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Title: Design of a Near-Optimum Waveguide Taper Transition

Authors: W. Grammer

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1.0 Summary

This memo covers the design and testing of a taper transition used to couple the new GBT C-band receiver to an existing C-band Cassegrain feed at the 140' telescope, for support of Space VLBI. A modified Dolph-Chebychev impedance taper, realized in circular waveguide, was chosen to achieve a good match in a short length, due to space limitations. The taper was measured with a vector network analyzer and a sliding load, and showed a return loss greater than 33 dB from 4.7-5.0 GHz, with more than 42 dB above 4.85 GHz. Overall length of the taper, excluding the mating flanges, was 7.5 inches.

2.0 Impedance Taper Design

The following are the design specifications/goals for the taper transition:

 a. C-band feed waveguide port diameter: 1.5285"
 b. C-band receiver waveguide input diameter: 1.9300"
 c. Design frequency band: 4.7-5.0 GHz
 d. Desired worst-case taper return loss: design for 40 dB ; minimum 30 dB
 e. Taper length: < 8.0"

The taper is designed based on the procedure outlined by Hecken [1], which modifies the optimum Dolph-Chebyshev impedance taper to eliminate the step discontinuities on the ends, for a slight increase in taper length. This "near-optimum" taper achieves a better match over the desired band for a specified length than other types of continuous tapers.

The first step in the design is to calculate the dominant-mode (TE_{11}) wave impedance versus frequency for the input and output waveguide diameters [2]; these are plotted in Figure 1. The impedance match is clearly the worst at 4.7 GHz, with a return loss of about 7.7 dB. Thus the desired improvement in match is (40-7.7) = 32.3 dB, or $\epsilon_{max} = 10^{(-32.5/20)} = 0.02371$. From this ϵ_{max}, the solution of the transcendental equation [1]

$$\epsilon_{max} = \frac{(0.21723)\,B}{\sinh{(B)}} \tag{1}$$

is used to calculate the minimum taper length, as expressed below in radians:

$$(\beta l)_{min} = \sqrt{B^2 + 6.523} \tag{2}$$

From [2], it is apparent that β varies in directly proportion to both waveguide diameter and frequency; hence the minimum value of β (and consequently the minimum *required* taper length l_{min}) is calculated from $D_{min} = 1.5285"$ and $f_{min} = 4.7$ GHz. From this, $l_{min} = 7.464"$.

Next, the taper impedance function is calculated in terms of normalized axial position ξ, where $\xi = 2x/l$ and $|x| < l/2$. From [1],

$$Z(\xi) = \exp\left[\frac{1}{2}\log_e(Z_2 Z_1) + \frac{1}{2}\log_e\left(\frac{Z_2}{Z_1}\right)G(B,\xi)\right]$$ (3)

where Z_1 and Z_2 are the input and output waveguide impedances (respectively) at 4.7 GHz, and

$$G(B,\xi) = \frac{B}{\sinh(B)}\int_0^\xi I_0\left(B\sqrt{1-\xi'^2}\right)d\xi'$$ (4)

The final step is to derive the physical taper profile from the taper impedance function. Assuming only dominant (TE_{11}) mode propagation, for circular waveguide [2],

$$k_c = k\sqrt{1-(\eta/Z_{TE})^2} = \frac{2p_{11}'}{D} \Rightarrow D(\xi) = \frac{2p_{11}'}{k\sqrt{1-(\eta/Z(\xi))^2}}$$ (5)

$$(k = 2\pi f_o\sqrt{\mu\epsilon})$$

where $D(\xi)$ is the diameter, $f_0 = 4.7$ GHz, $\eta = 376.7$ ohms, and $p_{11}' = 1.841$.

