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**Typical Heat Loads in a Cryogenic Receiver Dewar and a
Discussion of the Refrigerator Principles of Operation**

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TYPICAL HEAT LOADS IN A CRYOGENIC RECEIVER DEWAR AND A
DISCUSSION OF THE REFRIGERATOR PRINCIPLES OF OPERATION

J.R. Fisher

An important factor in the design of the R.F. input lines for the Australian Synthesis Telescope (AST) receiver is the thermal loading which these lines will present to the 4.2 K refrigerator. The initial receiver design calls for two sets of maser-plus-three-upconverters in one dewar with at least four R.F. input lines and four maser and upconverter pump lines plus miscellaneous temperature monitor wires and switch shafts. Before going too far with the input waveguide and orthomode transducer (OMT) design I have taken a rough look at the thermal loading problems associated with this system. To follow is a summary of the relevant heat loading equations which may be of use in other receiver designs along with specific calculations of thermal loads which can be expected in the AST receiver. The last section is an outline of the physics involved in the refrigeration mechanisms of the CTI 15 K systems and the 4 K J-T loop.

1. SUMMARY OF EQUATIONS

1.1 Radiation heat transfer

The equations in this section come mainly from Chapter I of "An Introduction to Heat Transfer" by M. Fishenden and O.A. Saunders, Oxford Univ. Press, London, 1950.

Three factors influence the radiative transfer of heat

between two objects or between one object and its surroundings:
(a) the temperature of the two objects, (b) the geometry of the objects, and (c) the emissivity or absorptivity of the objects. The latter can be temperature dependent, but for the moment let us consider the case where the emissivity is independent of temperature. At a given temperature emissivity is always equal to absorptivity for radiation of the same equivalent spectral temperature.

The total energy radiated from an object is proportional to its temperature to the fourth power. Hence, the net energy transfer between an object and perfectly absorptive surroundings is

$$H = 5.46 \times 10^{-12} E_1 (T_1^4 - T_2^4) \quad \text{W/cm}^2, \quad (1)$$

where E_1 is the emissivity of the object, and T_1 and T_2 are the temperatures of the object and its surroundings, respectively. The unit area (cm^2) in the dimensions refers to the surface area of the object assuming it is everywhere convex.

A situation which is closer to reality is the transfer of radiation between two parallel plates which have emissivities E_1 and E_2 . Then

$$H = 5.46 \times 10^{-12} \left[\frac{E_1 E_2}{E_1 + E_2 - E_1 E_2} \right] (T_1^4 - T_2^4) \quad \text{W/cm}^2. \quad (2)$$

An interesting point here is that the emissivities are not multiplicative. Let the term in brackets be called the effective emissivity E_{eff} . Then if $E_2 = 1$, $E_{\text{eff}} = E_1$, but if $E_2 = E_1 \ll 1$, $E_{\text{eff}} = E_1/2$ not E_1^2 . When the emissivities are small and equal, radiation gets trapped between the walls and has an equal chance of being absorbed on either wall. If

the emissivities of the two walls are substantially different the effective emissivity is roughly equal to the lower one, hence, it does not help much to partially polish one wall if the other is already polished.

Two other geometries which are useful approximations of dewar construction are infinitely long coaxial cylinders and concentric spheres. If r_1 and r_2 are the radii of the inner and outer cylinders with diffusely reflecting surfaces the radiative heat transfer per unit area of the small cylinder will be

$$H = 5.46 \times 10^{-13} \left[\frac{E_1 E_2}{E_2 + E_1 (1 - E_2) r_1 / r_2} \right] (T_1^4 - T_2^4) \quad \text{W/cm}^2, \quad (3)$$

and if r_1 and r_2 are the radii of spheres the heat transfer is

$$H = 5.46 \times 10^{-13} \left[\frac{E_1 E_2}{E_2 + E_1 (1 - E_2) (r_1 / r_2)^2} \right] (T_1^4 - T_2^4) \quad \text{W/cm}^2. \quad (4)$$

Note that if $r_1 \approx r_2$ the latter two equations are nearly the same as eq. (2) as is to be expected.

These equations assume that both walls are relatively smooth and do not contain cavities whose walls will intercept some of the radiation from their own interior. If such cavities exist they can be treated as areas where the effective emissivity is higher across their aperture. An illustrative model is an internally emitting sphere with a hole through which the radiation can escape. The effective emissivity of the hole is

$$E' = \frac{E}{1 - s(1-E)/4\pi r^2} \quad (5)$$

where E is the emissivity of the interior surface, s is the

area of the spherical surface excluding the hole and r is the radius of the sphere. For example, if the cavity is a hemisphere and $E = 0.1$, E' will be 0.18 or nearly double. Of course if the emissivity is already near unity the cavity cannot make it greater than one.

Some typical emissivities of common substances are given in Table 1 copied from "Experimental Techniques in Low Temperature Physics", G.K. White, Oxford, Clarendon, 1968, p. 190. A more extensive list can be found in the "American Institute of Physics Handbook", D.W. Gray Ed., Third Edition, McGraw Hill Book Co., New York, 1972, p. 6-201. Keep in mind that these are total emissivities over specified temperature ranges. If the temperature dependence is important the more complex equations given in the reference at the beginning of this section must be used. However, in most cases the experimental uncertainties in the measurement of emissivities of practical surfaces will be much greater than any effects due to surface temperature as can be seen in Table 1.

A practical variation on the two-wall radiative heat transfer problem is one where there is a reflective intervening shield which is allowed to seek its own temperature according to radiative energy flow balance. This might be the case if a heat shield were wrapped with reflective mylar. Let us assume that both sides of the intervening shield have the same emissivity, E_2 and take a look at a couple of combinations of E_1 and E_3 (see Fig. 1).

The heat flow from wall 1 to layer 2 must equal the flow from layer 2 to wall 3 so

$$H_{12} = H_{23} \quad (6)$$

Table 1 from "Experimental Techniques in Low Temperature Physics", G.K. White, Oxford, Clarendon, 1968.

Experimental Values of Emissivity

Material	Fulk, Reynolds, & Park (1955) 300° K radn. on 78° K surface	McAdams (1954) room temp.	Ramanathan (1952) 1½ μ radn. on 2° K surface	Blackman, Egerton, & Truter (1948) 293° K radn. on 90° K surface	Ziegler & Cheung (1957) 273° K radn. on 77° K surface
Al-clean po- lished foil .	0.02	0.04	0.011†	0.065	0.043‡
Al-plate .	0.03
Al-highly oxidized .	..	0.31
Brass-clean polished .	0.029	0.03	0.018†	0.040	0.10‡
Brass-highly oxidized .	..	0.6
Cu-clean polished .	0.015-0.019	0.02	0.0002-0.015†	0.019-0.035	..
Cu-highly oxidized .	..	0.6
Cr-plate .	0.08	0.08	..	0.065	0.084‡
Au-foil .	0.010-0.023	0.02-0.03	..	0.026	..
Au-plate .	0.026
Monel .	..	0.2	0.11‡
Ni-polished .	..	0.045
Rh-plate .	0.078
Ag-plate .	0.008	0.02-0.03	..	0.023-0.030	..
Stainless steel .	0.048	0.074
Zn-clean foil	0.013	0.06	0.013†	0.038	..
Soft solder .	0.03	0.047‡
Glass .	..	0.9	..	0.87	..
Wood's metal	0.18

† These surfaces were electro-polished (Ramanathan).
‡ These surfaces were neither highly polished nor heavily oxidized, but as encountered in normal practice (Ziegler). Ziegler observed that a thin layer of oil or Apiezon grease on a low-emissivity surface raised the emissivity to 0.2 or 0.3. He also found that varnishes such as GEC adhesive no. 7031 and bakelite lacquer gave an emissivity $\epsilon \approx 0.87$; similarly, Scotch tape (Sellotape) had an emissivity of about 0.88.

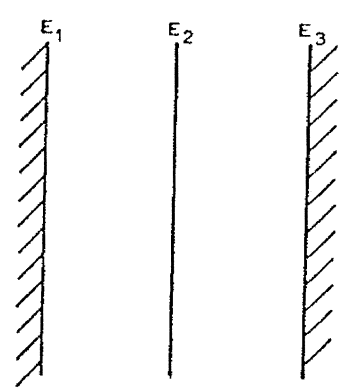


Fig. 1 - Geometry for radiation transfer between two walls with an intermediate shield.

or

$$\left[\frac{E_1 E_2}{E_1 + E_2 - E_1 E_2} \right] (T_1^4 - T_2^4) = \left[\frac{E_2 E_3}{E_2 + E_3 - E_2 E_3} \right] (T_2^4 - T_3^4) \quad (7)$$

suppose that $E_1 = E_3 = 1$, then

$$E_2 (T_1^4 - T_2^4) = E_2 (T_2^4 - T_3^4) , \quad (8)$$

$$T_2^4 = \frac{T_1^4 + T_3^4}{2} , \quad (9)$$

and

$$H_{12} = H_{23} = \frac{C E_2 (T_1^4 - T_3^4)}{2} \quad (10)$$

which is the same effect that would result if both walls had emissivity E_2 and the intervening layer were not there. The constant C is 5.46×10^{-13} when the units of H are W/cm^2 .

Another simple example might be that $E_3 = 1$ and

$E_1 = E_2 \ll 1$, then

$$\frac{E_2 (T_1^4 - T_2^4)}{2} = E_2 (T_2^4 - T_3^4) , \quad (11)$$

$$T_2^4 = \frac{T_1^4 + 2T_3^4}{3} , \quad (12)$$

and

$$H_{12} = H_{23} = \frac{C E_2 (T_1^4 - T_3^4)}{3} \quad (13)$$

which is 30% better than having two walls with emissivity E_2 without the intervening layer. The above examples are true if heat is transferred only by radiation. If the intervening layer were conductively shorted to one of the walls the heat transfer would be the same as if the intervening layer did not exist. The wall to which it is shorted would then have an effective emissivity equal to E_2 .

1.2 Conduction in a rarefied gas

A second avenue of heat transfer from the outside dewar walls to the refrigerator cold stations is conduction via gas in the dewar. In principle this form of conduction can be eliminated by evacuating the dewar, but in practice a perfect vacuum cannot be achieved, and it is useful to know how a good vacuum is necessary to make gas conduction negligible. Most of the equations in this section are found in C.A. Baily, "Advanced Cryogenics", Plenum Press, London, 1971, p. 140.

There are two regimes of heat conduction in gases. At relatively high pressures, say one atmosphere, the thermal conductivity of gases is nearly independent of pressure. As the pressure is increased the higher density of molecules which can carry thermal energy is balanced by the reduction in the mean free path of the molecules. At very low pressures, however, the mean free path can be much greater than the relevant dimensions of a vacuum container so the heat conduction is directly proportional to pressure. The mean free path between molecules (A. Sommerfeld "Thermodynamics and Statistical Mechanics", Academic Press, New York, 1950, p. 197) is given by

$$\ell = \frac{1}{n\pi s^2} \quad (14)$$

where

n is the number of molecules per unit volume, and

s is two times a molecular radius.

At 273 K and 1 atmosphere (760 torr)

$$n = 2.8 \times 10^{19}/\text{cm}^3, \text{ and}$$

$$s = 2 \times 10^{-8} \text{ cm for most diatomic molecules.}$$

Then if P is the pressure in torr

$$\ell = 2.16 \times 10^{-2}/P \quad \text{cm} . \quad (15)$$

Hence, if the distance between an outside dewar wall and a heat shield is 2 cm the transition from "high" to "low" pressure conduction regimes occurs at about 10^{-2} torr which is considerably higher than pressures commonly encountered in cryogenic receiver work.

The full expression for heat conduction through a low pressure gas between two walls is

$$H = \frac{a\bar{c}mC_V(T_2 - T_1)}{4} \quad \text{W/cm}^2 \quad (16)$$

where

a is an accommodation coefficient which states how efficiently energy is transferred from a molecule to a wall (typically 0.2 to 0.8),

\bar{c} is the mean molecular velocity,

m is the mass of one molecule, and

C_V is the specific heat at a constant volume.

If the constants are combined this equation becomes

$$H = .KaP(T_2 - T_1) \quad \text{W/cm}^2 \quad (17)$$

where

$K \approx 2.8 \times 10^{-2}$ for helium

$\approx 5.9 \times 10^{-2}$ for hydrogen

$\approx 1.6 \times 10^{-2}$ for nitrogen or air.

Again, P is in torr.

If the surface area of the outer 77 K heat shield is 2500 cm^2 (20 cm diameter \times 30 cm high) the conductive heat load from the outside dewar wall would be about 45 mW at a pressure of 10^{-5} torr ($a = 0.5$).

1.3 Conduction through solids

The equation for conductive heat flow per unit area across a surface in a solid is simply

$$H = k \frac{dT}{dx} \quad (18)$$

where x is measured perpendicular to the surface, and k is the conductivity of the solid. Conductivity is a quantity which depends in a rather complicated way on electron mobility and crystal structure in the solid and is usually a function of temperature. The conductivity of many common substances have been measured and are compiled in places like the "American Institute of Physics Handbook", D.W. Gray, Ed., Third Ed., McGraw-Hill Book Co., New York, 1972, p. 4-142.

Heat flow equations which assume that k is not a function of temperature are given in "An Introduction to Heat Transfer", M. Fishenden and O.A. Saunders, Oxford University Press, London, 1950, p. 39. For conduction through a plane slab of thickness d the heat transfer is

$$H = \frac{k}{d} (T_1 - T_2) , \quad (19)$$

and for a radial conduction through a cylinder of unit length

$$H = \frac{2k(T_1 - T_2)}{\ln(r_2/r_1)} \quad (20)$$

where r_1 and r_2 are the inner and outer radii of the cylinder, respectively.

