# ABSTRACT

Recently there have been several papers that address the issue of the morphology of the structures observed in molecular clouds (e.g. Blitz & Williams, ApJL, 488, 1997 and Elmegreen & Falgarone, ApJ, 471, 816, 1996). These papers have found conflicting results. We discuss several methodologies for determining the structural morphology of interstellar gas, paying particular attention to the biases of each method. We use models of gas clouds with varying morphologies to show how the various analyses are biased. Knowing the biases of the analysis in recent literature, we place reasonable limits on the morphology of molecular clouds.

# PREVIOUS ANALYSIS OF TURBULENCE IN MOLECULAR CLOUDS: CONFLICTING RESULTS

•Blitz & Williams (ApJL, 1997, 488) looked at the distribution of column densities in the Taurus Molecular Cloud with varying resolutions (all with $d > 0.1$ pc) and found no evidence of turbulent structures.

•Falgarone & Phillips (ApJ, 1996, 472, 191) looked at the line profiles of many CO transitions in the Perseus-Auriga complex and find that they can best explain the observations with a turbulent-like density power law extending over many decades of size scales.

•Elmegreen & Falgarone (ApJ, 1996, 471, 816) looked at the distribution of mass and density with size for many molecular clouds in the galaxy and find turbulent-like power laws for these distributions.

•Kleiner & Dickman (ApJ, 1987, 312, 837) looked at the variations of line centers over the Taurus Molecular cloud using correlation functions and structure functions. They find that the velocity fluctuations have a turbulent-like power law distribution on size scales less than 0.1 pc.

•Stutzki, Bensch, Heithausen, Ossenkopf & Zielinksy (to appear in A&A 1998) used the Allan variance to show that structure of the Polaris Flare cloud exhibits a power law spectrum.

# CONTRIBUTION OF TURBULENT DENSITY FLUCTUATIONS TO COLUMN DENSITY

- Turbulent fluctuations occur on all size scales between the energy injection scale (usually the largest size scale of the turubulence) and the energy dissipation scale.
- Turbulent fluctuations at a given size scale have a Gaussian distribution with exponential wings.
- The largest density fluctuations occur on the largest size scales ($l_o$) of the turbulence.
- We can approximate the density fluctuations as having a Gaussian distribution and a cell size of $l_o$.
- For a column density with depth $L$ there will be $N = L/l_o$ "turbulent cells".
- The standard deviation from the mean column density at a given point will be proportional to $\sqrt{N}$.
- If $N >> 1$ then the column density will be dominated by the mean density.
- If the beam resolution size scale $d$ is such that $d >> l_0$ then the beam will average over $N' = Nd/l_o$ turbulent cells which will further reduce the column density's response to turbulent fluctutations.
- Blitz & Williams' analysis is thus biased in such a way that it is **not sensitive** to the turbulent fluctuations.

## Structure Functions

- The structure function $D(\delta x)$ of $f(x)$ is defined as

$$D_f(\delta x) = \left\langle (f(x_o) - f(x_o + \delta x))^2 \right\rangle$$

where the angular brackets mean an ensemble average.

- Let $f(x) = f_o + \delta f(x)$ where $\langle \delta f(x) \rangle = 0$ but $\left\langle \delta f(x)^2 \right\rangle \neq 0$ then

$$D_f(\delta x) = 2 \left[ \left\langle \delta f(x)^2 \right\rangle - \langle \delta f(x_o) \delta f(x_o + \delta x) \rangle \right] .$$

Hence the structure function is only sensitive to the fluctuating component.

## Correlation Functions

- The correlation function $C(x)$ of $f(x)$ is defined as

$$C_f(\delta x) = \langle f(x_o) f(x_o + \delta f) \rangle .$$

- With the same definition of $f(x)$ as above we have

$$C_f(\delta x) = f_o^2 + \langle f(x_o) f(x_o + \delta f) \rangle .$$

Renormalizing $C'_f(\delta x) = C_f(\delta x)/C_f(0)$ results in the correlation function only depending on the fluctuating part of $f(x)$.

- Kleiner & Dickman's analysis thus allows the detection of turbulent fluctuations that may be present by eliminating any response to the mean.