With the physical taper profile known, its reflection coefficient may be calculated. From Collin [3], the approximate input reflection coefficient of a continuous taper of length l with a perfectly matched output is given by

$$\Gamma_i(l) \approx \frac{1}{2}\int_0^l e^{-j2\beta x'}\frac{d}{dx'}(\log_e Z(\xi))\,dx'$$ (6)

where $\xi = (2x'/l)-1$. Given a discrete vector for Z,

$$\Gamma_i(l) \approx 0.5\sum_{i=1}^N e^{-j2\beta_i x_i'}\frac{\Delta Z_i}{Z_i} \quad ; N \text{ is large}$$ (7)

Figure 2 shows the reflection coefficient computed for the waveguide taper at 4.7 GHz, versus overall taper length l. Clearly this is much worse than the expected response; it also lacks the

clearly defined peaks and nulls that are characteristic of this type of taper. What is wrong?

A crucial assumption made by Hecken is that β is independent of the axial position x along the taper (i.e., the line impedance) but dependent on frequency ω, as in a pure TEM-mode transmission line. However, this is not the case for waveguide tapers, especially at frequencies close to the lower cutoff frequency. Figure 3 shows a plot of β, computed at 4.7 GHz, versus ξ for the taper profile given in Eq. (5). There are significant deviations from the flat response expected with TEM-mode propagation.

Fortunately there is a way around this problem, outlined in Gould et. al. [4]. In the prior calculation of Γ_i, the physical taper profile calculated in Eq. (5) was assumed to be distributed uniformly; that is, on equally spaced points x_i' along the axis of symmetry. However, if the values in x_i' are scaled so as to offset the nonlinearity in β, the resulting taper should have the desired response, like the TEM case. Thus the new x-axis points become

$$
x_i = x_i' \left[\frac{\beta_{min}}{\beta_i} \right]
$$

(8)

Figure 4 shows the recalculated response of the original taper profile with non-uniform axial spacing. The response is now like that predicted for the TEM-line taper in [1].

The predicted frequency response of the taper over the desired band is calculated from Eq. (7) assuming a fixed length l_{min} (7.464"), and is shown in Figure 5. The return loss does not meet the 40 dB goal over most of the band, but still exceeds the minimum of 30 dB. This was deemed acceptable for the present; a longer taper may have improved the performance, but would have required fabrication in two pieces, and may have created mechanical problems with the installation of the receiver at the 140'.

As a final note, the above computations of the taper profile and predicted return loss were done using MATLAB, a numerical matrix math program. The MATLAB scripts used for these calculations are shown in the Appendix.

3.0 Fabrication and Testing

3.1 Fabrication

The physical taper profile calculated from Eqs. (5) and (8) above was machined in aluminum on a numerically-controlled lathe, and the correct mating flanges for the GBT receiver and 140' feed horn were added. After machining, the inside surface was lightly honed to smooth out the profile, and the entire part chromated. Figure 6 shows a drawing of the complete taper transition section in several views.

3.2 Measurement Methods

The characteristics of the taper can be measured using a network analyzer; this requires one or more coax-to-waveguide transitions and a set of waveguide calibration standards to properly de-embed the desired S-parameters. One obvious difficulty with getting full two-port parameters

of the taper is that the waveguide diameters at each end are different. A common way around this is to measure the combined response [S'] of a pair of back-to-back transitions. While a fair estimate of the taper insertion loss can be made, the return loss of either port still cannot be uniquely determined. From symmetry and reciprocity, $S_{11}' = S_{22}'$ and $S_{21}' = S_{12}'$, respectively, reducing the number of unique measurements from eight to four. As there are six unknowns (S_{11}, S_{22}, S_{21}), an additional measurement of a 'front-to-front' configuration will yield two additional measurements (S_{11}''), allowing a solution. It can be shown that the reflection coefficients are

$$S_{11} = \frac{S_{11}' - S_{21}'S_{11}''}{1 - (S_{21}')^2} \quad , \quad S_{22} = \frac{S_{11}'' - S_{21}'S_{11}'}{1 - (S_{21}')^2} \tag{9}$$

Nevertheless, this measurement approach has a number of drawbacks. First, it requires that two *identical* transitions be fabricated, difficult in this case due to machining tolerances. Second, the 'front-to-front' measurement would require recalibration with a different size waveguide, and hence a second set of two-port calibration standards.

