High Precision Calibration of Wide Bandwidth Observations with the GBT

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Typical Position-Switched Calibration Equation

$$S(\nu) = \left(\frac{2k}{\eta_A(\nu, Elev) \cdot Area_p} \right) \cdot T_A(\nu) \cdot e^{\tau(\nu,t) \cdot A(Elev,t)}$$

$$T_A(\nu) = \left(\frac{Sig(\nu) - Ref(\nu)}{Ref(\nu)} \right) \cdot T_{Sys}^{Ref}$$

$$T_{Sys}^{Ref} = \left\langle \frac{Ref(\nu)}{Ref_{On}(\nu) - Ref_{Off}(\nu)} T_{Cal}(\nu) \right\rangle_{BW}$$

A(Elev,t) = Air Mass
$\tau(\nu,t)$ = Atmospheric Zenith Opacity
T_{cal} = Noise Diode Temperature
Area = Physical area of the telescope
η_A (ν,Elev) =Aperture efficiency (point sources)
T_A (ν) = Source Antenna Temperature

S(ν) = Source Flux Density
Sig(ν), Ref (ν) = Data taken on source and
on blank sky (in units backend counts)
On,Off = Data taken with the noise diode on and off
T_{sys} = System Temperature averaged over bandwidth

Position-Switched Calibration Equation

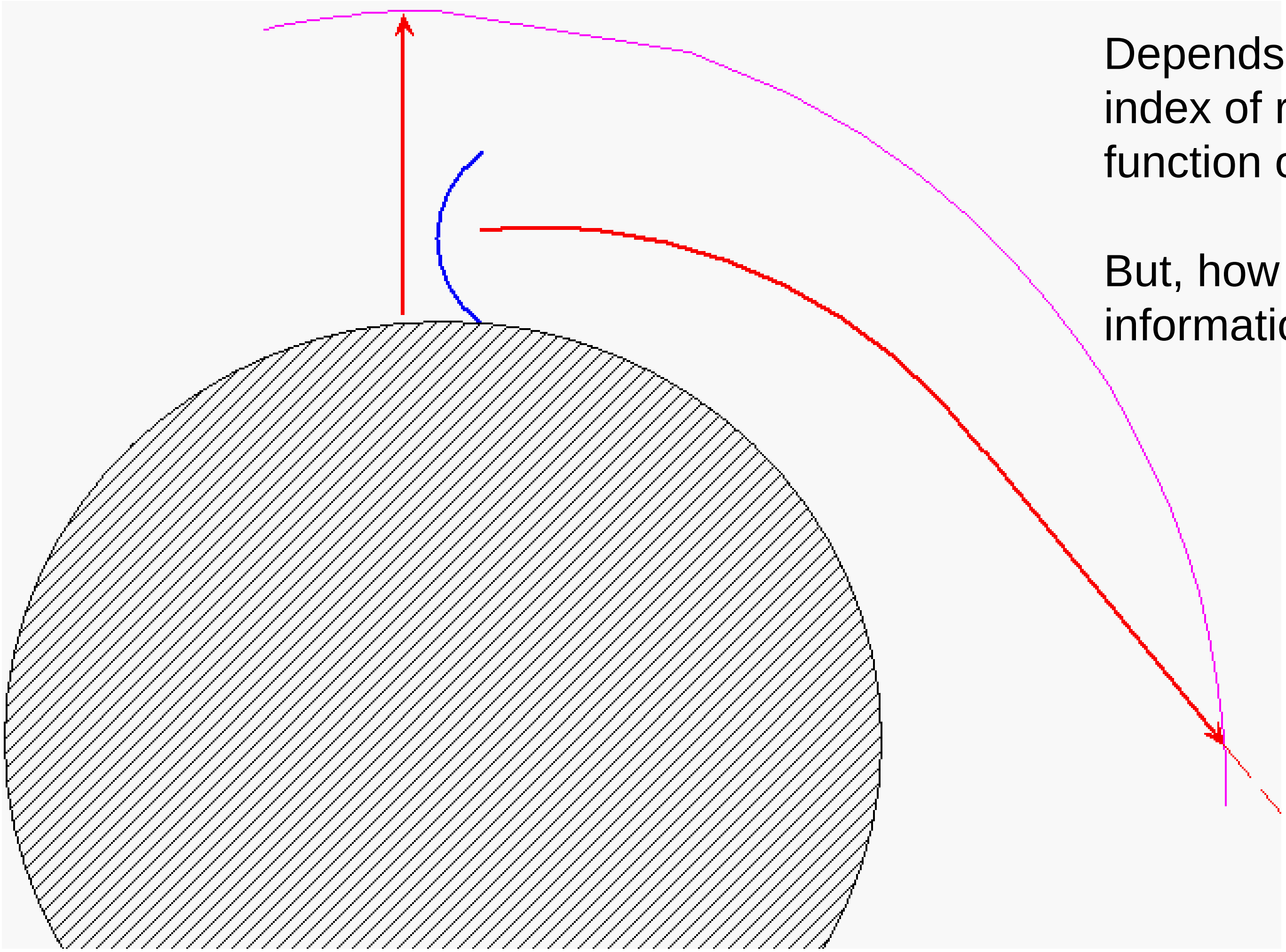
$$S(\nu)=\left(\frac{2k}{\eta_A(\nu, Elev)\cdot Area_p}\right)\cdot\left(\frac{Sig(\nu)-Ref(\nu)}{Ref(\nu)}\right)\left\langle\frac{Ref(\nu)}{Ref_{On}(\nu)-Ref_{Off}(\nu)}T_{Cal}(\nu)\right\rangle\cdot e^{\tau(\nu)\cdot A(Elev)}$$

Sources of uncertainties

$$\left(\frac{\Delta S}{S}\right)^2=(\tau\cdot\Delta A)^2+(A\cdot\Delta\tau)^2+\left(\frac{\Delta T_{cal}}{T_{Cal}}\right)^2+\left(\frac{\Delta\eta}{\eta}\right)^2$$

- 10-15% accuracy have been the ‘standard’
- Usually, errors in T_{cal} dominate
- Goal: To achieve 5% calibration accuracy without a significant observing overhead.

