Data Analysis Sceneraios for the Penn Array

v.07apr03 Brian S. Mason

Contents

1 Introduction

The purpose of this memo is to outline likely Penn Array data collection scenarios, and to describe the characteristics of some science targets, in order to clarify what we may need to support in the data analysis software.

2 Observing Sequence

A typical observing sequence will be as follows:

1. **load curve:** automated procedure used to correctly set the operating point (bias voltage) of the TES's. This together with total power information during the observation should give us a handle on where in the response curve we are.

2. **skydip**: to give a direct assesment of the opacity. The site tipper will also give some information; as yet we do not know if this will be astronomically useful.

3. **primary calibrator scan**: probably small nutated circles or "spirograph" type scans (see § 2.2). This could be an asteroid or a distant planet. The flux density or surface brightness of this source is the fundamental reference. In principle, the source could be resolved, *e.g.*, a disk,

and I don't think that's a problem except that it complicates a pointing solution.

4. **secondary calibrator scan**: used to monitor the calibration (pointing, sky opacity, load levels, *etc.*) through the observation. The observing strategy should be as similar as possible to that used in the primary calibrator observation.

5. **Mapping scans** for 10 to 30 minutes

6. **secondary calibrator**

7. go to 5— repeat until you have enough data

8. **secondary calibrator B**: near your second science target

9. **Mapping scans** for 10 to 30 minutes

10. the end

At this poin I imagine that the following types of distinct FITS files will be produced by the GBT M&C system: i) load curve; ii) skydip; iii) calibrator; iv) mapping. As we will discuss below (§ 2.2), another is possible, corresponding to spirograph-type mapping scans with a pointing reference. These can be "filled" as we desire/specify. The online (GBT M&C) system will determine pointing offsets for the primary and secondary calibrator scans, and apply them to the telescope pointing.

2.1 Sampling and Timestream Properties

We consider two basic scenarios: arcminute^2 mapping and degree^2 mapping. For the first case, a $1'/sec$ slew rate is assumed. Much faster than this is awkward given GBT acceleration limits of $12'/sec^2$. This implies a timestream sampling of 50 msec (see Appendix A). Deconvolution of the detector time response in this case is probably desirable but results will not be very sensitive to the assumed τ. In this scenario we assume a cal pulse every 20 samples or so.

For the degree^2 maps we assume a $40'/sec$ slew speed. This implies a timestream sampling of 1 msec. In this case accurate estimation and deconvolution of the detector response is crucial (see again Appendix A). In this scenario we assume a cal pulse every 1000 samples.

2.2 Spirograph Scans

We consider an observing strategy whose goal is to make deep, arcminute squared or larger maps under challenging seeing conditions. In this case we define a region of interest roughly circular, centered on a calibrator source several mJy or brighter. We then execute "spirograph" type scans about this calibrator source which encompass the entire region. One could, for instance, parameterize the pointing position $\vec{p}(t)$ as

$$\vec{p}(t) = r_o\,(\hat{x}\,sin\,\Omega t + \hat{y} cos\,\Omega t)\,sin\,\omega t \tag{1}$$

Here $\vec{p} = 0$ is the calibrator location, which is crossed every $t_{cyc} = \pi/\omega$ units of time. For the case $\Omega << \omega$ then the maximum radius that can be encompassed r_o for a given maximum acceleration a_o, if a calibrator crossing is required every t_{cyc} time units, is

$$r_o = \frac{1}{\pi^2} a_o t_{cyc}^2 \tag{2}$$

If $a = 12'/sec$ and $t_{cyc} = 10$ sec, then $r_o = 2°$. For $a = 3'/sec$, $t_{cyc} = 3$ sec, we have $r_o = 2'.7$. The crucial parameter is the cycle time. The sensitivity in this map will not be uniform, since there are many crossings of the map-center; this is mitigated somewhat by the fact that the array is moving fastest at this point. The maximum scan speed attained is

$$v_{max} = \frac{\pi r_o}{t_{cyc}} \tag{3}$$

which, for the two cases considered, has a value of $38'/sec$ and $2'.8/sec$ respectively. I don't see an efficient way to generalize this approach to significantly off-center calibrators, which would be nice— the acceleration limits limit the solutions. These calculations are only meant to be indicative of feasibility.

