Calibration

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Receiver calibration sources allow us to convert the backend’s detected voltages to the intensity the signal had at the point in the system where the calibration signal is injected.
Reference observations

- Difference a signal observation with a reference observation
- Types of reference observations
  - Frequency Switching
    - In or Out-of-band
  - Position Switching
  - Beam Switching
    - Move Subreflector
    - Receiver beam-switch
  - Dual-Beam Nodding
    - Move telescope
    - Move Subreflector
Position-Switched (On-Off) Observing

![Diagram of Position-Switched (On-Off) Observing]

- Signal
- Noise Diode
- Detector
Typical Position-Switched Calibration Equation for a Point Source

\[ S(\nu) = \left( \frac{2k}{\eta_A(\nu,Elev) \cdot Area_p} \right) \cdot T_A(\nu) \cdot e^{\tau(\nu,t) \cdot A(Elev,t)} \]

\[ T_A(\nu) = \left( \frac{Sig(\nu) - Ref(\nu)}{Ref(\nu)} \right) \cdot T_{Sys}^{Ref} \]

\[ T_{Sys}^{Ref} = \left\langle \frac{Ref(\nu)}{Ref_{On}(\nu) - Ref_{Off}(\nu)} T_{Cal}(\nu) \right\rangle_{BW} \]

A(Elev,t) = Air Mass
\( \tau(\nu,t) = \) Atmospheric Zenith Opacity
\( T_{cal} = \) Noise Diode Temperature
Area = Physical area of the telescope
\( \eta_A(\nu,Elev) = \) Aperture efficiency (point sources)
\( T_A(\nu) = \) Source Antenna Temperature
\( S(\nu) = \) Source Flux Density
Sig(\nu), Ref(\nu) = Data taken on source and on blank sky (in units backend counts)
On,Off = Data taken with the noise diode on and off
\( T_{sys} = \) System Temperature averaged over bandwidth
Position-Switched Calibration Equation

\[ S(v) = \left( \frac{2k}{\eta_A(v, \text{Elev}) \cdot \text{Area}_p} \right) \cdot \left( \frac{\text{Sig}(v) - \text{Ref}(v)}{\text{Ref}(v)} \right) \cdot \left( \frac{\text{Ref}(v)}{\text{Ref}_{on}(v) - \text{Ref}_{off}(v)} \right) T_{cal}(v) \cdot e^{\tau(v)A(Elev)} \]

Sources of uncertainties

\[
\left( \frac{\Delta S}{S} \right)^2 = (\tau \cdot \Delta A)^2 + (A \cdot \Delta \tau)^2 + \left( \frac{\Delta T_{cal}}{T_{cal}} \right)^2 + \left( \frac{\Delta \eta}{\eta} \right)^2
\]

- 10-15% accuracy have been the ‘standard’
- Usually, errors in \( T_{cal} \) dominate
- Goal: To achieve 5% calibration accuracy without a significant observing overhead.
Air Mass Estimates

Depends upon density and index of refraction as a function of height

But, how can one get this information?
Air Mass Estimate

- Air Mass traditionally modeled as $1/\sin(\text{Elev})$
- For 1% calibration accuracy, must use a better model below 15 deg.

$$A = -0.0234 + \frac{1.014}{\sin\left(\text{Elev} + \frac{5.18}{\text{Elev} + 3.35}\right)}$$

- Good to 1 deg
- Use $1/\sin(\text{Elev})$ above 60 deg
- Coefficients are site specific, at some low level
Typical Position-Switched Calibration Equation

\[ S(\nu) = \left( \frac{2k}{\eta_A(\nu, \text{Elev}) \cdot \text{Area}_p} \right) \cdot T_A(\nu) \cdot e^{\tau(\nu, t) \cdot A(\text{Elev}, t)} \]

\[ T_A(\nu) = \left( \frac{\text{Sig}(\nu) - \text{Ref}(\nu)}{\text{Ref}(\nu)} \right) \cdot T^{'\text{Ref}}_{\text{Sys}} \]

\[ T^{'\text{Ref}}_{\text{Sys}} = \left( \frac{\text{Ref}(\nu)}{\text{Ref}_{\text{On}}(\nu) - \text{Ref}_{\text{Off}}(\nu)} \cdot T_{\text{Cal}}(\nu) \right)_{\text{BW}} \]