However, at very low temperatures the temperature dependence of k must be taken into account so eqs. (19) and (20), respectively, become

$$H = \frac{1}{d} \int_{T_1}^{T_2} k(T) dT \quad (21)$$

$$H = \frac{2\pi}{\ln(r_2/r_1)} \int_{T_1}^{T_2} k(T) dT \quad (22)$$

Figure 2 shows the thermal conductivity of a number of copper

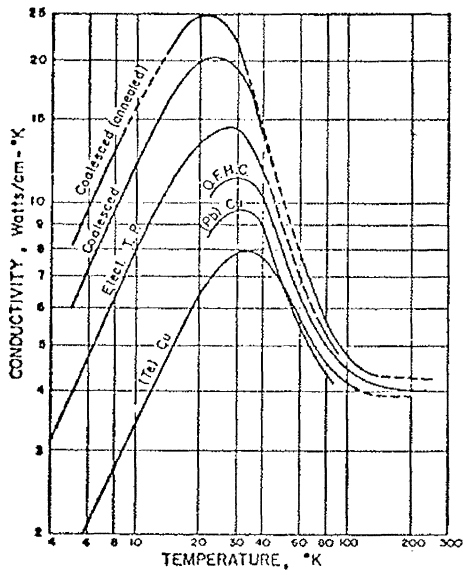


FIG. 2. Thermal conductivity of commercial coppers. The graph of the coalesced (annealed) copper is from reference 12; of O. F. H. C., from reference 10; of (1%) Cu, from reference 11. The dotted curves indicate interpolated region.

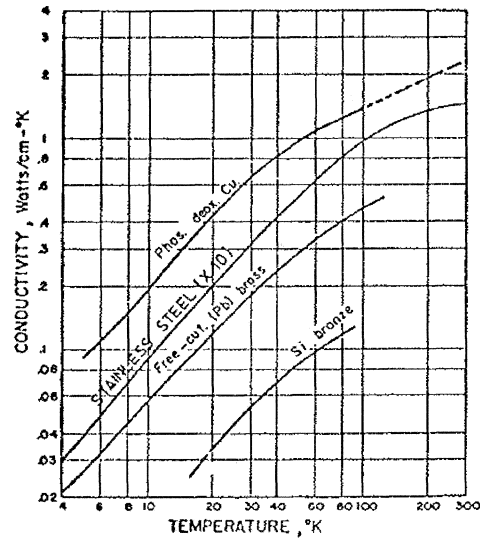


FIG. 6. Thermal conductivity of commercial copper alloys.

Fig. 2 - From Powell et al., Journal of Applied Physics, Vol. 28, p. 1282, 1957.

alloys, and it illustrates how the low temperature conductivity of a metal can be strongly affected by small amounts of impurities. The conductivity of nearly pure copper is about 100 W/cm K at both 4.2 K and 20 K. The highest thermal conductivity of any of the copper grades in Figure 2 is 25 W/cm K at 20 K and about 7 W/cm K at 4.2 K so even relatively pure commercial grades of copper have much lower conductivities

than pure copper.

Table 2 from the A.I.P. Handbook lists the thermal conductivities of a number of solid elements, alloys, and common compounds.

The rise in thermal conductivity at low temperatures of a metal such as copper produces two effects which must be taken into account when computing heat losses in a receiver dewar. First, the total heat load is higher than it would be if the conductivity were the same at all temperatures as it is at 300 K. For instance, if a uniform cross section electroformed tough-pitch copper line is tied between 300 K and 4.2 K stations the heat load will be about 40% higher than what one would calculate assuming the 300 K conductivity throughout. Second, the temperature distribution along a thermal conductor will be altered in the sense that a longer section will be held at low temperatures than would be the case if the conductivity were constant. The latter effect will be important if intermediate refrigerator stage heat sinks are to be tied to the input lines between 300 K and 4.2 K.

2. HEAT LOADS IN A TYPICAL MASER DEWAR

To get a feeling for where the greatest heat loads occur in a 4.2 K refrigerator/receiver system a typical dewar configuration will be analysed. The dewar outline shown in Figure 3 is roughly the same as the one used with the NRAO masers, one of which is being built at Radiophysics. Dimensions and compositions of the 22 GHz input and output waveguides and 50 GHz pump waveguide used in this system are shown in Figure 4.

Since the proposed AST receiver will have more and

Table 2 (continued)

TABLES 4g-II. THERMAL CONDUCTIVITY OF CRYSTALLINE DIELECTRICS AND OPTICAL MATERIALS (In watts/meter · kelvin)

Material	Ref.	T, K	Conduc-tivity	Material	Ref.	T, K	Conduc-tivity
AlF ₃	52	8	6.0	Glass (plastic perape).....	48	4.2	0.053
		10	3.7			20	0.074
		27	1.4			77	0.44
		77	0.31			184	0.88
AgCl.....	39	223	1.3	H ₂ (para + 0.5% ortho)	48	273	1.0
		323	1.2			2.5	100
		323	1.1			3.0	150
		373	1.1			4.0	200
Al ₂ O ₃ (sapphire) 38 deg to e axis.....	48	4.2	110	4.0	30	0.0	50
		20	3,500	7.0	3	10	3
Al ₂ O ₃ (sapphire) 1 to e axis.....	48, 72	55	6,000	H ₂ O (ice).....	72	273	3.3
		77	1,100			273	2.8
Al ₂ O ₃ (sintered).....	46, 48	373	2.6	Met.....	52	0.6	2
		523	3.9			1.0	25
		773	5.8			1.5	0.57
		773	5.5			2.0	0.21
BaF ₂	39	20	0.5	Met.....	52	0.6	42
		20	23			0.8	120
		104	48			2.0	0.18
		273	35			300	0.45
BaO.....	40, 48	77	150	I.....	62	325	0.42
		373	28			350	0.40
		573	8			2	170
		573	0.21			4.2	380
CaF ₂	39	373	0.27	KBr.....	39, 75	100	12
		225	20			100	17
		260	13.4			273	5.0
		305	10.9			323	4.8
CaCO ₃ 1 to e axis.....	49	370	10.5	KCl.....	39, 75	373	4.8
		4.2	0.3			4.2	500
		27	10			25	140
		77	3,400			50	35
CaCO ₃ 1 to e axis.....	72	77	210	K ₂	62	194	10
		191	870			273	3.1
		273	680			4.2	0.49
		83	25			77	0.2
CaCO ₃ 1 to e axis.....	72	194	6.5	LiF.....	5, 72	77	6.55
		273	4.6			20	1,800
		83	17			77	180
		373	3.6			77	190
CaF ₂	39	83	39	MgO-Al ₂ O ₃ (spinel).....	48, 68	373	13
		273	10			4.2	0.25
		323	9.2			40	45
		373	9.0			120	8.0
CaF ₂	39	223	1.2	MgO.....	39, 77	573	3.5
		273	0.94			4.2	440
		323	0.81			20	300
		373	0.77			77	30
CaI ₂	39	273	1.4	NaCl.....	39, 77	273	6.4
		273	1.2			323	5.6
		323	1.0			373	5.4
		373	0.95			5	1,100
Glass (pburcit).....	40	4.2	0.085	NaF.....	69, 75	50	250
		20	0.13			100	90
		77	0.37				

TABLE 4g-II. THERMAL CONDUCTIVITY OF CRYSTALLINE DIELECTRICS AND OPTICAL MATERIALS (Continued) (In watts/meter · kelvin)

Material	Ref.	T, K	Conduc-tivity	Material	Ref.	T, K	Conduc-tivity
Na.....	52	2	3.0	SiO ₂ (quartz) 1 to e axis.....	72	20	370
		3	4.8			194	10
		4.2	4.3			273	8.3
		10	0.8			4.2	0.25
NH ₄ Cl.....	40	20	0.7	SiO ₂ (fused).....	72	20	0.7
		177	17			77	0.8
		177	13			194	1.2
		273	29			273	1.4
NH ₄ H ₂ PO ₄ 1 to optic axis.....	72	315	0.71	TiO ₂	72	373	1.6
		339	0.71			873	1.8
NiO.....	72	342	1.34	TiO ₂ (rutile) 1 to optic axis.....	72	319	0.59
		4.2	5.9			311	0.75
		40	406			4.2	500
		104	83			273	13
SiO ₂ (quartz) 1 to e axis.....	72	20	730	TiO ₂ (rutile) 1 to optic axis.....	72	4.2	100
		194	20			20	69
		273	12			273	9

Various authors give from 5 to 25% on the experimental results for the conductivities of crystalline dielectric and optical materials. See ref. 40 for additional data.

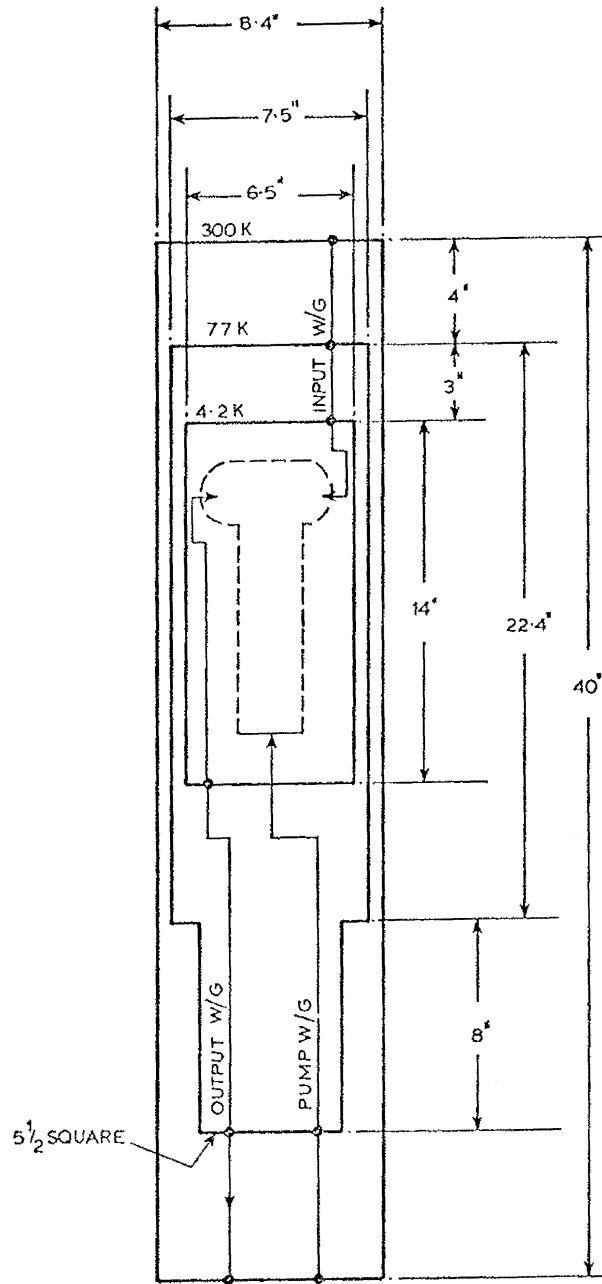


Fig. 3 - Dimensions of the 22 GHz maser dewar and heat shields.

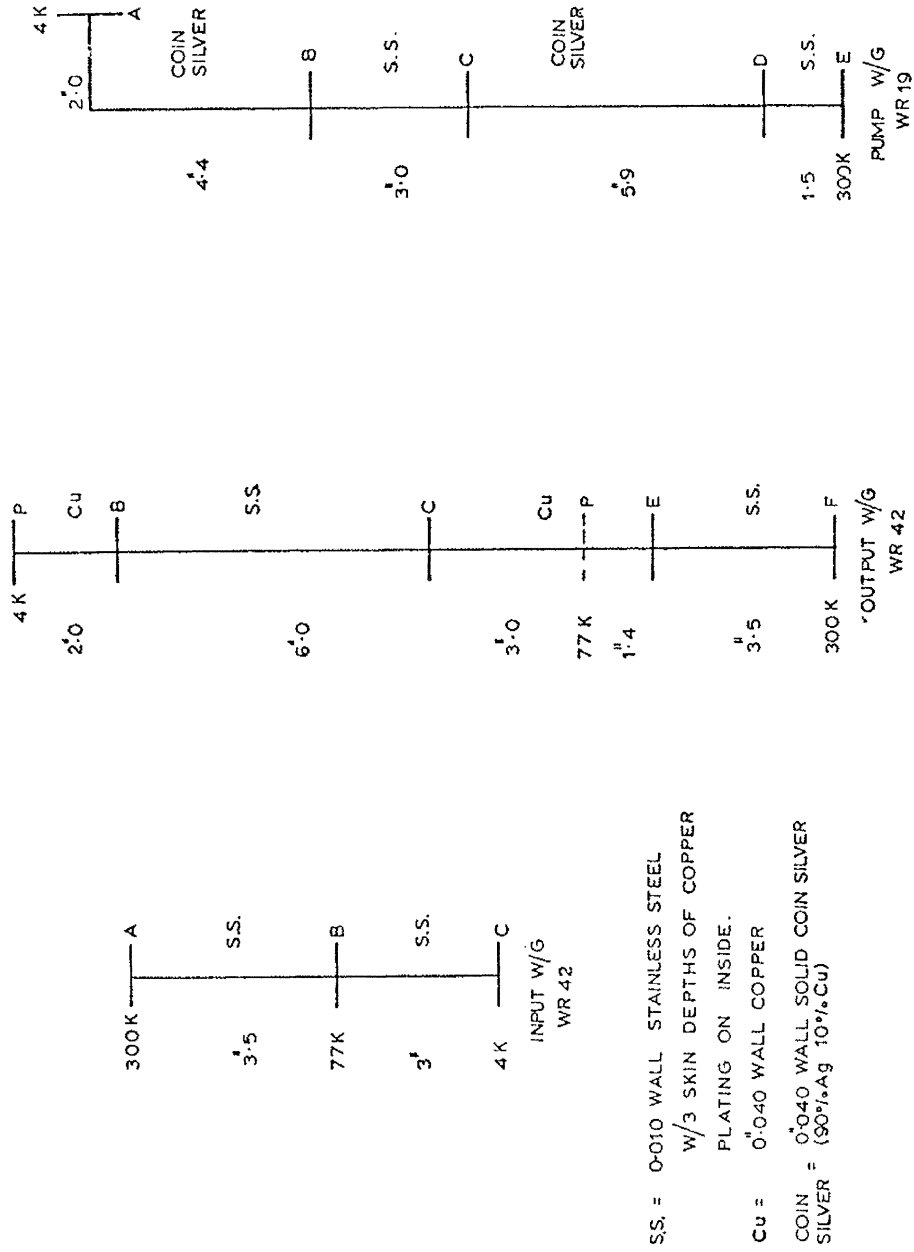


Fig. 4 - Dimensions and compositions of the input, output, and pump waveguides in the 22 GHz maser system shown in Fig. 3.

larger input and upconverter pump waveguides than the single maser system, calculations for L, C, and X-band waveguides are included later in this section along with a look at the use of waveguide gaps to help reduce the input waveguide heat loads.