With this approach, spirograph scans would effectively contain their own "tertiary" calibrator scans; secondary calibrator scans would probably not be needed except as an added cross-check. Some distinction is desirable— I don't think we want the online system determining pointing offsets from these data (???).

3 Case 1: Galactic SNR

Observations of Galactic SNRs will be excellent first-light/ prototype observations for the Penn Array. Consider the Crab nebula, which is about $4'$ in diameter, has an integrated $S_{32\,GHz} = 385\,$Jy, and a spectral index of $\alpha = -0.3$ through 3mm or so wavelength. This implies a surface brightness of 425 mJy per Penn Array $8''$ FWHM beam. Make a $6' \times 6'$ map: this will have 14400 Nyquist ($3''$) pixels, but only ~ 1800 statistically independent beams on the sky. The notional scan strategy is to slew $1''/sec$ in Az, completing a single row in six seconds. The back-scan would be offset by $16''$ ($0.5\,FOV$) in El. There are 24 rows, requiring 12 minutes to complete. In practice this should probably be a single scan, or a few scans. Following this an orthogonal pattern (scans in El, row offsets in Az) is executed; in total, 24 minutes are required. The theoretical thermal noise level in this map is $\sim 50\,\mu$Jy per beam. This gives an SNR $\sim 8 \times 10^3$. High-resolution maps of sources like this should easily be possible (if systematics do not prohibit it), but this will require an accurate knowledge of the beam. This is not a key scientific application, but could be a good proving ground for techniques.

Cas A is another obvious first light observation. It is similar in size to the Crab, but has a steeper spectrum. I estimate a mean 3mm surface brightness of $\sim 140\,\mathrm{mJy/beam}$. The mean SNR on-source would be $\sim 3 \times 10^3$.

In both of these cases it would be desirable to solve for residual pointing offsets from the mapping data.

4 Case 2: Survey of a Star Forming Region

Suppose we wish to survey a star forming molecular cloud core. This case is identical to § 3 except for the nature of the signal. Suppose again a $6' \times 6'$ map, requiring 24 minutes total to make, with a resulting $50\,\mu\mathrm{Jy}$ per beam RMS.

There are two signals of interest: one is the population of protostars and protostellar clumps. These have characteristic surface brightnesses of a few mJy to a few 10s of mJy per beam, and are unresolved or marginally resolved. Above $0.4\,M_{sun}$ (~ 4 mJy) these sources are distributed as

$$N(>S) = 1\,\mathrm{arcmin}^{-2}\left(\frac{S}{0.4\,\mathrm{mJy}}\right)^{-1} \tag{4}$$

There is also extended, nebulous, steeper-spectrum ($\beta \sim 2$) emission from the molecular cloud itself. At $1.2\,\mathrm{mJy}$ this has a typical surface brightness of $10\,\mathrm{MJy/Sr}$; this translates into $400\,\mu\mathrm{Jy}$ per GBT 3mm beam.

Clearly this map could be made larger to image an entire molecular cloud (tens of arcminutes on a side), or deeper, to more accurately image the fine-scale cloud structure. Due to the characteristic angular scales of interest and the presence of mJy sources, star-forming cores are also a good candidate for a spirograph scan pattern.

5 Case 3: Galactic Plane Survey

At the GBT maximum slew rate in Az of $40'/sec$ the PAR will be able to map $10\,\mathrm{deg}^2$ to an RMS of $1\,\mathrm{mJy}$ in 1 hour. In 50 hours one could map $500\,\mathrm{deg}^2$, a reasonable portion of the Galactic plane. The strategy is to slew as fast as possible, with $\sim 50\%$ overlap between scans as before. The "basketweaving" approach may need to be slightly modified due to the different slew speed limits in Az and El ($40'/sec$ and $20'/sec$, respectively)— by either a) abandoning crossed scans; b) adopting slower slew speeds; or c) not slewing purely in Az or El, but having two non-orthogonal axes which distribute the load between the two drives. Option (c) nominally allows faster slew speeds. The geometry of the Galactic plane is also a factor. For simplicity assume constant $20'/sec$ slew rate and scans crossed at an angle of $60°$.