A(\text{Elev}, t) = \text{Air Mass} \\
\tau(\nu, t) = \text{Atmospheric Zenith Opacity} \\
T_{\text{cal}} = \text{Noise Diode Temperature} \\
\text{Area} = \text{Physical area of the telescope} \\
\eta_A(\nu, \text{Elev}) = \text{Aperture efficiency (point sources)} \\
T_A(\nu) = \text{Source Antenna Temperature} \\
S(\nu) = \text{Source Flux Density} \\
\text{Sig}(\nu), \text{Ref}(\nu) = \text{Data taken on source and on blank sky (in units backend counts)} \\
\text{On,Off} = \text{Data taken with the noise diode on and off} \\
T_{\text{sys}} = \text{System Temperature averaged over bandwidth}
Opacities from the various components

- Dry Air Continuum
- Water Continuum
- Water Line
- Oxygen Line
- Hydrosols
Opacities from the various components

Zenith Opacity vs. Frequency
2008 Aug 03 17:00 UT

Total Opacity
Determining Opacities

\[ T_{SYS} = T_{Rcvr} + T_{Spillover} + T_{CMB}e^{-\tau\cdot A} + T_{ATM} \cdot (1 - e^{-\tau\cdot A}) \]
Typical Position-Switched Calibration Equation

\[ S(\nu) = \left( \frac{2k}{\eta_A(\nu, Elev) \cdot Area_p} \right) \cdot T_A(\nu) \cdot e^{\tau(\nu, t) \cdot A(Elev, t)} \]

\[ T_A(\nu) = \left( \frac{Sig(\nu) - Ref(\nu)}{Ref(\nu)} \right) \cdot T_{Ref} \]

\[ T_{Ref}^{Ref} = \left( \frac{Ref(\nu)}{Ref_{On}(\nu) - Ref_{Off}(\nu)} \cdot T_{Cal}(\nu) \right)_{BW} \]

A(Elev,t) = Air Mass  
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Determining $T_{\text{Cal}}$ from hot-cold load measurements in the lab

- Place black bodies (absorbers) of two known temperatures in front of the feed and record detected voltages.
  
  - $V_{\text{Hot\_Off}} = g \times T_{\text{Hot}}$
  - $V_{\text{Cold\_Off}} = g \times T_{\text{Cold}}$
  - $V_{\text{Cold\_On}} = g \times (T_{\text{Cold}} + T_{\text{Cal}})$

- $g$ and $T_{\text{Cal}}$ are unknown
Determining $T_{Cal}$ from hot-cold load measurements in the lab

$$T_{Cal} = \frac{V_{\text{Cold}_\text{On}} - V_{\text{Cold}_\text{Off}}}{V_{\text{Hot}_\text{Off}} - V_{\text{Cold}_\text{Off}}} \cdot (T_{\text{Hot}} - T_{\text{Cold}})$$

- Course frequency resolution
- Uncertainties in load temperatures
- Are the absorbers black bodies?
- Detector linearities (300 & 75 K)
- Lab $T_{Cal}$ may be off by 10%
- So… all good observers perform their own astronomical calibration observation
Noise Diode Estimates

- Instead, we recommend an On-Off observation
  - Use a point source with known flux -- polarization should be low or understood
  - Use the same exact hardware, exact setup as your observation. (i.e., don’t use your continuum pointing data to calibrate your line observations.)
  - Observations take ~5 minutes per observing run
  - Staff take about 2 hrs to measure the complete band of a high-frequency, multi-beam receiver.
  - Resolution sufficient: 1 MHz, sometimes better
  - Accuracy of ~ 1%, mostly systematics.
Noise Diode Estimates

\[
S(\nu) = \left( \frac{2k}{\eta_A(\nu, \text{Elev}) \cdot A_p} \right) \cdot \left( \frac{\text{Sig}(\nu) - \text{Ref}(\nu)}{\text{Ref}(\nu)} \right) \cdot \left( \frac{\text{Ref}(\nu)}{\text{Ref}_{\text{on}}(\nu) - \text{Ref}_{\text{off}}(\nu)} \right) T_{\text{cal}}(\nu) \cdot e^{\tau(\nu)A}
\]

Remove Averaging, Solve for T_{cal}

\[
T_{\text{cal}}(\nu) = \frac{\eta_A(\nu, \text{Elev}) \cdot \text{Area}_p}{2k \cdot e^{\tau(\nu)A}} \left( \frac{\text{Ref}_{\text{on}}(\nu) - \text{Ref}_{\text{off}}(\nu)}{\text{Sig}(\nu) - \text{Ref}(\nu)} \right) \cdot S(\nu)
\]
Noise Diode Estimates