2.1 Radiation loading

The largest uncertainty in computing the radiation heat transfer from the heat shields is the uncertainty in the emissivity of the shield surfaces. Both the 4.2K and 77 K heat shields are solid copper. Table 1 shows that the emissivity of copper ranges from 0.02 to about 0.5 depending on how well it is polished or how heavily oxidized it is. For the present let us assume that the copper emissivity is equal to 0.05 on all surfaces. The outside dewar wall is made of stainless steel which even when it is polished has an emissivity of 0.15 or higher so it will not help much in cutting down on the radiative load. For these calculations let us assume the emissivity of stainless steel is 0.2.

2.1.1 77 K heat shield load

It will be seen later that the amount of heat drawn from the 77 K heat shield by the 4.2 K shield is negligible so only the heat transfer from the outer dewar wall need be considered in this subsection.

If we think of the dewar in Figure 3 as having its long dimension vertical, the horizontal surfaces of the 77 K shield can be considered to be looking into shallow cavities whose effective emittance is about 50% higher ($E = 0.3$) than that of the stainless steel walls. The total area of the top and bottom surfaces of the 77 K heat shield is 570 cm^2 so the heat load is

$$H_1 = (570) (5.46 \times 10^{-12}) \frac{(.30)(.05)}{.30 + .05 - (.30)(.05)} (300^4 - 77^4)$$

$$= 112 \text{ mW .}$$

The sides of the 77 K shield can be analysed as two coaxial cylinder sections with a radius picked for the bottom square section such that the equivalent surface area of the cylinder is equal to that for the box. For the top section

$$H_2 = (3405 \text{ cm}^2) (5.46 \times 10^{-12}) \frac{(.20)(.05)}{.20 + .05 - (.20)(.05) (7.5/8.4)} (300^4 - 77^4)$$

$$= \overset{6220}{\cancel{722}} \text{ mW ,}$$

and for the bottom section

$$H_3 = (1135 \text{ cm}^2) (5.46 \times 10^{-12}) \frac{(.20)(.05)}{.20 + .05 - (.20)(.05) (7.0/8.4)} (300^4 - 77^4)$$

$$= \overset{2070}{\cancel{207}} \text{ mW .}$$

The total radiative heat load on the 77 K shield is then

$$H = H_1 + H_2 + H_3$$

$$= \overset{9410}{\cancel{941}} \text{ mW .}$$

This illustrates the value of the 77 K heat shield since without it this load would be imposed on the 4.2 K stage.

2.1.2 4.2 K heat shield load

To calculate the radiative transfer of heat from the 77 K to the 4.2 K heat shield let us assume that the bottom end of the 4.2 K shield is looking into a cavity with an effective emissivity of three times the wall emissivity (3×0.05) and that the top end sees a cavity of 50% higher emissivity (1.5×0.05). The inside of the 77 K shield might

be considered to be a good specular reflector instead of a diffuse reflector so that eq. (2) should be used instead of eq. (3) in computing the side wall radiation, but both equations will give nearly the same answer. For the top end

$$H_4 = (214 \text{ cm}^2) (5.46 \times 10^{-13}) \frac{(.05)(.075)}{(.05) + (.075) - (.05)(.075)} (77^4 - 4.2^4)$$

$$= \overset{1.3}{\cancel{0.13}} \text{ mW} .$$

For the bottom end

$$H_5 = (214 \text{ cm}^2) (5.46 \times 10^{-13}) \frac{(.05)(.15)}{.05 + .15 - (.05)(.15)} (77^4 - 4.2^4)$$

$$= \overset{1.6}{\cancel{0.16}} \text{ mW} ,$$

and for the sides

$$H_6 = (726 \text{ cm}^2) (5.46 \times 10^{-13}) \frac{(.05)(.05)}{.05 + .05 - (.05)(.05)} (77^4 - 4.2^4)$$

$$= \overset{3.6}{\cancel{0.36}} \text{ mW} .$$

The total heat transfer from the 77 K to 4.2 K shields is

$$H = H_4 + H_5 + H_6$$

$$= \overset{6.5}{\cancel{0.65}} \text{ mW}$$

which is quite small compared to other loads on the 4.2 K stage.

2.2 Gas conduction loading

The largest uncertainties in the gas conduction load calculations are in the accommodation coefficient and in the gas pressure in the dewar. The latter depends on the condition of the dewar seals and the efficiency of the vac-ion pump. For a start let us take a pressure of 10^{-5} torr, and the heat

loads can be scaled to other pressures later.

2.2.1 77 K heat shield to 300 K wall

The area associated with the calculation of heat transfer in eq. (17) is the outside surface area of the 77 K heat shield which is 5110 cm². If we assume an accommodation coefficient of 0.5

$$H = (2.8 \times 10^{-2})(0.5)(10^{-5})(300-77)(5110)$$

$$= 160 \text{ mW for helium gas, and}$$

$$H = (1.6 \times 10^{-12})(0.5)(10^{-5})(300-77)(5110)$$

$$= 91 \text{ mW for air.}$$

2.2.2 4.2 K to 77 K heat shields

The total surface area of the 4.2 K heat shield is 2272 cm² so

$$H = (2.8 \times 10^{-2})(0.5)(10^{-5})(77-4.2)(2272)$$

$$= 23 \text{ mW for helium (or 13 mW for air).}$$

If the pressure rises much above 10⁻⁵ torr gas conduction heat loading will become a problem.

2.3 Conduction heat loading through waveguides

A number of quantities which will be useful in computing thermal conductivity in waveguides are listed in Tables 3 and 4. Table 3 gives the thicknesses of copper and other materials which are required to provide three skin depths for RF conduction at a number of frequencies. To aid the evaluation of the integral in eq. (21) Table 4 lists the product of thermal conductivity and temperature interval (k(T)dT) for a number of copper alloys and for stainless steel between 4.2 and 300 K using the curves in Figure 2.

Table 3. Material thickness to provide three skin depths of several waveguide metals (μ metres)

Frequency (Ghz)	Copper	Gold	Silver	Brass		Aluminium
				65.8 Cu	34.2 Zn	
1.4	5.29	6.01	5.13	10.10		6.82
4.4	2.98	3.39	2.89	5.70		3.84
8.6	2.14	2.43	2.07	4.08		2.75
22.0	1.33	1.52	1.29	2.55		1.72
40.0	0.99	1.13	0.96	1.89		1.28

If waveguides of different wall thicknesses or of different metals are tied in series between two thermal potentials the heat loss equations must be solved iteratively because of the temperature dependence of the conductivity. For thermal conductors in series the thermal resistance concept is more useful. If we take the case which is typical in plated waveguides where two or more different metals are thermally tied in parallel we can define the resistance of such a composite conductor as

$$R = \frac{d(T_2 - T_1)}{A_1 \int_{T_1}^{T_2} k_1(T) dT + A_2 \int_{T_1}^{T_2} k_2(T) dT + \dots} \quad K/W \quad (23)$$

where T_1 and T_2 are the end point temperatures of the section, d is the length of the conductor and A_n is the cross sectional area of conductor n . To start an iterative solution of a number of conductors in series assume that the temperature distribution is a linear function of length to get the end point temperatures of each section of waveguide. Then compute the thermal resistance of each section from eq. (23). From this

Table 4. Thermal conductivity-temperature interval product for a number of waveguide metals.
 $\overline{k(T)}\Delta T$ (W/cm)

Temp. Range (K)	ΔT (K)	Elect. T.P. Copper	O.F.H.C.	(Te)Cu	Free-cut Brass	Stainless Steel
4.2-5	0.8	2.9	1.8	1.0	0.018	0.0026
5-6	1	4.3	2.7	1.9	0.029	0.0039
6-8	2	11.0	7.0	4.6	0.076	0.0106
8-10	2	14.4	8.8	6.2	0.092	0.015
10-12	2	17.4	11.0	7.6	0.126	0.019
12-14	2	20.0	13.0	8.8	0.15	0.023
14-16	2	23.0	14.6	9.4	0.165	0.026
16-18	2	25.0	16.4	10.6	0.19	0.031
18-20	2	26.0	18.0	12.2	0.22	0.036
20-25	5	70.0	52.0	35.0	0.70	0.110
25-30	5	72.5	56.0	38.0	0.85	0.130
30-35	5	67.5	56.0	40.0	1.00	0.165
35-40	5	63.0	54.0	39.0	1.08	0.19
40-50	10	100.0	90.0	71.0	2.70	0.48
50-60	10	80.0	72.0	63.0	3.10	0.58
60-70	10	63.0	58.0	53.0	3.50	0.68
70-77	7	40.0	38.0	34.0	2.60	0.52
77-80	3	16.5	15.5	14.0	1.20	0.24
80-90	10	50.0	48.0	44.0	4.20	0.85
90-100	10	47.0	46.0	42.0	4.30	0.90
100-120	20	90.0	86.0	80.0	9.80	2.00
120-150	30	129.0	125.0	117.0	16.20	3.45
150-200	50	210.0	205.0	195.0	31.0	6.25
200-250	50	205.0	200.0	195.0	42.0	6.60
250-300	50	205.0	200.0	195.0	50.0	6.80
		<u>Total Integrals</u>				
4.2-20	15.8	144.0	93.3	62.3	1.066	0.167
4.2-77	72.8	700.0	569.3	435.3	16.60	3.02
4.2-300	295.8	1652.5	1494.8	1317.3	175.30	30.11
20-77	57	556.0	476.0	373.0	15.53	2.85
20-300	280	1508.5	1401.5	1255.0	174.29	29.94
77-300	223	952.5	925.5	882.0	158.76	27.09

recompute the end point temperatures of each section, and repeat the thermal resistance calculation. Of course, if all waveguides in series are of the same composition the first thermal resistance solution is exact.

As an example of the procedure let's take a detailed look at the pump waveguide section in Figure 4.

2.3.1 Pump waveguide heat load

From Figure 4 it can be seen that the pump waveguide combination has only two fixed thermal potentials: 4.2 K and 300 K. There are four sections of waveguide with three intermediate joints. In the stainless steel waveguide the cross-sectional area of steel is $3.64 \times 10^{-2} \text{ cm}^2$, and the ideal cross-sectional area of copper is $1.42 \times 10^{-4} \text{ cm}^2$. The cross-sectional area of coin silver is $1.46 \times 10^{-1} \text{ cm}^2$.

The first real problem comes in deciding what the thermal conductivity properties of the metals are. Stainless steel seems to be well known so we can use the numbers in Table 4. The exact composition of the electroplated copper on the inside of the stainless steel waveguide is not known, but my guess is that it is similar to the electroformed tough-pitch copper in Table 4. The composition of coin silver is roughly 90% silver, 10% copper and 0.06% zinc. I have not found any thermal conductivity data on this material so again I can make only a guess. Since it is an alloy of a relatively high percentage of second metal its thermal conductivity probably starts at 300 K at roughly the conductivity of copper and silver (4 W/cm K) and monotonically decreases with decreasing temperature. As a model let us take the stainless steel curve and multiply its conductivities by 30 to bring the 300 K end up to the conductivity of silver. Hence.

all of the values in the last column can be multiplied by 30 for use with coin silver. Keep in mind that this is a guess!

Table 5. Joint temperatures and section thermal resistances as a function of iteration number for the pump waveguide in Fig. 4.