There will be two main classes of objects sought in such a survey:

1. **Isolated low-mass protostars**: These will have surface brightnesses of a few to 20 mJy per beam at 3mm; angular extents are 10s of arcseconds up to 2 arcminutes or so. To reliably find these, some pixel-averaging is desirable.

2. **High-Mass Star Forming Regions**: These will have surface brightnesses of a few 10s to a few 100 mJy per GBT 3mm beam, and typical angular extents of a few arcminutes.

6 Case 4: Deep High-z Galaxy Survey

Suppose we map a $3' \times 3'$ region, roughly twice the size of the Hubble Deep Field. As per our usual scan strategy there are twelve rows, 3 seconds per row are required, and 36 seconds gives one complete coverage of the field (one scan). The next scan would be in the orthogonal direction. We desire an RMS of $8\,\mu$Jy, requiring 360 such scans or 3.6 hours of integration total. In this region we expect $6-7$ sources above the 5σ detection threshold of $40\,\mu$Jy. If all point sources above $40\,\mu$Jy can be identified, then the RMS noise level in the residuals due to confusion is also $\sim 8\,\mu$Jy.

Variations

- **Wide Survey:** Suppose we map $1^\circ \times 1^\circ$ to an RMS of $190\,\mu$Jy. Scans will be at $40'/sec$; the region as a whole contains 240 rows, at 1.5 seconds per pass requiring 6 minutes for a single map of the whole region. The RMS in this map would be $960\,\mu$Jy. In two and a half hours (25 repeated maps) an RMS of $190\,\mu$Jy is expected. A square degree is expected to contain some 15 1 mJy sources and one 5 mJy source.

- **Spirograph Scans:** Suppose we do a single degree squared map (taking 6 minutes) to find a several mJy calibrator source. We then execute a $3' \times 3'$ deep map in the vicinity of this calibrator. We imagine a spirograph-type pattern centered on the calibrator, covering a region of radius $\sim 1'.7$. This map would have a dynamic range of about $5\,\mathrm{mJy}/8\,\mu\mathrm{Jy} = 625$.

7 Case 5: SZE of Distant Clusters

Suppose we want to map the SZE in a distant cluster. Consider a cluster of mass $M_{200} = 1.25 \times 10^{14}\, h^{-1}\, M_{sun}$, implying roughly $T_e \sim 3\,\mathrm{keV}$, $\tau \sim 4 \times 10^{-3}$, and a central flux density of $I_o = 42\,\mu$Jy per GBT beam. This will be distributed as

$$I(\theta) = I_o \left(1 + \frac{\theta^2}{\theta_o^2}\right)^{-\frac{3\beta}{2}+\frac{1}{2}} \tag{5}$$

with $\beta = 2/3$ typical, and $\theta_o \sim 30''$ characteristic of this case. With a $5' \times 5'$ map a 10% zero-level correction due to the cluster is required. We require $8\,\mu$Jy sensitivity in the map, implying 10 hours of integration with a strategy similar to that in § 6 (5 seconds per row; 20 rows; 100 seconds for a single complete coverage; 360 complete coverages required). It will be desirable to degrade this map to a lower resolution in order to image the extended cluster structure; before doing so it will be desirable to identify point sources and exclude them from further analysis.

A similar case is that of mapping the central region of a nearby (say, cooling flow) cluster. In this case, $I_o \sim 150 - 200\,\mu$Jy and $\theta \sim 1' - 2'$. A slightly larger map may be desired.

Variations

- **Coma cluster:** We wish to make a $1^\circ \times 1^\circ$ map of the Coma cluster or some other low-z cluster. Coma will have $I_o \sim 160\,\mu$Jy and $\theta_o = 9'$. A map with 80 μJy rms per beam is obtained in 14 hours. Again, a higher signal-to-noise, lower resolution estimate of the surface brightness will be desired.

- **Az/El Terms:** For this and other mapping applications, low-level, spatially variable ground spillover may be an issue. In this case one would want to fit for Az/El terms in the signal model. Care must be taken not to confuse the expected sky emission gradient in elevation with the ground signal— an external constraint will be needed.

