MUSTANG Sensitivities and MUSTANG-1.5 and -2 Sensitivity Projections

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This technical note explains the current MUSTANG sensitivity and how it is calculated. The MUSTANG-1.5 and MUSTANG-2 sensitivity projections are explained. First some general considerations are reviewed.

1 General Considerations: Point Source Sensitivity, Surface Brightness Sensitivity, and Photometry

Astronomical images by definition generally have units of specific intensity or surface brightness. In radio astronomy in particular it is quite common to use units of Janskies per beam; Janskies per steradian or Kelvin are other typical choices. It is also common that astronomical maps have noise which is white, *i.e.*, uncorrelated between pixels. This is approximately the case for *MUSTANG* when the maps are aggressively filtered. Here we work out some basic results for point source sensitivity for two different choices of units: Jy/bm, and Jy/pixel. We assume pixels of area $\delta\Omega$.

1.1 Point Source Sensitivity from Map Noise

Suppose the map noise (RMS of pixels) is σ_{pix} is uniform, and that we have observed an unresolved source and determine its flux density s_{est} by fitting the (assumed perfectly known) beam to this map. Further assume that the value of the beam at pixel i is B_i. For uncorrelated pixel noise it can be shown that the uncertainty $\sigma(s_{est})$ in the source flux density is:

$$\sigma(s_{est}) = \frac{\sigma_{pix}}{\sqrt{\sum B_i^2}} \tag{1}$$

Proof: The optimal estimate of point source flux density from a map when the point spread function and source location are known exactly is given by a least squares fit of the PSF to the map. We have

$$\chi^2 = \sum \left(\frac{y - y_m}{\sigma} \right)^2 \tag{2}$$

where y_m is the model and σ is the pixel noise, assumed uniform. Let $y = 0$ and $y_m = A \times b$ where $A = s_{est}$ is the source amplitude and b is the beam response. Then

$$\chi^2 = \frac{A^2}{\sigma^2} \sum b^2 \tag{3}$$

The minimum $\chi^2 = 0$ is at $A = 0$. The 1σ error bar on A is given by the region over which $\Delta\chi^2 = 1$, thus

$$\chi^2(A_{1\sigma}) - \chi^2(A = 0) = 1 = \frac{A_{1\sigma}^2}{\sigma^2} \sum b^2 \tag{4}$$

or

$$A_{1\sigma} = \sigma_A = \frac{\sigma}{\sqrt{\sum b^2}} \tag{5}$$

1.2 Janskies per Beam

Calibrating a map to Janskies per beam is equivalent to assuming a point spread function with a peak value of unity. In this case,

$$\int B\, d\Omega = \Omega_{bm} \tag{6}$$

The map pixel values are $I_i = B_i\, s_o$ and direct photometry (summing the pixel values p_i) will yield a value of

$$\sum p_i = s_o \sum B_i = s_o \frac{\Omega_{bm}}{\delta\Omega} \tag{7}$$

so the estimated source flux density is given by

$$s_{est} = \frac{\sum p_i}{\Omega_{bm}/(\delta\Omega)} \tag{8}$$

The uncertainty in this value is given by Eq. 1, with

$$\begin{aligned}
\sum B_i^2 &= \frac{1}{\delta\Omega} \sum B_i^2\, \delta\Omega &(9)\\
&= \frac{\int B^2\, d\Omega}{\delta\Omega} &(10)\\
&\sim \frac{\Omega_{bm}/2}{\delta\Omega} &(11)
\end{aligned}$$

where the last line uses the equality $\int B^2\, d\Omega = \frac{1}{2}\int B\, d\Omega$, which holds only for a Gaussian beam in two dimensions. The integral of the beam-squared is sometimes called the "variance beam".

Given this

$$\begin{aligned}
s_{est} &= \frac{\sigma_{pix}}{\sqrt{\frac{1}{\delta\Omega}\int B^2\, d\Omega}} &(12)\\
&\sim \frac{\sigma_{pix}}{\sqrt{\frac{\Omega_{bm}/2}{\delta\Omega}}} &(13)
\end{aligned}$$

Here σ_{pix} is in map units (Jy/bm) and the last line holds only for Gaussian beams.

