LEAST-SQUARES FREQUENCY SWITCHING (LSFS)

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HETERODYNE SPECTROSCOPY: THE BASIS OF RADIOS, TV'S, CELL PHONES, AND... *RADIO ASTRONOMY!!*

QUESTION: **For the human ear, how does sound carried by radio waves—say, at 91.7 MHz in the FM band—get converted to baseband? Ditto for the gab on your cellphone. And ditto for the 21-cm line, which is converted to baseband for our digital correlators.**

ANSWER: **TRIGONOMETRY!!!**

$$\underbrace{\cos(F_{RF}) \cdot \cos(F_{LO})}_{product} = \frac{1}{2}\cos\underbrace{(F_{RF} - F_{LO})}_{difference} + \frac{1}{2}\cos\underbrace{(F_{RF} + F_{LO})}_{sum}$$

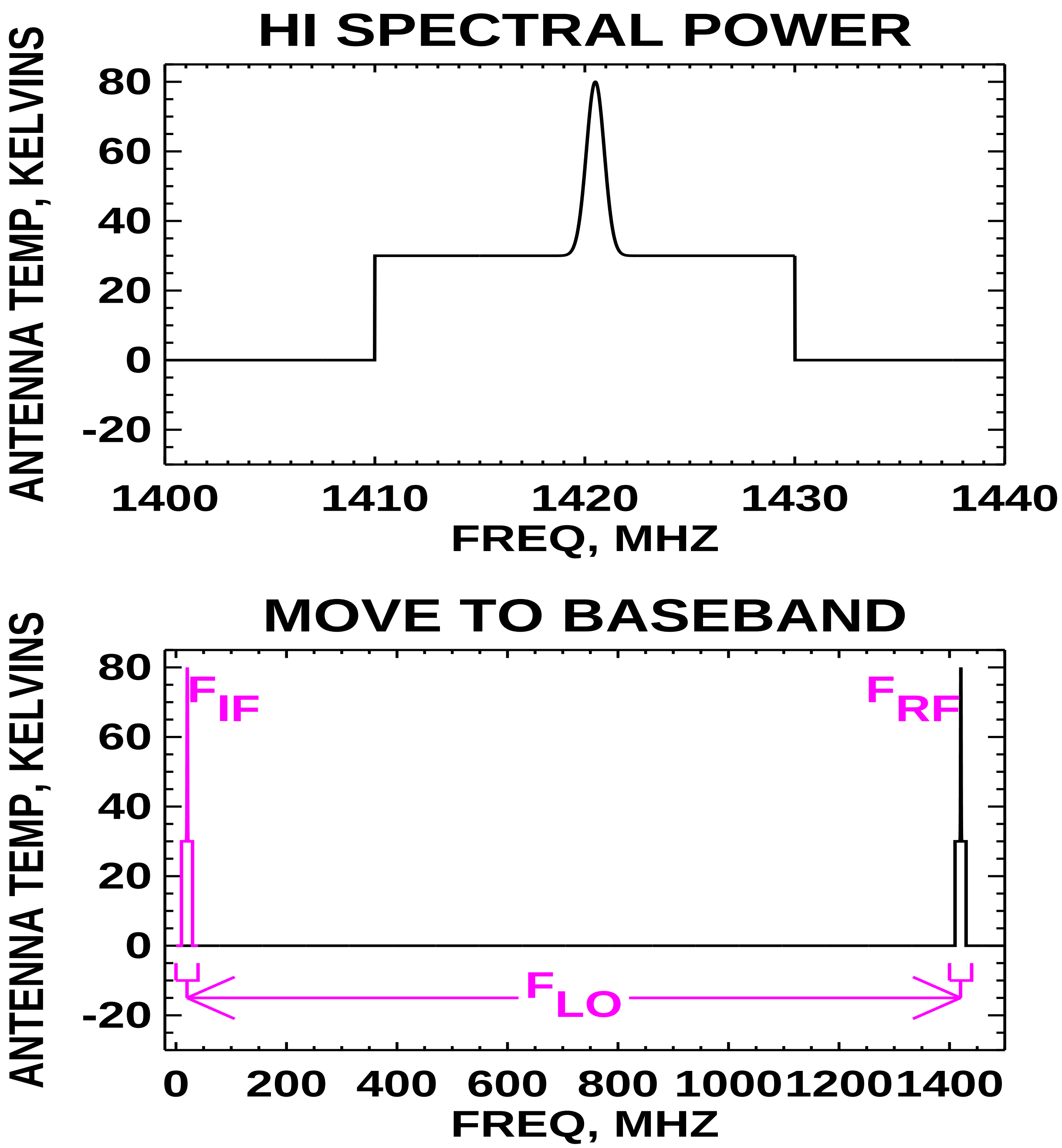
Discard the sum with a filter; then

$$F_{IF} = F_{RF} - F_{LO}$$

.

Graphically ...

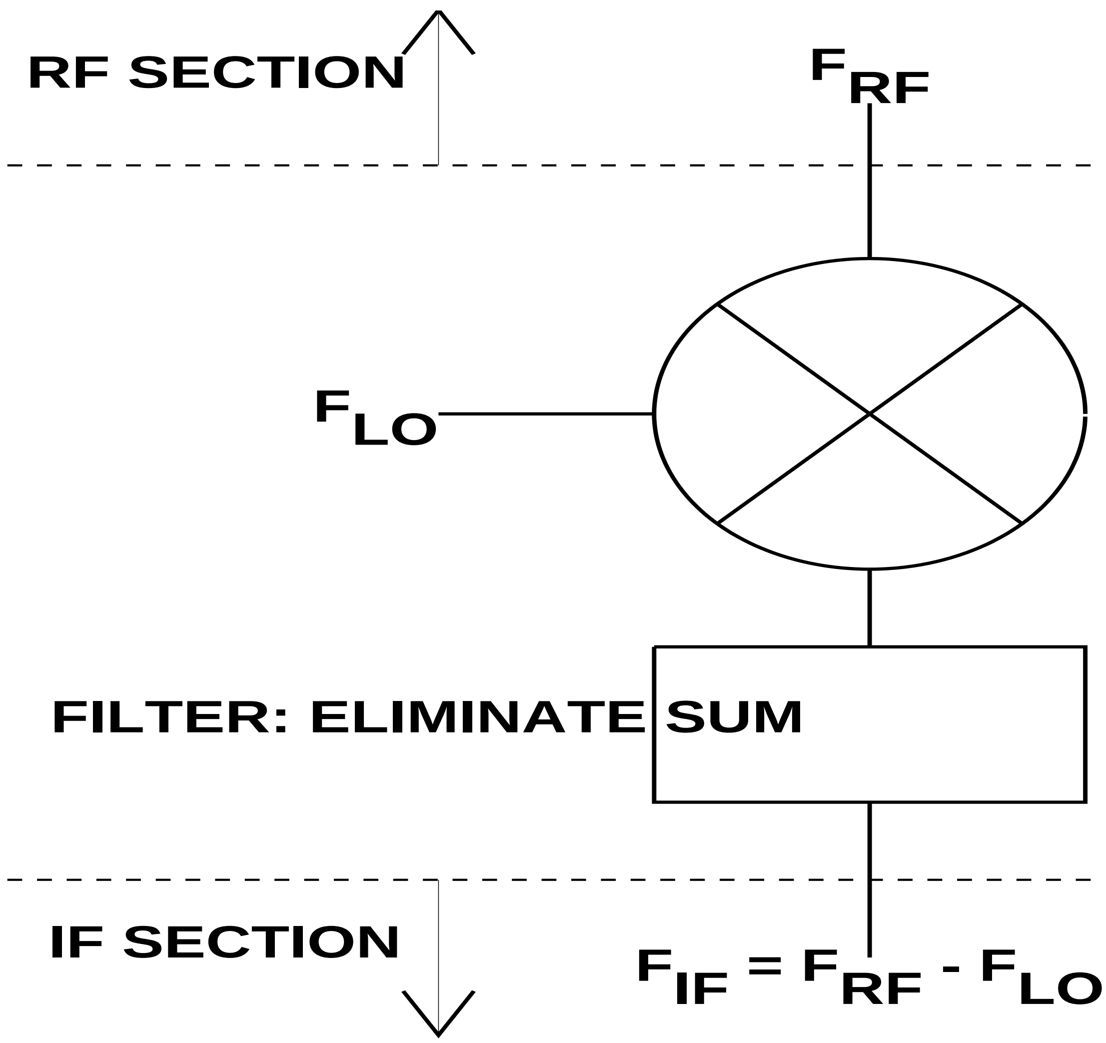
THE BASIS OF HETERODYNE SPECTROSCOPY



This multiplication process is called *mixing*. The multiplier is called a *mixer*.