# Allan Variance

•The Allan variance $A(\delta x)$ of $f(x)$ is defined as

$$A(\delta x) = \frac{1}{2}\left\langle\left[\frac{1}{\delta x}\left(\int_{x-\delta x}^{x} f(x')dx' - \int_{x}^{x+\delta x} f(x'')dx''\right)\right]^2\right\rangle .$$

•It can be shown that the Allan variance $A(\delta x)$ is the power spectrum of $f(x)$ weighted by $sin^4\,\pi k\delta x/\left(\pi k\delta x\right)^2$ where $k = 1/\delta x$. The Allan variance is only sensitive to the fluctuating component of $f(x)$.

• Stutzki et al.'s analysis is thus biased toward the detection of turbulent fluctuations.

# Analysis of the Serpens Molecular Cloud

•The $C^{18}O\ J = 1 \rightarrow 0$ line was observed with the NRAO 12m telescope (McMullin et al., 1998, in prep).

•Figures 1 and 2 are from Blitz & Williams (ApJL, 1997, 488) showing their analysis for the Taurus molecular cloud. Figure 3 shows the same analysis for the Serpens molecular cloud at a resolution of $20''$ and at $80''$. There is no difference in the results of Blitz & Williams' analysis for the Taurus and Serpens molecular clouds.

•Figure 4 shows the structure function for the Serpens molecular cloud. The structure function clearly shows that there is a turbulent power spectrum present in the column density of the Serpens molecular cloud.

•Figures 3 and 4 demonstrate, with certainty, that Blitz & Williams method of analysis is strongly biased toward the mean. Their method of analysis is thus insensitive to the existence of turbulent structures in interstellar gases.

# The Characteristics of Turbulent Structures in Molecular Clouds

•The results of Stutzki et al.and the structure function of the Serpens molecular cloud are consistent with the power spectrum of molecular cloud column densities having a spectral index of $-8/3$. These observations demonstrate that molecular clouds are comprised of turbulent structures.

•Stutzki et al.showed that all determinations of the various power law relationships are consistent with the same underlying turbulent power spectrum.

•The turnover of the structure function in Figure 4 indicates the largest size scale of the turbulence and their intensity in the Serpens Molecular cloud. It was found that $\delta T_A = 2.22 \pm 0.01\ K\ km/s$. With $\langle T_A \rangle = 3.6 \pm 2.1$ we find that $\frac{\delta T_A}{T_A} = 0.62 \pm 0.5$ and that $\frac{\delta T_A}{T_{A\ max}} = 0.250 \pm 0.001$.

•From Figure 3 it is seen that the largest turbulent feature is roughly $12.8\ arcsec$ which corresponds to $0.028\ pc$ at 150 pc.

•Kleiner & Dickman found that the largest size scale for the turbulence in the Taurus molecular cloud is $\approx 0.01\ pc$.

# Conclusions

- Methods that can detect turbulent fluctuations:
  - Allan Variance
  - Structure Functions
  - Correlation Functions
- Methods that are insensitive to turbulent fluctuations:
  - Blitz & Williams (ApJL, 1997, 488) histogram method

- **Molecular clouds have a turbulent component. The column density fluctuations are characterized by a power spectrum with an index of $\approx -8/3$. The largest scale size of the turbulent fluctuations appears to be $\approx 0.02\ pc$ for two different molecular clouds. The intensity of the turbulence is such that $\frac{\delta N}{N} \approx 0.25$ where $N$ is the column density.**

# Modeling Turbulent Molecular Clouds Using AIPS++

•A model turbulent molecular cloud was created with AIPS++ by using a theoretical power spectrum $PS(\vec{k})$ to create the three dimensional Fourier transform of the density structure of a molecular cloud $\eta(\vec{k}) = \Im\left[n(\vec{x})\right]$. The amplitude of $\eta(\vec{k})$ was chosen from a Gaussian distribution with a mean of $\sqrt{PS(\vec{k})}$. The turbulent strength was chosen such that $\frac{\delta n}{n} = 0.33$. The phases were chosen randomly between zero and $2\pi$ such that $\phi(\vec{k}) = -\phi(-\vec{k})$.

•Once $\eta$ was determined it was Fourier Transformed into $n$ which was subsequently integrated along the z-axis to form the column density of the simulated molecular cloud.

• The column density “depth” is 8 turbulent cells for this model.

•Figure 5 showa the modeled turbulent molecular cloud along with Blitz & Williams method of analysis for the cloud and the structure function of the cloud. It is readily seen that Blitz & Williams method of analysis cannot detect the *a piori* turbulent fluctations that are present in this model.