1.3 Janskies per Pixel

Calibrating to flux per pixel is equivalent to assuming a *normalized* point spread function B' such that

$$\int B' \, d\Omega = 1 \tag{14}$$

This beam is simply related to the other[1] by $B' = B/\Omega_{bm}$. In this case $I'_i = B'_i \, s_o$ and direct map photometry will yield

$$\sum p_i = s_o \sum B_i = s_o \tag{15}$$

Then for the point source sensitivity we have:

$$
\begin{aligned}
s_{est} &= \frac{\sigma_{pix}}{\sqrt{\sum (B'_i)^2}} && (16)\\
&= \frac{\sigma_{pix}\, \Omega_{bm}}{\sqrt{\sum B_i^2}} && (17)\\
&= \frac{\sigma_{pix}\, \Omega_{bm}}{\sqrt{\frac{1}{\delta\Omega} \int B^2 \, d\Omega}} && (18)\\
&\sim \sigma_{pix} \sqrt{2\Omega_{bm}\, \delta\Omega} && (19)
\end{aligned}
$$

where again σ_{pix} is in map units (this time Janskies per pixel, eg, Jy/arcsec-squared for 1 arcsecond pixels). The last line only holds for a Gaussian beam.

1.4 Integrated Flux and Surface Brightness Sensitivity

Supposing more generally we want to sum or average over an arbitrary larger area Ω_1 in the image. The number of independent samples is $\Omega_1/\delta\Omega$ and the total flux is $F = \sum p_i \, \delta\Omega$. For the sake of this example we assume a pixel noise of $(10\,\mu\mathrm{Jy/arcsec^2})$ in $\delta\Omega = 2.35 \times 10^{-11}$ Sr (1 arcsec^2) pixels. Then for an image in units of Jy/pixel the flux noise is

$$\mathrm{RMS(total\,flux)} = (10\,\mu\mathrm{Jy/arcsec^2}) \times \sqrt{\delta\Omega\, \Omega_1} \tag{20}$$

Putting $\delta\Omega$ in with the pre-factor, it is appropriate to define integrated flux density sensitivity in units of (for instance) Jy/arcsec

$$\mathrm{RMS(total\,flux)} = (10\,\mu\mathrm{Jy/arcsec}) \times \sqrt{\Omega_1} \tag{21}$$

Similarly the surface brightness noise is

$$\mathrm{RMS(average\,SB)} = (10\,\mu\mathrm{Jy/arcsec^2}) \times \sqrt{\frac{\delta\Omega}{\Omega_1}} \tag{22}$$

[1]Throughout we use B to denote the unity-peak normalized beam and B' to denote the unity-integral normalized beam.

or simply

$$\text{RMS(average SB)} = \frac{10\,\mu\text{Jy/arcsec}}{\sqrt{\Omega_1}} \tag{23}$$

If instead the image were calibrated to Kelvin with an example pixel noise for some given pixel size of 2 mK, then the uncertainty in the surface brightness is

$$\text{RMS(average surface brightness)} = \frac{2\,\text{mK}}{\sqrt{\Omega_1/(\delta\Omega)}} \tag{24}$$

Again putting $\delta\Omega$ in with the pre-factor, it is appropriate to define surface brightness sensitivity in units of (for instance) Kelvin-arcseconds, as:

$$\text{RMS(average surface brightness)} = \frac{2\,\text{mK arcsec}}{\sqrt{\Omega_1}} \tag{25}$$

2 MUSTANG vs MUSTANG-N Sensitivity

2.1 MUSTANG Beam & GBT 3.3 mm Gain

The point source data in the 2010/2011 observing season were analyzed to determine a typical MUSTANG+GBT beam (including the near-in error beam) with the following parameters: main beam FWHM = $9''.24$; main beam normalization = 92%; error beam FWHM = $22''.7$; error beam normalization = 8%. This beam has a volume of 136 arcsec^2 (97 arcsec^2 in the main lobe only); the volume of the beam-squared is 55 arcsec^2.

Holography maps show GBT primary reflector RMS's of $\sigma_{sfc} = 240\,\mu$m, predicting a 90 GHz surface efficiency of $\exp(-(4\pi\sigma_{sfc}/\lambda)^2) = 44\%$. With an 81% illumination efficiency the predicted total aperture efficiency is 36%. This corresponds to a gain $\Gamma = \eta_{surf}\eta_{illum}A_{geom}/2k_B = 1.01$ K/Jy (antenna temperature).

The beam efficiency is 81% including the characterized error beam (58% with respect to the main lobe only).

