Refraction Corrections for the GBT

J. J. Condon

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Abstract

Accurate refraction corrections that are independent of astronomical pointing data are needed by the GBT for both precise astronomical pointing and reliable measurements of the traditional pointing constants. The refraction corrections now used at the GBT do not meet these requirements. This project note reviews refraction calculations and proposes new refraction corrections which are separable from the traditional pointing constants and accurate enough for astronomical observations at $\lambda = 3$ mm above $E \approx 10^\circ$ elevation. We should add a caveat to the PTCS scientific requirements: refraction precludes $\lambda = 3$ mm observations at elevations below 10°.

Contents

History

35.1 2004 January 6. Original version (J. J. Condon).

35.2 2004 February 13. Revised (J. J. Condon).

1. Introduction

Refraction in the Earth's atmosphere lifts the apparent elevation E_0 of an astronomical source above its "true" elevation $E_{\rm t}$ by the angle of refraction $R \equiv E_0 - E_{\rm t}$. Since $R \approx 60'' \times \cot E_0$ is much larger than the tolerable pointing error of the GBT, refraction corrections must be included in the on-line pointing equations. Their accuracy concerns the PTCS for two independent reasons: (1) The PTCS group is broadly responsible for delivering good astronomical pointing at high frequencies, not just good telescope pointing. (2) Even the core PTCS program depends on astronomical measurements made with the GBT for determining the telescope pointing constants, some of which are quite sensitive to errors in R (Condon 2003). We won't be able to determine stable values for the those pointing constants until we have accurate estimates of R that are independent of those astronomical measurements.

All refraction calculations are based on (1) microphysics specifying the local refractive index of air at any point as a function of local meteorological data (Sec. 2) and (2) geometrical ray tracing through the atmosphere (Sec. 3). The refraction correction currently in use at the GBT (Sec. 5) contains two assumptions which should be questioned: (1) The local index of refraction is poorly determined by the microphysics and local meteorological data, so it must be multiplied by a scaling constant C determined by astronomical observations made with the GBT (Maddalena et al. 2002). Unfortunately, the asymmetric GBT is uniquely unsuitable for making the relevant observations. (2) The ray-tracing geometry is independent of local microphysics, so that the elevation dependence of R is independent of both meteorological conditions (temperature, humidity) and waveband (radio or optical) (Maddalena 1994). This assumption is correct at elevations above $E \approx 23^\circ$ but fails for observations at low elevations (Sec. 3.2), especially at radio wavelengths.

New refraction corrections for the GBT are proposed in Section 6. Even the best are likely to be increasingly inaccurate below $\approx 10^\circ$ elevation, the lowest elevation for which the PTCS should purport to deliver good pointing. In fact, the $D = 100$ m GBT is so large that *differential* refraction between the upper and lower edges of the primary will significantly degrade its $\lambda = 3$ mm beam at elevations near the $E_0 = 5^\circ$ mechanical pointing limit.

2. The Microphysics of Refraction

Ray bending is caused by changes in the real part of the refractive index n, which is defined as the ratio of the speed of light in a vacuum to the phase velocity in the medium:

$$n \equiv \frac{c}{v_{\rm p}} \ . \tag{1}$$

A related quantity is the refractivity N defined by

$$N \equiv 10^6 (n-1) \ . \tag{2}$$

In a vacuum, $n = 1$ and $N = 0$.

Thompson et al. (2001) provide a good overview of the relevant microphysics. Molecular resonances in a neutral dielectric gas produce both absorption lines (nonzero imaginary n) and changes in refractivity (changes in the real part of n). Each absorption line at frequency $\nu_{\rm abs}$ contributes a fixed amount to the refractivity at all frequencies below $\nu_{\rm abs}$. The refractivity of dry air is essentially the same at radio and optical wavelengths because the strongest dry-air absorption lines are in the ultraviolet. Dry-air refractivity is proportional to the molecular density, so the ideal gas law implies

$$N \propto (n-1) \propto \frac{P}{T} , \tag{3}$$

where P is the pressure and T is the absolute temperature (e.g., in K). The optical refractive index of dry air at the sea level is (Green 1985)

$$n_0 = 1.0002927 \tag{4}$$

when $P = 760$ mm Hg ≈ 1013 mbar and $T = 0$ C ≈ 273.15 K.

The strongest water-vapor transitions occur at infrared wavelengths, making the radio refractivity of water vapor much (about 22 times) higher than its optical refractivity. The only water line inside the GBT frequency range $\nu \leq 115$ GHz (at $\nu_{\rm abs} \approx 22$ GHz) is so weak that the refractivity of water vapor is essentially independent of radio frequency. Thus optical refraction data are useful predictors for the dry component of radio refraction but *not the wet component*.

Practical equations for calculating the local radio refractivity from meteorolical measurements of pressure, temperature, and relative humidity are clearly laid out in Section II of Maddalena (1994). The refractivity at the weather station has the form [Froome & Essen (1969) Eq. 3.9, as corrected in Maddalena (1994) Eq. 11]

$$\begin{aligned} N_0 \approx & \left(\frac{0.37884 P_{\rm dry}}{1 + 0.003661 T_{\rm C}}\right)[1 + (1.049 - 0.0157 T_{\rm C}) 10^{-6} P_{\rm dry}] \\ & + \left(\frac{86.24 P_{\rm w}}{273 + T_{\rm C}}\right)\left(1 + \frac{5748}{273 + T_{\rm C}}\right)(1 + 2.4 \times 10^{-5} P_{\rm w}) , \end{aligned} \tag{5}$$

where $T_{\rm C}$ is the Centigrade temperature, $P_{\rm dry}$ is the partial pressure of dry air in mm Hg, and $P_{\rm w}$ is the partial pressure of water vapor in mm Hg. The different refractivity of CO_2 has been ignored because it has a negligible effect on N_0. Froome & Essen (1969) claim that Equation 5 gives N_0 within $|\Delta N_0| = 0.1$ at frequencies up to $\nu = 30$ GHz for $-20^\circ < T_{\rm C} < +60^\circ$ and unsaturated $P_{\rm w} < 100$ mm Hg, the full range of meteorological conditions under which high-frequency observations will be made with the GBT. Equation 5 should remain accurate up to $\nu = 115$ GHz despite the O_2 absorption line at $\nu \approx 60$ GHz because that line is so weak (even though its absorption precludes astronomical observations) that its effect on the real part of radio refractivity is negligible (Thompson et al. 2001). Since $R \approx (n_0 - 1)\cot E_0$ rad (Sec. 3.1), an error $\Delta(n_0 - 1) \approx 10^{-7}$ changes the angle of refraction by only $\Delta R \approx 10^{-7}\,{\rm rad} \approx 0\rlap{.}''02$ at $E_0 = 45^\circ$ elevation. We can therefore treat Equation 5 as exact for all GBT observations.