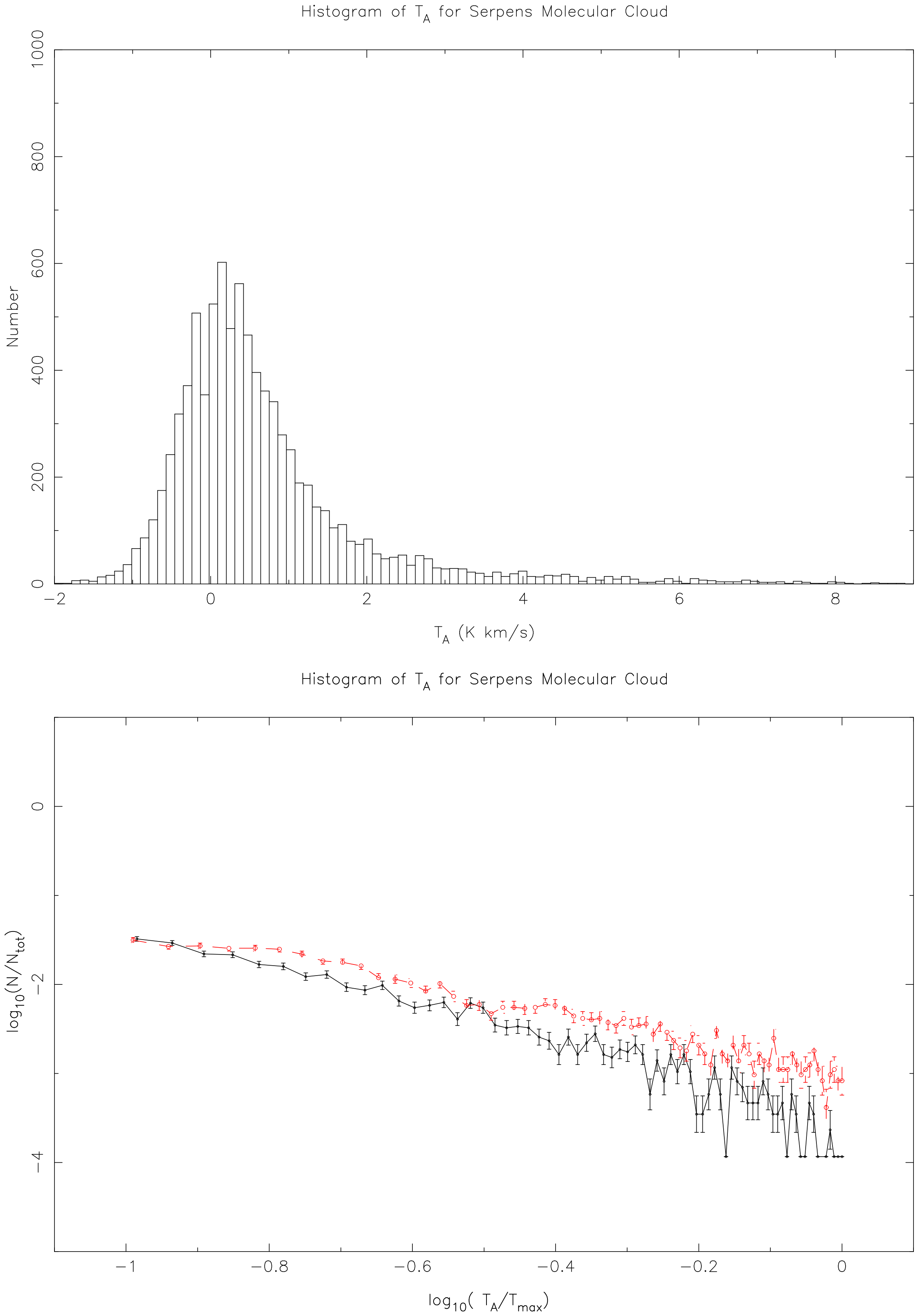

Figure 3. The top panel shows the histogram distribution of the Serpens Molecular cloud data. The lower panels shows the normalized histogram distribution for the Serpens molecular cloud at 20 arcsec resolution (black) and at 80 arcsec resolution (red). These panels are the same as Figures 1 and 2 except they are for the Serpens molecular cloud.

