# Estimating the GBT Pointing Model

Kim T. Constantikes

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**Abstract**

This document describes the process of estimating pointing model coefficients for the GBT pointing model. Data are collected from a variety of sources, including astronomical observations, weather stations, inclinometers, accelerometers, structural and air temperature sensors, encoders, and subreflector position and pose. These data streams are then preprocessed and registered. Model coefficients are then estimated by fitting the parametric model to the data, and finally the model fit is assessed.

**Contents**



## History

**n.0** 1 July 2008. Original version (Kim Constantikes).

## 1. Introduction

Given a linear model of pointing error composed of a subset of the terms in [2; 3] we need to determine the model coefficients either by minimizing a pointing error criterion or by direct measurement of the coefficients. For example, pointing errors due to azimuth track tilt can be measured directly via elevation axle inclinometers. Other coefficients can only be estimated via astronomical inference, *i.e.*, error due to gravitational distortions[1]. In this approach the offset in angular position between the measured position of an calibrated astronomical source and its predicted (via axis encoder) position is used to infer the coefficients of the model. The current implementation of model fitting uses a combination of direct measurement and astronomical inference.

The error model used in [1] assumes an underlying independent and identically distributed Gaussian error in order to establish a performance statistic and establishes thresholds using operational requirements (*e.g.* observing efficiency) and scientific requirements (*e.g* radiometric calibration). In practice the errors are noticeably long-tailed and asymmetric, but these deviations from assumptions can be handled by use of quantiles in the final assessment of pointing performance, and the assumptions considerably simplify the model fitting process.

There are errors in pointing due to model error and there are errors in inference due to randomness in the observations of pointing error. The distinction is made because the error in observation can be estimated in the peak fitting process (see below) while the model error cannot be. For example, Let $\hat{\phi}$ be the observation of elevation pointing error, and $\tilde{\phi}$ be the prediction of pointing error from the linear model composed of some subset of the effects in [3]. Let $\mathbf{x}$ be the vector of control variables, *e.g.*, temperature features, cosine and sine of azimuth and elevation angles, etc. Then the observed elevation is

$$\hat{\phi} = \mathbf{x}^T\mathbf{a} + \epsilon_{\text{model}}(\mathbf{x}) + \epsilon_{\text{measurement}}$$

where $\epsilon_{\text{model}}(\mathbf{x})$ is the error inherent in the model $\epsilon_{\text{measurement}}$ is the error in measurement of the peak location and and the linear prediction of elevation is

$$\tilde{\phi} = \mathbf{x}^T\mathbf{a}.$$

We seek the model coefficients that best minimize, in some sense, the error between the predicted and observed position, $\hat{\phi} - \tilde{\phi}$.

One choice is to find $\mathbf{a}$ such that the expected error $E[\hat{\phi} - \tilde{\phi}]$ is zero, *i.e* the estimate of pointing error is unbiased, and that the variance of the error,Var$[\hat{\phi} - \tilde{\phi}]$ is minimized. This is the BLUE (Best Unbiased Linear Estimator) of pointing error, and under general conditions (the error variance is bounded) it is (Gauss-Markov [5])

$$\hat{\mathbf{a}} = \mathbf{B}\hat{\phi}$$

where

$$\mathbf{B} = (\mathbf{X}^T\Sigma^{-1}\mathbf{X})^{-1}\mathbf{X}^T\Sigma^{-1}$$

and

$$\Sigma = \text{Cov}[\epsilon_{\text{model}} + \epsilon_{\text{measurement}}].$$

[1]The original concept of a large scale metrology system using laser ranging and trilateration was intended to measure these contributions directly. It was not implemented.

The matrix $\mathbf{X}$ has a row of the control variables for each observation, and $\hat{\phi}$ is a column vector of the observations. In the case that $(\mathbf{X}^T\Sigma^{-1}\mathbf{X})$ is not invertible then the minimum norm solution is used. Implicitly the model error has been assumed to be a random variable: This is justified by assuming that the observations are a random sample of elevation angles. Model errors and measurement errors are independent so that $\Sigma$ is diagonal and with entries of $\sigma^2_{\text{measurement,i}} + \sigma^2_{\text{model,i}}$. In the implementation of the model fitting routine an approximate variance for model errors, based upon preliminary fitting, is used while bootstrap estimates of measurement error is used.