Air Mass Estimates



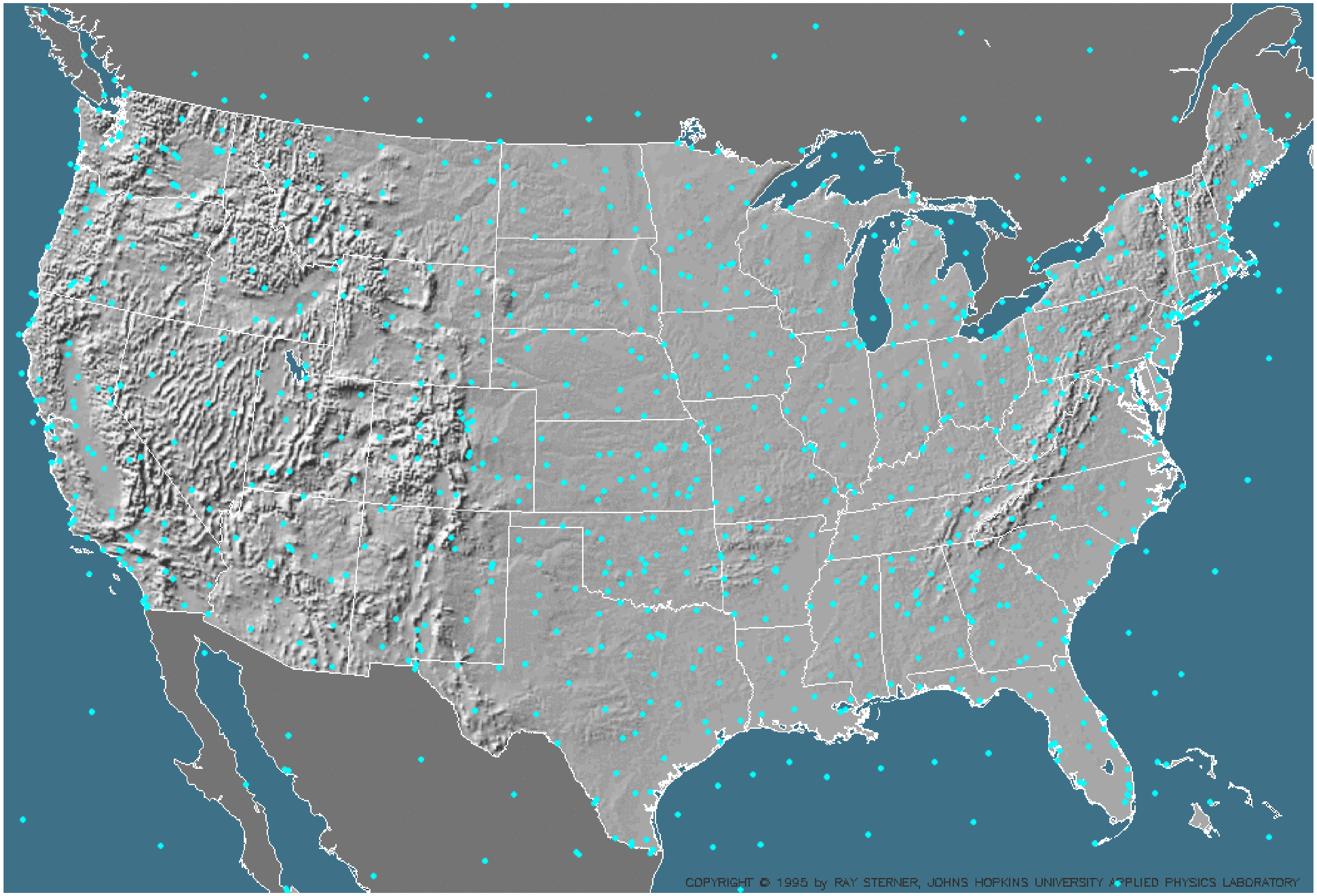
Depends upon density and index of refraction as a function of height

But, how can one get this information?

Vertical Weather Data

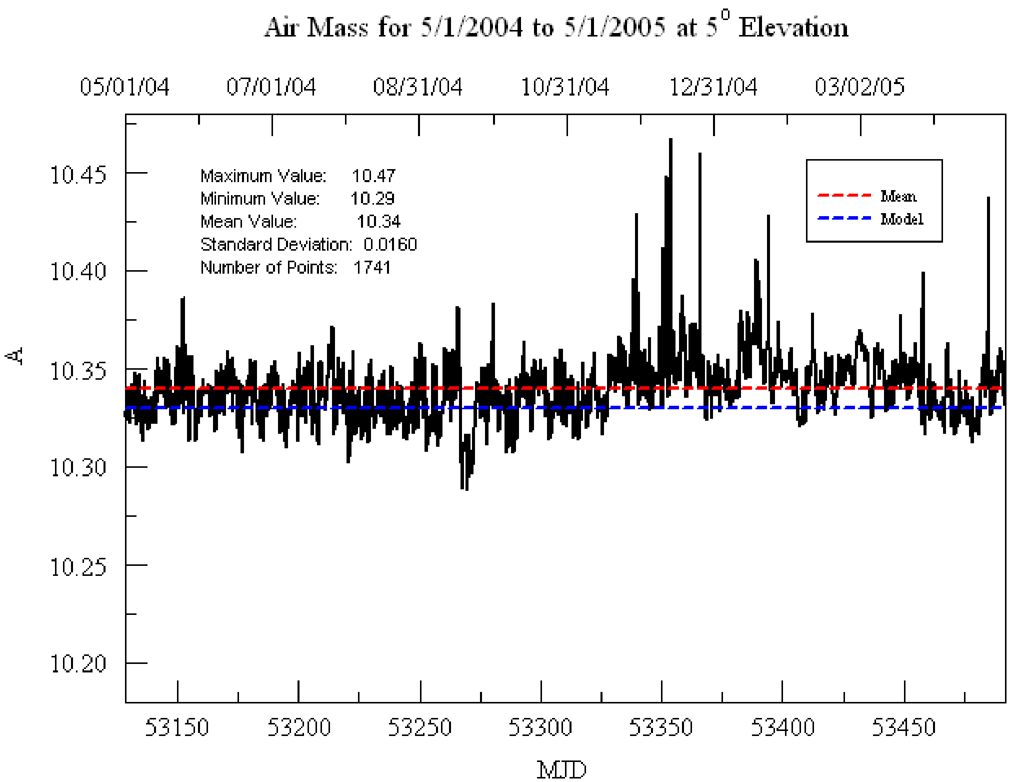
- Provided by the national weather services via FTP
- 60 hr forecasts (ETA model), updated every 12 hrs
- For each hour, provides as a function of ***height above the ground***
  - Temperature, Pressure, Dew Point, Cloud Cover, ….
- ~40 heights that extend well into the stratosphere
- One can derive ***as a function of height***:
  - Density
  - Index of refraction
  - Absorption coefficient (dry air, water – continuum & line, oxygen line, hydrosols) (Liebe model)

