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GBT MEMO

Pseudo-Correlation Radiometer Observing Modes & Calibration

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Abstract

We describe the principle of operation of a pseudo-correlation radiometer and describe observing modes appropriate for calibritaing beamswitched data, total power data, and polarimetric data.

1 Introduction

The GBT 3mm and 1cm heterodyne receivers under construction use a MAP-style psuedo-correlation radiometer design, shown in figure 1. This design consists of two radiometers (I and II), each of which, using magic T junctions, multiplexes the signals from opposite polarizations of opposite horns[1] over a pair of amplifier chains, and de-multiplexes the mixed signals after the amplifiers. Between the magic T's each radiometer also has a phase switch, allowing a relative sign to be applied to the complex (voltage) gains of the two arms of the radiometer. This design has the advantage of minimizing the contribution of gain variations to the system noise and allowing very rapid switching via the phase switches to cancel $1/f$ noise in other components of the system, maximizing the radiometric sensitivity.

In this memo I tabulate some results for the likely operational modes of these receivers, and consider how to use noise diodes to calibrate them in each of these modes. Given engineering and financial constraints in connection with the 3mm receiver, one of the main goals of this effort is to determine whether there is a calibration scheme involving a single noise diode which suffices to calibrate the desired modes, and to see what impact this scheme has on the operating modes given existing system constraints.

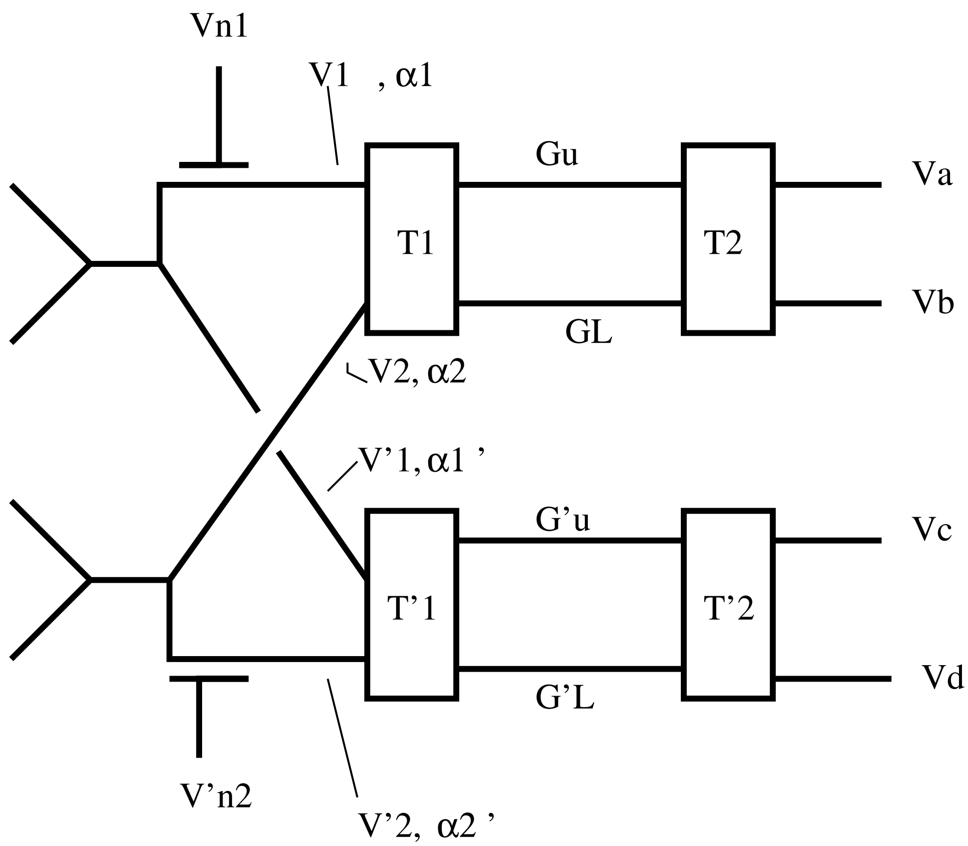

Figure 1: Balanced radiometer and proposed 3mm Rx calibration scheme.

