha \approx \frac{11}{3}$. The power spectrum and the density are related as follows: $\langle \delta n(x,y,z)\delta n(x+\delta x, y+\delta y, z+\delta z) \rangle = \int_{-k_i}^{k_i}\int_{-k_i}^{k_i}\int_{-k_i}^{k_i} PS_n(\vec{k}) e^{2\pi\imath\vec{k}\cdot\vec{\delta x}} d^3k$ where $\langle \delta n(x,y,z)\delta n(x+\delta x, y+\delta y, z+\delta z) \rangle$ is the spatial auto-correlation function of the density. Since the observed intensity is assumed to be proportional to the column density $I(x,y) = CN(x,y)$ for optically thin line emission such as the H I 21 cm line and the CO (J 2 → 1) line, we can write

$$\left\langle I(\vec{x})I(x \vec{+} \delta x)\right\rangle = C^2 n_o^2 z_o^2 + C^2 \left\langle \int_0^{z_o} \delta n(x,y,z)dz \int_0^{z_o} \delta n(x+\delta x, y+\delta y, z')dz' \right\rangle = C^2 n_o^2 z_o^2 + C^2 \int_0^{z_o}\int_0^{z_o} \langle \delta n(x,y,z)\delta n(x+\delta x, y+\delta y, z')\rangle dz dz' = C^2 n_o^2 z_o^2 + C^2 \int_0^{z_o}\int_0^{z_o}\int_{-k_i}^{k_i}\int_{-k_i}^{k_i}\int_{-k_i}^{k_i} C_n^2 \left(k^2 + k_o^2\right)^{-\alpha/2} e^{2\pi\imath\left(k_x\delta x + k_y \delta y + k_z\left(z - z'\right)\right)} dk_x dk_y dk_z dz dz'.$$

When the integration depth is much larger than the largest turbulent size scale ($z_o \gg l_o$) we are in a regime where we can let $z_o \rightarrow \infty$ in the limits of integration. This is equivalent to assuming that the fluctuations fill all of space, resulting in

$$\left\langle I(\vec{x})I(x \vec{+} \delta x)\right\rangle = C^2 \int_{-k_i}^{k_i}\int_{-k_i}^{k_i} C_n^2 \left(k_x^2 + k_y^2 + k_o^2\right)^{-\alpha/2} e^{2\pi\imath\left(k_x\delta x + k_y\delta y\right)} dk_x dk_y.$$

Thus we see that when $z_o \gg l_o$ the spectral index of the density power spectrum is the same as the spectral index of the column density power spectrum.

When the integration depth is within a factor of a few or smaller of the largest turbulent size scale we cannot use the above approximation. This case may arise under several circumstances: 1) the emitting gas is confined spatially to a filament or thin sheet, 2) opacity effectively allows you to see only a short distance into the emitting region, and 3) spectral line observations combined with a velocity-distance relationship confine the emitting gas to be from a limited region of space. Under these circumstances the assumption that $z_o \rightarrow \infty$ is no longer valid. In the following we will assume that z_o is finite, *i.e.* $z_o \in (-\infty, \infty)$. Integrating over z and z' we have $\left\langle I(\vec{x})I(x \vec{+} \delta x)\right\rangle = C^2 n_o^2 z_o^2 + C^2 \int_{-k_i}^{k_i}\int_{-k_i}^{k_i}\int_{-k_i}^{k_i} C_n^2 z_o^2 \left(k^2 + k_o^2\right)^{-\alpha/2} \mathrm{sinc}\left(z_o k_z\right) e^{2\pi\imath\left(k_x\delta x + k_y\delta y\right)} dk_x dk_y dk_z$ where $\mathrm{sinc}\left(x\right) = \frac{\sin \pi x}{\pi x}$. This integral is not solvable in a closed analytic form. We can estimate its behavior by expanding the *sin* term in a Taylor series, giving $\left\langle I(\vec{x})I(x \vec{+} \delta x)\right\rangle = 2C^2 \sum_{j=0}^{\infty}\sum_{l=0}^{\infty} \frac{(-1)^{j+l}}{(2j+1)!(2l+1)!} \int_{-k_i}^{k_i}\int_{-k_i}^{k_i}\int_0^{k_i} \left(k^2 + k_o^2\right)^{-\alpha/2} \left(\pi z_o k_z\right)^{2j+2l} dk_z dk_x dk_y$ where we have used the fact that the power spectrum in an even function of k_z. The integral over k_z can be solved in terms of sums of hyper-geometric functions. If we are in the region where $k \ll k_i$ (*i.e.* when $\sqrt{x^2 + y^2} \gg l_i$) then we can let $k_i \rightarrow \infty$. The leading terms where $j + l \leq 1$ can then be solved into a direct analytic equation. This results in

$$\left\langle I(\vec{x})I(x \vec{+} \delta x)\right\rangle = 2C^2 \sum_{j=0}^{1}\sum_{l=0}^{1} \frac{(-1)^{j+l}}{(2j+1)!\,(2l+1)!} \left(1 - \delta(j + l - 2)\right) \int_{-k_i}^{k_i}\int_{-k_i}^{k_i} \frac{\left(k_x^2 + k_y^2 + k_o^2\right)^{-\alpha/2}}{\pi z_o} \left[\pi^2 z_o^2 \left(k_o^2 + k_y^2 + k_x^2\right)\right]^{j+l+1/2} \frac{\Gamma(j + l + 1/2)\Gamma(\alpha/2 - j - l - 1/2)}{\Gamma(\alpha/2)} dk_x dk_y$$

from which it is easy to see that the leading term in the column density power spectrum is $PS_N(k_x, k_y) \propto \left(k_x^2 + k_y^2 + k_o^2\right)^{(1-\alpha)/2}$. Thus, under common observing circumstances the column density power spectrum can have an index that is one plus the index of the density power spectrum: $\alpha_N = 1 - \alpha_n$ for $z_o \in (-\infty, \infty)$ and $l \gg l_i$.