Finally, note that the BLUE estimate makes no assumption about the distribution of observation and model noise. In the case that both are Gaussian distributed, the BLUE estimate is also the maximum likelihood estimate of the model.

## 2. Preprocessing

Some data are generated using the PrePoint package[2]. Track map data and astronomical data are processed as below. Both data streams are injested into the point model database and make-table queries are used to construct inputs to the fitting routines that use the appropriate data sources.

### 2.1. Track Data

The influence of tilt and irregularity of the track on pointing is discussed in [3]. The model terms in Table 4 are estimated as follows. The telescope is rotated at approximately 150"/sec in azimuth from $0^\circ$ to $360^\circ$ and then back to $0^\circ$. The data streams from encoders, inclinometers, and accelerometers are injested and then linearly interpolated to a common, regular sampling in time. Correlation analysis between accelerometers, inclinometers, and azimuth acceleration (derived via a Savitski-Golay filter) is used to register the data streams in time (accelerometer data are not further used). Then the inclinometer data are filtered with a zero group delay (Matlab filtfilt function) FIR filter with passband from DC to 0.05 Hz to remove structural resonances from inclinometer response. The resulting inclinometer data is segmented into forward and backward passes and interpolated to a regular sampling in azimuth angle. The symmetries in [3] are then applied by symmetrically or anti-symmetrically averaging forward and backward passes, and an alignment check is made between the manlift side and opposite side sensors. Finally the tabular functions for $\lambda_{2,t}$, $\Delta\xi_1$, and $\Delta\xi_{19}$ are calculated from the processed inclinometer signals. These data are injested into the fitting code for fitting and are decimated in azimuth angle to a $0.1^\circ$ resolution for use in the GBT `M&C` implementation of the pointing model.

### 2.2. Astronomical Data

Data from the DCR, Antenna, and Go samplers are collated into a common data structure and then processed to identify the type of scan, *i.e.* elevation, azimuth, or focus, and the direction of scan where

[2] http://wiki.gb.nrao.edu/bin/view/Software/ModificationRequest5C706

increasing (unwrapped) azimuth, elevation, or focus position are denoted "Forward" scans. This is accomplished by least-squares fitting of a polynomial of order one (linear) as a function of time to the reported positions and then comparing the linear coefficient to a threshold, currently set at 10 arcsec/sec and 2 mm/sec. Note that the focus scan check tests the subreflector Y axis rate. If the azimuth or elevation rates exceed the threshold, then the scan is forward in that axis. If the negative of the rate exceeds the threshold, then the scan is backward. Focus is tested only for forward scans. If none of these conditions are met the scan is marked as in error and added to a list of scans to ignore in further processes.

After the scan direction has been discovered the center time of each DCR integration is calculated by adjusting for blanking times and the sampling times recorded for calibration diode on and off, and for left and right circular polarization. Finally a structure is constructed that contains, amongst other supporting data, the offset in spherical coordinates of the ideal main beam center (via mount angles) with respect to the refraction-corrected angular position of the calibration source. This is

$$\begin{aligned}\text{Offset}_{\text{Az}}(n) &= [\text{Az}_{\text{mount}}(n) - \text{Az}_{\text{OBSC}}(n)]\cos(El(n))\\ \text{Offset}_{\text{El}}(n) &= \text{El}_{\text{mount}}(n) - \text{El}_{\text{OBSC}}(n)\end{aligned}$$

where the argument $n$ is the sample index.