2 Notation & Total Power Modes

Refer to figure 1 and consider radiometer I (T_1, T_2). There are two input voltages V_1 and V_2; the RMS's of these two voltages correspond to antenna temperatures as $V_1^2 = \alpha_1 T_1$ and $V_2^2 = \alpha_2 T_2$. We do not consider the gains

[1] S. Padin chose this particular scheme because the dominant expected leakage is a near-field coupling between like polarization modes of adjacent horns, instead of reflections.

α beyond noting that are likely to differ by a few percent due to different RF plumbing, which gives rise to an offset which for differenced observations will have to be removed by a higher-order differencing scheme. For the 3mm Rx, (V_1, V_2') are similar linear polarizations in different directions on the sky, and (V_2, V_1') are the other polarization; for the 1cm Rx the situation is the same, except the polarizations are circular.

In the design illustrated a *single* noise sources is injected before the T's at two places (V_1, V_2'— see the figure). This particular arrangement is in fact optimal for calibration. We will show below that it is important for the cross-correlation calibration that this noise source be coherent. Optionally, a second coherent source can be coupled into the other two paths (V_2, V_1').

The magic T assembly multiplexes the signals, passes them both over the upper and lower amplifier chains of each radiometer, and demultiplexes them, giving output voltages:

$$\begin{aligned} V_A &= \frac{1}{2}(G_U + G_L)V_1 + \frac{1}{2}(G_U - G_L)V_2 \ (radiometer I) && (1) \\ V_B &= \frac{1}{2}(G_U + G_L)V_2 + \frac{1}{2}(G_U - G_L)V_1 \ (radiometer I) \\ V_C &= \frac{1}{2}(G_U' + G_L')V_1' + \frac{1}{2}(G_U' - G_L')V_2' \ (radiometer II) \\ V_B &= \frac{1}{2}(G_U' + G_L')V_2' + \frac{1}{2}(G_U' - G_L')V_1' \ (radiometer II) \end{aligned}$$

Without loss of generality we take the complex gains to be

$$G_U = g_1 \quad (2)$$
$$G_L = \pm g_2 e^{i\Delta q} \quad (3)$$
$$G_U' = g_1' \quad (4)$$
$$G_L' = \pm g_2' e^{i\Delta q'} \quad (5)$$

where $g_{1,2}$ are real and the Δq are differential passband errors; the signs indicate the effect of the phase switch. The algebra which follows is simlified if we introduce

$$G_{\pm} = \frac{1}{2}(G_U \pm G_L) \quad (6)$$

and its equivalent $G_{\pm}'$ for radiometer II. Note that if the gains are equal, the output voltages are are proportional to the input voltages and there is no mixing. An advantage of this notation is that a change of state of the phase switch (whose states are 0° and 180°) can be taken to correspond to $G_{\pm} \leftrightarrow G_{\mp}$. This permits rapid beam switching.

Since all complex products are magnitudes, I omit modulus signs; *e.g.*, $G_+^2 = |G_+|^2$, $V_1^2 = |V_1|^2$, *etc.* With this in mind a useful relation is

$$G_{\pm}^2 = g_1^2 + g_2^2 \pm 2g_1g_2cos(\Delta q). \quad (7)$$

A fundamental observable is the time-averaged total power of a single output voltage, which also roughly corresponds to the autocorrelation of that voltage. Introducing detection gains d_{A-D} we have

$$\mathbf{A}_0 \equiv d_A\langle V_A^* V_A\rangle = d_A(G_+^2V_1^2 + G_-^2V_2^2) \quad (8)$$
$$\mathbf{A}_\pi \equiv d_A\langle V_A^* V_A\rangle = d_A(G_-^2V_1^2 + G_+^2V_2^2) \quad (9)$$
$$\mathbf{B}_0 \equiv d_B\langle V_B^* V_B\rangle = d_B(G_+^2V_2^2 + G_-^2V_1^2) \quad (10)$$
$$\mathbf{B}_\pi \equiv d_B\langle V_B^* V_B\rangle = d_B(G_-^2V_2^2 + G_+^2V_1^2) \quad (11)$$
$$\mathbf{C}_0 \equiv d_C\langle V_C^* V_C\rangle = d_C(G_+^{'2}V_1^{'2} + G_-^{'2}V_{'2}^2) \quad (12)$$
$$\mathbf{C}_\pi \equiv d_C\langle V_C^* V_C\rangle = d_C(G_-^{'2}V_1^{'2} + G_+^{'2}V_2^{'2}) \quad (13)$$
$$\mathbf{D}_0 \equiv d_D\langle V_D^* V_D\rangle = d_D(G_+^{'2}V_2^{'2} + G_-^{'2}V_1^{'2}) \quad (14)$$
$$\mathbf{D}_\pi \equiv d_D\langle V_D^* V_D\rangle = d_D(G_-^{'2}V_2^{'2} + G_+^{'2}V_1^{'2}) \quad (15)$$
$$(16)$$