RF=Radio Frequency,
IF=Intermediate Frequency,
LO= Local Oscillator.

The block diagram:



CONTRIBUTIONS FROM THE RF SECTION

Contributions to the observed spectrum from the RF section include the object of interest and everything else. The other stuff includes both spectrally-dependent added noise and gain. They include:

- Continuum and spectral radiation from the sky. This is what you want to measure!
- Continuum radiation from the sky reflected from structural components into the feed. This may well be far from the direction the telescope points.
- Ground emission making its way into the feed, both directly and via reflections.
- Spectrally-dependent noise added by RF electronics.
- The spectrally-dependent gains of the feed and the RF electronics.

REFLECTING ON REFLECTIONS...

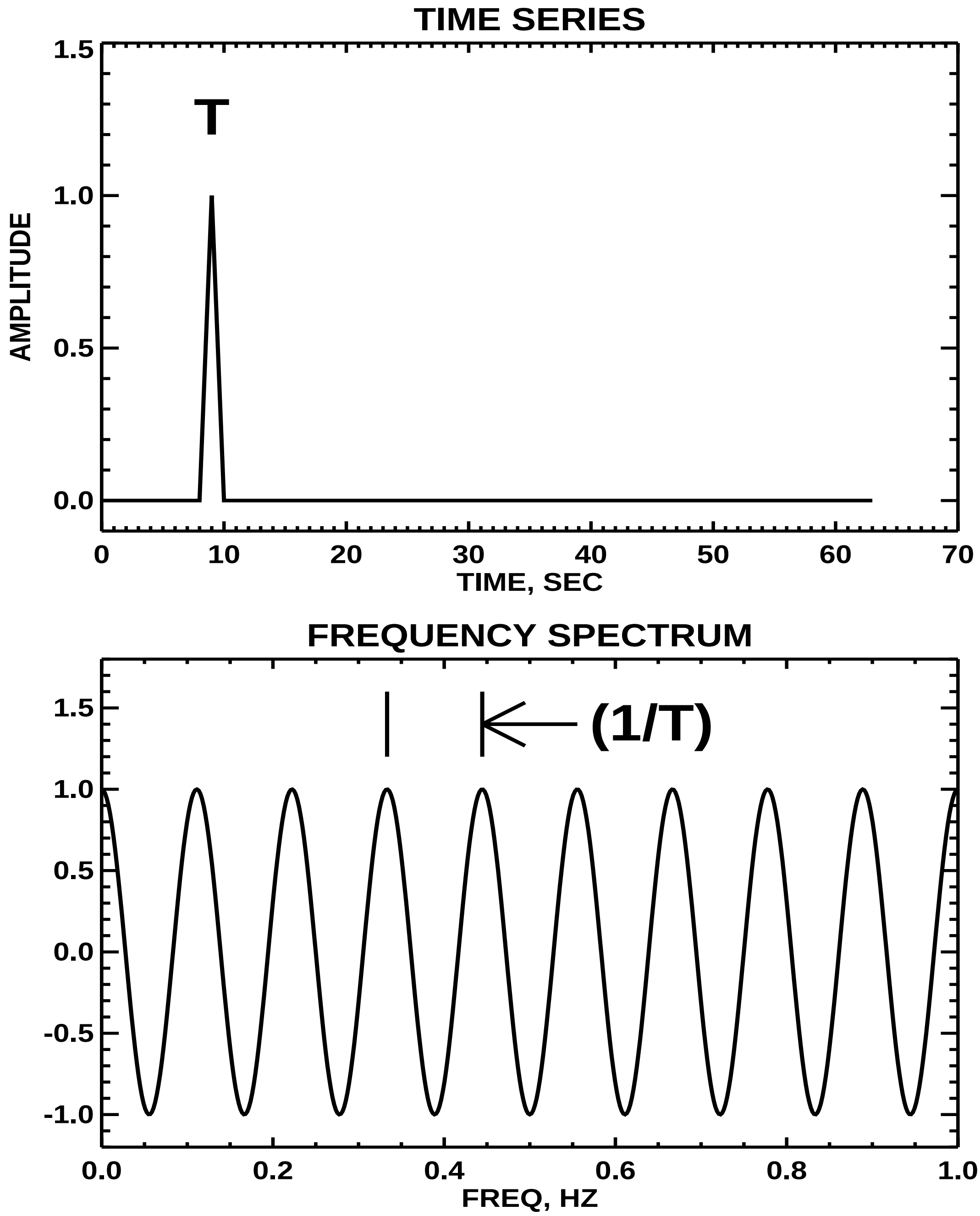
Suppose a signal enters the feed from two paths whose lengths differ by L. Then the signal adds to a delayed copy of itself; the time delay is $\frac{L}{c}$. This means its autocorrelation function has a sharp spike at time delay $\frac{L}{c}$.

Remember the correlation theorem: The power spectrum is the Fourier transform of the Autocorrelation function. In Fourier space, a spike in one coordinate leads to a sinusoidal ripple in the conjugate coordinate. The spike at delay $\frac{L}{c}$ gives a sinusoidal ripple in the spectrum with period $\frac{c}{L}$.

- At Arecibo, the feed-to-bowl distance is about 130 meters. The round-trip path is 260 meters. The ripple period is 1.1 MHz.
- At the GBT the feed-reflector distance is about 80 meters. The ripple period is 1.6 MHz.

At both telescopes there are additional paths for reflection, both RF and IF.

ILLUSTRATING THE CORRELATION THEOREM...



CONTRIBUTIONS FROM THE IF SECTION

In contrast to the RF section, the IF section adds no noise in a well-designed system. But it has severe spectrally-dependent gain. This is easily dealt with as long at it is time-independent. Unfortunately, this is not always the case: time-dependent effects are always difficult.

MATHEMATICAL DESCRIPTION

Let:

- $T_*(f_{RF})$ **be the sky spectral line contribution [indicated by** (f)**].**
- T_* **be the sky continuum contribution (no frequency dependence).**
- $T_R(f_{RF})$ **be the front end section spectral contribution.**
- T_R **be the front end section continuum contribution.**
- $G_{RF}(f_{RF})$ **be the RF gain (dependent on RF frequency).**
- $G_{IF}(f_{IF})$ **be the IF gain (dependent on IF frequency).**
- $M(f_{IF})$ **be the measured power spectrum.**
- **unprimed quantities indicate the ON measurement.**
- **primed quantities indicate the OFF measurement.**

Then we have

$$M(f_{IF}) = G_{IF}(f_{IF})G_{RF}(f_{RF}) \left\{ \underbrace{(T_*(f_{RF}) + T_R(f_{RF}))}_{spectral} + \underbrace{(T_* + T_R)}_{continuum} \right\}$$

Our goal: find $\boldsymbol{T_*(f_{RF})}$.

This is normally by switching either in position or frequency. That is, we take ON (unprimed) and OFF (primed) spectra. The OFF spectrum evaluates the various contaminating terms. We assume the system contributions are identical for ON and OFF. We assume the system spectral contribution is fractionally small compared to its continuum contribution, i.e. $T_R(f_{RF}) \ll T_R$, so we can make a Taylor expansion.