All of the data in this structure are linearly interpolated to the time tags of the Antenna sampler, *i.e.*, DCR data are interpolated to the same sampling intervals as the Antenna sampler. The DCR time tag is also adjusted for blanking and channel sampling time[3], which may be experiment dependent. The supplied code is appropriate for the observations to date with two "phases". The offset from DCR time tag to center of the calibrate-on phase is, using DCR FITS field names, $\text{BLANKTIM}(1) + \text{PHASETIM}(1)/2$. The offset for the calibrate-off phase is $2 * \text{BLANKTIM}(2) + 3 * \text{PHASETIM}(1)/2$.

Other data included in the structure are the time in seconds from the first scan in a project, DMJD, scan number, mount angles, DCR data, subreflector positions and pose, rest frequency[4], and Project ID.

This structure is then used for a bootstrapped peak fit for each scan and each DCR channel within the scan. First the scan is segmented into locations close to the peak, within $\pm 1.5\sigma$ of the DCR peak intensity, and locations that are affected only by baselines, more than $3\sigma$ away from the peak intensity, where $\sigma = \frac{740}{2.354 f_r}$ arcsec and $f_r$ is the rest frequency in giga-Hertz. Note that $\sigma$ is the standard deviation of an assumed Gaussian peak profile. Then, for the union of these two sets of offset angles, we fit the observed intensity with the function

$$y(n) = a_1 e^{\frac{-(x(n)-a_2)^2}{2a_3}} + a_4 + a_5 x(n) + a_6 x(n)^2 + a_7 x(n)^3 \tag{1}$$

where $x$ is the angular offset associated with observed intensity $y$. The fit uses a non-linear Levenberg Marquart curve fitting routine with appropriate initial values and with upper and lower limits of $\pm 0.1$ of the observed peak value, peak location, and baseline average. It is almost certainly the case that the

[3]The definition of the phase timings are a bit arcane, and the equations below are via private communications with the GB staff.

[4]Rest frequency is later used to determine approximate beam width, in turn used to select parameters for peak fitting

same peak fitting performance would be yielded by using a quadratic peak shape. Then the peak fit could be implemented as a generalized least-squares, *i.e.* linear, problem which in turn would speed the fitting and bootstrapping process by orders of magnitude.

This process is bootstrapped[4] (sampling with replacement of the intensity,location pairs) for each scan in order to estimate the mean and standard deviation of the fitted parameters.A similar process is applied to focus scans with the exception that a $4^{\mathrm{th}}$ order polynomial fit is applied to the entire record rather than a segmented portion of the record.

Structural temperature data is injested and the model features (linear combinations of temperatures) are computed, then estimates of the time derivatives of the features are computed. These data included in the output dataset, and where used to determine if temperature trends could be used to improve the thermal corrections of pointing. The results were inconclusive and no trend corrections are currently being used.

Finally a dataset is written that can be injested into the pointing model database. The column headers are generally self-explanatory. Note that the column ”OffsetError” is the estimate of the offset standard deviation and the offset is the mean offset, both via bootstrap. The column ”SNR” is the ratio of the observed peak minus average baseline to the RMS fit residual for azimuth and elevation data, and the ratio of the observed peak minus the average of the first and last $1/10^{\mathrm{th}}$ of the record to the RMS fit residual in the case of focus scans.

## 3. Model PFM5d

Azimuth, elevation, and focus are independently fit. For each, first a fitting dataset is constructed in the modeling database by selecting date ranges, rejecting low signal-to-noise measurements, etc. These data are then injested into the model fitting code either as CSV data or using the Matlab ODBC interface. Further selections can be made, *e.g.* use of only one polarization. Then a weighted least squares estimate of model coefficients is made. The fit is bootstrapped ($n = 1000$) in order to provide data for bootstrap confidence interval estimation of the individual coefficients. Various diagnostics and performance assessments are calculated and displayed.

The following sections describe PFM Model 5d. Data selection is the PFM database includes azimuth and elevation data from September 2007, with SNR greater than 60, bootstrap standard deviation of offset error less than 0”.5, and with calibration diode off. Focus data are selected similarly but without an SNR criterion and with bootstrap standard deviation of offset less than 0.8 mm. Selections in the model fit code include: Restriction to elevations between $20^\circ$ and $80^\circ$ elevation in order to avoid potential problems with refraction correction (less than $20^\circ$) and high azimuth rates (greater than $80^\circ$), wind range from 0 to 3 m/s to avoid wind distortion, both polarizations, and both forward and backward scans, and all times of day. Track map data are from the September 10, 2007 measurement.