Iteration number	T _B	T _C (K)	T _D (K)	R _{AB} (K/W)	R _{BC} (K/W)	R _{CD} (K/W)	R _{DE} (K/W)	ΣR
1	112.7	165.5	269.4	64.1	1739.	26.2	689.	2518
2	11.7	216.0	219.1	577.0	1808.	25.9	696.	3107
3	59.1	231.3	233.7	115.3	1630.	25.9	693.	2437
4	18.0	213.7	216.8	391.1	1782.	25.9	696.	2895
5	44.2	226.2	228.9	154.1	1680.	25.9	694.	2554
6	22.0	216.6	219.6	313.2	1761	25.9	696.	2796
7	37.3	223.6	226.4	186.3	1703.	25.9	694.	2609
8	31.1	220.8	223.6	235	1725.	25.9	695.	2681

Table 5 summarizes the values derived for the temperatures at the waveguide joints and the thermal resistances of the individual sections at each iteration. R_{AB}, for instance, is the resistance of the section between joints A and B. The values for R_{AB} and T_B converged rather slowly so in iteration 8 these values were forced to convergence by taking the midpoint between step 6 and the average of steps 5 and 7.

The total resistance is 2681 K/W across the 295.8 K temperature differential so the heat load on the 4.2 K stage is 110 mW. In sections BC and DE the very thin layer of copper required at 40 GHz caused most of the heat to flow in the stainless steel (83% in BC and 89% in DE).

2.3.2 Output waveguide heat load

The 22 GHz output waveguide must be analysed in two parts since there are three fixed thermal potentials. The composition of the two waveguide types are O.F.H.C. in the solid copper guide, and we shall assume electrolytic tough-pitch plated on to stainless steel. Lengths are given in Figure 4. The cross-sectional area of the copper waveguide is $3.46 \times 10^{-1} \text{ cm}^2$, and in the stainless steel waveguide the steel cross-section is $7.87 \times 10^{-2} \text{ cm}^2$, and the copper cross-section is $4.0 \times 10^{-4} \text{ cm}^2$.

Tables 6 and 7 are tabulations of the iterative calculations for the 4.2 K to 77 K and 77 K to 300 K waveguide sections, respectively. The heat transfer from the 77 K stage to the 4.2 K stage via this waveguide is 34 mW, and from 300 K to the 77 K stage it is 282 mW. The net load on the 77 K stage is then 248 mW. In section BC 46% of the heat is carried in the steel while in section EF 85% of the conduction is in the steel.

Table 6. Joint temperatures and section thermal resistances as a function of iteration number for the 4.2 to 77 K section of output waveguide in Fig. 4.

Iteration number	T_B (K)	T_C (K)	R_{AB} (K/W)	R_{BC} (K/W)	R_{CD} (K/W)	ΣR (K/W)
1	17.4	57.1	2.75	1977.9	3.75	1984
2	4.3	76.9	6.52	2137.2	4.06	2148
3	4.4	76.9	6.53	2138.2	4.06	2149

Table 7. Joint temperatures and section thermal resistances as a function of iteration number for the 77 to 300 K section of output waveguide in Fig. 4.

Iteration number	T_E (K)	R_{DE} (K/W)	R_{EF} (K/W)	ΣR (K/W)
1	140.7	2.32	747.5	750
2	77.7	1.99	788.4	790
3	77.6	1.99	788.4	790

2.3.3 Input waveguide heat load

Calculation of the heat load through the 22 GHz input waveguide is straightforward since only one type of waveguide is involved. The copper plated on to the stainless steel has a cross-sectional area of $4.0 \times 10^{-4} \text{ cm}^2$ and the steel cross-section is $7.87 \times 10^{-2} \text{ cm}^2$.

The heat load on the 4.2 K station from the 77 K stage is 68 mW (46% carried in the steel), and the load from 300 K to the 77 K stage is 283 mW (net 215 mW with 85% carried in the steel).

2.4 Heat load summary

The heat loads on the 77 K and 4.2 K refrigerator stages of the maser system in Figure 3 are summarized in Table 8. A refinement on these calculations would be to compute the heat load on the 4.2 K stage assuming that its temperature is at the point where the refrigerator capacity is lowest to make sure that this stage can get cold. The various uncertainties which go into the above calculation make this refinement unwarranted, however.

Table 8. Heat loads on the refrigerator system shown in Fig. 3 excluding electronic power dissipation.

Source	Load	
	77 K (mW)	4.2 K (mW)
1. Radiation (emissivity = 0.05 on copper surfaces)	941 941.0	0.65 6.5
2. Gas conduction ($P = 10^{-7}$ torr, $a = 0.5$ with helium gas)	1.4	0.23
3. Pump waveguide	-	110.
4. Output waveguide	248.	34.
5. Input waveguide	215.	68.
	1405 7874	213 320

3. OTHER HEAT LOADS OF POSSIBLE INTEREST

3.1 Input waveguides or coax at lower frequencies

The AST receiver will have many more sources of heat load. The necessity of keeping the total load within the limits of the refrigerator may influence the design of some of the RF components so Table 9 has been compiled to indicate the heat loads that can be expected for a number of coaxial cables and waveguides that might be used in this receiver. To compute the heat load of a particular line divide the tabular value by the line length in centimetres.

It is clear that straight waveguide runs to the 4.2 K stage from either the dewar walls or the 77 K stage are not going to be feasible at C or L-Band. A 10 cm run of round C-band stainless steel waveguide from 77 K to 4.2 K would produce a 430 mW load on the 4.2 K stage. The stainless steel 0".141 semi-rigid coax might be worth considering to connect the upconverters at 4.2 K to waveguide to coax transitions

Table 9. Thermal heat loads of a number of RF transmission media that might be used in the AST receiver

Material	Heat load (mW cm)*		
	4.2-77 K	77-300 K	4.2-300 K
0".141 O.D. stainless steel coax with 0".012 thick outer wall and 0".0359 diameter copper clad steel inner conductor.	412(38)	1,770(8)	2,170(72)
Same as above except O.F.H.C. outer conductor.	18,950	31,200	50,200
7/8" air filled coax with 0".010 stainless steel inner and outer tubes with three skin depths of copper at L-Band (5.3 μ m)	4,480(17)	11,900(58)	16,400(47)
Round waveguide of 0".010 stainless steel with three skin depths of copper.			
K-Band 0.95 cm dia.	507(45)	2,430(84)	2,940(78)
X-Band 2.4 cm dia.	1,710(34)	6,690(77)	8,440(68)
C-Band 4.8 cm dia.	4,300(27)	14,700(71)	19,000(61)
L-Band 15.0 cm dia.	21,100(17)	56,200(58)	77,300(47)
Standard rectangular waveguide of 0".010 stainless steel wall with three skin depths of copper.			
K-Band WR-42	509(45)	2,440(84)	2,950(78)
X-Band WR-90	1,490(34)	5,890(77)	7,390(68)
C-Band WR-187	3,850(27)	13,100(71)	16,900(61)
L-Band WR-650	22,100(17)	59,100(58)	81,200(47)

*Numbers in () are percentages of heat carried in the steel.

held at 77 K. Ten centimetres of this coax would have a loss of about 0.07 dB at L-band and 0.12 dB at C-band which would result in input temperature increases of about 1 K and 2 K respectively. These losses are computed from room temperature conductivities of copper and stainless steel so they are probably upper limits.

3.2 Waveguide gaps as insulators

One way of reducing the heat conduction in a waveguide is to put a transverse gap in the guide wall which is too small to affect the RF properties of the waveguide but stops heat conduction along the metal walls. The only forms of heat transfer directly across the gap are radiation and gas conduction both of which are independent of gap spacing but are directly proportional to the gap face area. Radiation and gas conduction also takes place between the interiors of the two waveguide sections which are at different temperatures.

A rough estimate of the gap face area in circular waveguide can be made by realizing that the longitudinal currents on the waveguide walls must be carried by a series capacitor whose impedance should be small compared to the waveguide impedance ($\sim 500 \Omega$). Figure 5 shows the gap geometry. We shall assume that $W \ll \lambda/4$ so the phase across the gap is essentially constant. If we take the effective length of the capacitor to

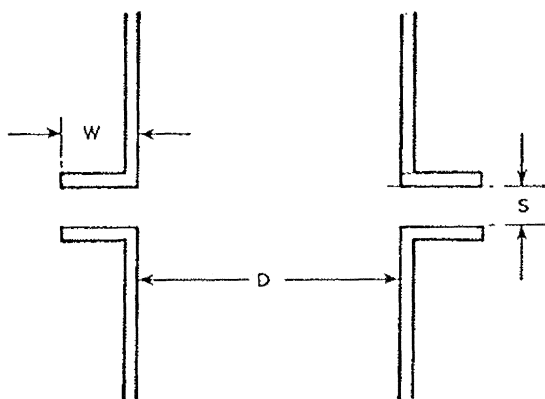


Fig. 5 - Geometry of a thermally insulating waveguide gap.

be $1/\pi$ of the waveguide circumference then the capacitance will be

$$C = \frac{0.0885 DW}{s} \quad \text{pF} \quad (24)$$

where D, W and s are in centimetres. The capacitive reactance is

$$X = \frac{1}{2\pi fc} = \frac{159}{F C} \quad (25)$$

where F is in GHz and C in pF. Solving for W from eqs. (24) and (25) we get

$$W = \frac{1.8 \times 10^3 s}{DFX} . \quad (26)$$

The product DF will be roughly constant (~ 20) so

$$W \approx \frac{905}{X} . \quad (27)$$

If X is required to be $< 10 \Omega$ and the gap spacing, $s = 0.5 \text{ mm}$, then

$$W \approx 0.45 \text{ cm} .$$

This width meets the $W \ll \lambda/4$ requirement at 1.4 and maybe 4.4 GHz but not at 8.6 GHz where the gap spacing and width would have to be made smaller.

The total face area of the 0.45 cm wide gap which can pass heat by radiation or gas conduction is $0.45\pi D$ which is 21 cm^2 at L-Band and 6.8 cm^2 at C-Band. Assuming the emissivity of both faces to be 0.05 and the accommodation

coefficient to be 0.5 the heat transfer across the gap will be as given in Table 10. The heat transfer is quite small so the gap is a good insulator, however, heat transfer will also take place between the waveguide interiors so this, too, must be estimated.

Table 10. Heat transfer across a 0.45 cm wide flange gap in circular waveguide (mW).

		L-Band (D = 15 cm) 4.2-77 K 77-300 K		C-Band (D = 4.8 cm) 4.2-77 K 77-300 K	
Gas conduction (Helium at 10^{-5} torr)	Across gap	0.21	0.65	0.069	0.21
	Through aperture	2.0	11.0	.37	1.13
Gas conduction (Air at 10^{-5} torr)	Across gap	0.12	0.37	0.040	0.12
	Through aperture	2.1	6.3	.21	.64
Radiation	Across gap (E = 0.05)	0.01	2.3	0.0033	0.75
	Through aperture		2.3	.033	7.5
	(E' = 0.15)	2.7 0.27	6.3 63.1	.28 0.028	6.5 6.5
	Total (Helium)	4.1	77.1	0.47	8.6
		2.6	66.6	.75	7.9

If we know the emissivity and accommodation coefficients of the interiors of the waveguides the only factors affecting the heat transfer are the diameter and depths of the guides. For the sake of estimation assume that the waveguide lengths are such that the effective emissivity of the apertures of diameter, D, are three times the emissivity of the interior metal, and assume that the effective accommodation coefficient is unity. Taking the interior metal emissivity to be 0.05, eq. (2) will give a radiative heat transfer of

$$H = 5.46 \times 10^{-\cancel{12}^{13}} \left[\frac{(0.15)(0.15)}{(0.15)+(0.15)-(0.15)^2} \right] (T_1^4 - T_2^4) \quad \text{W/cm}^2$$
$$= 4.43 \times 10^{-\cancel{14}^{13}} (T_1^4 - T_2^4) \quad \text{W/cm}^2 .$$

With a 15 cm diameter waveguide (L-Band) and the opposite gap faces at 4.2 K and 300 K the radiated power across the aperture will be 63 mW. Radiation heat loads for other parameters are given in Table 10.

Gas conduction through the aperture is computed from eq. (17) with $a = 1.0$ and $P = 10^{-5}$ torr.

$$H = 2.8 \times 10^{-7} (T_2 - T_1) \quad \text{W/cm}^2 \quad (\text{helium})$$
$$= 1.6 \times 10^{-7} (T_2 - T_1) \quad \text{W/cm}^2 \quad (\text{air}) .$$

A temperature differential of 295.8 K will produce a heat load of 14.6 mW with helium filling the 15 cm aperture. Again, see Table 10 for other conditions. Heat transfer is far greater through the aperture than across the gap although the heat loads are much less than they would be if the gap did not exist.

4. THE 4 K REFRIGERATOR

Since we are computing heat loads on the refrigerator it might be interesting to take a look at how the refrigerator works and see what limits its capacity. It is important to realize that the 4 K system consists of two distinct refrigeration processes even though both use helium gas.

4.1 The CTI Cryostat

Many refrigerators work on the principle that work is done on a gas by compressing it while it is in thermal contact with a good heat sink at, say, room temperature and letting the gas do work by expanding while it is in contact with an object to be cooled. Work and heat are both forms of energy so one can be traded for the other.

Figure 6 is a simplified schematic diagram of one stage of the Cryogenic Technology Inc. (CTI) refrigerator.