POSITION SWITCHING.

The best OFF is off in *position*. We combine the ON and OFF as follows. We assume the OFF position has no line, i.e. $T'_*(f_{RF}) = 0$)

$$\frac{M(f_{IF}) - M'(f_{IF})}{M'(f_{IF})} = [T_*(f_{RF}) + (T_* - T'_*)] \left[\frac{1 - \frac{T_R(f_{RF})}{(T'_* + T_R)}}{T'_* + T_R} \right]$$

This gives the ON-OFF spectrum, normalized by the right-hand factor. The frequency-dependence of this factor is fractionally small, so it changes the line shape only a bit and is not serious.

Contaminating influence: **The possibility for *real problems* arises if there is a large continuum difference between ON and OFF, i.e. if $(T_* - T'_*)$ is large. This affects weak line measurements on continuum sources.**

FREQUENCY SWITCHING.

If the spectral line is spatially extended you can't move off in position. Galactic HI is the prime example! So here your OFF is taken off frequency, which means the OFF and ON $G_{RF}(fRF)$ **aren't identical. Retaining only first-order terms, we obtain the previous result plus a complicating term: first factor is replaced by**

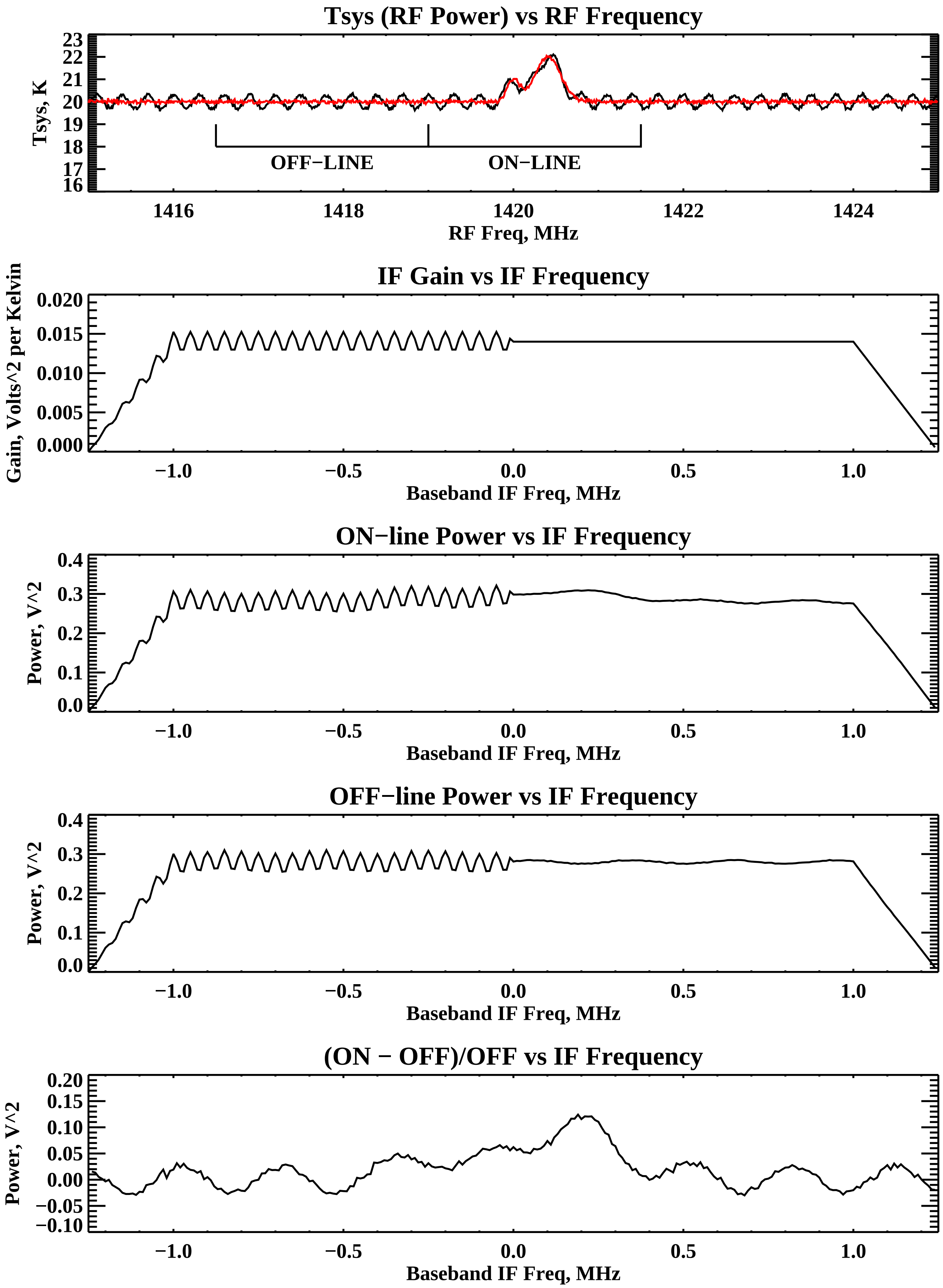
$$\left[T_*(f_{RF}) + (T_* - T_*') + \frac{\delta G}{G}(T_R + 2T_*)\right]$$

where $\frac{\delta G}{G} = 1 - \frac{G'_{RF}(f_{RF})}{G_{RF}(f_{RF})}$**. Even though** $\frac{\delta G}{G} \ll 1$**, it operates on the** *receiver temperature***, which is large, and produces serious baseline contamination.**

TOO MUCH ALGEBRA!! TRY SOME GRAPHS. . .

We illustrate the math for frequency switching with a graphical example. We include. . .

- **A weak HI line (1420 MHz)**
- **A severe RF sinusoidal ripple**
- **An IF Gain spectrum that any engineer would be ashamed of.**