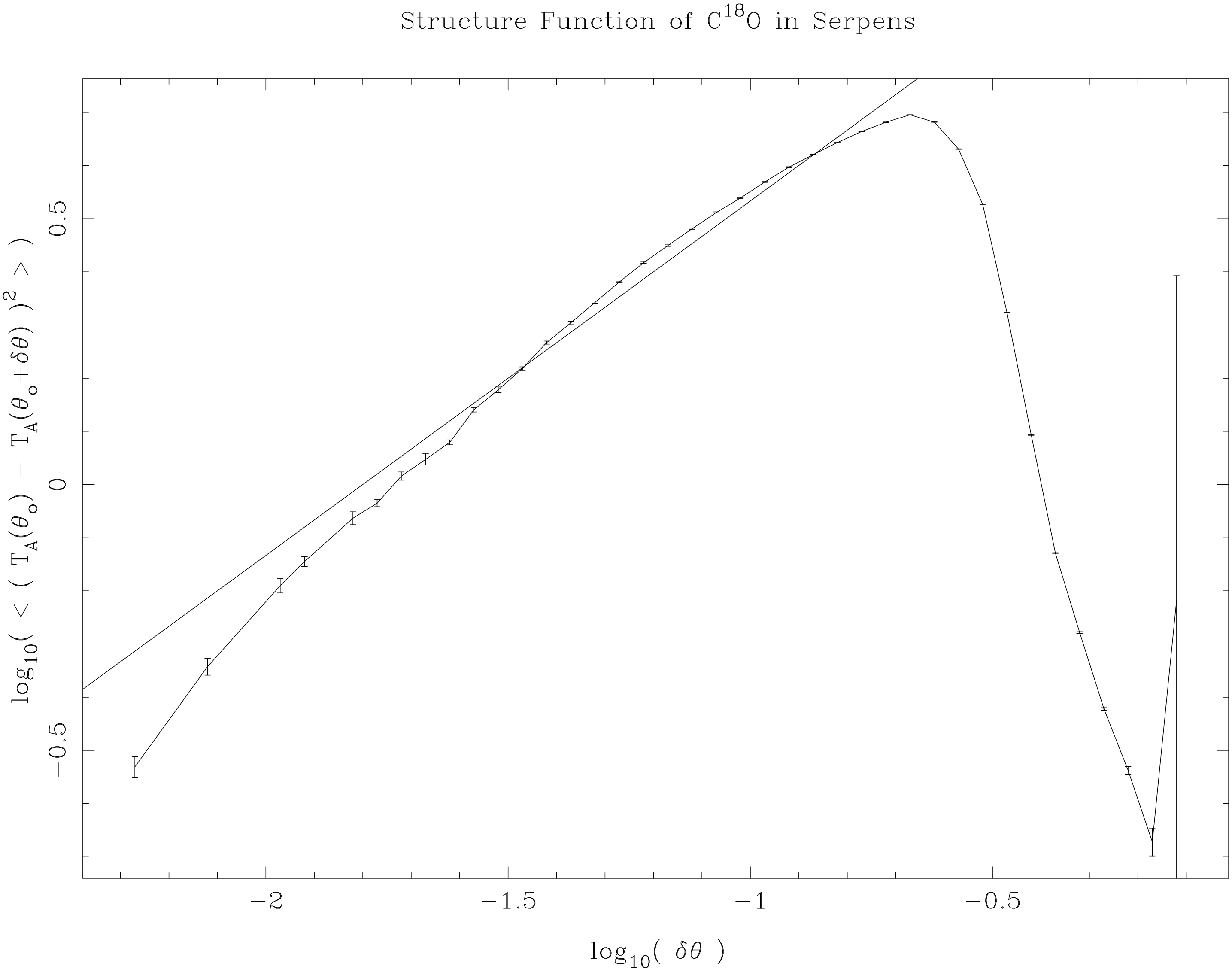

Figure 4. The structure function of the Serpens molecular cloud. A power law with an index of 2/3 is shown for comparison. (An index of 2/3 is what is expected from a power spectrum with an index of 8/3.) The largest size scale of the turbulence is where the structure function peaks at 20 arcmin.