The azimuth model includes the temperature features from [2] and model terms ([3] Table 4) for horizontal collimation, elevation axle collimation, azimuth axis zero, East and North Tilt, and hysteresis. Note that the elevation axle collimation and azimuth zero terms include alidade distortions due to track unevenness. The $1\sigma$ model error is set to 2”. The elevation model includes the temperature features from [2] and elevation zero, asymmetric gravity, East and North tilt, and hysteresis. The $1\sigma$ model error is set to 5”. The focus model includes temperature, zero, and gravity terms from [2]. An ordinary least squares fit is performed on the focus data.

**REFERENCES**

James Condon. Refined scientific requirements for the GBT. Technical Report PTCS/PN28.4, NRAO Green Bank, 2003.

Kim Constantikes. Thermally-neutral tradition pointing models and thermal corrections to pointing and focus. Technical Report PTCS/PN25.4, NRAO Green Bank, 2007.

Kim Constantikes. The GBT pointing model. Technical Report PTCS/PN53.1, NRAO Green Bank, 2008.

Morris H. DeGroot and Mark J. Schervish. *Probability and Statistics*. Addison-Wesley, 2002.

Curtis R. Vogel. *Computational Methods for Inverse Problems*. SIAM, 2002.

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This preprint was prepared with the AAS LaTeX macros v5.2.

## 4. Model PFM5d Coefficients, Confidence Intervals, and Performance

In the following the significance column is the product of the range of the model control variable, *e.g* T1E, and its model coefficient. This is the range of the correction to pointing or focus correction associated with that model term. The quantities p50, p68, and p95 are the $50^{th}$, $68^{th}$, and $95^{th}$ quantiles of error in pointing or focus.

```
X-El
      p: [3847x1 double]
    err: [3847x1 double]
    plo: [3847x1 double]
    phi: [3847x1 double]
    p50: 1.7397
    p68: 2.6263
    p95: 6.4592

El
      p: [3502x1 double]
    err: [3502x1 double]
    plo: [3502x1 double]
    phi: [3502x1 double]
    p50: 3.5476
    p68: 5.0465
    p95: 10.6127

Focus
      p: [2610x1 double]
    err: [2610x1 double]
    plo: [2610x1 double]
    phi: [2610x1 double]
    p50: 1.5782
    p68: 2.4242
    p95: 6.7731


X-El: Name, Coeff, [95% CI], Boot Std,Significance

 1)           T2A    -2.4512 [   -3.1201    -1.7743]     0.3459     6.5937
 2)           T3A     0.0630 [   -0.3306     0.4570]     0.2070     0.1576
 3)           T4A    -1.0435 [   -1.4554    -0.6070]     0.2138     2.8489
 4)           T5A     1.7411 [    0.7804     2.6921]     0.5007     2.0873
 5)           T6A     0.0742 [   -0.1082     0.2417]     0.0879     1.0083
 6)         Const   -33.2393 [  -35.8927   -30.3825]     1.4052     0.0000
 7)        El Sin     3.5994 [    1.3414     5.7431]     1.1263     2.3066
```