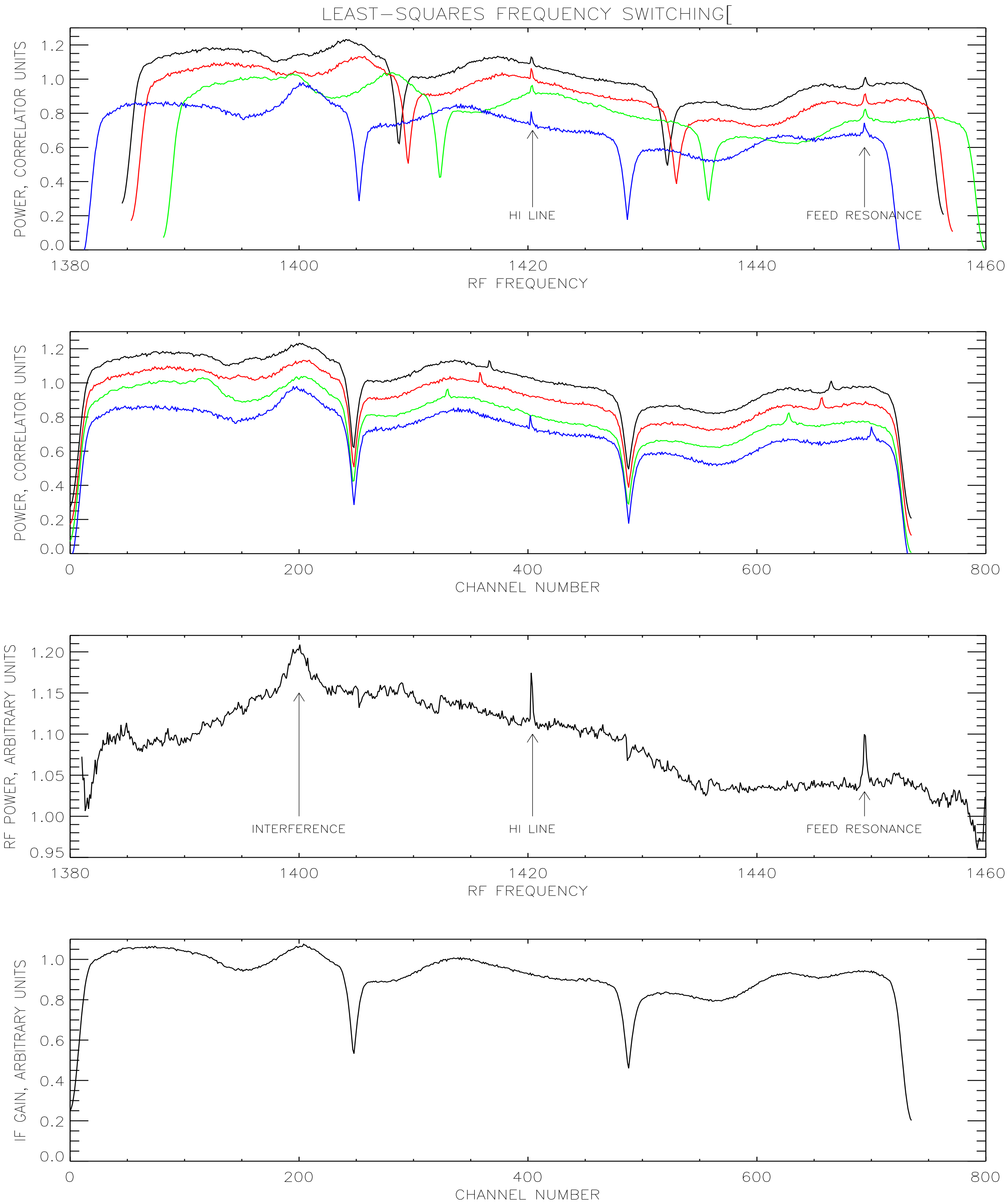

Tsys (RF Power) vs RF Frequency
Tsys, K
23
22
21
20
19
18
17
16
OFF−LINE
ON−LINE
1416
1418
1420
1422
1424
RF Freq, MHz
IF Gain vs IF Frequency
Gain, Volts^2 per Kelvin
0.020
0.015
0.010
0.005
0.000
−1.0
−0.5
0.0
0.5
1.0
Baseband IF Freq, MHz
ON−line Power vs IF Frequency
Power, V^2
0.4
0.3
0.2
0.1
0.0
−1.0
−0.5
0.0
0.5
1.0
Baseband IF Freq, MHz
OFF−line Power vs IF Frequency
Power, V^2
0.4
0.3
0.2
0.1
0.0
−1.0
−0.5
0.0
0.5
1.0
Baseband IF Freq, MHz
(ON − OFF)/OFF vs IF Frequency
Power, V^2
0.20
0.15
0.10
0.05
0.00
−0.05
−0.10
−1.0
−0.5
0.0
0.5
1.0
Baseband IF Freq, MHz

This illustrates that:

- **The IF Gain spectrum totally cancels—as long as you measure it accurately!**
- **The RF ripple (which, below, we call** *fixed pattern noise*, **contaminates the final spectrum** *because you subtract one part of it from another*.

It would be really nice to eliminate subtracting one part of the RF spectrum from another, because this exacerbates the pattern noise (ripple).

LEAST-SQUARES FREQUENCY SWITCHING: NEW!!



LSFS uses measurements at multiple LO frequencies and performs a least-squares fit to obtain the IF gain and RF power spectral dependencies as *independent entities*.

Recall that, for any individual measurement,

$$M(f_{IF}) = \underbrace{G_{IF}(f_{IF})}_{IF\ Gain}\,\underbrace{G_{RF}(f_{RF})\left\{(T_*(f_{RF}) + T_R(f_{RF})) + (T_* + T_R)\right\}}_{RF\ Power}$$

LSFS gives us, as independent entities, the IF Gain

$$M(f_{IF}) = G_{IF}(f_{IF})$$

and the RF power $P_{RF}(f_{RF})$

$$P_{RF}(f_{RF}) = G_{RF}(f_{RF})\left\{(T_*(f_{RF}) + T_R(f_{RF})) + (T_* + T_R)\right\}$$

LSFS DETAIL ARE... COMPLICATED

- **The equation-of-condition matrix is large ($\sim NI \times 2I$) [I channels, N LO frequencies].**
- **The number of unknowns is $\gtrsim 2I$; this matrix must be inverted.**
- **The inverse matrix can be degenerate if the LO frequencies are not chosen properly. Best to use SVD.**

However... no problem in the end. See my PASP paper for these and many other details, including the *most important one...*

THE SINGLE MOST IMPORTANT CONSIDERATION: HOW TO CHOOSE THE LO FREQUENCIES, AND HOW MANY TO USE?

HOW MANY LO FREQUENCIES TO USE? **the conflict is. . .**

- **The smallest LO frequency interval limits the resolution**
- **The largest LO frequency interval should be a nontrivial fraction of the total bandwidth**

So you want a large *range* **of LO frequency intervals.**

But for practical reasons you don't want to use a *large number* **of LO settings. At the GBT the observing script is set to use** *eight* **LO settings. At Arecibo, for GALFA, we use** *seven***.**

HOW TO SPACE THE LO SETTINGS? **The problem is precisely akin to the minimum-redundancy 1-d array! Call this the MRN, where N is the number of LO frequencies.**

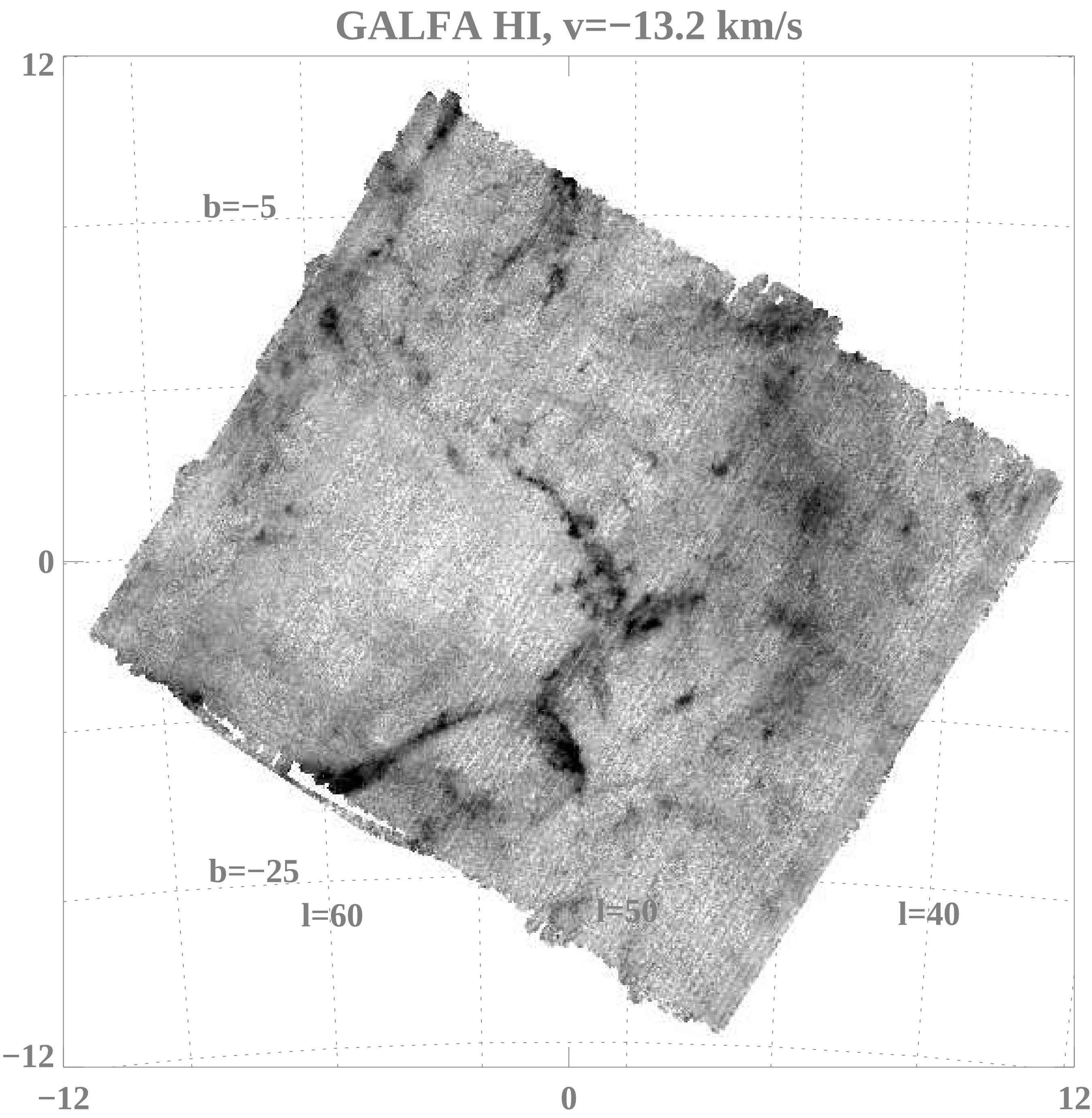
For MR7, the spacings range continuously from 1 to 18 units and the largest spacing is 31 units. For a large number of channels, the fractional bandwidth is small.