Vertical Weather Data



Air Mass Estimates

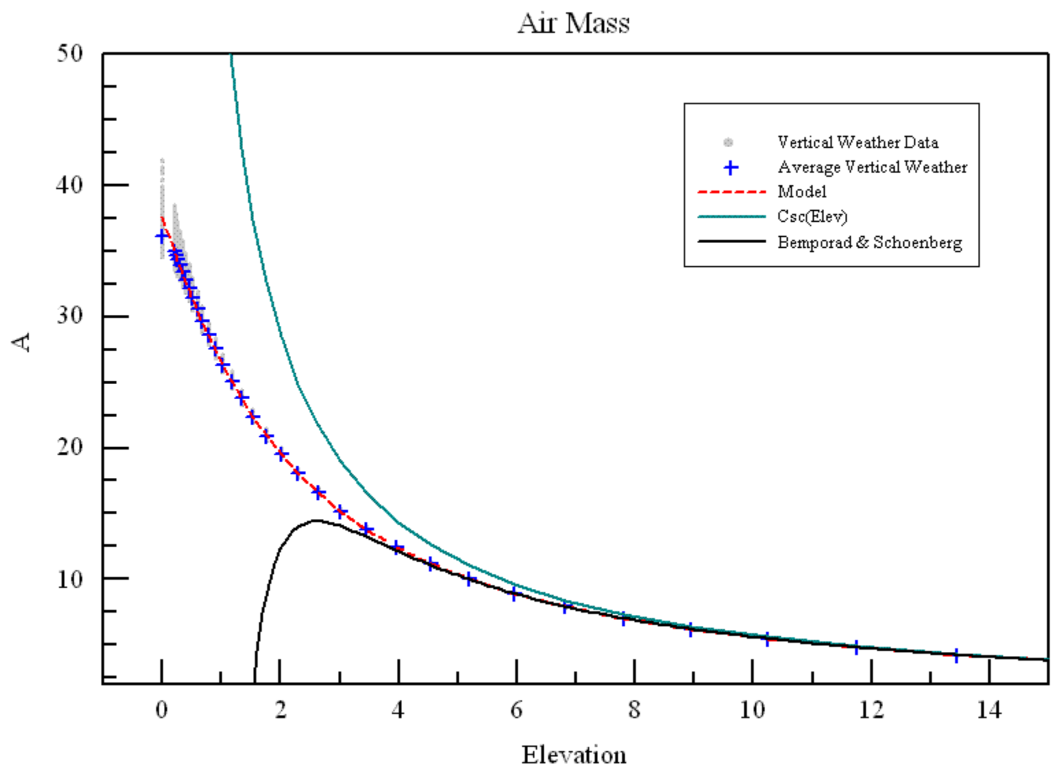
- Air Mass derived from the vertical values of density & index of refraction.
- For 1% calibration error, require A to ~0.1



- Probably should use weather dependent Air Mass for elevations below 5 deg
- Probably can ignore weather dependency above 5 deg.