The refractivity code currently in use at the GBT (Brandt, private communication) evaluates Equation 5 using the absolute temperature $T = (273.15 + T_{\rm C})$ in K, the total pressure $P = (P_{\rm dry} + P_{\rm w})$ in mB, and the relative humidity $\varkappa$ (dimensionless) as meteorological inputs. After conversion of units, the saturation pressure of water vapor in mm Hg is calculated from (Rueger 1990)

$$P_{\rm sat} = 4.5841(1.0007 + 4.61 \times 10^{-6} P) \exp\left(\frac{17.502\, T_{\rm C}}{240.97 + T_{\rm C}}\right) , \tag{6}$$

where the pressures are in mm Hg. Then (Crane 1976)

$$P_{\rm w} = P_{\rm sat}\varkappa\left[1-(1-\varkappa)\frac{P_{\rm sat}}{P}\right]^{-1} . \tag{7}$$

Uncertainties in the weather data lead to errors in N_0 estimated via Equation 5, but they are small. Maddalena (1994) presented a thorough error analysis showing that errors in N_0 cause errors $|\Delta R| < 1''$ in the angle of refraction if the data uncertainties satisfy $\Delta P < 1$ mm Hg (pressure), $\Delta T < 0.5$ C (temperature), and $\Delta T < 0.2$ C (dew point).

At low radio frequencies, the ionosphere is also refractive. Its refractive index is

$$n = \left(1 - \frac{\nu_{\rm p}^2}{\nu^2}\right)^{1/2} , \tag{8}$$

where $\nu_{\rm p}$ is the highly variable plasma frequency, typically a few MHz. Fortunately, the refractive bending of the ionosphere can be ignored for geometric reasons (see Sec. 3).

The bottom line of this section is that atmospheric refractivity is well understood theoretically and the local refractivity can be calculated accurately from meteorological data. The uncertainty in angle of refraction R caused by uncertainties in the local refractivity N is much too small for us to measure with the GBT, so *we should not use GBT measurements of astronomical sources to rescale the refractivity predicted by Equation 5.*

3. Geometrical Ray Tracing

A ray crossing the boundary between regions having different refractive indices n_1 and n_2 is bent. If the ray is incident at angle z_1 from the normal to the boundary, the exit angle z_2 is given by Snell's law:

$$n_1 \sin z_1 = n_2 \sin z_2 . \tag{9}$$

3.1. Plane-parallel Atmosphere

The simplest geometrical model for atmospheric refraction is a stack of plane-parallel layers. Snell's law implies that

$$n_0 \sin z_0 = \sin z_{\rm t} \;\; \text{or} \;\; n_0 \cos E_0 = \cos E_{\rm t} \tag{10}$$

where n_0 is the refractive index near the ground, $E_0 = (\pi/2) - z_0$ is the source elevation observed on the ground, and $E_{\rm t} = (\pi/2) - z_{\rm t}$ is the "true" source elevation above the atmosphere. Then

$$\frac{(n_0-1)\cos E_0}{\sin E_0} = (n_0-1)\cot E_0 = \frac{\cos E_{\rm t} - \cos E_0}{\sin E_0} . \tag{11}$$

For $R \ll 1$ rad, $\Delta \cos E = \cos E_{\rm t} - \cos E_0 \approx -R \sin E_0$ and

$$R \approx (n_0 - 1)\cot E_0 . \tag{12}$$

Equation 12 implies that the angle of refraction in the plane-parallel or "flat Earth" model depends *only* on the local refractivity at the telescope and not on conditions elsewhere along the ray path. Since the ionosphere does not extend to the ground, it contributes *nothing* to R in this approximation. The ionospheric R resulting from ionospheric curvature is normally so small [see Fig. 13.19 in Thompson et al. (2001)] that it can be ignored at the GBT (for all $\nu > 300$ MHz and $E_0 > 5°$).

3.2. Spherical Atmosphere

Equation 12 fails at low elevations because the atmospheric thickness is a finite fraction of the Earth's radius. If the Earth's atmosphere were perfectly spherical, the exact equation for refraction (Green 1985) would be

$$R = r_0 n_0 \sin z_0 \int_1^{n_0} \frac{\mathrm{d}n}{n(r^2 n^2 - r_0^2 n_0^2 \sin^2 z_0)^{1/2}} \ , \tag{13}$$

where

- $r_0 \approx 6.37 \times 10^7$ m is the radius of the Earth,
- n_0 is the index of refraction at the Earth's surface,
- $z_0 = \pi/2 - E_0$ is the zenith angle observed at the Earth's surface,
- $r = r_0 + h$ is the distance from the center of the Earth at altitude h, and
- n is the index of refraction at altitude h.

Not too close to the horizon, this integral may be expanded in odd powers of $\cot E_0$ (Green 1985):

$$R \approx (n_0 - 1)\left(1 - \frac{H_0}{r_0}\right)\cot E_0 - (n_0 - 1)\left[\frac{H_0}{r_0} - \frac{(n_0 - 1)}{2}\right]\cot^3 E_0 \ , \tag{14}$$

where H_0 is the effective thickness of the atmosphere defined by integrating the density along a vertical column:

$$H_0 \equiv \frac{1}{\rho_0}\int_0^\infty \rho \, \mathrm{d}h \ . \tag{15}$$

An immediate consequence of Equation 14 is that *the elevation dependence of the refraction angle R depends on weather conditions and waveband (radio or optical)* through the effective thickness H_0. In an exponential atmosphere, H_0 is just the density scale height. The scale height of the dry atmosphere is proportional to absolute temperature (Cox 2000)

$$H_0 \approx 8000\,\mathrm{m}\left(\frac{T}{273.15\,\mathrm{K}}\right)(\mathrm{dry}) \ . \tag{16}$$

Thus the elevation dependence of R varies with temperature. The water-vapor scale height is much less (Cox 2000),

$$H_0 \approx 2000\,\mathrm{m}\ (\mathrm{wet}) \ , \tag{17}$$

so the elevation dependence of R also varies with humidity. The large ratio of dry to wet scale heights is not particularly important at optical wavelengths because water vapor contributes very little to refractivity. Water vapor is 22 times more refractive at radio wavelengths, so the effective thickness (Eq. 15) is lower and declines more rapidly as humidity increases. Consequently, the elevation dependence of refraction measured optically by Allen (1955) cannot be relied on to predict the elevation dependence of radio refraction under different weather conditions, particularly at low elevations.

The differences ΔR between the plane-parallel model (Eq. 12) or expansion approximation (Eq. 14) and the "exact" solution obtained by integrating Equation 13 numerically through an exponential atmosphere are plotted in Figure 1. For a dry atmosphere with surface refractive index $n_0 = 1.0002824$ (the optical value when $P = 760$ mm Hg and $T = 10$ C), the plane-parallel model (dashed curve) agrees within $\Delta R = 1''$ down to $E_{\rm t} \approx 23^\circ$. Thus at higher elevations, only *local* atmospheric measurements of pressure, temperature, and humidity are needed to compute $(n_0 - 1)$ and specify the refraction correction completely. Plane-parallel atmospheric structures, such as temperature inversions or differing wet and dry scale heights, will have negligible consequences for R at elevations above $E_{\rm t} \approx 23^\circ$. There are no free parameters (e.g., refraction constants) to be determined empirically.