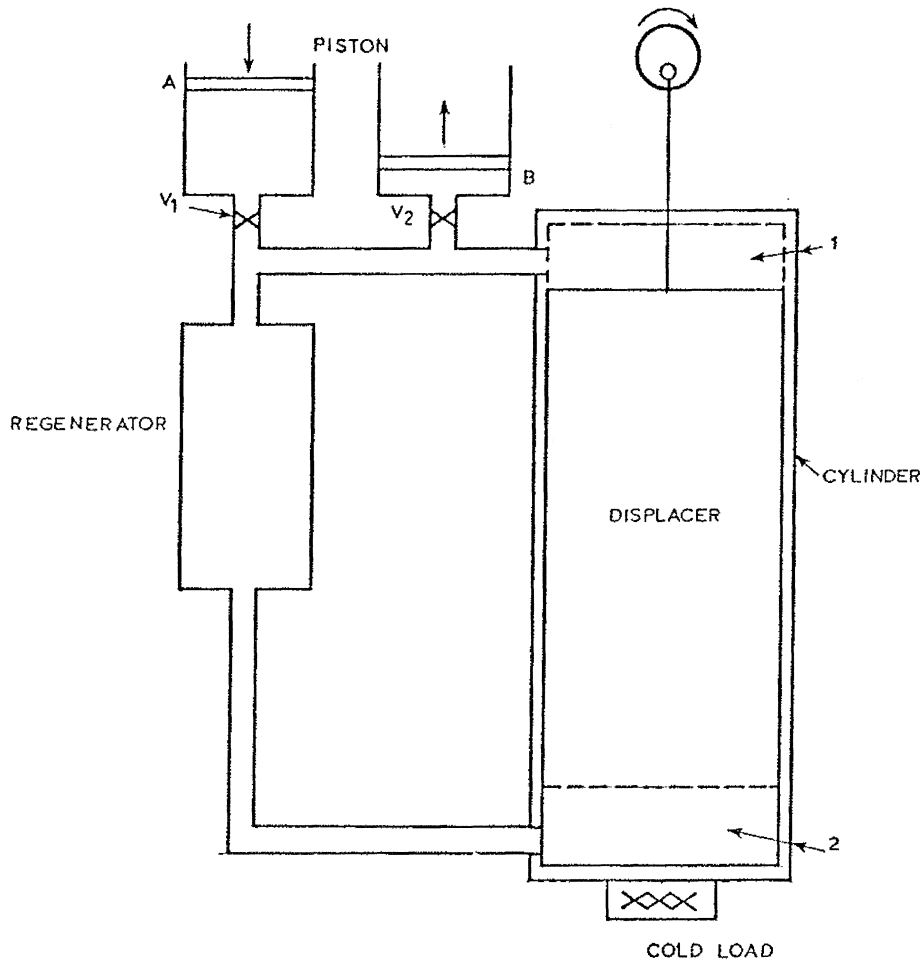


Fig. 6 - Schematic diagram of one stage of the CTI Cryostat.

Pistons A and B do not actually exist in the refrigerator but they are shown here to show where work is put in and given up. The refrigerator is constructed so that there is very little heat conduction along the walls or in the displacer. The regenerator is a device which has a large heat capacity, but a very poor longitudinal conduction, very little gas volume, and very low resistance to gas flow. Assuming that everything in the system starts out at room temperature let us follow the history of the gas through the first displacer cycle.

Initially the displacer is at the bottom of the cylinder, piston A is high and B is low, and valve V_1 is open. The first thing that happens is that work is added to the system by pushing piston A down forcing gas into chamber 1 and converting the work into heat in the gas. The gas, now above room temperature, is then moved through the regenerator by pulling the displacer to the top of the cylinder. Because of the large heat capacity of the regenerator the gas emerges at the bottom of the refrigerator again at room temperature but at a higher pressure since piston A continues to be forced down. Now valve V_1 is closed and V_2 is opened, and work is done on piston B by the high pressure gas in chamber 2. This expenditure of work comes at the expense of heat in the gas so the cold load and the bottom of the regenerator are cooled and the gas emerges at the top of the regenerator at slightly higher than room temperature since it will collect the heat put in on the way down. Finally the displacer is pushed down to expell the rest of the gas in chamber 2, and then valve V_2 is closed. Our little piece of the world has been temporarily thrown out of equilibrium by making one end of the cylinder colder than the other. It can be pushed even further out by repeating the cycle faster than heat can

flow back through the cylinder or through equipment attached to the cold end.

The CTI system, as well as being a two-stage device, is more clever than the one shown in Figure 6. Figure 7 is a more accurate picture of the real cryostat. In the CTI machine the regenerator is inside the displacer and a compressor replaces the pistons A and B in Figure 6. With the external compressor most of the heat added by compression of the gas is extracted before it enters the cylinder, and the heat generated in compression of the gas already in chamber 1 is carried out by the exhaust gas.

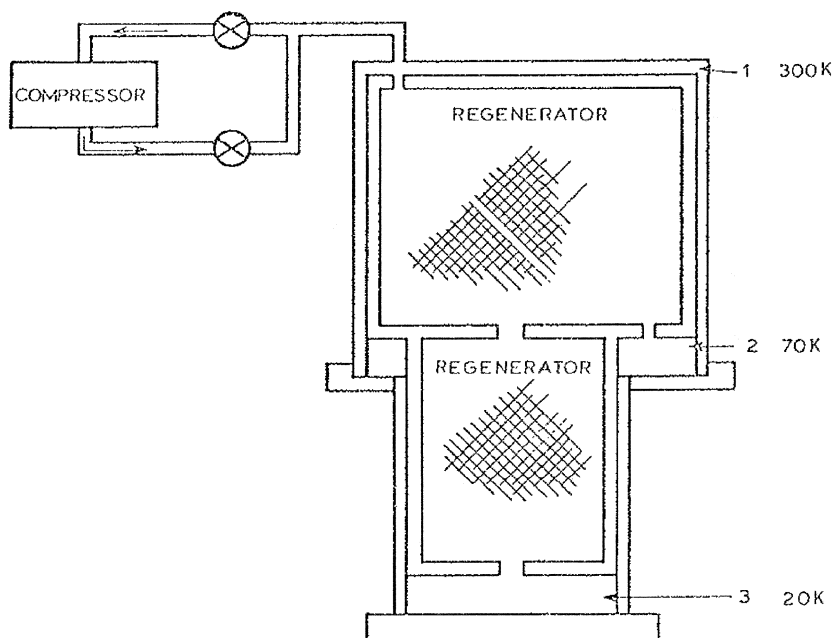


Fig. 7 - The two-stage CTI cryostat.

The refrigerator cycle described above has often been referred to as a "no work" cycle, but this label must have been invented by a myopic thermodynamicist. The term comes from the fact that no work is needed to move the displacer since

the pressure is always the same on both ends. Work is done in the compressor, and just because it is put out of hearing range does not mean that it is not part of the system.

More detailed descriptions of the refrigeration cycle are given by Gifford and McMahon, after whom the cycle is named, in *Advances in Cryogenic Engineering*, Vol. 5, pp. 354-372, 1959 and Vol. 11, pp. 152-159, 1965 and *Progress in Cryogenics*, Vol. 3, pp. 51-73, 1961. The first of the three papers is particularly readable and gives a good rundown on the sources of inefficiency in a practical refrigerator.

The above references show that the ideal refrigerator will handle a heat load of

$$Q_L = (p - p_o)V$$

where p is the input gas pressure, p_o is the exhaust gas pressure, and V is the volume of gas moved by the displacer. If the pressure differential is 200 pounds/in² (1.38×10^7 dynes/cm²) a displaced volume of 0.72 cm³/sec would be required for a 1 Watt heat load.

In the 1959 reference Gifford and McMahon divide the sources of inefficiencies in a real refrigerator into three categories: regenerator inefficiency, pressure-volume inefficiency, and heat transfer loss. The first is due to the finite heat capacity and transverse thermal resistance of the regenerator. Its effect on overall refrigerator efficiency can be quite pronounced when the refrigeration load is below 100 K in a single-stage system as shown in Figure 8 which is taken from McMahon and Gifford (1959). If gas entering the colder chamber has not been completely cooled by the regenerator less gas will be available for cooling when

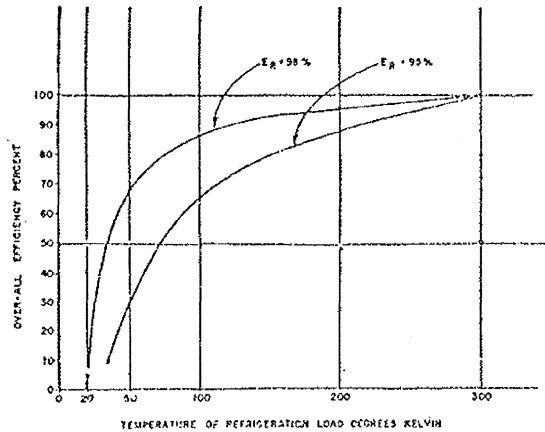


Fig. 4. The effect of regenerator efficiency on over-all efficiency.

Fig. 8 - From McMahon and Gifford, *Advances in Cryogenic Engineering*, Vol. 5, pp. 354-372, 1959.

expansion occurs. Regenerator efficiencies of 98% or higher are fairly easily achieved, except at very low temperatures where the heat capacities of most materials become very small. The latter fact limits the useful temperature range of these refrigerators to about 15 K and above.

The pressure-volume inefficiency is due to volume such as piping and space in the regenerator which is outside of the displacer cylinder. If some portion of the gas in the system does not flow all of the way from chamber 1 to chamber 2 it will not participate fully in the cooling process. Wasted space is much more deliterious when it is at the cold end of the system.

Finally, the heat transport loss includes conduction through the cylinder walls, the regenerator and the displacer. A more subtle loss comes from reciprocating action of the displacer. When it is at the warm end it picks up heat from the cylinder wall which will be deposited on a colder part of the cylinder when the displacer moves to the cold end.

In principle a single-stage refrigerator could be built for cooling to 15 K just as efficiently as a two-stage device, but as can be seen from Figure 8 the requirements on the regenerator efficiency would be severe. If two or more stages are used the regenerator problem is greatly relaxed plus a second cold station at an intermediate temperature is available for sinking radiation and other heat loads which otherwise would be imposed on the 15 K station. Since for a given heat load the volume of gas required from the compressor is about five times higher at 15 K than at 77 K there is a savings to be had in compressor power if some of the thermal load can be taken by an intermediate stage.

4.2 The Joule-Thompson 4 K stage

After the helium gas has been cooled to about 15 K a different type of refrigeration mechanism is used to cool the helium to 4.2 K at which point some of it will liquify. This mechanism, called the Joule-Thompson effect, also takes advantage of the fact that net work is done on the outside world by the refrigerator, but instead of cyclically compressing and expanding the gas in different chambers the gas is forced through an orifice called a J-T valve.

Consider the Joule-Thompson experiment in Figure 9 as

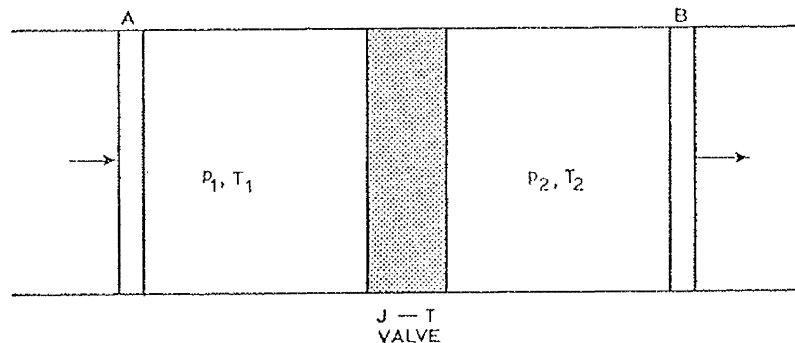


Fig. 9 - The Joule-Thompson experiment.

described by Sears in "An Introduction to Thermodynamics", Addison-Wesley, Reading, Mass, 1966. Here we have a cylinder which contains a porous plug (J-T valve) which offers some resistance to gas flow and has a very low thermal conductivity. On the left there is a piston A moving to the right which holds the gas to the left of the plug at pressure p_1 . Assume that the experiment has reached equilibrium so the walls of the cylinder are at the same temperature as the adjacent gas, hence no heat is transferred to the walls. The cylinder has very low longitudinal thermal conductivity. To the right of the plug let the pressure p_2 , which is lower than p_1 , be held constant with the appropriate velocity of piston B. Any difference in temperatures T_1 and T_2 will be due to the difference between work put in by piston A and work done on piston B. For the moment let us assume that $T_1 = T_2$ and see if any net work is done to or by the gas. If not, the system is in equilibrium, and the temperatures will remain equal.

Suppose we are using an ideal gas whose state is defined by the product of its pressure times its volume equal to a constant, R , times its temperature:

$$PV = RT .$$

Work is the product of force, F , times distance, d , but the force on a piston is the pressure, p , times the piston area, A , and the distance it moves is the volume swept up, ΔV , divided by the area:

$$W = Fd = pA \frac{\Delta V}{A} = p\Delta V .$$

Thus, the work in pistons A and B, respectively, are

$$W_A = p_1 \Delta V_1 \quad \text{and}$$

$$W_B = p_2 \Delta V_2 ,$$

but since the temperature has been assumed to remain constant for the gas pushed through the plug

$$W_A = p_1 \Delta V_1 = RT = p_2 \Delta V_2 = W_B .$$

Hence, no net work has been done on the gas and the constant temperature assumption is valid. If we had assumed $T_2 > T_1$ then $W_A = p_1 \Delta V_1 = RT_1 < RT_2 = p_2 \Delta V_2 = W_B$, or work would have been done on piston B taking energy out of the gas to the right of the plug and forcing $T_2 = T_1$. The same argument holds if the initial assumption had been $T_2 < T_1$.

