TWO WAY TIMING RESIDUALS: DATA CHARACTERISTICS AND METHODS OF HANDLING

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As described in OVLBI-ES #11, the two-way timing link to each OVLBI spacecraft results in a measurement of the residual timing error on the uplink; that is, it determines the difference between the uplink delay assumed in real time for Doppler compensation and the true uplink delay. The present memo concerns the statistical properties of this residual as a function of time. The objective is to find a way to transmit the measured function in digital form at a reasonable bit rate without losing anything significant.

The known physical processes which cause the residual to be non-zero are the following: (1) the predicted orbit differs from the actual orbit; (2) the excess delay through the troposphere and ionosphere differs from that of the model used in real time; (3) the effective delay in earth station and spacecraft electronics is different from nominal; and (4) since quasi-monochromatic signals are being used, the effective phase delay may be affected by multipath propogation. Some of these processes may be non-reciprocal, causing errors in estimation of the uplink residual; such errors are a cause of fluctuations in the measured values in addition to fluctuations in the residual itself. In this report, we consider mainly processes (1) and (2), and we assume that the others can be made sufficiently well behaved by proper design.

The orbit prediction error is a much larger effect than the propogation medium error. The former is expected to be the order of 1000 m, whereas the medium delay is the order of 1 m total and its modeling error is the order of several cm. Both are sufficient to cause significant decorrelation (indeed, complete decorrelation at high frequencies) if not corrected. On the other hand, the orbit error is expected to vary on a much longer time scale than the medium error. Fluctuations in the medium delay occur because of turbulence, and significant variations occur on time scales less than one second. There is a shortage of experimental evidence on this, but it is believed that the fluctuation spectrum contains little energy above a few Hz. The orbit error must be much slower; it should be nearly periodic with a period of one orbit (2.1×10^4 sec or 5×10^{-5} Hz for VSOP), but may have important components in its Fourier series up to many times the orbital frequency.

Suppose, for the sake of discussion, that we bandlimit the measured residuals to 5 Hz. This probably encompasses all of the significant medium fluctuations and it includes orbital harmonics up to order 10^5. It seems safe to assume that this bandlimited signal contains all the information of interest. Now suppose that we sample the bandlimited signal at the Nyquist rate, 10 Hz. The sampling theorem shows that all information is preserved, but exact recovery of the bandlimited signal requires sinc interpolation. Whereas such interpolation cannot be implemented exactly, it is possible that significant errors will arise if only Nyquist-sampled data is transmitted. Oversampling might need to be considered; alternatively, methods of parameterizing the signal without sampling might be employed (such a method has been proposed by JPL [1]). To put it another way, the problem is not

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*The National Radio Astronomy Observatory (NRAO) is operated by Associated Universities, Inc., under a cooperative agreement with the National Science Foundation.

Typeset by $\mathcal{A}\mathcal{M}\mathcal{S}$-TEX

aliasing of high-frequency components, but rather dynamic range: interpolated corrections are needed to a precision of <1 mm (27 deg of phase at 22 GHz) when the uncorrected errors are as large as 1000 m, a ratio of 10^6.

A. Simple Quantitative Estimate

To get a rough quantitative idea of the possibilities, we now attempt to estimate the error introduced by a simple interpolation scheme under realistic conditions. For lack of any better data, consider the simulations on Radioastron orbit prediction presented at the 11th Radioastron Review Meeting [2]. These show, under certain assumptions, that predictions 4.0 to 5.0 days following the end of a 1-day, 2-station tracking session have an rms position error Δx with a peak value of 880 m and an rms velocity error Δv with a peak value of 0.35 m/s, although these do not occur at the same time. If we plot $2\Delta x/\Delta v$ vs. time, we get, for each point in the orbit, the amount of time it is likely to take for the position error to evolve from one end of its range to the other; we take this to be the characteristic time of the worst fluctuation. Next, since it is only the error in range that matters for time transfer, we take its rms uncertainty to be $\Delta x/\sqrt{3}$, and it peak uncertainty to be three times this. In this example, using the cited simulation, we find that the shortest characteristic time is 1140 sec, at which time the peak range error is 470 m. Using this, we suppose that the worst error component can be modeled as

$$\Delta R(t) = a \sin(2\pi t/T)$$

where $a = 470$ m and $T = 1140$ sec.

Now suppose that a measured signal of this form is sampled with period $\delta t = 0.1$ sec, and that the samples are simply *linearly* interpolated. The maximum interpolation error can be shown to be

$$\epsilon_{\mathrm{max}} = (3/8)a(2\pi\delta t/T)^2$$

which evaluates to 5.3×10^{-5} m. This is certainly small enough to be acceptable.

Of course, what we really need is a detailed understanding of the possible temporal evolution of the orbit prediction errors. The above is a simple-minded interpretation of available data, and it is not intended to be anything more that rough; it is hoped that the experts in this area will provide appropriate corrections.

C. onclusions

From simple arguments, it appears that the transmission of the time corrections derived from the two-way link can be accomplished as a time series of samples at a reasonable rate. Even very simple interpolation of the samples is adequate to represent the very large corrections needed for the orbit error because these errors should be slowly varying compared to the sample rate. The required sample rate is set by the faster fluctuations in the correction caused by the propogation medium, but fortunately these are small in magnitude, so that simple interpolation should be adequate for them also.

REFERENCES

[1] Annon., "Design Requirements: DSN Orbiting VLBI Subnet." JPL Document No. DM515606A, 16 May 1991, Appendix B.

[2] J. Ellis and J. Estefan, "NASA Navigation Support for Radioastron," paper presented at 11th Radioastron Review Meeting, Moscow, November 1990.