- **Source Cleaning:** (noted above)

A Appendix: Sampling the Sky

A single dish telescope measures spatial frequencies on the sky from zero up to

$$\nu_{sp,max} = \frac{D}{\lambda}, \tag{6}$$

where D is the antenna diameter and λ is the observing frequency. With $D = 100\,m$ and $\lambda = 3\,mm$, $\nu_{sp,max} = 0.16/arcsec = 1/(6.2\,arcsec)$. To Nyquist sample this spatial frequency requires a sample every $3.1\,arcsec$. This is a conservative criterion, since most realistic illumination tapers (and simple geometry) ensure that there are "not very many baselines of length D". If one were to desire to map a region of ths sky with full spatial frequency sampling, this pixels should be spaced no further apart than this spacing. In practice the ~ 4 arcsecond spacings provided by the $0.5f\lambda$ optics are probably close enough (**??**).

The Nyquist criterion translates into two constraints on scanning observations, both related to the time sampling and slew speed. First consider an ideal bolometer with an instantaneous response; this bolometer gives rise to a time-stream S which bears a one-to-one correspondence with beam-sized samples of the sky brightness. Let the samples in the timestream be separated by δt_{min}. Consider also that the telescope is slewing with a velocity v (units of, say, arcseconds per second). In this case a spatial frequency mode on the sky ν_{sp} corresponds to a frequency $\nu_{ts} = v\nu_{sp}$ in the timestream S. We require

$$\begin{aligned} \nu_{ts,max} &> v\nu_{sp,max} && (7) \\ \frac{1}{2\delta t_{min}} &> && (8) \\ & && (9) \end{aligned}$$

or

$$v < \frac{1}{2\nu_{sp,max}\,\delta t_{min}} \tag{10}$$

For the above-quoted $\nu_{sp,max} = 0.16/arcsec$ and a timestream sampling of 1 msec, the slew-speed limitations are $52'/sec$. 1 msec is a standard "highest time resolution" spec for GBT high frequency receivers and is derived from $40'/sec$ at 115 GHz. The GBT max slew rate of $40'/sec$ corresponds to a single-pass mapping speed of 20 square arcminutes per second, and the same number of square degrees per hour. For typical slew speeds of $1'/sec$ and $40'/sec$ the required time samplings are 1.3 and 52 msec.

Consider now the effect of a finite bolometer time-response, $R(t)$. The observed timestream S_{obs} is

$$S_{obs} = R \otimes S \tag{11}$$

where $\otimes$ is the convolution operator. Then in fourier space

$$\tilde{S}_{obs} = \tilde{R} \times \tilde{S}. \tag{12}$$

Bolometers typically have a time response like

$$R(t) = e^{-t/\tau} \tag{13}$$

so

$$\frac{\mid \tilde{R}(\omega) \mid}{\mid \tilde{R}(0) \mid} = \frac{1}{\sqrt{\omega^2 + 1/\tau^2}} \tag{14}$$

where ω is 2π times the frequency ν of the timestream fourier mode in question. The effect of the bolometer time-response is thus to low-pass filter the timestream. As before the critical frequency is $\omega = 2\pi\, v\, \nu_{sp,max}$, this being the highest frequency you would ever try to work with in the timestream. For 5 ms bolometer time constants (design target) and the GBT's spatial Nyquist frequency at 3mm, a slew speed of $5.7'/sec$ gives a reduction of a factor of two in the (timestream) frequency response. At $40'/sec$ the reduction is a factor of 12. This has two consequences:

1. The impact of noise sources which are *not* convolved with the detector time-response filter, such as Johnson noise in the detectors and post-detector electronics, are then effectively amplified by $\sqrt{1 + \omega^2\tau^2}$ (in this example, a factor of two). In practice the white noise component should be a factor of a few below photon noise. Note that photon noise looks like sky signal, *i.e.*, is convolved with $R(t)$.

2. At scan rates of a few $'/sec$ and faster it becomes important to know accurately the detector response $R(t)$. Errors in the time constants translate into systematic over or under estimates of the spatial frequency amplitudes, with these errors becoming greater as the scan rate increases and worst at the highest spatial frequencies. These translate into image artifacts along the direction of the scan. At a scan rate of $5.7'/sec$ and faster a 5% error in τ corresponds to a $\sim$ 5% error in the measurement of the Nyquist spatial frequency mode along the scan direction. $R(t)$ is probably best estimated from the data themselves using the decay of the detector response to the cal pulses.