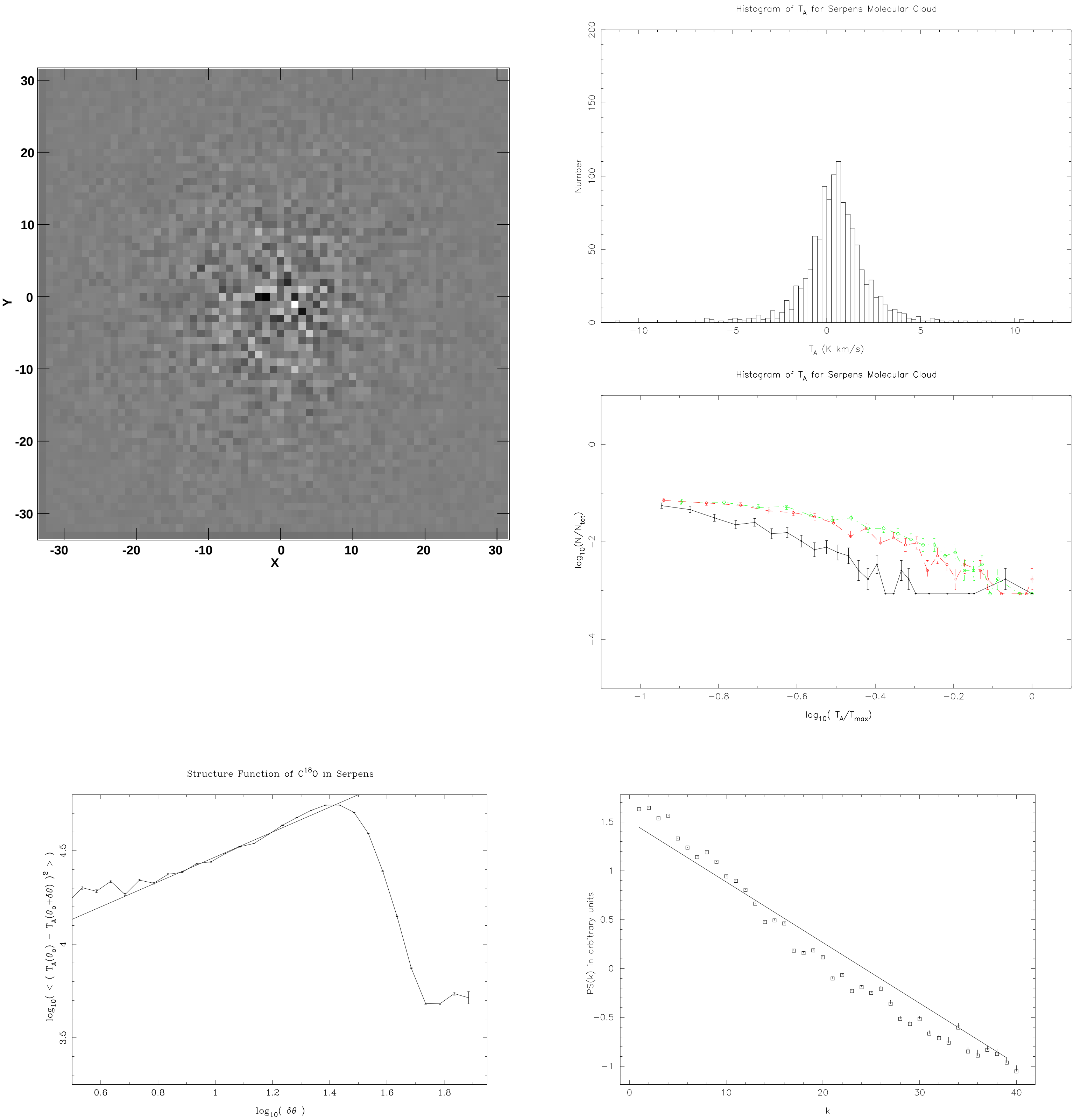

Figure 5. The top left panel shows a grey scale map of the modeled turbulent molecular cloud with rms/mean = 1/3. The top right two panels show the results of Blitz & Williams method of analysis for this cloud. Red is half the initial resolution and green is three times the initial resolution. It is easily seen that there is no difference in this analysis and those for the Taurus and Serpens molecular clouds using this method. The bottom panel shows the structure function of the modeled turbulent molecular cloud. The straight line is the expected result from the input power spectrum for the density fluctuations. The fit is extraordinarily good. The power spectrum input into the model is shown in the lower right panel.