where the subscripts $(0, \pi)$ indicate the two states of the phase switch for that radiometer. By default these averages are taken to be against the sky; averages with the CAL on or against an external hot load are denoted as $\mathbf{A}_{0,CAL}$, $\mathbf{B}_{\pi,LOAD}$, *etc.*.

We see in Eqs. 8 - 16 that the total power response of a given channel (*e.g.* $\mathbf{A}$) potentially significant leakage (G_-^2) from the other horn: the gains in the two arms are not generally equal in any precision sense across the band, or on average, and it is unclear *a priori* what the typical values of G_- will be. It may easily be that this level of confusion is tolerable for many spectral line observations.

In this approximation we can easily calibrate the total power with the noisecal. The observed noise cal signal is

$$\mathbf{A}_{0,CAL} - \mathbf{A} = d_A G_+^2 V_N^2 \tag{17}$$

which can be used to scale the astronomy data and will capture the dominant part of the signal which is changing with the gains.

However by using the noise cal and phase switches in concert, we can retrieve the original inputs V_1^2, V_2^2 and calibrate the signals all in a go. For the record, the V_A outputs with the noise cal are

$$\mathbf{A}_{0,CAL} = d_A(G_+^2 V_1^2 + G_+^2 V_N^2 + G_-^2 V_2^2) \tag{18}$$
$$\mathbf{A}_{\pi,CAL} = d_A(G_-^2 V_1^2 + G_-^2 V_N^2 + G_+^2 V_2^2) \tag{19}$$

where V_N^2 is the noise cal signal. Using these, their equivalents for the other outputs, and Eqs. 8 - 11, it is not too hard to show that the input power V_1^2 is

$$V_1^2 = \frac{\mathbf{A}_0 - \frac{\mathbf{A}_{\pi,CAL} - \mathbf{A}_\pi}{\mathbf{B}_{\pi,CAL} - \mathbf{B}_\pi}\mathbf{B}_0}{(\mathbf{A}_{0,CAL} - \mathbf{A}_0) - (\mathbf{A}_{\pi,CAL} - \mathbf{A}_\pi)\frac{\mathbf{B}_{0,CAL} - \mathbf{B}_0}{\mathbf{B}_{\pi,CAL} - \mathbf{B}_\pi}} V_N^2 \tag{20}$$

Similar relations can be derived for the other 3 inputs. Here V_N^2 is an arbitrary quantity, which will ultimately be referenced to a celestial calibrator; thus if V_N^2 is stable, we have also taken out system gain variations. For spectral line observations which do not require beam switching, this would allow unambiguous total power mapping using both beams separately and simultaneously. This is also the really correct way to calibrate the total power modes for gain fluctuations.

A potential problem with this approach is that it involves multiple instances of dividing a large quantity by a small noisy quantity. Further analysis is needed to determine under what circumstances and with what procedures– if any– this is practical for astronomy.

An example of the switch states needed to allow for the above observations are in Table 1. For the sake of the example an astronomical duty cycle of 50% has been assumed, although this can be increased. This illustrated cycle provides all necessary information for the full solution, Eq. 20. The two radiometers' phase switches need not have any particular relation to each other, but they do need to be synchronized, in so far as they both must be synchronized with the CAL.

phase switch	0	0	0	0	π	π
CAL	0	0	0	1	1	0

Table 1: Sample total power switch state cycle.