Air Mass Estimate

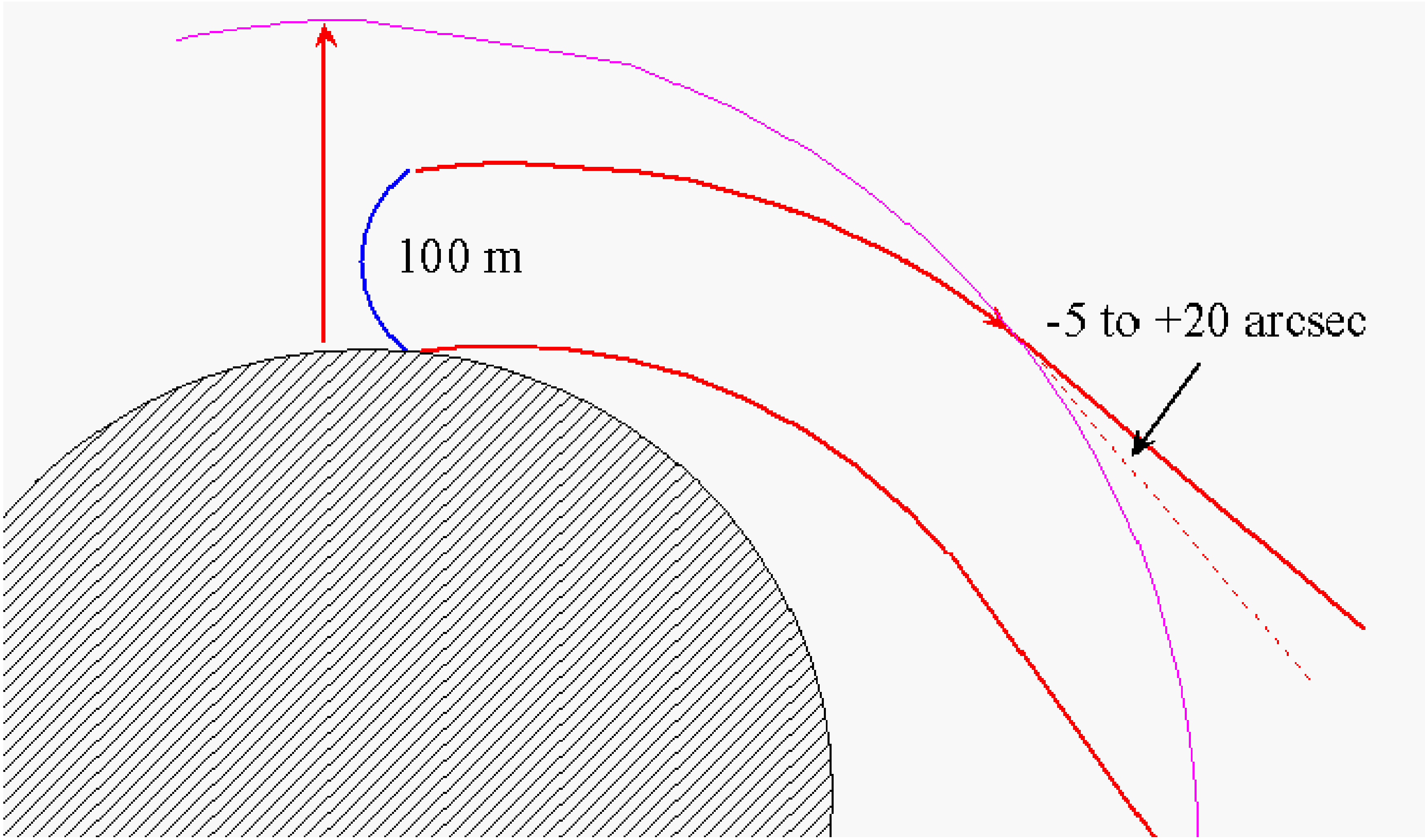
- Air Mass traditionally modeled as 1/sin(Elev)
- For 1% calibration accuracy, must use a better model below 15 deg.



$$A = -0.0234 + \frac{1.014}{\sin\left(Elev + \frac{5.18}{Elev + 3.35}\right)}$$

- Good to 1 deg
- Use 1/sin(Elev) above 60 deg
- Coefficients are site specific, at some low level

Air Mass Estimates

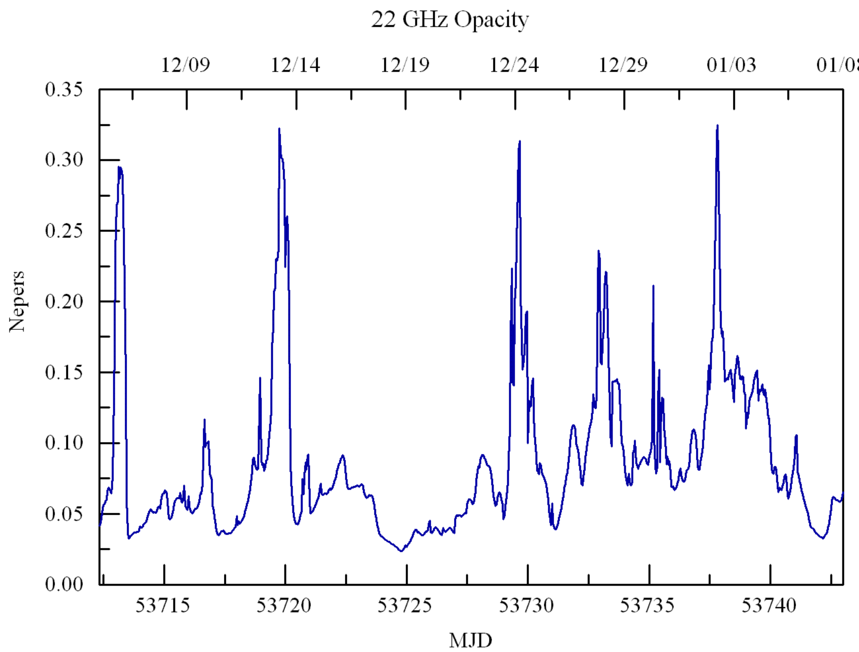


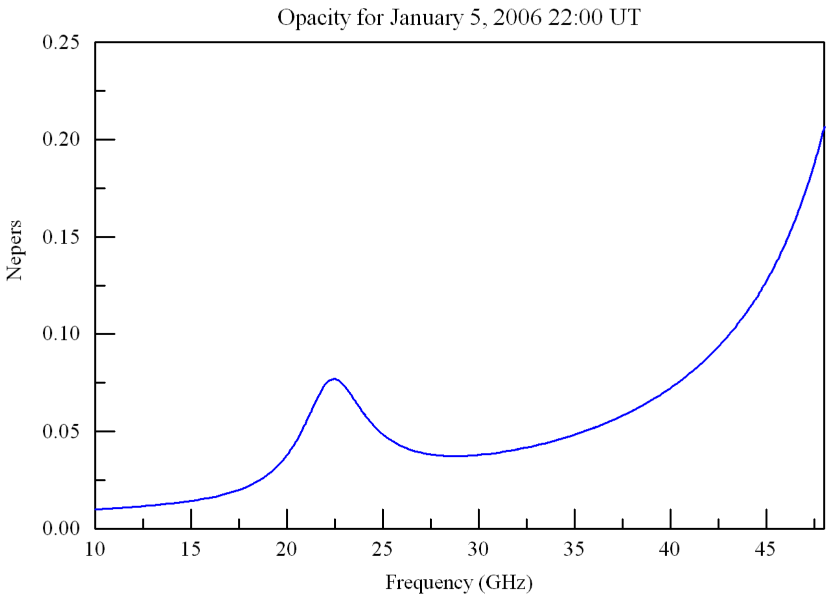
Opacity Estimates

- Vertical weather data provides absorption as a function of height

$$\tau(\nu,t) = \int_0^\infty \left(\kappa_{Dry}(\nu,t) + \kappa_{O2}(\nu,t) + \kappa_{Water\_cont}(\nu,t) + \kappa_{water\_line}(\nu,t) + \kappa_{hydrosols}(\nu,t)\right) \cdot dH$$

$$T_{Sys}(Elev,\nu,t) \cong T_{rcvr}(\nu) + T_{spill}(Elev) + T_{cmb}\, e^{-\tau(\nu,t)\cdot A(Elev)} + \int_0^\tau T(H,t)\cdot e^{\tau(h,\nu,t)}\, d\tau$$