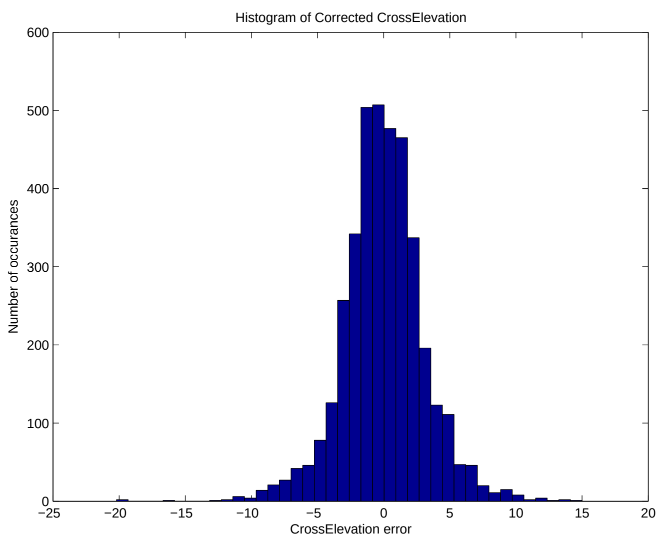
| | | | | | | |
|---|---|---|---|---|---|---|
| 8) | El Cos | 7.2317 | [ 5.3518 | 9.0205] | 0.9367 | 5.5294 |
| 9) | El Sin Az Cos | -2.8765 | [ -3.0940 | -2.6716] | 0.1102 | 5.5792 |
| 10) | El Sin Az Sin | -4.0852 | [ -4.2667 | -3.9184] | 0.0875 | 7.9045 |
| 11) | dz1 cEl | 0.4105 | [ -1.5190 | 2.1520] | 0.9322 | 0.2488 |
| 12) | dz1 sEl | 1.0004 | [ -0.7219 | 2.6577] | 0.7986 | 0.6425 |
| 13) | dir cEl | -0.6114 | [ -0.7655 | -0.4560] | 0.0772 | 1.1481 |

```
El: Name, Coeff, [95% CI], Boot Std,Significance
```

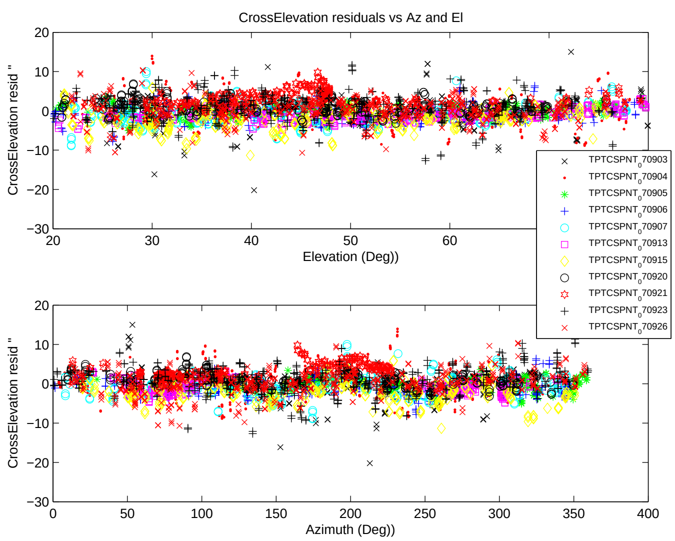
| | | | | | | |
|---|---|---|---|---|---|---|
| 1) | T1E | -5.1097 | [ -6.2149 | -3.9727] | 0.3459 | 9.0953 |
| 2) | T2E | 1.4698 | [ 1.4026 | 1.5245] | 0.2070 | 22.6126 |
| 3) | T3E | 4.7077 | [ 4.4606 | 4.9759] | 0.2138 | 36.7673 |
| 4) | T4E | -6.7295 | [ -7.6399 | -5.7958] | 0.5007 | 10.0269 |
| 5) | Const | -697.4402 | [ -704.1911 | -691.3730] | 0.0879 | 0.0000 |
| 6) | El Sin | 638.4686 | [ 633.3940 | 643.9481] | 1.4052 | 407.2872 |
| 7) | El Cos | 773.1129 | [ 769.5282 | 777.2743] | 1.1263 | 590.0750 |
| 8) | Az Cos | -5.9195 | [ -6.1807 | -5.6598] | 0.9367 | 11.8387 |
| 9) | Az Sin | 2.5932 | [ 2.3375 | 2.8375] | 0.1102 | 5.1865 |
| 10) | dir | -1.0560 | [ -1.2429 | -0.8691] | 0.0875 | 2.1120 |