3 Switched Power Modes

S. Padin has considered these in detail, so we only summarize the results. An instantaneous measurement of the antenna temperature difference is

$$\mathbf{A}_0 - \mathbf{B}_0 = [(d_A V_1^2 - d_B V_2^2)G_+^2 + (d_A V_2^2 - d_B V_1^2)G_-^2]. \tag{21}$$

Note that the "naively" observed CAL signal is

$$(\mathbf{A}_{0,CAL} - \mathbf{B}_{0,CAL}) - (\mathbf{A}_0 - \mathbf{B}_0) \propto (d_A G_+^2 - d_B G_-^2) V_N^2 \tag{22}$$

which is not quite the gain you want to measure. If so inclined you could go through the exercise analagous to that we did for the total powers.

For the continuum backend, detector gain flucutations are an issue, and so we impose a further difference via the phase switch modulation:

$$\mathcal{D}_{\mathbf{AB}} \equiv \frac{1}{2}[(\mathbf{A}_0 - \mathbf{B}_0) - (\mathbf{A}_\pi - \mathbf{B}_\pi)] = \frac{1}{2}(d_A + d_B)(V_1^2 - V_2^2) g_1 g_2 cos(\Delta q). \tag{23}$$

This is also a more easily calibrated combination than the instantaneous difference. We have

$$\mathcal{D}_{\mathbf{AB},CAL} - \mathcal{D}_{\mathbf{AB}} = \frac{1}{2}(d_A + d_B)\, V_N^2\, g_1 g_2 cos(\Delta q) \tag{24}$$

and so forth. Table 2 shows a sample arrangement of switch states for beam-switched observing such as these.

phase switch	0	π	0	π
CAL	0	0	1	1

Table 2: Sample switched power switch state cycle.

Note that while the requirements for gain-fluctuation suppression are not as stringent for spectroscopic observations as they are for the continuum (*e.g.*, the correlator gains are stable to $\sim \frac{1}{2}\%$ on 15 minute timescales), the relative simplicity of calibrating the phase switched differenced observations (eqs. 23 and 24) suggests that for beam-switched spectroscopy, this mode is adopted. To reduce the quantity of data, much longer phase switching periods could be used; in this case the sky cancellation will still be excellent because the $\mathbf{A}_0$ and $\mathbf{B}_0$ (and $\mathbf{A}_\pi$ and $\mathbf{B}_\pi$) differences are collected continuously.

There is also an interesting total power mode obtained by averaging successive phase states from a single detector which results in a clean measurement of the average total power between the beams. That is,

$$\frac{1}{2}(\mathbf{A}_0 + \mathbf{A}_\pi) = d_A(G_+^2 + G_-^2)(V_1^2 + V_2^2). \tag{25}$$

This is trivially calibratable with the noise diode, and allows the diodes (for this mode anyway) to be referenced to an external load. This can be constructed and calibrated from data collected in the differential mode of Table 2.

4 Polarimetry

Polarimetry is a special case in that you will be measuring correlations between different outputs rather than autocorrelations. In particular, the correlations between orthogonal polarizations coming from a single horn are of interest. In order to calibrate this, coherent noise must be injected into both arms, and this requires the phase switches in the two radiometers to have the opposite settings. Other than this the calibration is straightforward. A sample switch state cycle is shown in table 3.

Note that in this case the calibration cycle signal consists of nothing but noise from the diode, *i.e.*, the sky and system contributions are uncorrelated.

5 Odds & Ends

5.1 Gain Fluctuation Suppression & Strong Source Observations

Consider a phase-switched differential power measurement D_1 and neglect the gains after the second magic T. We have:

$$D_1 \equiv V_A^2 - V_B^2 = g_1 g_2\, (V_1^2 - V_2^2). \tag{26}$$

phase switch I	0	π	0	π
phase switch II	0	π	π	0
CAL	0	0	1	1

Table 3: Sample polarimetric beam-switched switch state cycle.

Compare this to the case of the equivalent measurement D_2 made by passing V_1 and V_2 over two *independent* amplifier chains (in which case $V_A^2 = g_1 V_1^2$ and $V_B^2 = g_2 V_2^2$). In that case the differential measurement is

$$D_2 \equiv V_A^2 - V_B^2 = g_1^2 V_1^2 - g_2^2 V_2^2. \tag{27}$$

Consider now the effect of fluctuations in the gains g_1, g_2 on the measurements made with these two arrangements. We assume the fluctuations are Gaussian random. Then

$$\sigma^2[D_1] = (\sigma_{g_1}^2 g_2^2 + \sigma_{g_2}^2 g_1^2)\,(V_1^2 - V_2^2)^2 \tag{28}$$

and

$$\sigma^2[D_2] = 4(g_1^2 \sigma_{g_1}^2 V_1^4 + g_2^2 \sigma_1^2 V_2^4). \tag{29}$$

It is clear that for weak signals $\sigma^2[D_1] << \sigma^2[D_2]$, because for the correlation radiometer the gain fluctuations multiply the signal difference rather than the total power (although note that balanced antenna temperatures do not generate balanced input voltages, so there will be some gain fluctuation contribution).