For $H_0 \approx 8300$ m, the dry scale height when $T = 10$ C (Eq. 16), the continuous curve in Figure 1 shows that the simplest spherical approximation (Eq. 14) remains accurate within $|\Delta R| = 1''$ down to $E_{\rm t} \approx 10^\circ$. Realistically, millimeter-wave observations in Green Bank cannot be made at lower elevations because atmospheric absorption and emission are too strong. The difference between the dashed and solid curves above $E_{\rm t} \approx 10^\circ$ in Figure 1 is roughly proportional to the atmospheric thickness H_0 and shows how important it is to use the correct value of H_0 for refraction corrections below $E_{\rm t} \approx 23^\circ$. *At radio wavelengths, accurate corrections for R in the elevation range $10^\circ < E_{\rm t} < 23^\circ$ require simple atmospheric models and cannot be based on empirical functions that are independent of temperature and humidity.* Below $E_{\rm t} \approx 10^\circ$, R depends on the detailed structure of the atmosphere (Garfinkel 1944), which is not purely exponential primarily because (1) the temperature changes at the adiabatic lapse rate $\Delta T / \Delta h \approx -6.5$ K per km altitude in the troposphere and (2) the stratospheric temperature is nearly constant above $h \sim 10^4$ m.

3.3. Differential Refraction Across the GBT Aperture

The GBT primary diameter $D \approx 100$ m is a finite fraction of the atmospheric scale height $H_0 \approx 8000$ m. Thus the refractivity changes by

$$\frac{\Delta N}{N} \approx \frac{D}{H_0} \approx 0.0125 \tag{18}$$

between the lower and upper edges of the dish at low elevations. Near the GBT mechanical elevation limit $E_0 = 5^\circ$, $R \approx 600''$, so the refraction angle difference ΔR between the lower and upper edges is

$$\Delta R \approx \frac{D}{H_0} R \approx 7''\!.5 \ , \tag{19}$$

an amount comparable with the beam size at $\lambda = 3$ mm. *Differential refraction will significantly degrade the GBT beam shape and aperture efficiency when $\lambda = 3$ mm and $E_0 \approx 5^\circ$.* In principle, this degradation could be mitigated by adjusting the primary shape.

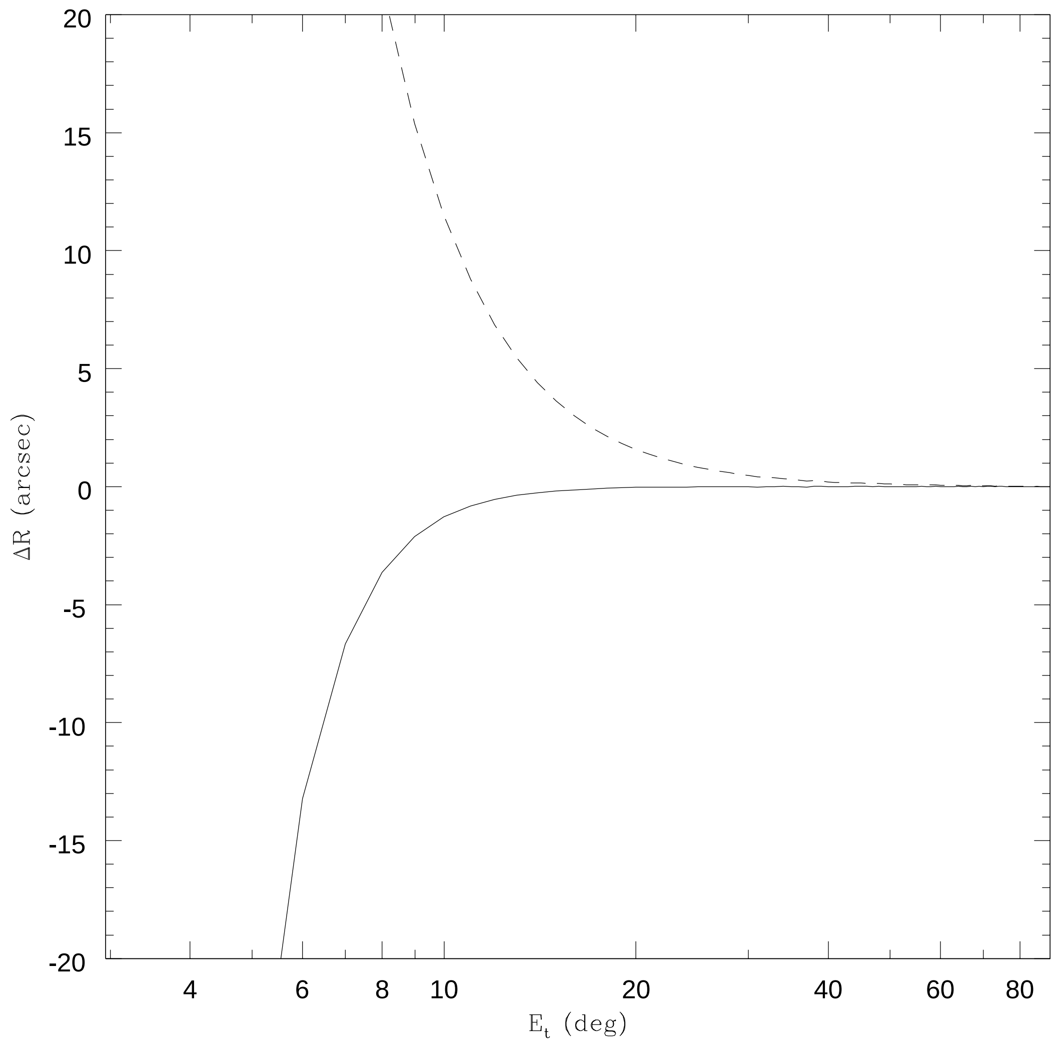

Fig. 1.— Departures of the plane-parallel model (Eq. 12) (dashed curve) and the simplest spherical model (Eq. 14) (continuous curve) from the "exact" integration of Equation 13 are $< 1''$ above elevations $E_{\rm t} \approx 23^\circ$ and $E_{\rm t} \approx 10^\circ$, respectively. Abscissa: True elevation (deg). Ordinate: Refraction angle differences from the "exact" model (arcsec).

4. Practical Equations for the Elevation Dependence of Refraction

Equations 12, 13, and 14 all specify R as functions of apparent elevation E_0; that is,

$$R = (n_0 - 1) f(E_0) , \tag{20}$$

where the function $f(E_0) \approx \cot E_0$ at high elevations. Equations 12 and 14 can be solved quickly but diverge at low elevations, while the "exact" Equation 13 may take too long to evaluate on-line. Also, the true elevation $E_{\rm t}$ of a source is normally known and the apparent elevation E_0 is not, so practical refraction corrections should have the form

$$R = (n_0 - 1) g(E_{\rm t}) . \tag{21}$$

Practical ways around these difficulties include (1) using simple empirical fits to measured $g(E_{\rm t})$ functions (Sec. 4.1) or (2) iterative solutions of equations having the form of Equation 20.