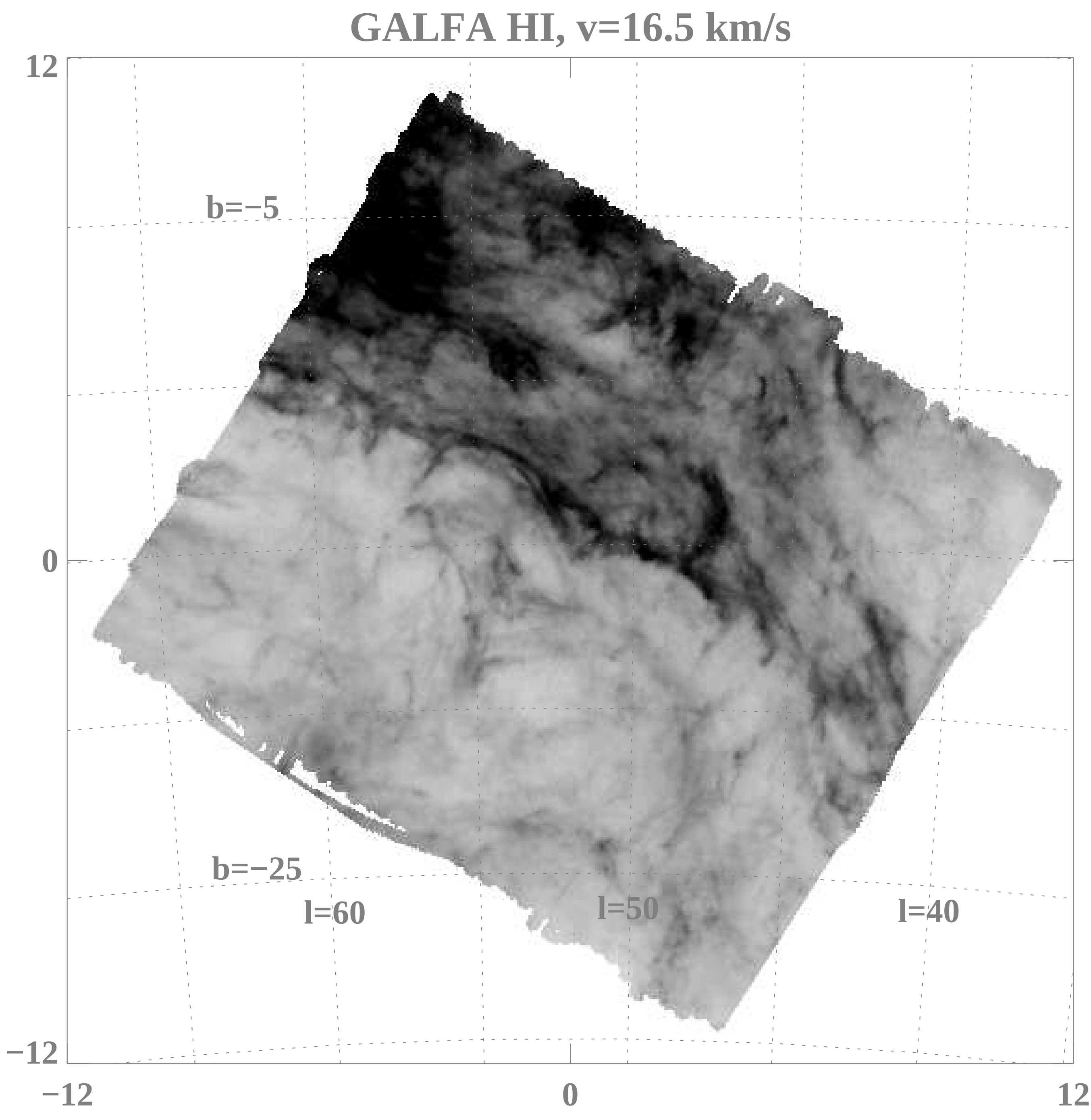
Numerical experiments show that you get pretty good results with the MRN^2 schema, for which the spacings are the square of the MRN spacings. For N=6, this provides spacings ranging from 1 to 108 units, but they are not continuously covered.

PRACTICAL, REAL-LIFE APPLICATION AND EXPERIENCE

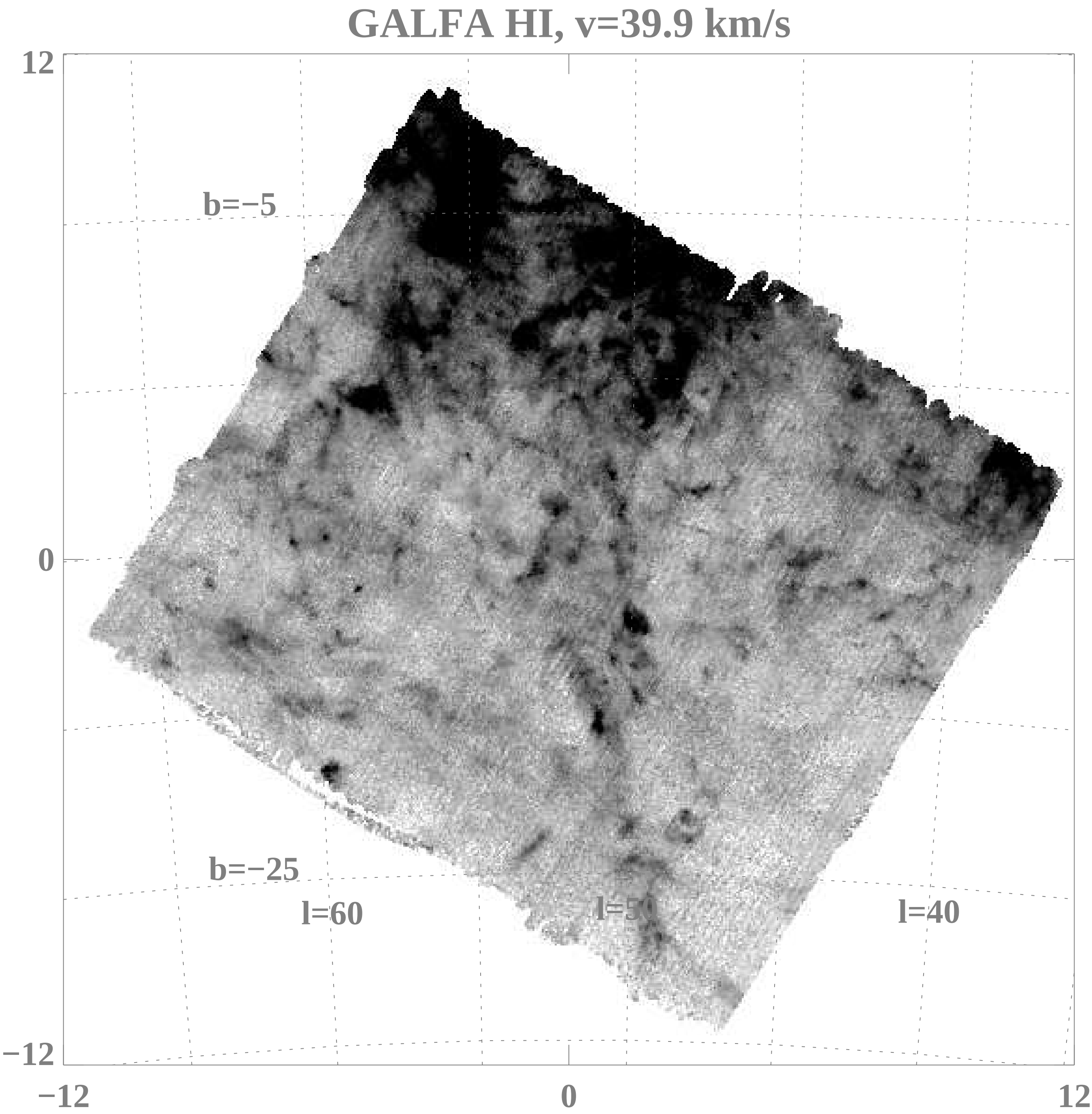
- **The GALFA survey at Arecibo. We use LSFS to derive the IF gain $G(_{IF}(f_{IF})$. We apply this to thousands of measured profiles. $G(_{IF}(f_{IF})$ is very stable and if one day's calibration fails we use a previous one with no problem. (The IF setup is always forced to *precisely* identical from day to day).**
- **We measured the Arecibo "fixed-pattern noise" for both polarizations for all seven feeds. This $P_{RF}(f_{RF})$ arises from the various contributions we discussed above. It is 100% polarized and totally uncorrelated from feed-to-feed. It varies slowly with telescope position and, also, slowly (if at all) with time.**
- **We observed OHMM for Zeeman splitting using the GBT. It worked very well and, as a by-product, we rediscovered the $\sim$ 1.8 MHz ripple produced by the feed-to-reflector reflection.**