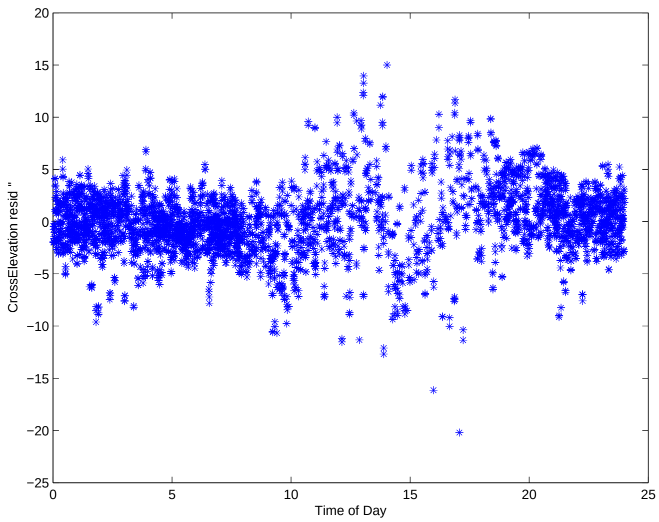
```
Focus: Name, Coeff, [95% CI], Boot Std,Significance
```

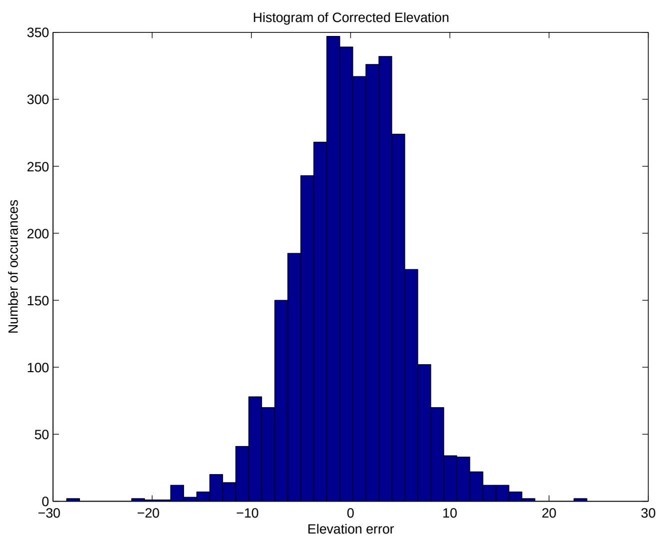
| | | | | | | |
|---|---|---|---|---|---|---|
| 1) | Pri-SR (T1F) | -0.0166 | [ -0.3016 | 0.2621] | 0.3459 | 0.3412 |
| 2) | Pri-VFA (T2F) | 4.0645 | [ 3.3132 | 4.8459] | 0.2070 | 36.0522 |
| 3) | HFA (T3F) | 0.3972 | [ -0.3652 | 1.1801] | 0.2138 | 6.1072 |
| 4) | BUS1 (T4F) | -8.1747 | [ -10.0802 | -6.3087] | 0.5007 | 14.5714 |
| 5) | BUS2 (T5F) | 1.2075 | [ -0.7251 | 3.2192] | 0.0879 | 14.2544 |
| 6) | BUS3 (T6F) | 0.0701 | [ -2.1151 | 2.0695] | 1.4052 | 0.8105 |
| 7) | Const | -167.9376 | [ -172.0457 | -163.9924] | 1.1263 | 0.0000 |
| 8) | El Sin | 61.7080 | [ 58.4250 | 64.9762] | 0.9367 | 39.5853 |
| 9) | El Cos | 190.8441 | [ 188.0720 | 193.6542] | 0.1102 | 145.6019 |