4.1. The Elevation Dependence of Optical Refraction Under Standard Conditions

The elevation dependence of the observed optical refraction angle was tabulated by Allen (1955) for a standard atmosphere with $P = 760$ mm Hg and $T = 10$ C, so $n_0 \approx 1.0002824$ and $H_0 \approx 8300$ m. Since the elevation dependence of these data is not well approximated by Equation 14 below $E_{\rm t} \approx 10^\circ$, von Hoerner (1976) suggested using the empirical approximation

$$R = (n_0 - 1) \times g(E_{\rm t}) \approx 0\farcm973 \times \frac{\cos(E_{\rm t})}{\sin(E_{\rm t}) + 0.00175 \cot{(E_{\rm t} - 2.5^\circ)}} \tag{22}$$

to the Allen (1955) data to specify the elevation dependence of refraction at the 140-foot telescope and later at the GBT (von Hoerner 1993). A new empirical formula for $g(E_{\rm t})$ (Maddalena et al. 2002), also constructed to match the elevation dependence of the Allen (1955) data, was implemented in the GBT antenna control system during 2001 December:

$$g(E_{\rm t}) = S - 0.1185 \sin(14.69 S + 7.57) \tag{23}$$

where

$$S = 1.02 \cot \left(E_{\rm t} + \frac{10.3}{5.11 + E_{\rm t}} \right) \tag{24}$$

and all angles are expressed in degrees. Equation 23 approaches $0.973 \times \cot E_{\rm t}$ at high elevations, so I have divided it by 0.973 for comparison with other versions of $g(E_{\rm t})$.

Figure 2 plots the differences ΔR between the various models and the data in Allen (1955) as functions of true elevation $E_{\rm t}$. The filled symbols are based on numerical integration of Equation 13 with $n_0 = 1.0002824$ and scale height $H_0 = 8300$ m. The crosses indicate the errors in $R = (n_0 - 1) g(E_{\rm t})$ from von Hoerner's approximation, and the open symbols show the errors in $R = (0.973)^{-1} (n_0 - 1) g(E_{\rm t})$ from Maddalena's new formula. The "exact" spherical model and both empirical formulae are quite satisfactory for predicting the elevation dependence of the optical angle of refracton R at $T = 10$ C over all elevations accessible to the GBT, $E_0 > 5^\circ$. However, the empirical formulae do not follow variations in $g(E_{\rm t})$ at radio wavelengths caused by changing temperatures and nonzero humidity. Ron Maddalena (2003, private communication) has estimated that we can expect elevation errors up to $|\Delta R| = 8''$ at $E_0 = 5^\circ$ from the differences between the spherical model with "standard" inputs (or its empirical approximation) and the real atmosphere.

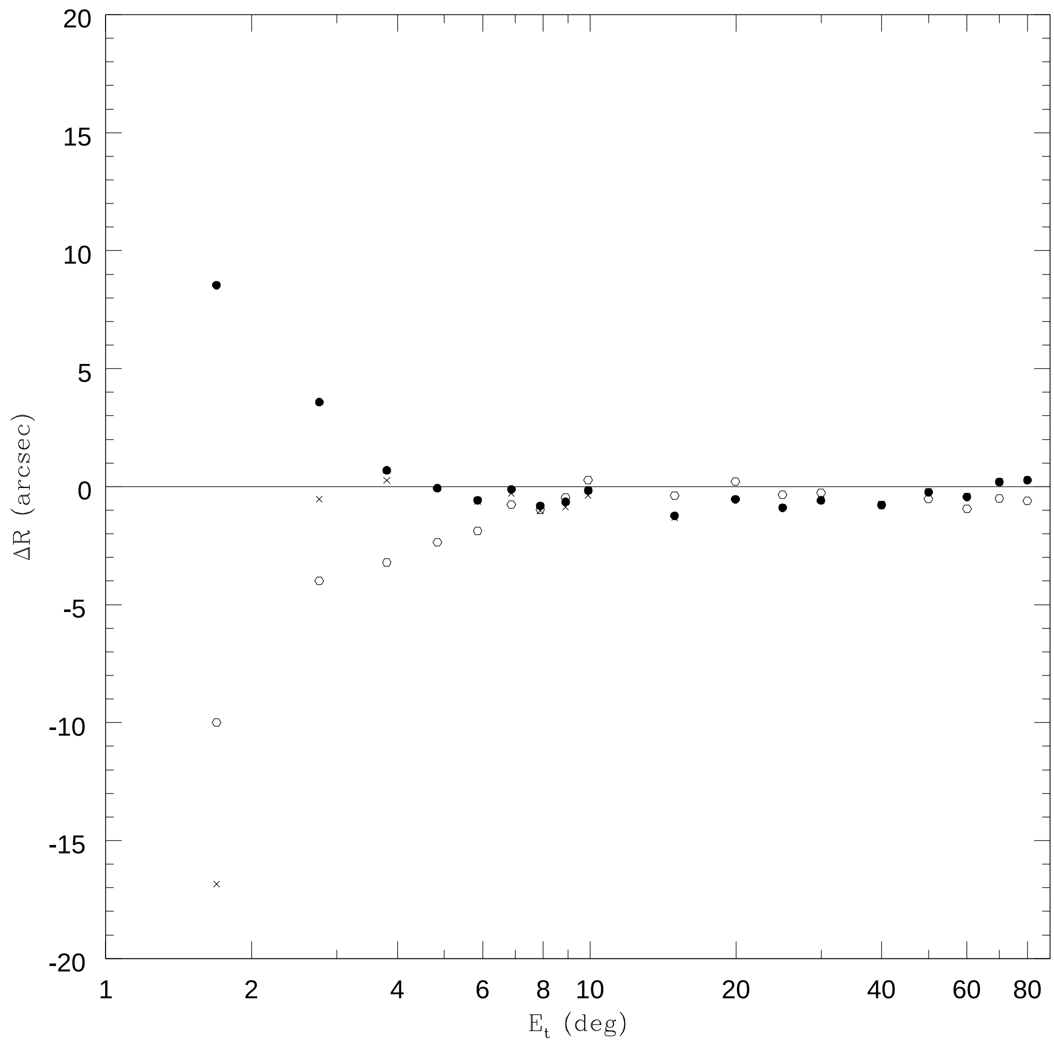

Fig. 2.— Differences between models for the elevation dependence of refraction and the optical data for a standard ($P = 760$ mb Hg, $T = 10$ C) dry atmosphere. The filled circles come from the "exact" spherical model (Eq. 13) for an exponential atmosphere with $H_0 = 8300$ m. The crosses show the von Hoerner (1976) empirical model, and the open circles are from the rescaled empirical model (Maddalena et al. 2002) currently in use at the GBT. All three agree well with the optical data at all elevations $E_0 \approx E_\mathrm{t} > 5^\circ$ accessible to the GBT. Abscissa: True elevation (deg). Ordinate: Refraction angle differences between the models and the optical data (arcsec).