Now consider a real gas which differs from an ideal gas in two ways. First, the molecules occupy a finite amount of volume so that the pressure essentially goes to infinity at this volume rather than at zero volume. The gas equation would then look like

$$p(v-b) = RT \tag{28}$$

where v is the volume per unit mass, V/m . Secondly, the molecules have an attraction for one another which can absorb some of the thermal energy which would normally go into the gas pressure. If we add this property eq. (28) becomes

$$\left(p + \frac{a}{v^2}\right)(v-b) = RT \tag{29}$$

which is called van der Waals' equation of state. Life is a bit more complicated than was assumed by van der Waals in deriving this equation, but the constants \underline{a} and \underline{b} are often listed in handbooks for various gases, and the concepts are very useful in explaining the J-T refrigeration process.

Suppose the finite volume of the molecules is the most important difference between a real and an ideal gas so that eq. (28) is a good approximation. Then the assumption of no temperature change gives

$$p_1(\Delta v_1 - b) = RT = p_2(\Delta v_2 - b) , \quad (30)$$

but the work per unit mass is

$$w_A - w_B = p_1 \Delta v_1 - p_2 \Delta v_2 = p_1 b - p_2 b , \quad (31)$$

and since

$$p_2 < p_1 .$$

more work is put into the gas than is taken out ($w_B > w_A$) so T_2 must be greater than T_1 to be in equilibrium. This certainly will not work as a refrigerator!

What happens if the intermolecular forces are dominant so that eq. (29) can be used with $b = 0$. Then

$$p_1 \Delta v_1 + \frac{a}{\Delta v_1} = RT = p_2 \Delta v_2 + \frac{a}{\Delta v_2} \quad (32)$$

and

$$w_A - w_B = p_1 \Delta v_1 - p_2 \Delta v_2 = \frac{a}{\Delta v_2} - \frac{a}{\Delta v_1} \quad (33)$$

but

$$\Delta v_2 > \Delta v_1$$

so $w_A < w_B$, and net work is done by the gas. Hence, T_2 must be lower than T_1 to reach equilibrium. Nearly all real gases will work in a J-T refrigerator, but in some gases like helium the molecular attractions dominate only at rather low temperatures and moderate pressures. Therefore helium must first be cooled to less than 30 K or so at pressures less than 40 atmospheres (~600 psi) to work in a J-T system.

For low temperature refrigerator engineering the van der Waals' equation is not an accurate enough description of the relationships between the pressure, volume, and temperature of helium. We must resort to the tabulated properties of helium. Whether using an equation or tables a very useful gas property in refrigerator work is enthalpy. Enthalpy, H, at a particular gas pressure, volume, and temperature is simply the amount of energy which would have to be removed to cool the gas to absolute zero. Strictly speaking, it is the sum of the internal energy, U, tied up in molecular attraction potentials and kinetic energy and the energy which would go into work in raising the gas from $T = 0\text{K}$ at zero pressure and volume to higher temperature, pressure and volume. Thus

$$H = U + PV . \quad (34)$$

When these quantities are written in lower case they usually mean values per unit mass or "specific" quantities. In practice one is not terribly worried about how the energy is distributed between U and PV, and, in fact, it is often

traded back and forth as the gas goes through a system. The passage of gas through a J-T valve is a constant enthalpy (isenthalpic) process where internal energy is traded for work.

Table 11 gives the enthalpy of helium over a wide range of temperatures and pressures. The units are joules/gm-mole where a gm-mole of helium is equal to 4 grams. Notice that below 40 K the enthalpies are lower at higher pressures. Helium at 20 atmospheres and 15 K has an enthalpy of 314.5 J/gm-mole, and if it is expanded through a J-T valve to 1 atmosphere pressure it will have the same enthalpy but a lower temperature (~12.5 K). This gas has a potential cooling or heat sinking capacity of 53.0 J/gm-mole (367.5-314.5) if we assume that the gas is to be exhausted at 15 K. The heat capacity, C , is just the derivative of the enthalpy and can be stated for constant pressure, C_p , or constant volume, C_v .

Armed with the enthalpy table let us take a look at an actual refrigerator shown schematically in Figure 10. Heat exchangers X_1 , X_2 and X_3 have the job of transferring enthalpy from the incoming gas to the outgoing gas. Each has an efficiency E which is the ratio of incoming ΔH to outgoing ΔH , e.g. $E_1 = (H_0 - H_1)/(H_{10} - H_9)$. Since the incoming gas has a slightly higher heat capacity than the lower pressure outgoing gas it also is necessary to exchange heat with the 77 K and 15 K* stations of the expansion refrigerator. These stations also make up for any inefficiencies in X_1 and X_2 .

*In the sample calculations to follow we shall assume that the second cryostat stage is always at 15 K, however, in practice this temperature will depend on the load on this stage.

Table 4. Enthalpy of gaseous helium 4 (⁴He) joule mole⁻¹ (Pressure, atmospheres)

Temperature °K	1	2	5	10	15	20	25	30	40	50	60
4.213 {	39.04	(Liquid)									
5	121.2	(vapour)									
6	147.5										
7	172.2	154.8	85.0	82.5	89.8	93.2	107.3	116.7	136.0	155.4	174.6
8	195.1	183.1	135.6	105.8	109.2	114.9	122.5	130.8	148.9	167.6	186.2
9	217.3	207.8	174.9	135.8	130.4	134.2	140.1	147.4	163.7	181.4	199.3
10	239.0	231.0	206.3	169.1	166.7	166.4	169.1	166.1	160.6	156.9	154.1
11	260.6	253.8	233.5	201.7	183.6	181.1	182.3	186.7	169.3	164.0	160.2
12	282.2	276.3	258.7	231.5	214.6	207.4	203.4	208.9	191.2	182.6	177.8
13	303.7	298.4	282.7	259.3	243.0	234.5	231.4	238.5	214.2	203.4	196.3
14	325.0	320.3	308.1	285.7	270.3	261.6	257.7	265.2	238.4	223.4	213.8
15	346.2	342.0	329.2	311.1	297.1	288.3	284.7	292.3	262.6	243.1	229.0
16	367.5	363.6	352.1	335.8	322.0	314.5	310.7	318.3	284.8	261.4	243.4
17	388.7	385.2	374.8	350.0	337.0	330.2	326.5	334.1	297.7	270.7	248.3
18	409.8	406.6	397.2	373.8	360.2	354.2	350.6	358.0	309.7	278.7	252.3
19	430.8	428.0	419.5	407.3	393.9	389.7	386.2	393.2	333.0	297.9	268.6
20	451.8	449.3	441.0	430.5	420.9	413.8	409.7	417.3	352.8	313.4	280.0
20	472.9	470.6	463.6	453.5	444.7	437.7	433.6	441.3	407.3	363.4	325.4
30	631.8	630.9	625.2	624.1	624.7	627.2	632.0	638.1	630.8	602.0	569.3
40	890.1	889.9	889.3	889.4	887.5	886.8	886.0	885.7	883.7	862.0	828.5
50	1098.8	1098.8	1099.1	1099.9	1100.9	1101.8	1102.7	1103.0	1102.4	1079.0	1043.5
60	1306.8	1307.0	1308.5	1310.3	1312.0	1314.1	1316.3	1318.6	1323.1	1295.7	1258.0
70	1514.5	1515.0	1517.3	1520.2	1522.9	1525.7	1528.6	1531.7	1538.3	1506.1	1464.0
80	1722.9	1723.6	1725.9	1729.3	1732.5	1735.7	1738.6	1742.2	1751.1	1715.7	1669.2
80	1831.0	1831.9	1834.3	1838.3	1843.3	1848.3	1853.2	1858.4	1864.0	1825.0	1775.2
100	2079.5	2080.4	2083.1	2087.7	2093.3	2098.9	2104.6	2110.3	2116.9	2073.0	1998.8
120	2495.4	2496.4	2499.6	2504.9	2510.3	2515.6	2520.9	2526.3	2532.7	2485.6	2398.3
140	2911.2	2912.3	2915.8	2921.5	2927.3	2932.9	2938.7	2944.4	2950.1	2900.7	2800.9
160	3327.0	3328.2	3331.7	3337.7	3343.7	3349.7	3355.6	3361.6	3367.6	3315.4	3203.8
180	3742.7	3743.9	3747.0	3753.7	3759.9	3765.9	3772.1	3778.2	3784.3	3729.7	3607.6
200	4158.4	4159.7	4163.4	4169.6	4175.9	4182.1	4188.3	4194.5	4200.7	4141.9	4015.0
220	4574.2	4575.4	4579.2	4585.5	4591.8	4598.1	4604.4	4610.7	4617.0	4553.9	4413.5
240	4990.0	4991.2	4995.0	5001.3	5007.7	5014.0	5020.4	5026.7	5033.0	5000.4	4854.8
260	5405.6	5406.9	5410.8	5417.1	5423.5	5429.9	5436.3	5442.7	5449.0	5438.3	5431.1
280	5821.4	5822.7	5826.5	5832.9	5839.4	5845.8	5852.3	5858.7	5865.1	5884.5	5897.3

The enthalpy zero is that of the liquid at 0° K under zero pressure.
 Based on data from: Reeburk, I. R., and Osterker, H. 1931, *Phys. Rev.* 43, 69, 1931, 65, 317; Reeburk, I. R., and Merz, T. A., 1941, *Temperature, Its Measurement and Control in Science and Industry*, P. 61, New York; Academic Publishing Corporation; Hill, R. V., and Lounsbury, O. V., 1937, *Phil. Mag.*, 2, 143; 1960, *Phil. Trans. Roy. Soc. A*, 252, 337.

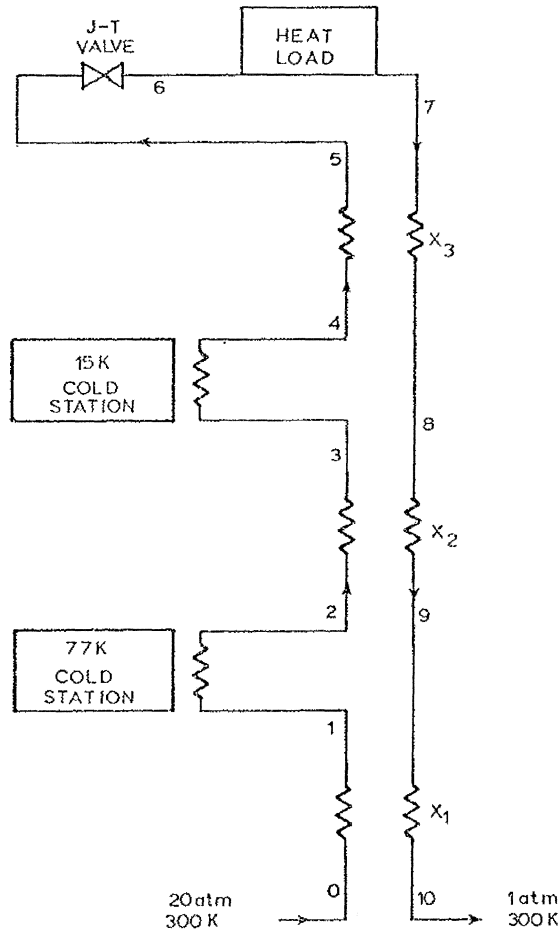


Fig. 10 - Schematic diagram of the J-T loop and heat exchangers in the 4K refrigerator.

Since the enthalpies are in tabular form it is easiest to follow the heat flow through the refrigerator by making a table of the enthalpies at various points in the system. It can be very confusing to try to account for all of the effects of the heat exchanger inefficiencies on the first analysis of the system so we will make a few assumptions which can be modified later. Table 12 is a list of temperatures and enthalpies at points labelled in Figure 10 assuming a gas flow

Table 12. Temperatures and enthalpies at various points in the system shown in Fig. 10 with a mass flow rate of 0.32 gm/sec (3.8 scfm). The "second approximation" columns take second order effects of the heat exchanger inefficiencies into account ($E_1 = E_2 = 0.96$).

Point in Fig. 10	First approx.		Second approx.		Difference equals
	T (K)	H (J)	T (K)	H (J)	
0	<u>300</u>	<u>505.3</u>	<u>300</u>	<u>505.3</u>	
1	88.1	149.4	83.8	145.3	} 77 K load
2	<u>77</u>	<u>134.0</u>	<u>77</u>	<u>134.0</u>	
3	20.0	34.76	19.5	33.97	} 15 K load
4	<u>15</u>	<u>24.88</u>	<u>15</u>	<u>24.88</u>	
5	(4.2)	6.99	(4.2)	6.99	} 4.2 K capacity
6	(4.2)	6.99	(4.2)	6.99	
7	4.2	9.64	4.2	9.64	
8	<u>15</u>	<u>29.32</u>	14.5	28.53	
9	<u>77</u>	<u>132.7</u>	74.5	128.6	
10	<u>300</u>	<u>503.4</u>	291.1	488.6	

rate of 0.32 gm/sec. This is approximately the flow rate in the 3 Watt 4 K system. The "per second" part allows us to convert enthalpies into power since 1 W = 1 J/sec. Later these numbers can be scaled to any flow rate. Numbers in boxes are fixed by high capacity heat sinks at 300, 77, and 15 K (points 0, 2 and 4), and the underlined numbers are set on the first go-round to simplify the analysis (points 8, 9 and 10). No substantial resistance to gas flow is offered by the heat exchangers in our model so all of the pressure drop occurs at the J-T valve.

In heat exchanger X_1

$$H_1 = H_0 - E_1(H_{10} - H_9) \quad (35)$$

which if E_1 were unity would be 134.6 joules. This is only 0.6 joules greater than H_2 so the 77 K station would have to sink only 0.6 W. However, E_1 is more typically 0.96 in which case $H_1 = 149.4$ which on first analysis leaves 15.4 W to be carried away by the 77 K station. A much less efficient X_1 would swamp this station.