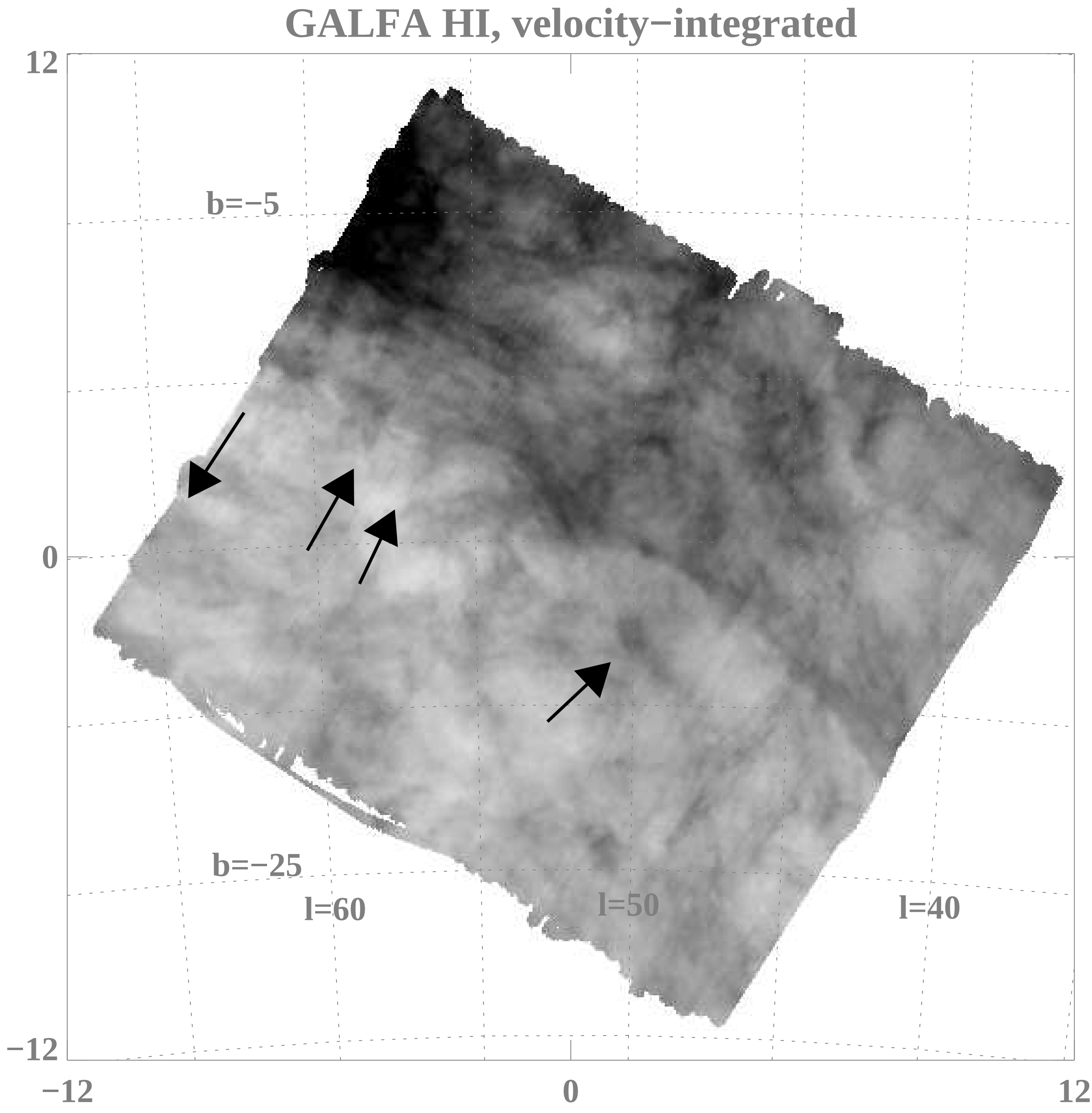
OK, LET’S TAKE A LOOK AT SOME DATA!!


GALFA HI, v=−13.2 km/s
12
0
−12
−12
0
12
b=−5
b=−25
l=60
l=50
l=40



GALFA HI, v=16.5 km/s
12
0
−12
−12
0
12
b=−5
b=−25
l=60
l=50
l=40


GALFA HI, v=39.9 km/s
12
0
−12
−12
0
12
b=−5
b=−25
l=60
l=50
l=40


GALFA HI, velocity−integrated
12
0
−12
−12
0
12
b=−5
b=−25
l=60
l=50
l=40

SOME CHARACTERISTICS OF ALFA's FIXED PATTERN NOISE (FPN)

Carl Heiles (October 27, 2005)

Contents

1. INTRODUCTION

Accuracy of the spectra taken with ALFA and LBW are limited by systematic baseline wiggles that change slowly, or perhaps not at all, with time. After a few second of integration, this *fixed pattern noise*, or *FPN*, dominates the uncertainty in the spectrum. It tends to have frequency structure of width $\gtrsim 1$ MHz, which is comparable to time delays of $\lesssim 1$ μsec in the autocorrelation function. We have strongly believed, and prove it here, that the structure results from reflections, the path differences are $\lesssim 300$ m. 1 MHz corresponds to 200 km/s at the HI line—a velocity scale in which lots of interesting things happen, scientifically speaking, so this FPN signifantly affects the science. We performed some experiments to determine some of its characteristics, of which we report in this document.

2. SUMMARY OF IMPORTANT FINDINGS

In this section we briefly summarize our most interesting findings. We document each in later sections with data and more discussion.