4.2. The Elevation Dependence of Radio Refraction Under Nonstandard Conditions

If the temperature varies, the scale height changes (Eq. 16) from $H_0 \approx 8000$ m at $T = 0$ C to ≈ 7500 m at -15 C and ≈ 8500 m at $+15$ C. Even it the local refractivity N_0 is unchanged, this causes R to change at low elevations. Such differences ΔR in the refractive angles for sources at true elevations $E_{\rm t}$ were calculated by iterating Equation 13 for a dry atmosphere ($\varkappa = 0$), and the results are plotted in Figure 3. Temperature variations experienced in Green Bank can cause changes up to $|\Delta R| \approx 2''$ at $E_{\rm t} = 10^\circ$ and $10''$ at $E_{\rm t} = 5^\circ$, in agreement with Ron Maddalena's recent estimate.

Increasing humidity also affects the elevation dependence of R for fixed local refractivity. Figure 4 shows the effect of increasing the relative humidity from $\varkappa = 0$ to $\varkappa = 0.5$ at fixed $T = 10\ C$. The angle of refraction R at radio wavelenths increases by about $1''$ at 10° elevation and by $6''$ at 5° elevation primarily because the scale height of water vapor is much less than the scale height of dry air. This increase does not occur at optical wavelengths and cannot be corrected by $g(E_{\rm t})$ functions fitted to optical observations (Allen 1955).

If we want a radio refraction correction accurate to $|\Delta R| \approx 1''$ at $E \approx 10^\circ$, its elevation dependence must depend on temperature and humidity.

5. The Current Refraction Correction at the GBT

The refraction correction in use at the GBT since 2001 December is

$$R = C(n_0 - 1)g(E_{\rm t}) \ , \tag{25}$$

where $C = 2.338 \times 10^5$ arcsec $= 1.13$ rad is the "refraction constant" measured with the GBT (Maddalena et al. 2002) and $g(E_{\rm t})$ is the weather-independent empirical function of elevation given by Equation 23. Comparison of this correction with Equation 12 indicates that the refraction constant in Equation 25 should be precisely $C = 1\,{\rm rad} \approx 206264\overset{''}{.}81$ if $g(E_{\rm t}) \approx \cot E_0$ at high elevations. Section 2 concluded that C is not a "scaling coefficient that is only roughly known and must be determined empirically" (Maddalena et al. 2002); at most, it might be used to absorb small systematic errors in $(n_0 - 1)$ resulting from inaccurate weather data. If we rescale Equation 23 by 0.973^{-1} so $g(E_{\rm t} \approx \cot E_0$ at high elevations, then the current value of C is too high by about 10% (corresponding to $\Delta R \approx 6''$ at $E_{\rm t} = 45^\circ$). This explains the discrepancy suggested by Pat Wallace (see `http://www.gb.nrao.edu/ptcs/IPR/IPRReport.pdf`) and (roughly) confirmed by recent PTCS pointing data (Condon 2003).

The function $\cot E_0$ can be remarkably well approximated by the "traditional" elevation pointing terms $d_{0,0}$, $b_{0,1} \sin E_0$, and $d_{0,1} \cos E_0$ except at very low elevations (Condon 2003), so the observed value of C is entangled with the GBT pointing constants $d_{0,0}$, $b_{0,1}$, and $d_{0,1}$. Indeed, the asymmetric GBT is especially bad for estimating C because it has a nonzero asymmetric gravity coefficient $b_{0,1}$. Symmetric telescopes are much better at measuring C because the symmetric gravity term $d_{0,1} \cos E_0$ is nearly independent of elevation at low elevations where R is changing rapidly.

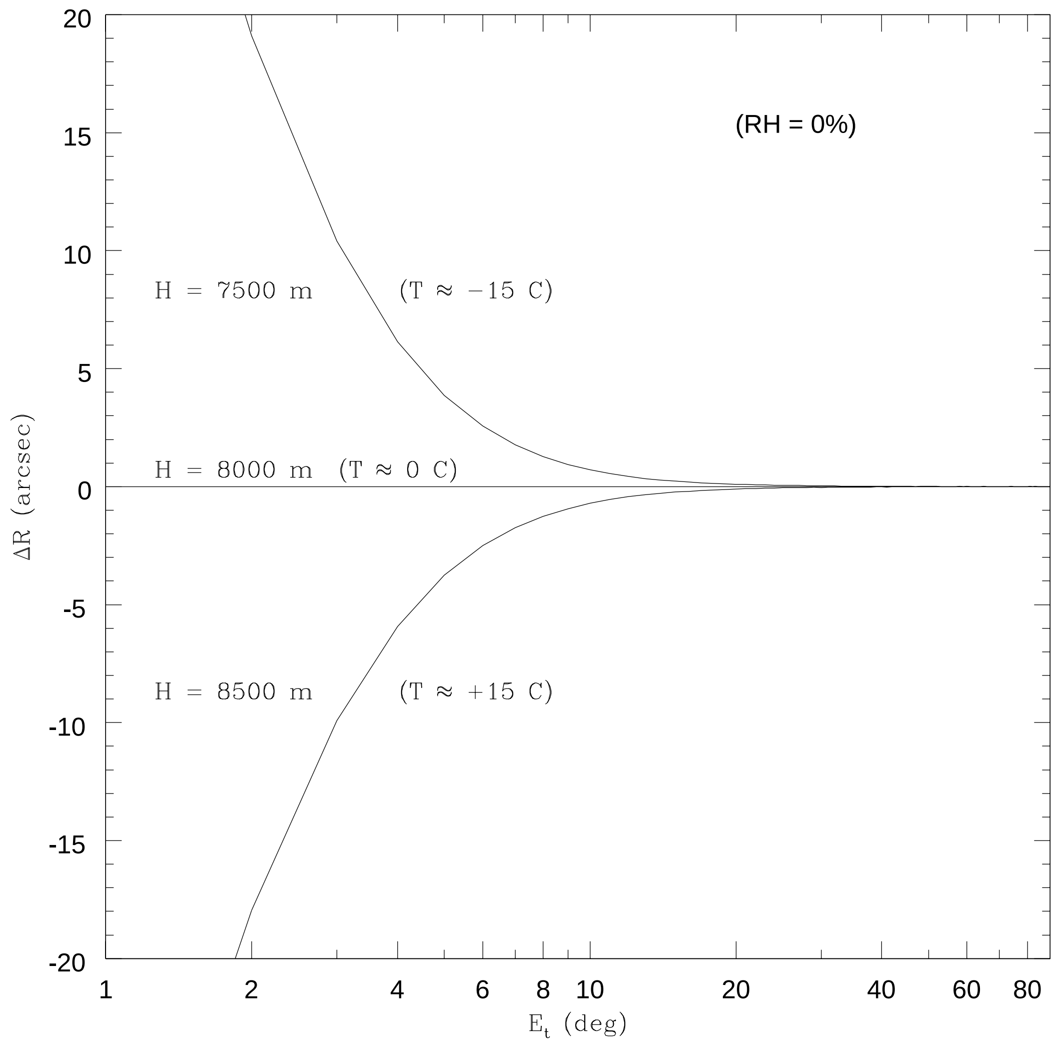

Fig. 3.— Iterative integration of Equation 13 shows how scale height H affects refraction at low elevationsn. The scale heights $H = 7500$, 8000, and 8500 m correspond to dry ($\varkappa = 0$) atmospheres with temperatures $T \approx -15$, 0, and $+15$ C, respectively. Abscissa: True elevation (deg). Ordinate: Refraction angle differences from the refraction angles when $H = 8000$ m (arcsec).