In heat exchanger X_2

$$H_3 = H_2 - E_2(H_9 - H_8) \quad (36)$$

which if E_2 were unity would be 30.6 joules leaving 5.7 W to be dissipated by the 15 K station. If $E_2 = 0.95$, $H_3 - H_4$ will be 9.9 W which is roughly the capacity of the CTI 1020 refrigerator on which the 3 W machine is based. We now have a hint as to where the ratio of roughly 3:1 comes from for the ratio between 15 K and 4 K stage capacities. In principle, this ratio could approach 2:1 if E_2 were closer to 1.0. This would not be an easy task!

We can now look at the J-T loop as a box with an input at point 4 and an output at point 8. The cooling capacity of this loop must be $H_8 - H_4 = 29.32 - 24.88 = 4.44$ W if $E_3 = 1.0$. For the moment suppose that point 7 is held at 4.2 K by adjusting the heat load so that it is exactly equal to the loop capacity. Then $H_7 = 10.4$ joules, and the capacity not available to the heat load due to the inefficiency of exchanger X_3 will be

$$\Delta H(X_3) = (1 - E_3)(H_8 - H_7) \quad (37)$$

which if $E_3 = 0.96$ would be 0.79 W. Taking this into account the loop capacity would be $4.44 - 0.79 = 3.65$ W.

The quantity $\Delta H(X_3)$ above has the effect of making the gas a point 8 colder than 15 K, the corrected H_8 and H_3 will be smaller by ΔH , and the load on the 15 K station will be reduced by ΔH to 9.1 W.

In the same way the inefficiency of X_2 will result in lower H_1 and H_9 by

$$\Delta H(X_2) = (1-E_2)(H_9-H_8) = 4.1 \text{ W} \quad (38)$$

resulting in a heat load reduction on the 77 K stage to 11.3 W. The inefficiency of X_1 will result in a bit of cooling in the output line.

So far the analysis has assumed that the 4 K stage is already cold and that everything has reached equilibrium. Suppose now that we start the J-T loop from scratch or, in other words, at the point where everything in the J-T loop has been cooled to 15 K by a combination of reduced efficiency J-T cooling and by the constant pressure heat capacity of the helium (~ 6 J/gm K) which has been cooled by the 15 K stage.

Given the input and output enthalpies of the J-T loop and a perfectly efficient heat exchanger the cooling capacity of the loop is only a function of the mass flow rate. All of the above numbers assume that the J-T valve has been designed to give a mass flow rate of 0.32 gm/sec (3.8 scfm) at a J-T valve output temperature of about 4.2 K. To find out what happens to the flow rate at higher input temperatures we have to look in more detail at the J-T valve.

Instead of a porous plug as in Figure 10 the J-T valve is a small orifice or nozzle consisting of a hypodermic

needle with a wire through the centre to reduce its effective area. With the pressures, densities and mass flow rates of interest here the orifice will be so small that the gas velocity will reach the speed of sound. In this case the flow rate is not controlled by the normal resistance to flow in a pipe* but by the amount of gas at a particular density which will flow through an opening at the sonic velocity since the gas cannot be made to travel faster than the speed of sound in the nozzle. All of the pressure drop does not occur in the nozzle. As much as half of this drop happens in a shock front at the nozzle outlet, but the details of this are not important here.

It seems reasonable to assume that the mass flow rate will be lower at higher J-T valve input temperatures since the input density will be lower. To say how much lower we will have to look at the physics of flow through a nozzle.

Life gets a bit complicated because there is no a priori way of knowing what the temperature, pressure and density of the gas will be when it reaches the smallest part of the nozzle. We have to take the gas at a given set of initial conditions T_0 , P_0 and ρ_0 at the wide input to the nozzle and follow what happens to it as it approaches the constriction. For a fairly detailed discussion of nozzles you are referred to "Thermodynamics" by G.J. van Wylen, 1959, John Wiley & Sons, Inc., New York, Chapter 13, but the essence of what has to be done to design a nozzle is as follows.

First we know from the conservation of mass that the

*In practice frictional resistance does play a part in determining the flow rate through the needle, but this will not qualitatively affect the arguments to follow.

mass flux, $\dot{m} = dm/dt$ must be the same at all points along the nozzle and that this mass flux is given by

$$\dot{m} = A\bar{V}\rho \quad (39)$$

where A is the cross-sectional area at any point, and \bar{V} and ρ are the average gas velocity and density at that point. Since we are going to have to follow the gas through the nozzle on a computer using the tabulated properties of helium we would prefer to have eq. (39) in differential form which can be accomplished by differentiating by the three variables and dividing through by $A\bar{V}\rho$.

$$\frac{\delta A}{A} + \frac{\delta \bar{V}}{\bar{V}} + \frac{\delta \rho}{\rho} = 0 \quad (40)$$

Another equation that must be obeyed is one that states the conservation of energy. Because of the conservation of mass the velocity of the gas must increase as it approaches the smallest part of the nozzle which means that its kinetic energy ($m\bar{V}^2/2$) will increase. We are not adding any energy to the gas as it flows through the nozzle so the only source of energy is the gas itself. Hence,

$$h + \frac{\bar{V}^2}{2} = h_0 \quad (41)$$

where \underline{h} is enthalpy per unit mass and \underline{h}_0 is a constant equal to \underline{h} at $\bar{V} = 0$. In differential form this is

$$\delta h = -\bar{V}\delta\bar{V} \quad (42)$$

By the same token, the only source of a pressure change in the gas is its internal energy so

$$\delta h = v \delta P = \frac{\delta P}{\rho} \quad (43)$$

(remember that $vP = \text{work}$) where v is the volume per unit mass which is $1/\rho$. It is not intuitively obvious that the pressure will change on the input side of the nozzle, but when we combine eqs. (42) and (43) we get

$$\frac{\delta P}{\delta \bar{V}} = -\bar{V} \rho \quad (44)$$

In other words, the pressure will decrease as the velocity increases. Combining eqs. (40) and (44) we get

$$\frac{\delta \Lambda}{\delta \bar{V}} = \frac{\Lambda}{\bar{V}} \left[\bar{V}^2 \frac{\partial \rho}{\partial P} - 1 \right] \quad (45)$$

but from van Wylen's book (p. 354) we find that

$$\frac{\delta P}{\delta \rho} = c^2 \quad (46)$$

where c is the sound velocity. Therefore,

$$\delta \Lambda = \frac{\Lambda}{\bar{V}} \frac{\bar{V}^2 - c^2}{c^2} \delta \bar{V} \quad (47)$$

This equation is best integrated by a computer with the aid of eq. (39) since c is a function of density. Start with the initial conditions A_0 , ρ_0 and \dot{m} at the input to the J-T valve. Compute \bar{V}_0 from eq. (39). Determine c from ρ_0 and solve for δA for a small change in velocity $\delta \bar{V}$ with eq. (47). Compute new values $A = A + \delta A$ and $\bar{V} = \bar{V} + \delta \bar{V}$, solve for new ρ and c in eq. (39) and go back to eq. (47). The point where $\bar{V} = c$ determines the area of the throat of the J-T valve for the assumed value of \dot{m} . The mass flow rate is then

ρ_0 determined from T_0 and P_0 .

The trick to finding the relationship between c and ρ is to assume that the gas flow into the J-T valve is an isentropic process. Isentropic means adiabatic and reversible. Adiabatic, in turn, means no energy or work is added or taken away from the system - a condition which we have already used in connection with eq. (42) and (43). Reversible means that the gas could be made to return to its original state with no net gain or loss of energy or work. If we made our nozzle symmetric so that the output port area equalled the input port area the gas would end up with the same temperature, pressure and density with which it started. Hence the process is reversible and isentropic. In reality, our J-T valve is not symmetric so only the flow toward the smallest part of the nozzle can be considered isentropic.

Entropy is a quantity which can be tabulated for any gas as a function of temperature and pressure, so if we know the initial state of a gas at the beginning of an isentropic process we know exactly how P , T and ρ are going to react if any one of them is caused to change. There are only two independent variables in the equation of state of a gas, and if the entropy is fixed there is only one. At the risk of getting lost on our way to computing the properties of the J-T valve it might be worth a diversion to see how entropy is defined and calculated.

The concept of entropy is a device used to quantitatively state that, left to themselves, all thermodynamic processes will proceed in only one direction, e.g. heat only flows from hot to cold, gas only flows from high to low pressure, etc. This is the second law of thermodynamics. A refrigerator works only because work is being done to force heat to flow

from cold to hot, but somewhere in the refrigerator system a lot more heat is flowing from hot to cold than is flowing from cold to hot. Chapter 8 in the reference by Sears gives the mathematical definition of entropy in differential form as

$$ds = \int_{T_1}^{T_2} \frac{dQ}{T} \quad (48)$$

where dQ is the heat gained or lost by a system at temperature T . The entropy change, ds , is larger for a given dQ in a low temperature system than in a high temperature system, therefore, heat running from high T to low T results in a net increase of entropy. The entropy of a closed system can go only one way, up.

Entropy change at a constant pressure can be written as

$$ds = \int_{T_1}^{T_2} C_p \frac{dT_p}{T} \quad (49)$$

where C_p is the heat capacity at constant pressure (dQ/dT). It is a measurable quantity. Entropy is defined as being zero at absolute zero so the entropy at a given temperature, T_2 , can be found by integrating eq. (49) from $T = 0$ to T_2 at constant pressure. The heat capacity is zero at $T = 0$ so the integral goes to zero as $T \rightarrow 0$. One might think that simply not adding any heat to a system would suffice to define constant entropy, but, for example, if you let gas out of a pressurized container you would be hard pressed to get it all back in again without losing any heat to the outside world. Thus, the requirement for reversibility.

Suffice it to say that we can plot, T , P and ρ for helium as functions of one another along lines of constant entropy as has been done in Figure 11. Also plotted in

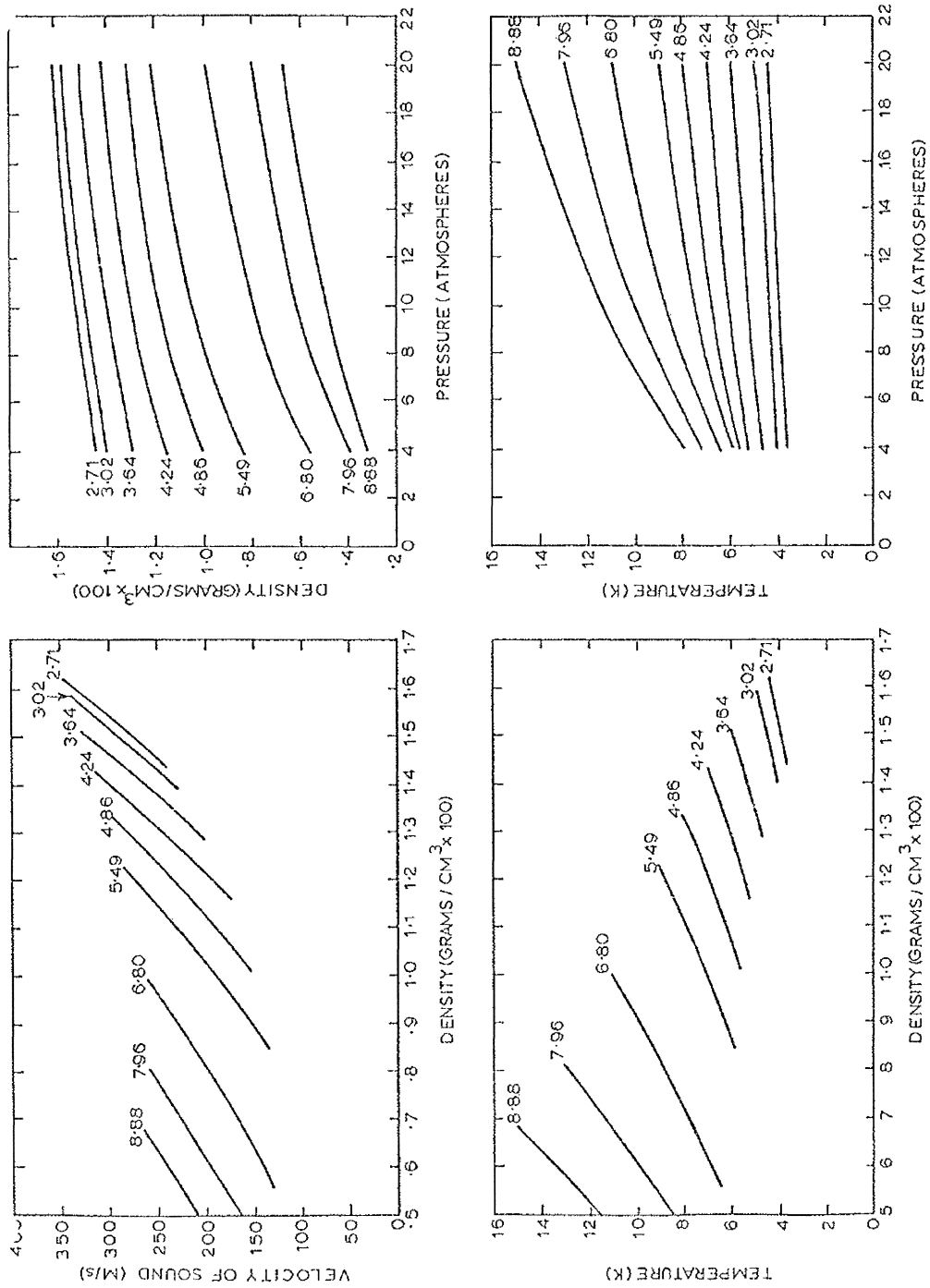


Fig. 11 - Isentropic curves for low temperature He4. The parameter is entropy in joules/gm.