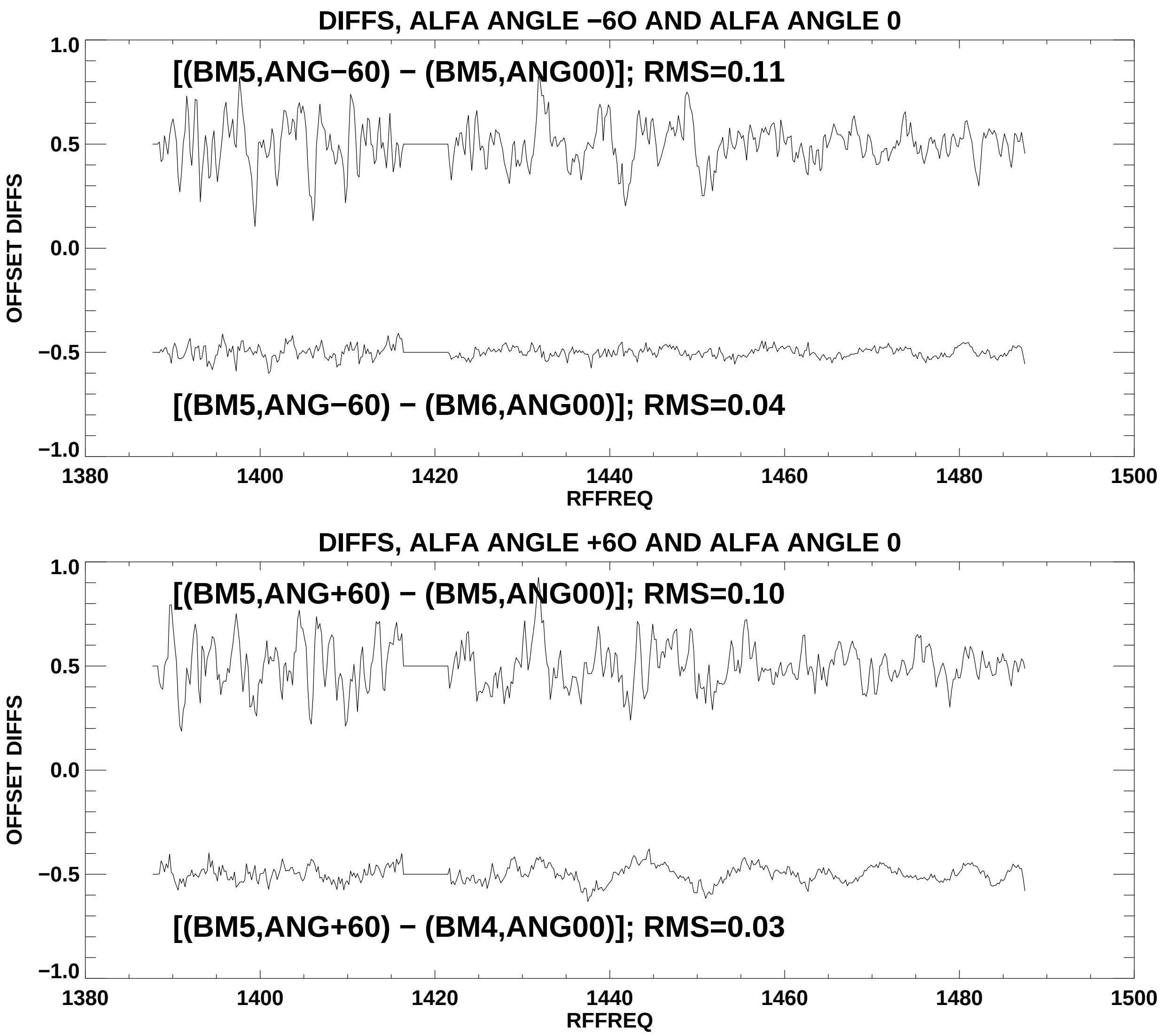
1. The FPN arises from reflections and is 100% polarized. (§4, 5, 7)
2. Most of the FPN arises from reflections involving shorter path lengths. The spectrum of delays is essentially continuous and looks random. (§8; figures 9, 10, 12, 13, 15)
3. A small component of the FPN arises from reflections between the bowl and the feed and associated structures; this component depends on the feed and the ALFA turret angle. This component of the FPN has sharp time-delay features in the 0.90, 1.04, and 1.16 μs. There seem to be additional time delays but the results look suspicious and should be repeated. (§8; figures 9, 10)
4. The FPN changes when the ALFA turret is rotated. The further the rotation, the larger the change. (§6)
5. The FPN changes with zenith angle za. The further the move, the larger the change. (§9)
6. Over the limited range in azimuth (az) that we tested, the FPN changes: the further the move, the larger the change. (§10)
7. The FPN amplitude increases noticeably towards lower frequencies. (§11)

3. EXPLANATION OF THE PLOTS IN FIGURES 1, 2, and 3

The formats of these three figures are identical. Each figure shows a pair of spectral plots in two panels to save space. The spectra are polynomial-flattened RF powers obtained from the

ARECIBO'S FIXED PATTERN NOISE

It depends only on the *location in the focal plane.* **When you interchange one feed for another (by rotating the turret by** 60°**), the new feed has the identical pattern to the old.**