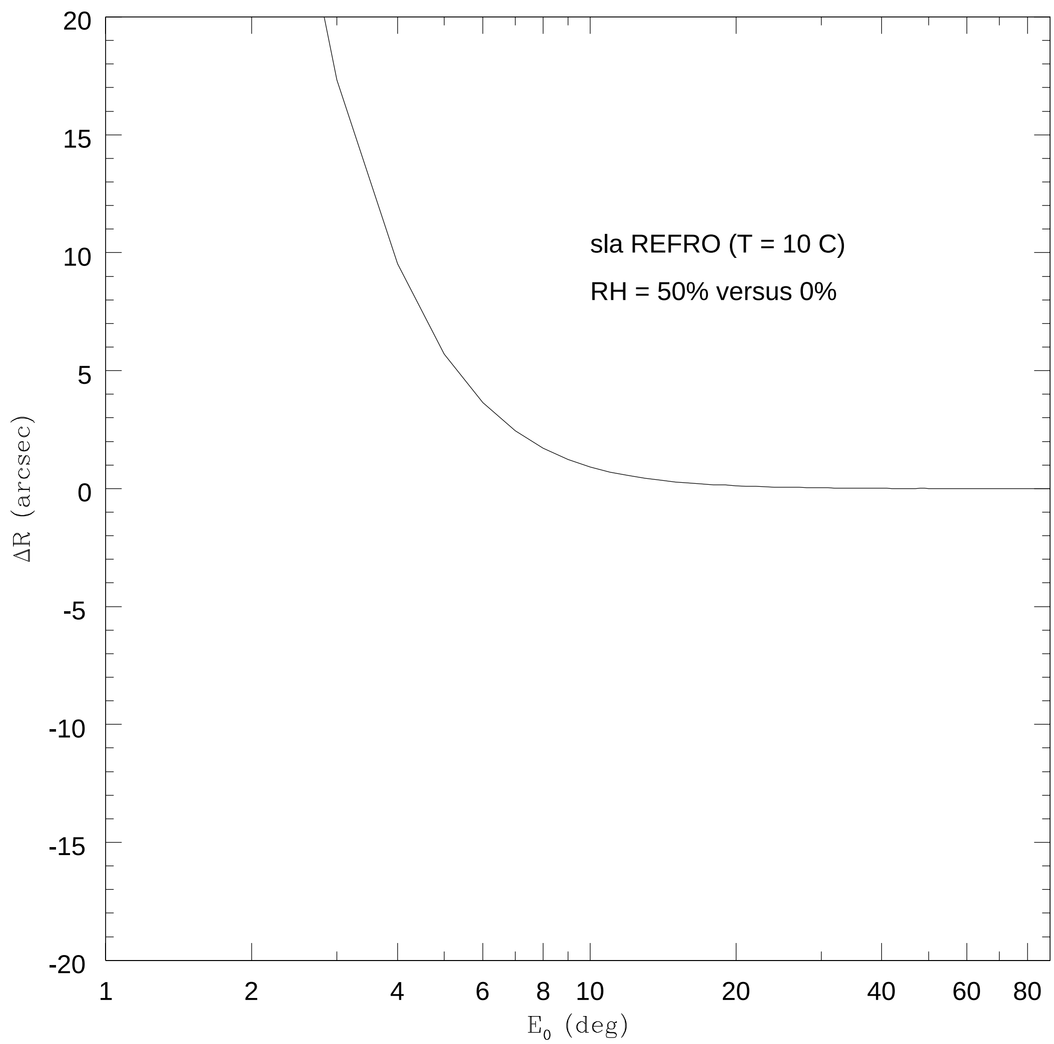

Fig. 4.— Increasing the relative humidity from $\varkappa = 0$ to $\varkappa = 50\%$ at fixed local refractivity N_0 causes the refractive angle to increase at low elevations. Abscissa: Apparent elevation (deg). Ordinate: Change in refractive angle (arcsec).

6. Proposed Refraction Corrections for the GBT

For the short term, I recommend that we simply replace the refraction constant $C = 2.338 \times 10^5$ arcsec in the current code by $C = 206265''/0.973 = 2.120 \times 10^3$ arcsec. This simultaneously removes the arbitrary part of the scaling factor C and and rescales Equation 23 to be proportional to $\cot E_\mathrm{t}$ at high elevations. At the same time, three traditional pointing constants will have to be updated as follows: $\Delta d_{0,0} = -50''$, $\Delta b_{0,1} = +45''$, and $\Delta d_{0,1} = +18''$ (Condon 2003). The resulting refraction correction will be very accurate for elevations above $E \approx 10^\circ$ in all weather conditions. It will be accurate down to $E \approx 5^\circ$ in very dry conditions when $T \approx 10$ C but will be in error by $|\Delta R| \leq 10''$ in colder, wetter weather.

For the long run, I recommend using the Starlink (`http://star-www.rl.ac.uk/`) refraction package in `SLALIB`, possibly with a small modification. Its most accurate refraction calculation `REFRO` integrates Equation 13 at either radio or optical wavelengths with a realistic model atmosphere, but it is probably too slow for use while the GBT is actually scanning. `REFRO` accurately reproduces the Allen (1955) optical refraction data for $T = 10$ C down to the GBT elevation limit (Fig. 5). For other temperatures, the variations of R predicted by `REFRO` (Fig. 6) agree with mine (Fig. 3). Maddalena (1994) initially did not recommend using the Starlink code at the GBT because the comments that came with that code were insufficient for him to understand the model assumptions and because of weather-dependent discrepancies between `REFRO` and models based on empirical fits to optical data for a standard atmosphere. I now believe that the weather dependence of `REFRO` is understood and is more accurate than empirical models in which the elevation dependence of R is independent of weather.

For fast on-line refraction corrections, the `SLALIB` program `REFCO` calls `REFRO` to estimate the weather-dependent constants $A \approx 60''$ and $B \approx -0''.06$ used in the approximation

$$z_\mathrm{t} \approx z_0 + A \tan z_0 + B \tan^3 z_0 \ . \qquad (26)$$

Only one time-consuming call to `REFCO` would be needed prior to each scan because the weather changes slowly. Then the program `REFZ` uses the Newton-Raphson iterative approximation

$$z_0 \approx z_\mathrm{t} - \frac{A \tan z_\mathrm{t} + B \tan^3 z_\mathrm{t}}{1 + (A + 3B \tan^3 z_\mathrm{t}) \sec z_\mathrm{t}} \ . \qquad (27)$$

to compute R as a function of z_t. `REFZ` is fast enough that it can be used to update the refraction corrections as the elevation changes during a scan.