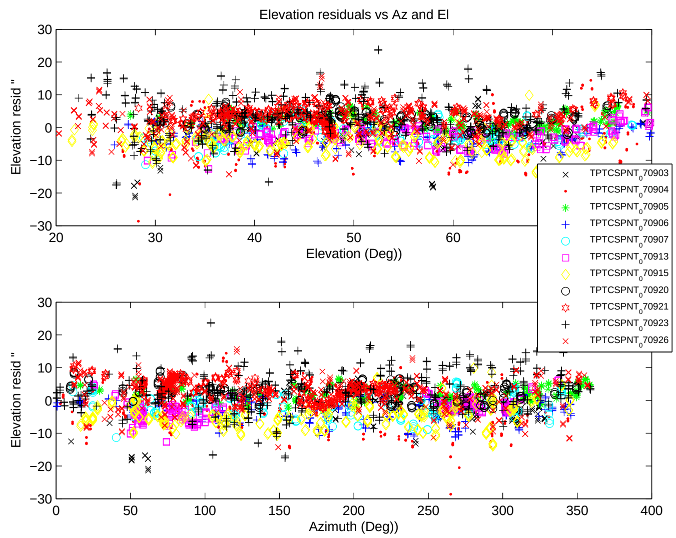


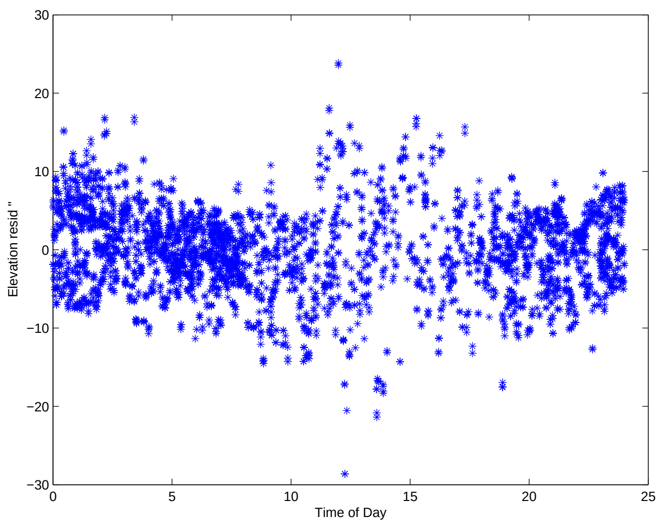
Histogram of Corrected CrossElevation
Number of occurances
CrossElevation error
600
500
400
300
200
100
0
−25
−20
−15
−10
−5
0
5
10
15
20

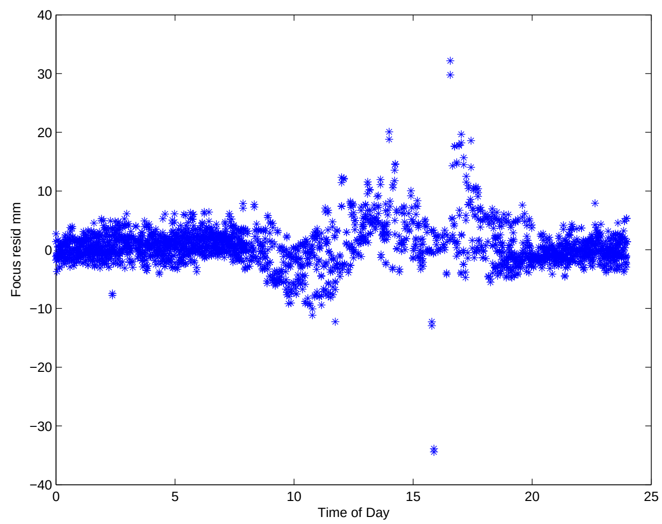

CrossElevation residuals vs Az and El
CrossElevation resid ''
Elevation (Deg))
Azimuth (Deg))
TPTCSPNT_0_70903
TPTCSPNT_0_70904
TPTCSPNT_0_70905
TPTCSPNT_0_70906
TPTCSPNT_0_70907
TPTCSPNT_0_70913
TPTCSPNT_0_70915
TPTCSPNT_0_70920
TPTCSPNT_0_70921
TPTCSPNT_0_70923
TPTCSPNT_0_70926