Main beam RJ Temperature to flux conversion is given by 0.795 K_{RJ}/Jy, i.e., 795 mK in the main beam produces a signal equivalent to one Jansky. In CMB units this is 0.98 K_{cmb}/Jy. An SZ surface brightness of $y = 10^{-5}$ is equivalent to 28.6 μJy. All of this includes the full main beam volume (136 arcsec^2).

2.2 MUSTANG

The standard MUSTANG sensitivity number is stated as: one can make a $3' \times 3'$ map to 0.4 mJy RMS in one hour of integration time. The meaning of this number is: the RMS of 2 arcsecond pixels gridded a certain way (cloud in cell), in a map fully calibrated to Jy/bm. Preliminary investigation showed that smoothing reduced the noise by a further factor of $\sim FWHM/4''$. For example, a $9''$ FWHM smoothing would give $\sim 170\,\mu$Jy/bm RMS sensitivity on $3' \times 3'$ in one hour. Here we recap where the standard sensitivity number comes from in detail and give a better current number:

Technical comment: gridmap()) defaults to 2″ pixels and uses a cloud-in-cell gridding kernel which has effects similar to those of smoothing the data by a $2\times$ pixel size $= 4''$ FWHM Gaussian. *mapfromtod()* grids data into the nearest cell (no smoothing) and defaults to $1''.25$ pixels. This is all complicated because we calibrate our maps to *Jy/bm*, which is useful and standard, but our noise is mostly white noise and lives in surface brightness (Jy/pixel or Kelvin) space.

The MUSTANG sensitivity numbers are derived as follows:

- The data: 8a56-13 (25feb09) scans 40-42. One quarter hour (integration time) mapping $4'.5 \times 4'.5$ with box scans. Spot checked with several other datasets.
- Method: *mustangmap()* with /dataflip, /gflip, /faint, poltime=5. Deafult calibration from closest cal scan. default τ corrections applied. 47 detectors were in operation at the time.
- calibration factor that night was very close to unity (calibrator = Ceres; telescope surface file version in use = $v2.35$ - about 20% improvement to the telescope gain was subsequently made).
- Map RMS's are summarized in Table 1. Numbers were obtained by Gaussian-normalized mean absolute deviation of pixels having greater than half the median (non-zero) weight – you can think of this as a more robust way to measure the RMS. *mapfromtod()* numbers are more useful since they are a measure of the true white noise in the map.
- Taking the *gridmap()* 2″-pixel number and scaling it gives the standard 0.43 mJy RMS on $3' \times 3'$ in one hour (integration). Results depend strongly on gridding method for "pixel noise".

A sample of more recent data (several sessions of the MACSJ0717+3745 dataset) were examined to check these sensitivity numbers in light of surface improvements since 8a56-13 and get an idea how much they vary night to night. We find fully calibrated timestream RMS's (43 msec integrations) of 6.4 to 9.4 mJy with a mean of 7.5 mJy. Allowing for this gain, but also the number of detectors (33 typical now, vs 47 typical previously) gives point source sensitivities of **121 − 178** μJy (RMS) currently, with a **typical values of 142** μJy (RMS). All of this is for $3' \times 3'$ in one hour (integ.).

Table 2 summarizes the current MUSTANG sensitivity several different ways.

2.3 MUSTANG-N Improvement

MUSTANG is detector noise limited. Supposing we do nothing but migrate the existing detectors into feedhorns, the SNR will increase by a factor of 3 for the same (18 GHz) bandwidth due to the increase in sky power[2]. We plan to

[2] A 0.5 f_λ detector array, each detector only measures $\sim 25\%$ of the power in one diffraction-limited PSF. The current MUSTANG optics measure $\sim 33\%$ of the power in one PSF.

Gridding Method	Pixel Size	RMS
mapfromtod()	$1''$	3.66 mJy/bm
"	$2''$	1.85 mJy/bm
gridmap()	$1''$	2.46 mJy/bm
"	$2''$	1.29 mJy/bm

Table 1: Measured map noise— RMS of pixel values— in representative $4'.5 \times 4'.5$ dataset with 15 minutes integration time (GBT 8A56-13 scans 40 - 42).