Equations 26 and 27 become inaccurate below $E_\mathrm{t} \approx 7^\circ$, so `REFZ` uses an empirical (but weather dependent through A and B) formula at lower elevations. I compared the results from `REFZ` for radio wavelengths with accurate calculations based on iterative (to get solutions as functions of z_t) calls to `REFRO` for a wide range of temperatures ($T = -15$, 0, and $+15$ C) and relative humidities ($\varkappa = 0.2$, 0.5, and 0.8). `REFZ` proved to be quite accurate down to $E_\mathrm{t} \approx 10^\circ$, but the low-elevation empirical formula does not work very well at radio wavelengths in the elevation range $5^\circ < E_\mathrm{t} < 10^\circ$. A simple "fix" for this problem is to multiply the `REFZ` value of R by the factor

$$1 + 0.00195[10.8 - E_\mathrm{t}(\mathrm{deg})] \qquad (28)$$

when $E_\mathrm{t} < 10.8$ deg. Then the differences between the quick `REFZ` calculation and the accurate iterated `REFRO` calculation are $|\Delta R| \lesssim 1''$ for all elevations $E_0 > 5^\circ$ accessible to the GBT (Fig. 7).

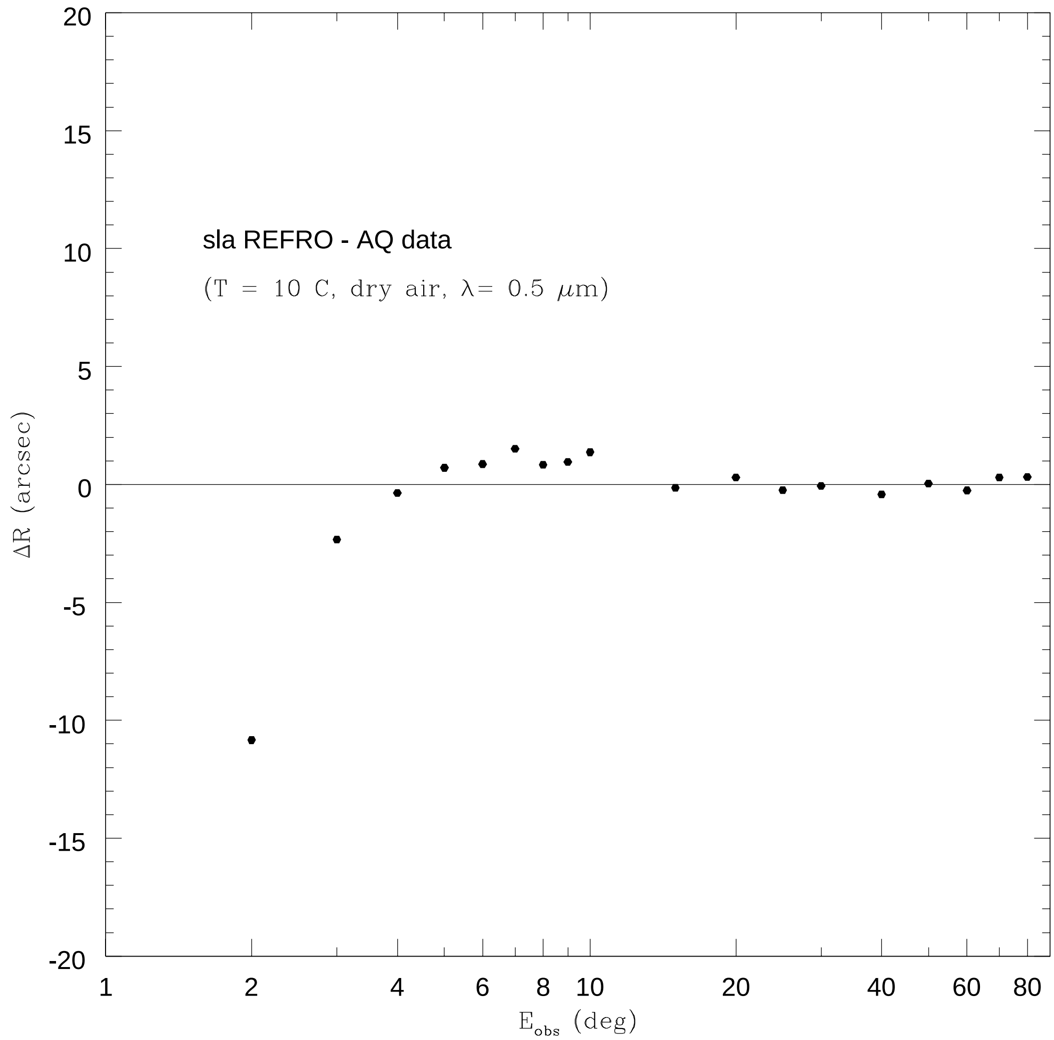

Fig. 5.— The slow but accurate Starlink refraction subroutine `REFRO` agrees very well with the optical data for a standard dry atmosphere (Allen 1955) down to the GBT elevation limit $E_0 = 5°$.