If $V_1^2 - V_2^2 > V_{sys}^2$ (where V_{sys}^2 is a system-temperature equivalent RMS noise voltage) then gain fluctuations in the signal are larger than the system noise, as Martin Shepherd has pointed out. Even in this case, however, the correlation radiometer is a factor of two better than two independent total power radiometers which are differenced (in this case, the latter actually acts like a *single* total power radiometer). Consider the case where the gains and their RMS's are equal, and $V_1^2 >> V_2^2$; then eq. 28 becomes

$$\sigma^2[D_1] = 2\sigma_g^2 g^2 V_1^4 \tag{30}$$

whereas eq 29 is

$$\sigma^2[D_2] = 4\sigma_g^2 g^2 V_1^4. \tag{31}$$

This is because the correlation radiometer averages the gain fluctuations in the two amplifier chains, reducing the amplitude by a factor of $\sqrt{2}$ (of the power gain, in power).

Note that *unlike* two independent radiometers, the receiver gain in differential mode is independent of the relative signal strengths. That gain is simply $g_1 g_2$. There is thus no problem transferring the calibration from bright sources to faint sources. The increased noise on bright sources with $T_{ant} > T_{sys}$ will require longer integration times than the radiometer equation would predict. To reduce this, magic T assemblies could be "nested", but that is complicated— a better solution would be to use more gain-stable devices. However bright source observations are not a driving consideration for the design and so we do not consider this further.

5.2 Summary of the Case for Noise Diode Calibration

Noise diodes allow the receiver to be calibrated in the same mode as the observations are taken, which makes the calibration simpler and more robust. They also greatly reduce the minimum timescale for calibration, reducing the impact of $1/f$ gain fluctuations (which still exist for the signal, although the balanced arrangement minimizes their contribution to the noise). An item of concern for the scenario of relying exclusively on hot load calibrations for the calibration is the fact that the 3mm Rx is likely to be somewhat susceptible to compression, as well as the fact neither of these receivers contain variable attenuators. Transferring a total power calibration to a differential power measurement is susceptible to corruption by imperfections in the magic T's, whose power balance specification is only $\sim 10\%$; these imperfections are probably measurable and correctible, but this would also add complication to the calibration scheme. Correctly handled, noise diodes also allow instrumental calibration of the cross-correlation ("spectro-polarimetric") modes, which complements the astronomical calibration of these modes. With loads that would require an auxilliary mechanical assembly, *e.g.*, a wire mesh insertable before the the load.

6 Conclusions

The main modes in which the receiver will be operated are:

1. **Total Power**
2. **Phase-Modulated, Beam Switched**
3. **Cross-Correlation**

Calibration and switch cycle schemes for these modes have been outlined. In connection with them our main conclusions are:

- A single noise diode, doubly coupled as shown here, allows the primary observing modes of the correlation radiometer receivers to be calibrated robustly. A second noise diode so coupled to the other arms will add some flexibility, but also complexity.
- The raw measured total powers have cross-horn leakage, but with the phase switches and noise diodes the calibrated input voltages are fairly easily retrieved.
- Since the spectrometer can only control four external signals, and the noise diodes need to be coordinated with the actions of two phase switches in each receiver, each receiver should have no more than two noise diodes (each doubly coupled).
- The cross-correlation mode needs coherent noise signals injected into different arms of the receiver, and requires that the phase switches be commandable to opposite states.
- Although phase switching is not always necessary for cancelling gain fluctuations, it usually yields the mostly easily calibrated combination of observables for the correlation radiometers.

I think Jim Braatz, Rick Fisher, Glen Langston, Roger Norrod, Mike Stennes, and Galen Watts for helpful conversations in connection with this work.