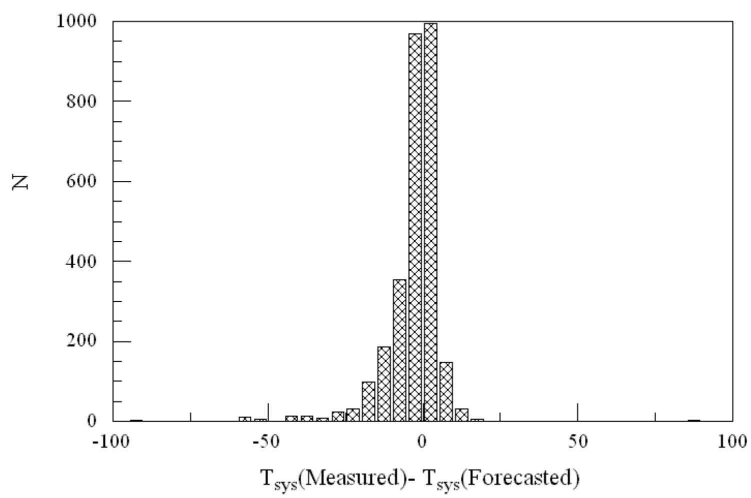


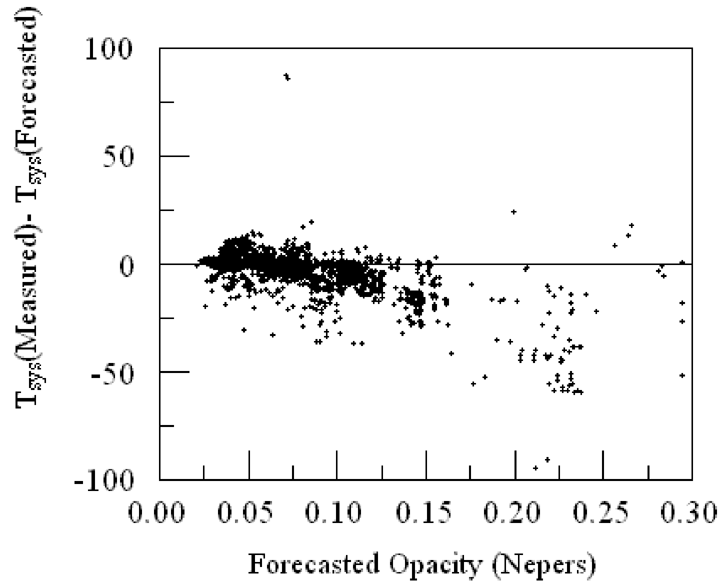
Opacity Estimates

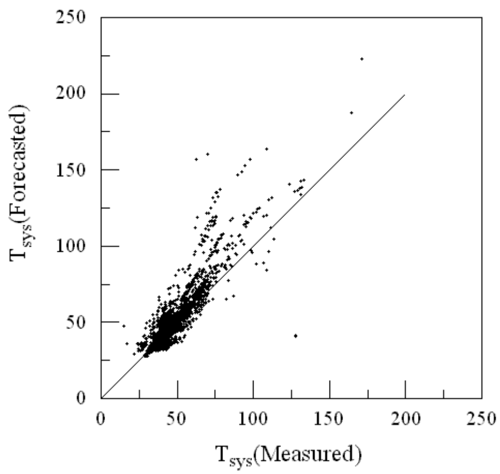
- Are derived opacities accurate? Comparisons using tipping radiometers have difficulties
  - Must do multiple tips for wideband observations
  - Tips take up telescope time
  - Requires knowing T_{cal} to high accuracy, which requires knowing τ.
  - Some dedicated tippers do not provide enough information to estimate τ near the 22 GHz water line
  - Requires a representative T_{Atm} that is good to ~5 K

$$T_{Atm} \approx \frac{\int \kappa(H)\cdot T(H)\cdot dH}{\int \kappa(H)\cdot dH}$$

Comparison of measured and estimated 22 GHz Tsys







- For 1% calibration accuracy, requires τ to 0.01
- Implies forecasted and actual Tsys should be within 3 K
- Current model sufficient at $\tau < 0.1$
- Overestimates contribution from hydrosols. Not unexpected.

Noise Diode Estimates

- Traditionally used hot-cold load measurements
  - Provide ~10% accuracy
  - Frequency resolution sometimes wider than frequency structure in T_{rcvr} or T_{cal}
  - Time consuming
  - Systematics/Difficulties
    - Loads must be opaque
    - Frost forming on LN_2 loads
    - Linearity (T_{Hot} >> T_{Cold})
  - Observers can’t do there own Hot-Cold tests

Noise Diode Estimates

- Instead, we recommend an On-Off observation
  - Use a point source with known flux -- polarization should be low or understood
  - Use the same exact hardware, exact setup as your observation. (i.e., don't use your continuum pointing data to calibrate your line observations.)
  - Observations take ~5 minutes per observing run
  - Staff take about 2 hrs to measure the complete band of a high-frequency, multi-beam receiver.
  - Resolution sufficient: 1 MHz, sometimes better
  - Accuracy of ~ 1%, mostly systematics.