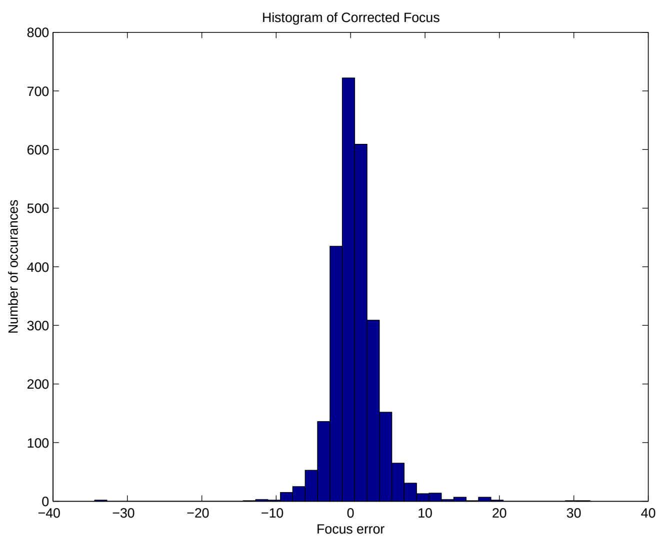

CrossElevation resid ''
Time of Day
20
15
10
5
0
−5
−10
−15
−20
−25
0
5
10
15
20
25

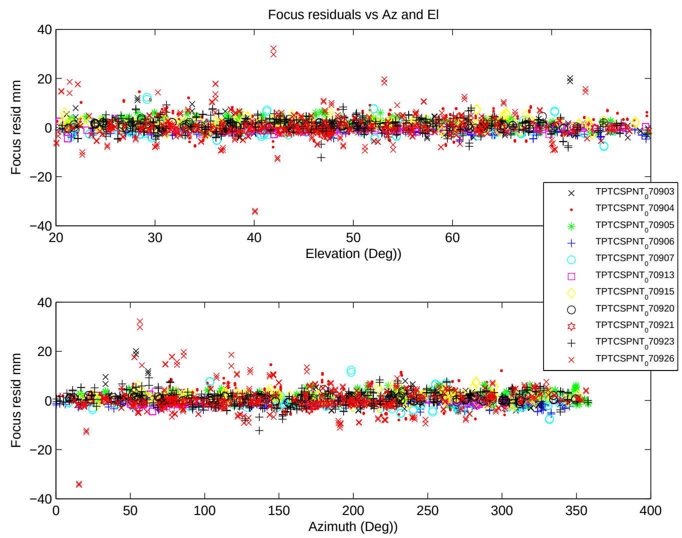

Histogram of Corrected Elevation
Number of occurances
Elevation error
350
300
250
200
150
100
50
0
−30
−20
−10
0
10
20
30


Elevation residuals vs Az and El
Elevation resid ''
Elevation (Deg))
Azimuth (Deg))
TPTCSPNT_0 70903
TPTCSPNT_0 70904
TPTCSPNT_0 70905
TPTCSPNT_0 70906
TPTCSPNT_0 70907
TPTCSPNT_0 70913
TPTCSPNT_0 70915
TPTCSPNT_0 70920
TPTCSPNT_0 70921
TPTCSPNT_0 70923
TPTCSPNT_0 70926


30
20
10
0
−10
−20
−30
Elevation resid ''
0
5
10
15
20
25
Time of Day


40
30
20
10
0
−10
−20
−30
−40
0
5
10
15
20
25
Focus resid mm
Time of Day


Histogram of Corrected Focus
Number of occurances
Focus error
800
700
600
500
400
300
200
100
0
−40
−30
−20
−10
0
10
20
30
40


Focus residuals vs Az and El
Focus resid mm
Elevation (Deg))
Azimuth (Deg))
TPTCSPNT_0_70903
TPTCSPNT_0_70904
TPTCSPNT_0_70905
TPTCSPNT_0_70906
TPTCSPNT_0_70907
TPTCSPNT_0_70913
TPTCSPNT_0_70915
TPTCSPNT_0_70920
TPTCSPNT_0_70921
TPTCSPNT_0_70923
TPTCSPNT_0_70926