Quantity	Value	Notes
a) Map White Noise	1.05 mJy/bm	$1''$ pixels
b) Map White Noise	$7.72\,\mu$Jy/arcsec^2	$1''$ pixels
c) Map White Noise	$1.32\,\mathrm{mK_{RJ}}$	$1''$ pixels
d) Point Source Sensitivity	$142\,\mu$Jy	$(9''.32'', 22''.7)$ (beam, error beam)
e) Surface Brightness	$178\,\mu\mathrm{K_{RJ}}\sqrt{\mathrm{beam}}$	
f) Surface Brightness	$22\,\mu\mathrm{K_{RJ}}$ arcmin	
g) Compton y	$4.96 \times 10^{-5}\sqrt{\mathrm{beam}}$	
h) Compton y	6.13×10^{-6} arcmin	

Table 2: Current MUSTANG sensitivity for 1 hour integration over $3' \times 3'$, with 33 good detectors and a mean detector noise level of 7.5 mJy (RMS of fully calibrated 43 msec integrations).

increase the bandwidth from 18 to 30 GHz, resulting in a conservative projected improvement of 5.0 in SNR per detector (assuming dual-pol detectors). Further gains come from achieving BLIP, increasing the yield, reducing vibrational susceptibility, and reducing the amount of data to be thrown out.

An independent argument is as follows:

1. At 3mm in Green Bank, the photon occupation number n is greater than unity at the detector, and the noise is therefore described by the radiometer equation (Bose noise). We can estimate the effective system temperature of the current MUSTANG detectors from the observed RMS noise level of the fully calibrated detector data. **The RMS of fully calibrated 43 millisecond integrations is** $8.5\,\mathrm{mJy} = 1.8\,\mathrm{mJy}\sqrt{\mathrm{sec}}$. This is after common mode subtraction, and is an accurate reflection of the white noise in a typical, individual detector.

2. The GBT aperture efficiency at 90 GHz is about 35%, yielding a gain of about $\Gamma = 1.0\,\mathrm{K/Jy}$. The aperture efficiency has been measured directly from MUSTANG data (using load curves to absolutely calibrate them and compare with observations of celestial sources known from WMAP-7); it is consistent with Ruze efficiency estimates from holographic measurements of the dish surface.

3. The expected noise level is $\Delta S_\nu = T_{sys}/\sqrt{\Delta\nu\tau}/\Gamma$. Plugging in 43 msec, 18 GHz, the gain and observed data RMS from above, **we determine an effective system temperature of** $190\,\mathrm{K}$**.**

4. By way of comparison the sky, CMB, ground (8 K), and instrument loading set a limiting background $T_{sys} = 45\,\mathrm{K}$ or so, for observations at 45 degrees elevation. Including the planned bandwidth increase, **achieving BLIP will result in a** $5.4\times$ **decrease in per-(dual-pol)-detector noise.**

5. This does not include improvements from less flagging and vibrational susceptibility.

Table 4 shows the time required for several map scenarios.

	Detectors	Relative	Relative	Map Integration Time (hrs)		
— — ———	#	Sensitivity	Speed	Current	Survey	Deep
Depth Map size				40 μJy $4'.25 \times 4'.25$	20 μJy $8' \times 8'$	5 μJy $8' \times 8'$
MUSTANG	33	1	1	25	358	5,736
M-1.5 (32)	29	5	22	1.2	16	261
M-1.5 (64)	58	5	44	0.6	8	130
MUSTANG-2	332	5	250	–	1.4	23
ALMA Band 1	-	-	-	0.7	10.0	160
ALMA Band 3	-	-	-	1	15.0	240

Table 3: ALMA Band 1 is 40 GHz, Band 3 is 90 GHz. Band 3 beam in its highest surface-brightness sensitivity configuration is $4''$ so the comparisons made here effectively assume an SZE feature that is unresolved on that scale. For the constant surface brightness (on $9''$) case, ALMA B3 will take $6.3\times$ longer than shown. Number of detectors for MUSTANG-1.5 and MUSTANG-2 assumes 90% yield.

	Detectors	Relative	Relative	Map Integration Time (hrs)		
— — ———	#	Sensitivity	Speed	Current	Survey	Deep
Depth Map size				40 μJy $4'.25 \times 4'.25$	20 μJy $8' \times 8'$	5 μJy $8' \times 8'$
MUSTANG	50	1	1	50	717	11,471
M-1.5 (32)	29	5	22	2.3	33	522
M-1.5 (64)	64	5	44	1.2	16	261
MUSTANG-2	332	5	250	n/a	2.9	46

Table 4: Comparison of time required for several map scenarios. For the MUSTANG current case point source sensitivity of 142 μJy (in $3' \times 3'$, 1 hr integrating) was assumed. All integration times include 2x overheads.