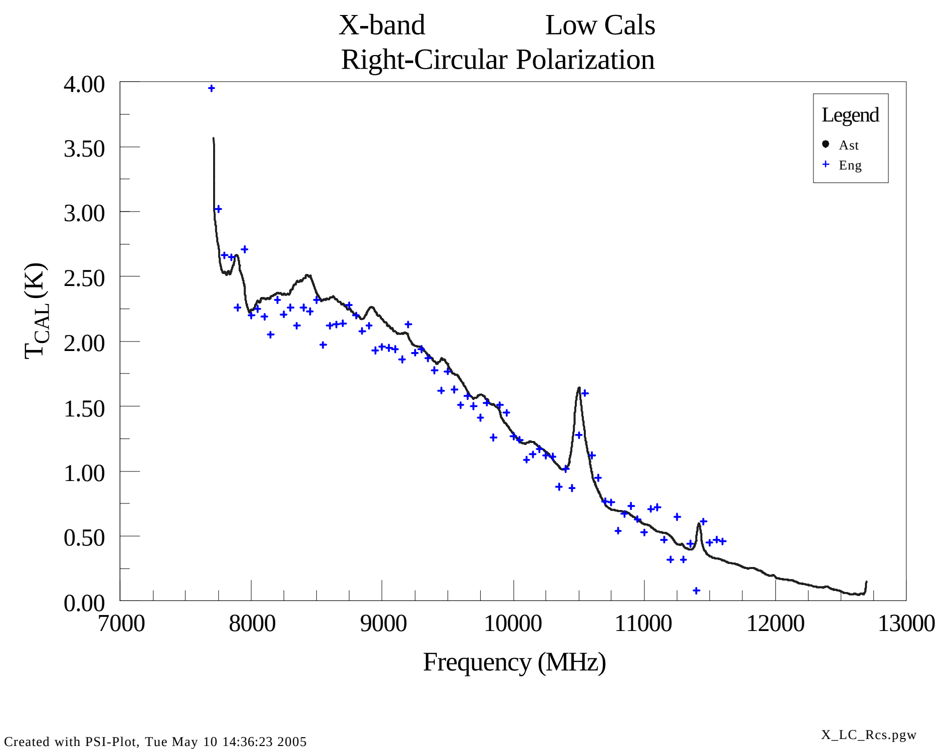
Noise Diode Estimates

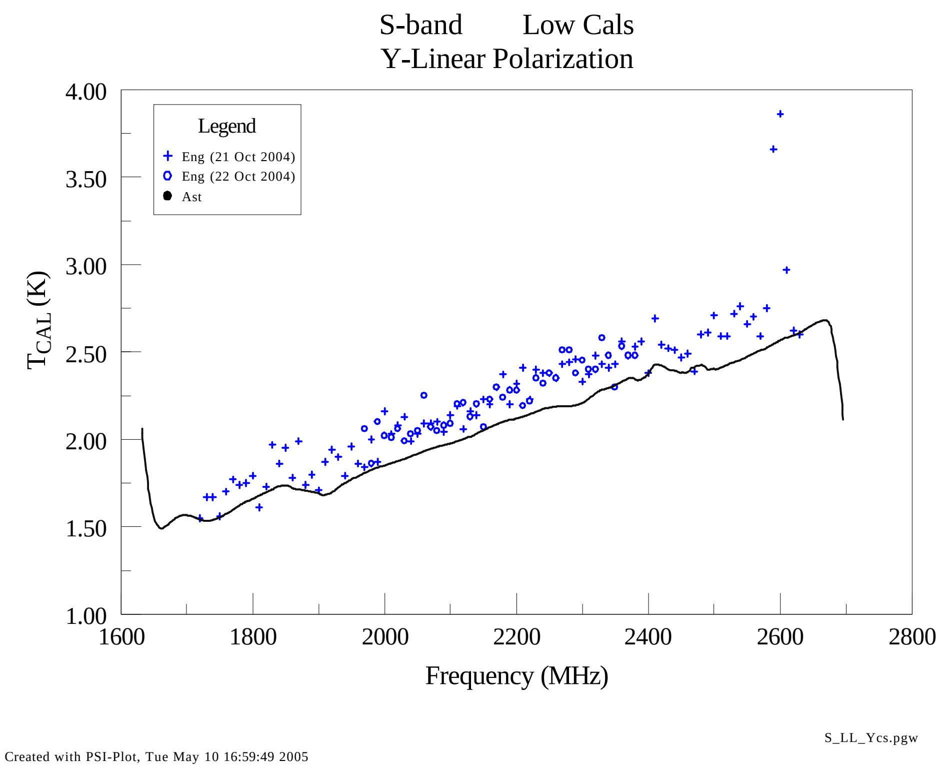
$$S(\nu) = \left(\frac{2k}{\eta_A(\nu, Elev) \cdot A_p} \right) \cdot \left(\frac{Sig(\nu) - Ref(\nu)}{Ref(\nu)} \right) \left\langle \frac{Ref(\nu)}{Ref_{On}(\nu) - Ref_{Off}(\nu)} T_{Cal}(\nu) \right\rangle \cdot e^{\tau(\nu) \cdot A}$$

Remove Averaging, Solve for Tcal

$$T_{Cal}(\nu) = \frac{\eta_A(\nu, Elev) \cdot Area_p}{2k \cdot e^{\tau(\nu) \cdot A}} \left(\frac{Ref_{On}(\nu) - Ref_{Off}(\nu)}{Sig(\nu) - Ref(\nu)} \right) \cdot S(\nu)$$