Figure 11 is the speed of sound as a function of density at constant entropy. All of these data and the data used in Table 12 are from U.S. N.B.S. Technical Note 631 by R.D. McCarty, 1972, "Thermophysical Properties of He4 from 2 to 1500 K with Pressures to 1000 Atmospheres".

Getting back to the J-T valve, we now have enough information to integrate eq. (47) and determine the nozzle openings for given mass flows and initial conditions. The results are shown in Figure 12 for an input pressure of

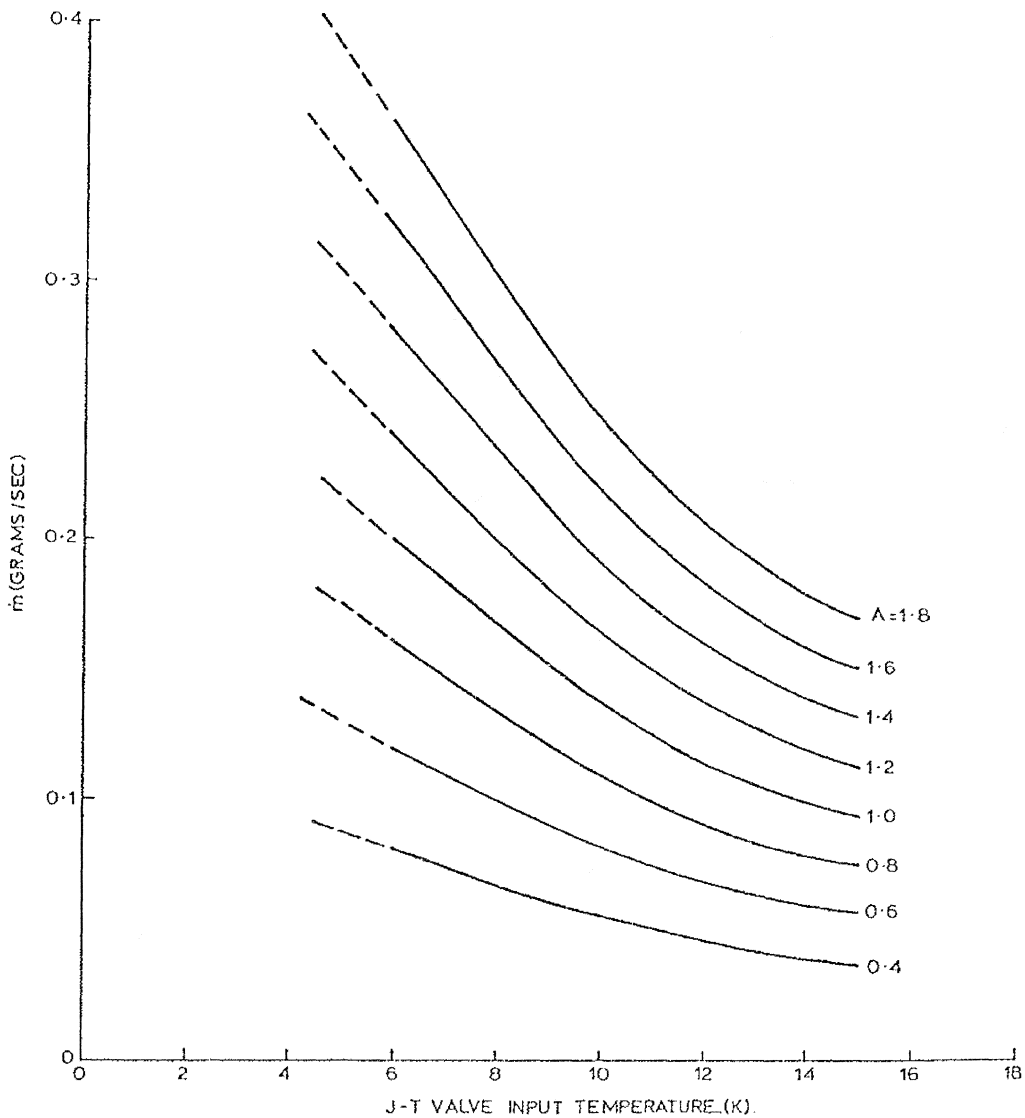


Fig. 12 - J-T valve mass flow rate as a function of input helium temperature for a sonic nozzle. The parameter is nozzle

20 atmospheres (294 psi). As we guessed, the mass flow rate increases with decreasing temperature of the gas entering the J-T valve. Since the cooling capacity of the J-T loop is directly proportional to the mass flow rate it is apparent that the J-T station can handle a larger heat load once it is cold than it can in the process of cooling itself from 15 K to 4.2 K. (I am uncertain about what happens to the curves in Figure 15 with input temperatures below about 6 K because the helium can turn to liquid in the J-T nozzle.) If heat exchanger, X_3 , were perfectly efficient the vertical axis of Figure 12 could be relabelled "refrigeration capacity" with a suitable scale factor, but, of course, the heat exchanger is not perfect.

To a first approximation heat exchanger, X_3 , will not be required to pass any heat from the output to input gas streams when the cold load is still at 15 K. Hence, the exchanger inefficiency is of no consequence when cool-down from 15 K begins. As the load gets colder more and more heat is exchanged in X_3 , and since the heat lost in X_3 is proportional to the total heat exchanged times $(1-E_3)$ the inefficiency will have a larger effect. The cooling power available to the load is

$$Q_L = [H_7(1-E_3) - H_4 + E_3 H_8] \dot{m} . \quad (50)$$

The value H_7 gets smaller as the load gets colder. For an efficiency of $E_3 = 0.96$, at $T_L = 5$ K, Q_L will be 87% of its value at $T_L = 15$ K if the mass flow rate is constant. If eq. (50) is combined with the curves in Figure 12 the approximate cooling capacity of the J-T loop as a function of J-T valve input temperature will be as shown in Figure 13.

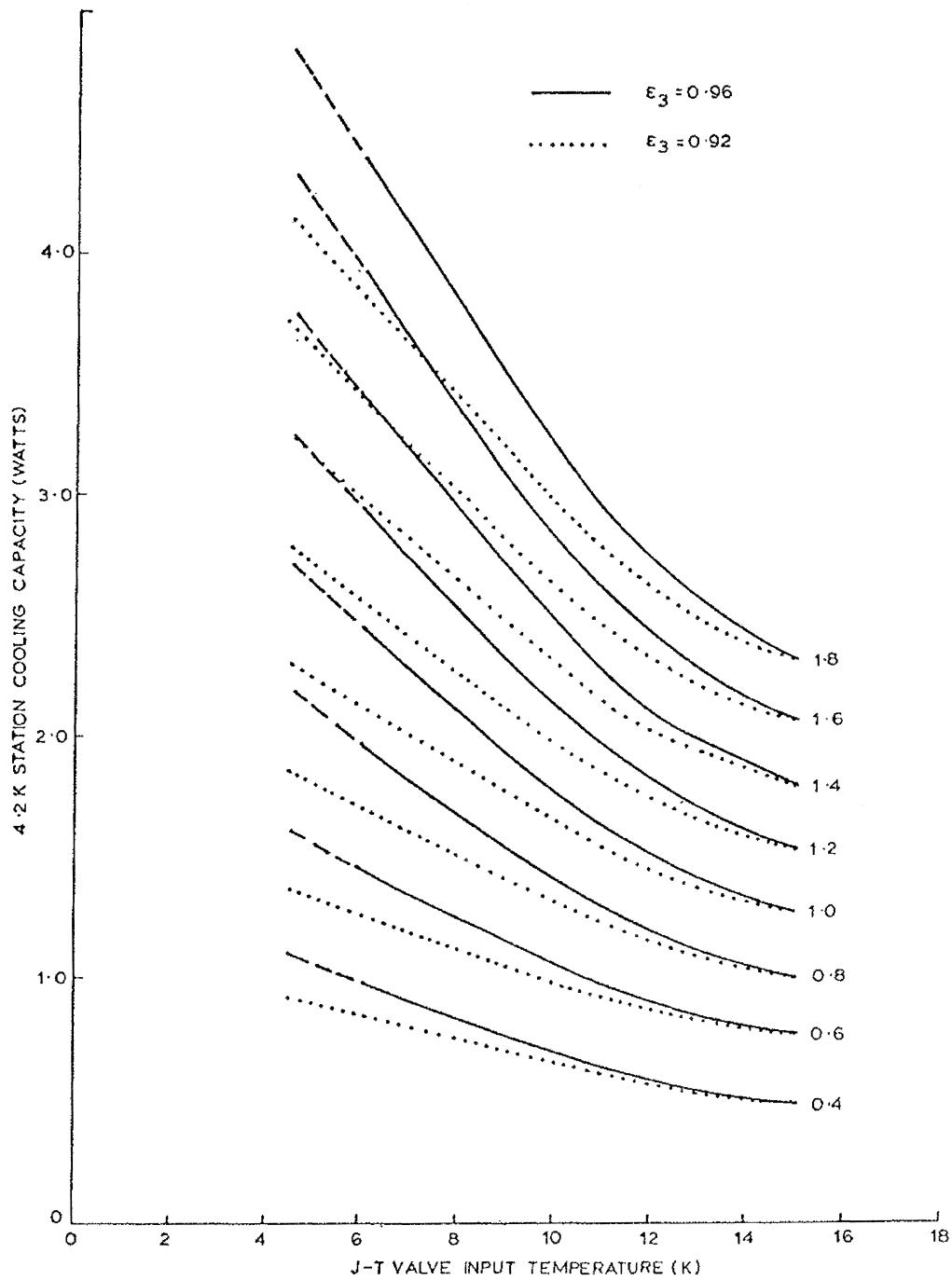


Fig. 13 - Refrigeration capacity as a function of J-T valve input helium temperature. The parameter is J-T nozzle area.

The curves in Figure 13 can never be verified in practice because the load temperature will not stabilize anywhere on the curves. Since the capacity increases at lower temperatures the load will continue to get colder until it reaches the boiling point of helium. The most important feature of the curves is the cooling capacity at 15 K. If the thermal load tied to the 4 K station is higher than this value the J-T loop will never get below 15 K. A 3.0 W 4 K refrigerator with $\dot{m} = 0.25$ gm/sec and $E_3 = 0.96$ will have a "start-up" capacity of about 1.6 W. The extra 1.4 W of load can be added after the 4 K station gets cold. If the 3 W load is ever exceeded the refrigerator will quickly warm up to 15 K. With $E_3 = 0.92$ a 3.2 W machine would have a mass flow rate of 0.32 gm/sec (3.8 scfm) and a start-up capacity of 1.8 W. This is very close to what has been measured on the Green Bank system.

Another effect which could increase the ratio of start-up to 4.2 K capacity is a pressure drop on the high pressure side of the heat exchangers due to resistance to gas flow. This would tend to decrease the slope of the mass flow curves in Figure 12 since the J-T flow is very nearly directly proportional to its input pressure. A pressure drop in heat exchanger, X_3 , would not be as harmful as flow resistance in X_1 or X_2 because X_3 is in the J-T loop, and much of the cooling due to the pressure drop would still be available to the heat load.

A bit of mass flow rate information is available from one of the NRAO 1 Watt refrigerators which shows that the flow increases from 0.064 gm/sec to 0.139 gm/sec as the cold station temperature drops from 18.7 K to 5.2 K. This is a somewhat weaker \dot{m} vs T dependence than shown by the curves in

Figure 12 even taking into account the fact that the J-T input temperature will be higher than the cold station temperature. This disagreement is probably due to the fact that there is a pressure drop in the heat exchangers which will increase as the mass flow rate increases and to the fact that there is a frictional component in the mass flow resistance in the J-T valve which has not been considered here. Another measurement on the same refrigerator shows that with a load on the 4 K station which is 100 mW higher than the maximum start-up capacity (~600 mW) the system stalled at 26.8 K at a flow rate of 0.052 gm/sec. We have assumed a stall temperature of 15 K, but since the curves in Figure 13 become rather flat at the high temperature end it is not surprising that a 15% excess load produces a considerably higher stall temperature. Apparently there is not enough heat carried away by the helium flowing through the J-T loop and by the hydrogen vapour switch* to pull the 4 K station temperature down to the cryostat second-stage temperature.

A final point of possible interest is that when there is no load on the 4.2 K station a large portion of the helium downstream of the J-T valve is in liquid form. The latent heat of vaporization of helium at 4.2 K is 20.6 joules/gm, and we know that in a perfect system operating with a J-T loop input of 20 atm at 15 K the cooling capacity available is 13.25 joules/gm so if all of the cooling capacity went into liquifying helium 64% of it could be turned into liquid. Hence, even in the no-load condition the piping after the J-T valve

*The hydrogen vapour switch is a conduction path between the 4 K and 15 K stations which opens up when the 4 K station reaches about 20 K.

contains a mixture of gas and liquid. The advantage of having a large amount of heat absorbed by the gas-to-liquid phase change is that the temperature of the helium is held constant over the full capacity range of the refrigerator. Since the boiling point of helium varies from about 2.5 K at 0.1 atmospheres pressure (absolute) to 5.2 K at 2.2 atmospheres the temperature of the J-T stage can be controlled by varying the exhaust pressure without affecting the refrigeration capacity very much.

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