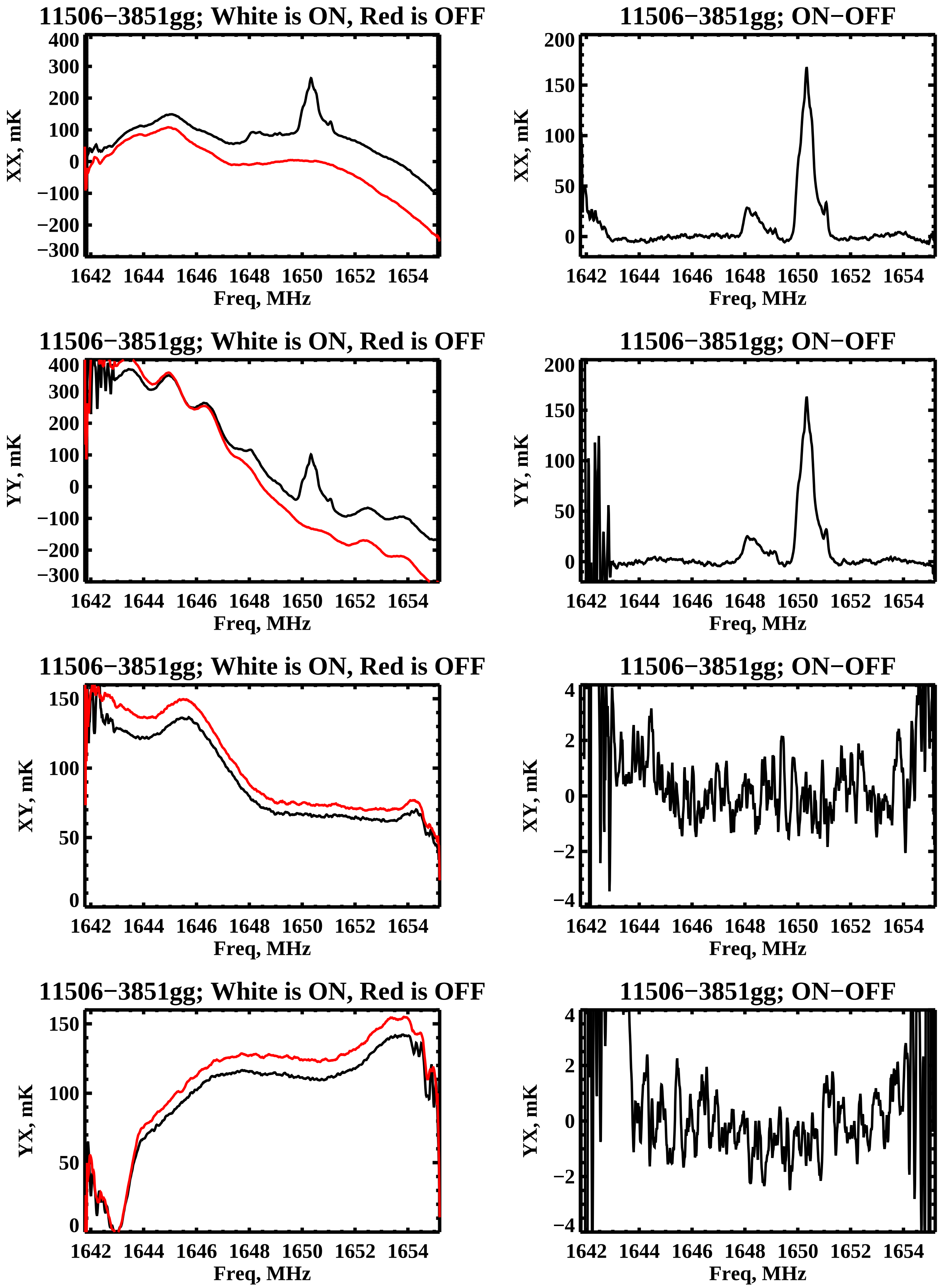
USING LSFS ON OHMMs AT THE GBT

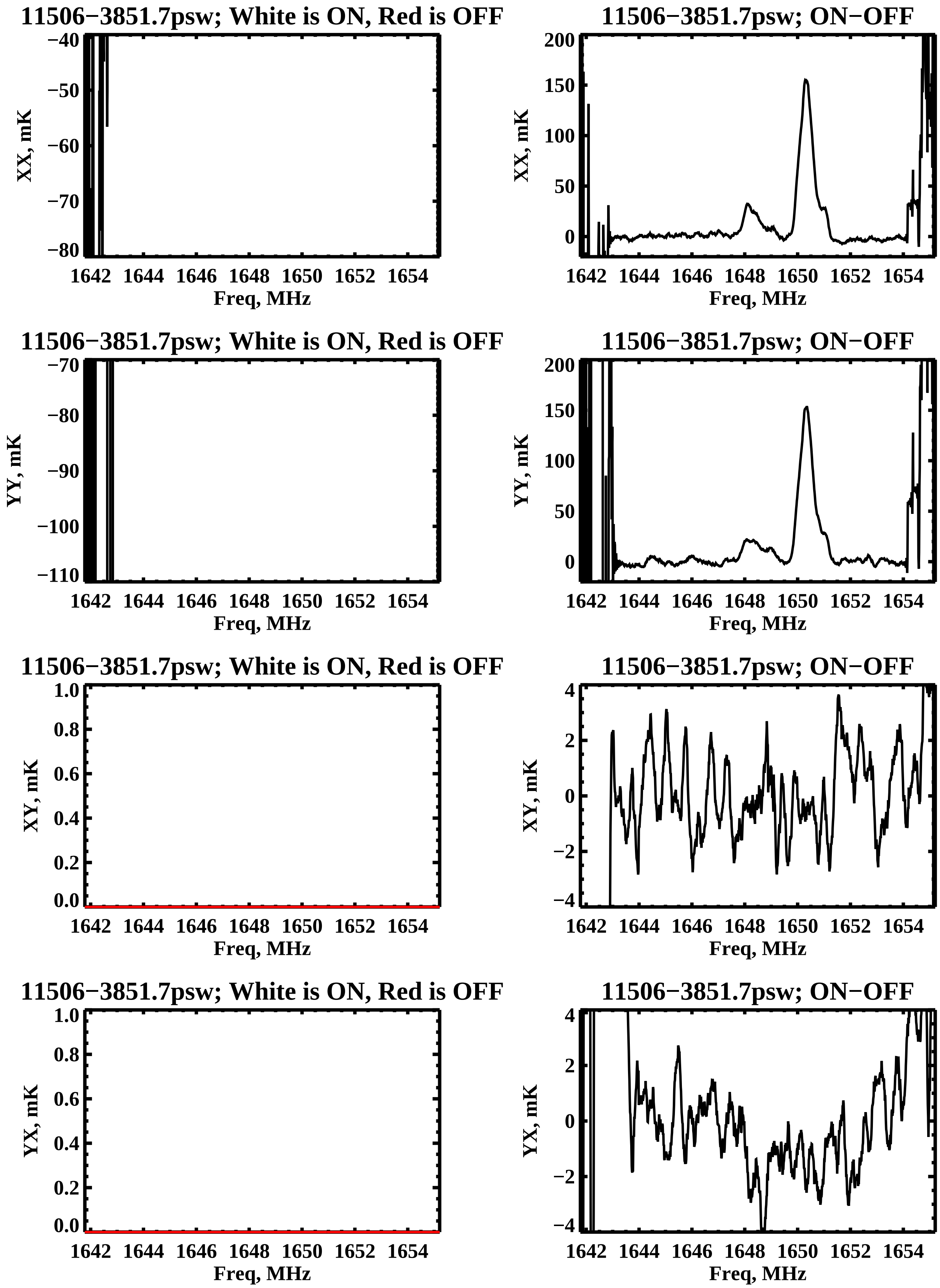
Tim Robishaw and I used a combination of LSFS and position switching to observe Zeeman splitting of OHMMs at the GBT. We spent 4 times less time on the OFF-positions and smoothed the OFFs to regain the S/N.

We reduced the data in two ways:

- **LSFS to derive separate ON-source and OFF-source spectra; then subtract to get the ON-OFF. The unswitched spectra reveal the GBT's 1.8 MHz baseline ripple, level about 100 mK. Position-differenced spectra are of excellent quality but needed low-order polynomials subtracted.**
- **Using a "standard" position-switching reduction. XY and YX spectra are noisier than for LSFS reduction (probably because the interval for positions switching was long, about 30 minutes).**

First, the LSFS results. . .


11506−3851gg; White is ON, Red is OFF
XX, mK
Freq, MHz
11506−3851gg; ON−OFF
XX, mK
Freq, MHz
11506−3851gg; White is ON, Red is OFF
YY, mK
Freq, MHz
11506−3851gg; ON−OFF
YY, mK
Freq, MHz
11506−3851gg; White is ON, Red is OFF
XY, mK
Freq, MHz
11506−3851gg; ON−OFF
XY, mK
Freq, MHz
11506−3851gg; White is ON, Red is OFF
YX, mK
Freq, MHz
11506−3851gg; ON−OFF
YX, mK
Freq, MHz


11506−3851.7psw; White is ON, Red is OFF
XX, mK
Freq, MHz
11506−3851.7psw; ON−OFF
XX, mK
Freq, MHz
11506−3851.7psw; White is ON, Red is OFF
YY, mK
Freq, MHz
11506−3851.7psw; ON−OFF
YY, mK
Freq, MHz
11506−3851.7psw; White is ON, Red is OFF
XY, mK
Freq, MHz
11506−3851.7psw; ON−OFF
XY, mK
Freq, MHz
11506−3851.7psw; White is ON, Red is OFF
YX, mK
Freq, MHz
11506−3851.7psw; ON−OFF
YX, mK
Freq, MHz

LSFS: WHAT'S IT GOOD FOR?

- **LSFS is excellent for investigating fixed pattern noise, e.g. caused by reflections, because it provides the RF power spectrum unambiguously.**
- **Even the GBT has fixed pattern noise, so even with LSFS you should still use position switching (except, of course, for HI).**
- **With position-switched spectra, the LSFS-derived OFF spectrum should be featureless; in particular, it does *not* contain the IF bandpass shape. Thus, you can greatly smooth the OFF spectrum. This gains as much as a full factor of 2 in sensitivity. This is the main advantage of LSFS.**
- **LSFS position-differenced spectra are not flat, but are well-represented by a low-order polynomial. This suggests that pure position switching is better for broad lines.**

WE NEED *MORE EXPERIENCE* WITH LSFS—BEST, A FEW WELL-CONCEIVED AND CONTROLLED EXPERIMENTS