Noise Diode Estimates





Position-Switched Calibration Equation

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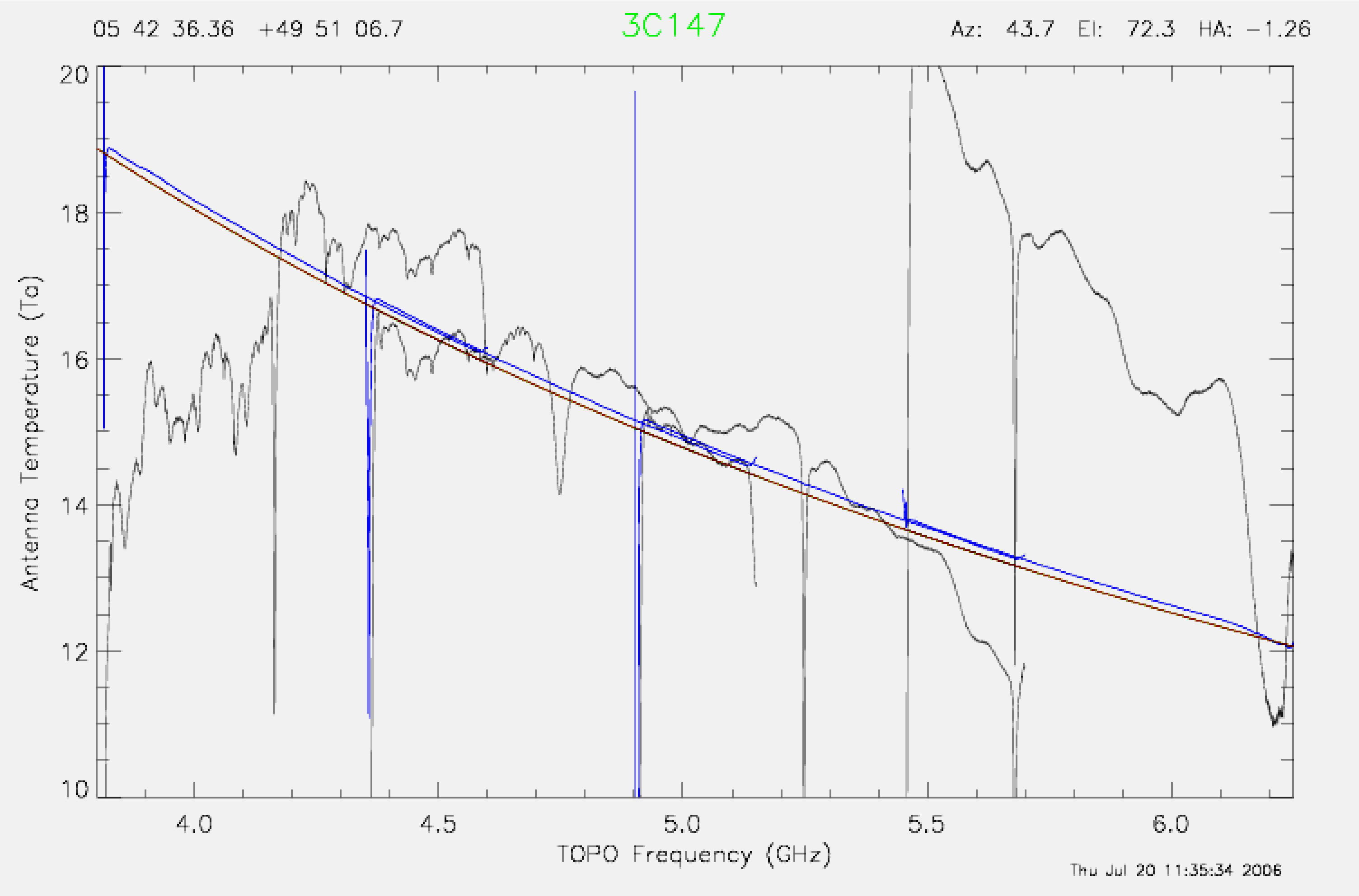
Baseline structure

$$Baselines(\nu)=\frac{T_A+T_{atm}^{Sig}-T_{atm}^{\mathrm{Ref}}}{\left\langle T_{sys}\right\rangle_{BW}}\cdot\left\{\left\langle T_{rcvr}+\frac{T_{cal}}{2}\right\rangle_{BW}-T_{rcvr}(\nu)-\frac{T_{cal}(\nu)}{2}\right\}$$

Assumption of linearity

$$S\propto Sig-Ref$$

Baseline Structure



Baseline Shapes

$$S(\nu) = \left(\frac{2k}{\eta_A(\nu, Elev) \cdot A_p} \right) \cdot \left(\frac{Sig(\nu) - Ref(\nu)}{Ref(\nu)} \right) \left\langle \frac{Ref(\nu)}{Ref_{On}(\nu) - Ref_{Off}(\nu)} T_{Cal}(\nu) \right\rangle \cdot e^{\tau(\nu) \cdot A}$$

Remove Averaging – Vector Calibration

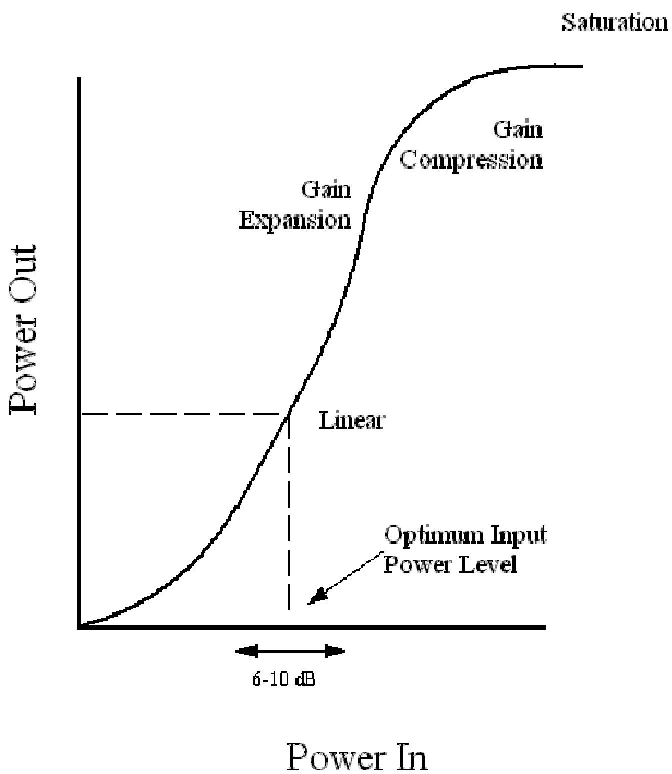
$$S(\nu) = \left(\frac{2k}{\eta_A(\nu, Elev) \cdot A_p} \right) \cdot \left(\frac{Sig(\nu) - Ref(\nu)}{Ref_{On}(\nu) - Ref_{Off}(\nu)} \right) \cdot T_{Cal}(\nu) \cdot e^{\tau(\nu) \cdot A}$$