X-band                Low Cals
Right-Circular Polarization

S-band                Low Cals
Y-Linear Polarization

Legend
   Ast
   Eng

Created with PSI-Plot, Tue May 10 14:36:23 2005
X_LC_Rcs.pgw

Legend
   Eng (21 Oct 2004)
   Eng (22 Oct 2004)
   Ast

Created with PSI-Plot, Tue May 10 16:59:49 2005
S_LL_Ycs.pgw
Typical Position-Switched Calibration Equation

\[ S(\nu) = \left( \frac{2k}{\eta_A(\nu, Elev) \cdot Area_p} \right) \cdot T_A(\nu) \cdot e^{\tau(\nu,t) \cdot A(Elev,t)} \]

\[ T_A(\nu) = \left( \frac{Sig(\nu) - Ref(\nu)}{Ref(\nu)} \right) \cdot T_{Ref} \]

\[ T_{Ref}^{Ref} = \left( \frac{Ref(\nu)}{Ref_{On}(\nu) - Ref_{Off}(\nu)} \right) T_{Cal}(\nu) \right\}_{BW} \]

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Telescope efficiencies – Part 1

Not shown: $\eta_r$ and $\eta_{\text{illumination}}$

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GBT Gain Curve

Ruze Equation – Surface errors

\[ G(\epsilon) = G_0 \ e^{-\left(\frac{4\pi\epsilon}{\lambda}\right)^2} \]
GBT Gain Curve
Non-linearity

- If system is linear,
  - \( P_{out} = B \times P_{in} \)
  - \( (\text{Sig}_{\text{On}} - \text{Sig}_{\text{Off}}) - (\text{Ref}_{\text{On}} - \text{Ref}_{\text{Off}}) = 0 \)

- Model the response curve to 2nd order:
  - \( P_{out} = B \times P_{in} + C \times P_{in}^2 \)

- Our ‘On-Off’ observations of a calibrator provide:
  - Four measured quantities: \( \text{Ref}_{\text{off}}, \text{Ref}_{\text{on}}, \text{Sig}_{\text{off}}, \text{Sig}_{\text{on}} \)
  - \( T_A \) From catalog
  - Four desired quantities: \( B, C, T_{\text{cal}}, T_{\text{sys}} \)

- It’s easy to show that:
  - \( C = \frac{(\text{Sig}_{\text{on}} - \text{Sig}_{\text{off}}) - (\text{Ref}_{\text{on}} - \text{Ref}_{\text{off}})}{2T_A T_{\text{cal}}} \)

- Thus:
  - Can determine if system is sufficiently linear
  - Can correct to 2nd order if it is not
Non-linearity

\[(\text{SigOn-SigOff}) - (\text{RefOn-RefOff})\]
Reference observations

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Position switching

- Move the telescope between a signal and reference position
  - Overhead
  - $\frac{1}{2}$ time spent off source
- Difference the two spectra
- Assumes shape of gain/bandpass doesn’t change between the two observations.
- For strong sources, must contend with dynamic range and linearity restrictions.
Frequency switching

- Eliminates bandpass shape from components after the mixer
- Leaves the derivative of the bandpass shape from components before the mixer.
In-Band Frequency Switching

\[ F_{\text{Sky}} \]
\[ F_1 \]
\[ F_2 \]
\[ F_{\text{Sky}} - F_1 \]
\[ F_{\text{Sky}} - F_2 \]

\[ V_1 - V_2 \]
\[ V_2 - V_1 \]

Shift and Average to decrease noise by \( \text{sqrt}(2) \)
Out-Of-Band Frequency Switching

$F_{\text{Sky}}$

$F_1$

$F_2$

$F_{\text{Sky}} - F_1$

$F_{\text{Sky}} - F_2$

$V_1 - V_2$
Beam switching – Internal switch

- Difference spectra eliminates any contributions to the bandpass from after the switch.
- Residual will be the difference in bandpass shapes from all hardware in front of the switch.
- Low overhead but ½ time spent off source.
Atmosphere is in the near field

- Common to all feeds in a multi-feed receiver
Beam Switching – Subreflector or tertiary mirror

- Optical aberrations
- Difference in spillover/ground pickup
- Removes any ‘fast’ gain/bandpass changes
- Low overhead. ½ time spent off source
Nodding with dual-beam receivers - Telescope motion

- Optical aberrations
- Difference in spillover/ground pickup
- Removes any ‘fast’ gain/bandpass changes
- Overhead from moving the telescope. All the time is spent on source
Nodding with dual-beam receivers - Subreflector or tertiary mirror

- Optical aberrations
- Difference in spillover/ground pickup
- Removes any ‘fast’ gain/bandpass changes
- Low overhead. All the time is spent on source
References

- Rohlfs & Wilson, “Tools of Radio Astronomy”