Note from Joe Brandt and Rick Fisher on Antenna position error and position sampling rate

Date: Feb 7, 2012

Since there seems to be renewed interest in the sample rate
of antenna encoder positions, I am forwarding the attached
email thread from 1999. Rick Fisher's email estimated the
possible error for a given sample rate.

I should point out that at that time, we were assuming the use of the
original acceleration of 0.2 deg/sec^2 for observing.
This has since been reduced to a maximum of 0.05-0.1 deg/sec^2.
I show Rick's original calculation, with this revised value below.

I should also point out that the PTCS servo system upgrade increases
the sample rate to 1000Hz; this data is then band-limited using
a FIR filter for the ACU. The point here is that even under periods
of high acceleration, we will likely record encoder positions even better
than Rick's calculations show.

Rich Fisher wrote:

The positions recorded at the sample times can be assumed to be correct so
what we are worried about is the error that we make when
interpolating between points.

If we take the simplest case of the telescope accelerating in the y
coordinate from a standing start, where y(t=0) = 0, the true track of the
telescope in the y coordinate will be

y = 0.5*a*t^2, where a is the acceleration.

If we approximate this track with a linear interpolation we will get

yi = m*t,

where m is the slope, and y == yi at the next sample point, t1, hence,

0.5*a*t1^2 = m*t1, or

m = 0.5*a*t1

The error that we are worried about is maximum of y - yi
between t=0 and t=t1.

dely = y - yi = 0.5*a*t^2 - m*t

This error is maximum when d(dely)/dt = 0.

d(dely)/dt = d(y - yi)/dt = 0 = a*t - m

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i.e. at time t = m/a.  Substituting into the equation for dely

       dely = 0.5*a*(m/a)^2 - m^2/a = -0.5*m^2/a = -0.125*a*t1^2
or
       t1 = sqrt(8*dely/a)

If we take dely = 0.5/3600 degrees and
             a = 0.2 deg/sec^2,
then the maximum sample period is
            t1 = 0.0745 seconds.

With an updated value of a = 0.1, t1 = 0.105 seconds

The current value of t1 is 0.10 seconds.

-- jbrandt
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