- Traditional equation OK for narrow bandwidth observations
- Traditional provides good calibration only at band center
- Vector algorithm provides good calibration across wide bandwidths
- Vector algorithm is substantially nosier when $T_A \neq 0$

$$\sigma^2 \approx \frac{1}{BW \cdot t} \left(T_A^2 + T_{Sys}^2 + \frac{T_{Sys}^2 T_A^2}{T_{Cal}^2} \right)$$

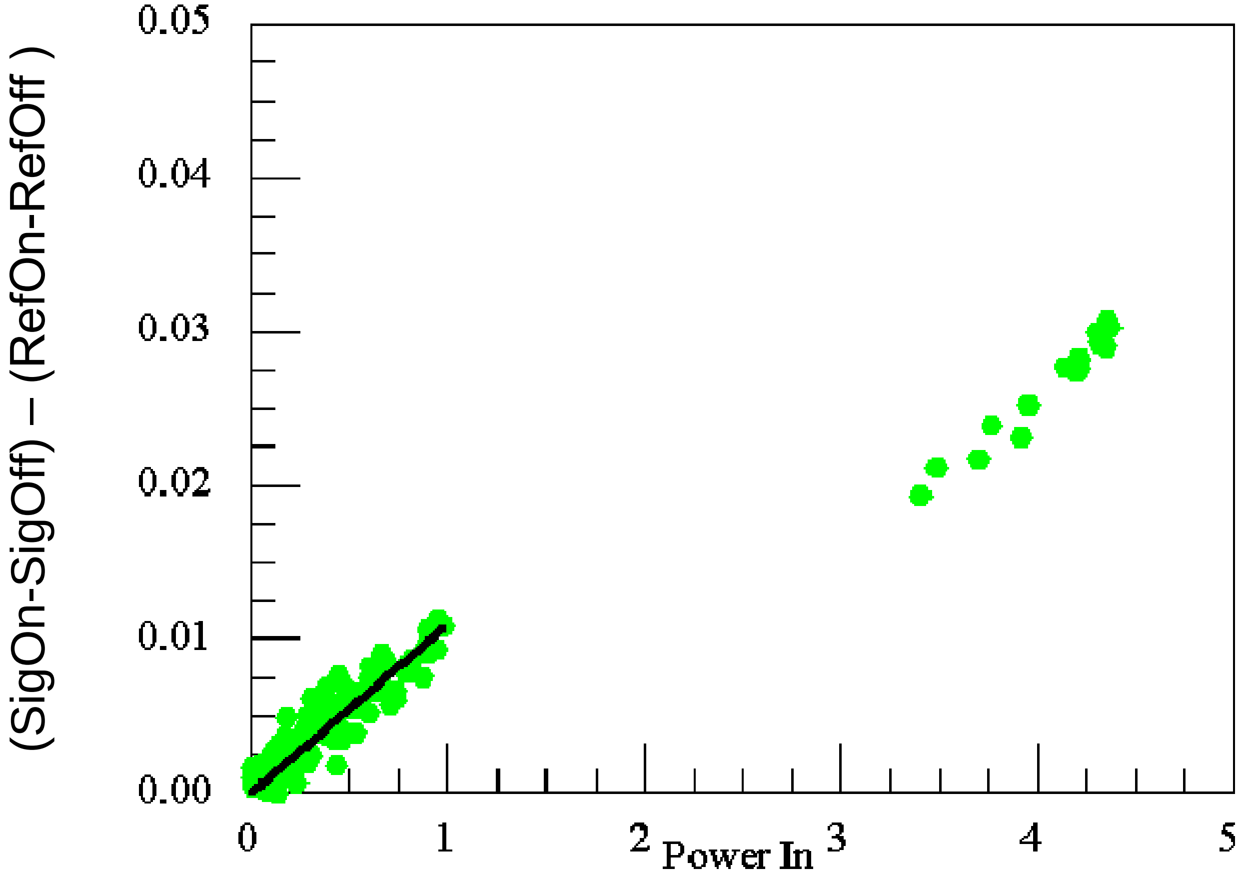
- Smooth Ref_{on}-Ref_{Off} – use Savitzky-Golay smoothing

Non-linearity



- If system is linear,
  - $P_{out} = B * P_{in}$
  - $(Sig_{On}\text{-}Sig_{Off}) - (Ref_{On}\text{-}Ref_{Off}) = 0$
- Model the response curve to 2nd order:
  - $P_{out} = B * P_{in} + C * P_{in}^2$
- Our 'On-Off' observations of a calibrator provide:
  - Four measured quantities: Ref_{off}, Ref_{on}, Sig_{off}, Sig_{on}
  - T_A From catalog
  - Four desired quantities: B, C, Tcal, Tsys
- It's easy to show that:
  - $C = [(Sig_{on} - Sig_{off})-(Ref_{on} - Ref_{off})]/(2T_A T_{cal})$
- Thus:
  - Can determine if system is sufficiently linear
  - Can correct to 2nd order if it is not

Non-linearity



Summary

- To obtain few percent calibration accuracy
  - Wide bandwidths require frequency dependent opacities, efficiencies, Tsys, andTcal.
  - New weather-independent model for air mass is usually sufficient
  - Opacities from vertical, forecasted weather data sufficient for medium to low-opacity conditions
  - Simple observation of calibrator provides high accuracy, high frequency resolution Tcal.
  - Assumptions of traditional on-off calibration algorithm introduces baseline shapes for wide bandwidth observations. Should use 'vector' algorithms.
  - 2nd order non-linearity -- measurable (by-product of Tcal observation) and correctable.   Might be significant.

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T_A (ν) = Source Antenna Temperature
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On,Off = Data taken with the noise diode on and off
T_{sys} (ν,Elev,t) =System Temperature
T_{CMB} = Cosmic Microwave Background
T_{rcvr} (ν) = Receiver Temperature
T_{spill}(Elev) = Antenna Spillover
T_{Atm} (ν,t) =Representative temperature of the atmosphere