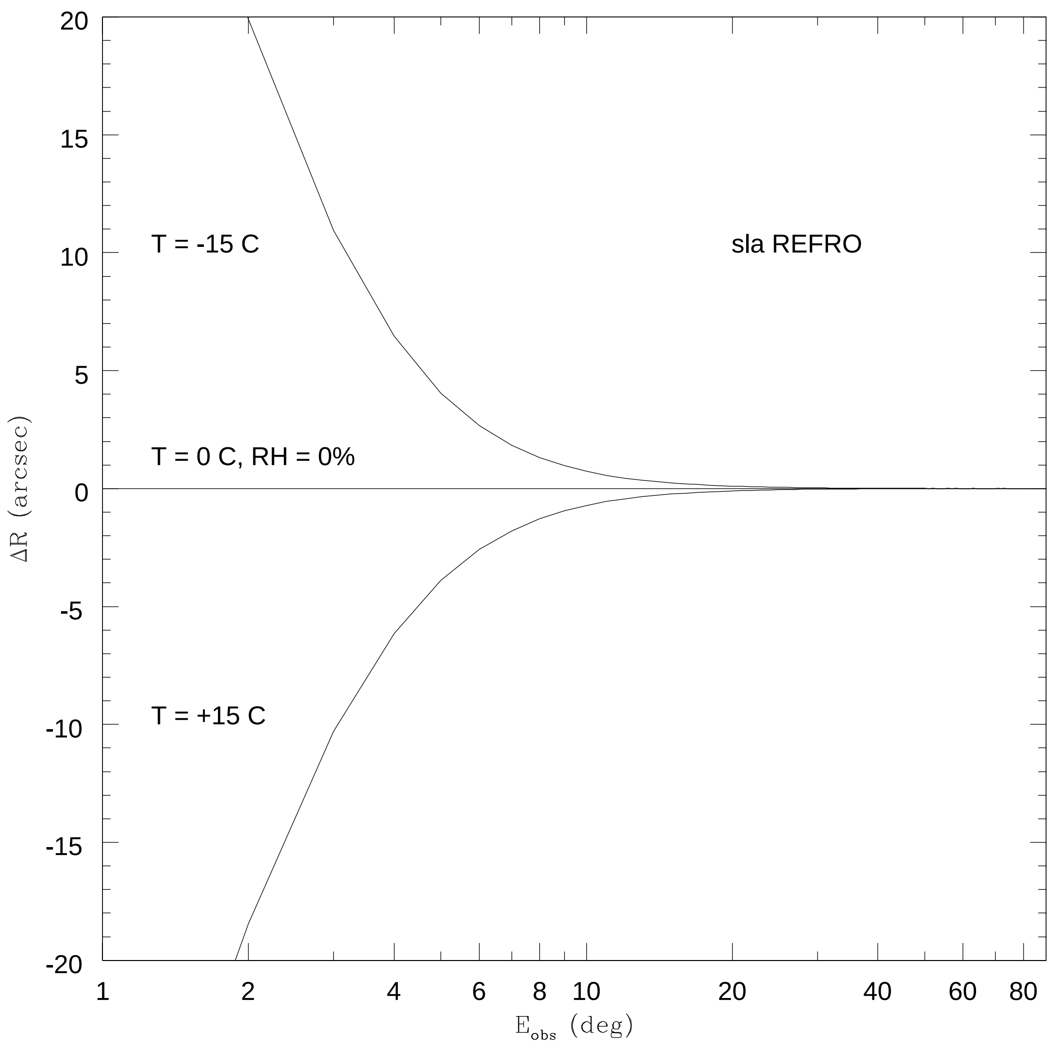

Fig. 6.— The refraction changes with dry air temperature estimated by the accurate Starlink refraction routine `REFRO` agree well with those predicted from changing scale heights. Abscissa: Observed elevation (deg). Ordinate: Refraction change compared with refraction at $T = 0$ C (arcsec).

It probably isn't worthwhile trying to make more sophisticated corrections than this. Even the best calculations (e.g., `REFRO`) based on local weather data are likely to be inaccurate below 10° elevation. In his painfully detailed analysis of astronomical refraction, Saastamoinen (1972) noted that "horizontal temperature gradients in the surroundings of the observing station probably make one of the most important sources of error in the determination of refraction" and concluded "It would appear that refraction corrections for geodetic astronomy can be adequately attained up to zenith distances of 75 to 80 degrees. Beyond that limit, the determination of refraction from simple meteorological observations seems to be impossible."

I thank Ron Maddalena for many helpful comments and suggestions.

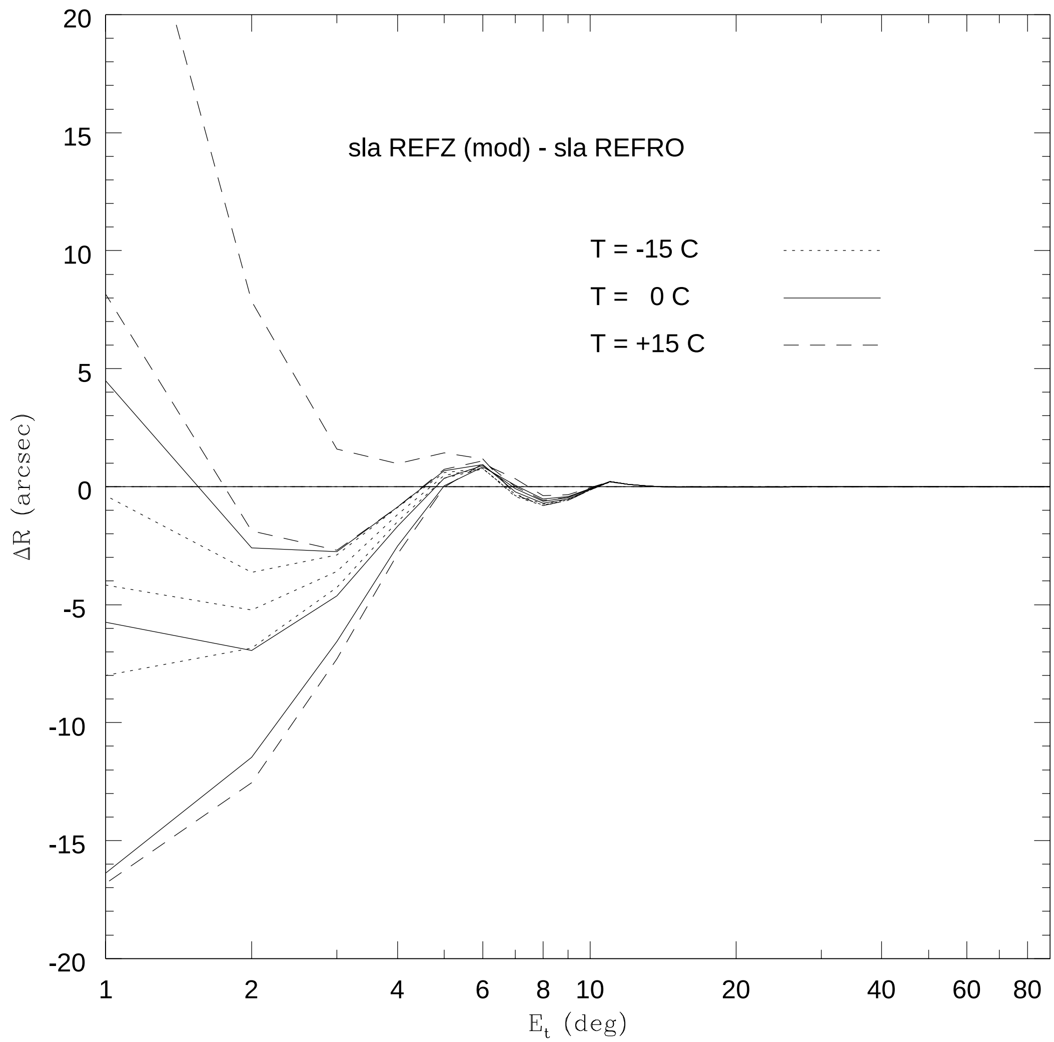

Fig. 7.— The faster Starlink refraction subroutine REFZ agrees very well with the more accurate routine REFRO down to $E_{\rm t} \approx 10^{\circ}$ and, when modified by Equation 28, is acceptable down to the GBT elevation limit. The effects of temperature are shown at -15 C (dotted curves), 0 C (continuous curves), and $+15$ C (dashed curves). For each temperature, there are three curves corresponding to relative humidities $\varkappa = 0.2$, 0.5, and 0.8. Abscissa: True elevation (deg). Ordinate: Differences between the modified REFZ and REFRO refraction corrections (arcsec).

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