If the full two-port response is not needed, an accurate measurement of either S_{11} or S_{22} can still be done by terminating the opposite port with a sliding load and taking multiple one-port measurements at different slide positions, as illustrated in Figure 7. The sliding load is designed to be adjustable over a length $\lambda_g/2$ at the lowest frequency (4.7 GHz). Assuming the change in S_{21} is insignificant over this length (valid for a waveguide above cutoff), and the sliding load is well-matched ($|\Gamma_L| \ll 1$), the measured points will trace out a circle in the reflection coefficient plane as the slide is moved. It is easily shown that this reflection coefficient is expressed by

$$\Gamma_i \approx S_{11} + (S_{21})^2 \Gamma_L \tag{10}$$

If three measurements are taken at well-spaced intervals on this circle, its center and radius, and from this both S_{11} and $|\Gamma_L|$ can be uniquely determined. It can be shown that for three points A, B and C on a circle in the complex plane, the center ($x_0 + iy_0$) is the solution of the linear system

$$\begin{bmatrix} A_x - B_x & A_y - B_y \\ A_x - C_x & A_y - C_y \end{bmatrix} \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} |A|^2 - |B|^2 \\ |A|^2 - |C|^2 \end{bmatrix} \tag{11}$$

The solution is much simpler in the special case of two measured points $180°$ apart and collinear with the center and origin; Eq. (10) then reduces to a pair of simultaneous scalar equations:

$$\begin{aligned} |\Gamma_{i,max}| &\approx |S_{11}| + |S_{21}|^2 |\Gamma_L| \\ \pm|\Gamma_{i,min}| &\approx |S_{11}| - |S_{21}|^2 |\Gamma_L| \end{aligned} \tag{12}$$

$|\Gamma_{i,min}|$ and $|\Gamma_{i,max}|$ are readily measured by adjusting the sliding load for the maximum and minimum return loss, respectively. From simultaneous solution of the above equations, the magnitude of S_{11} can be obtained as:

$$|S_{11}| \simeq (|\Gamma_{i,max}| \pm |\Gamma_{i,min}|) / 2 \tag{13}$$

Note there are two possible solutions to the above equation; the positive sign is used for $|S_{11}|>|\Gamma_L|$, the negative sign for $|S_{11}|<|\Gamma_L|$. The two cases are shown graphically in Figure 8. The important thing to note is that $\Gamma_{i,min}$ and $\Gamma_{i,max}$ will always have the same phase for case (a), while for case (b) they will be $180°$ out of phase. This allows correct determination of the sign in Eq. (13) directly from the measured data.

3.3 Measured Data

The taper transition was measured using the setup in Figure 7. An HP8720A network analyzer is used, and a 1-port calibration done with circular waveguide standards (short, offset short, and sliding load), from 4.7 to 5.3 GHz. Data was then taken over this frequency range, at 50 MHZ intervals. At all frequencies except 4.70 GHz, the sliding load was adjusted for $|\Gamma_{i,min}|$ and $|\Gamma_{i,max}|$, and the magnitude and phase of Γ_i recorded at these points. At 4.70 GHz, the sliding load could not be positioned to obtain $|\Gamma_{i,min}|$, so two other points at approximately $120°$ intervals were measured, and Eq. (11) used to solve for $|S_{11}|$. The measured raw data and calculated $|S_{11}|$ of the taper transition is shown in Table 1; the measured $|S_{11}|$ is plotted against the theoretical response in Figure 9.

4.0 Conclusions

The measured data shows the taper to have better than 33 dB return loss from 4.7-5.3 GHz band, and more than 42 dB above 4.85 GHz. Several fairly sharp nulls (4.9 and 5.3 GHz) were also observed. While the performance of the taper was good, it nevertheless does not agree well with the predicted frequency response. One possibility is that there is significant coupling to higher-order modes in the waveguide. This would introduce errors in the waveguide calibration, and also violate one of the main assumptions made by Hecken. Another possibility is that this taper operates sufficiently close to cutoff to make the perturbational term neglected in the theoretical development in [1] large enough to create errors of the magnitude seen. Yet another explanation may be due to machine tolerances during fabrication, especially at the small end of the taper. The profile is very gradual in this area, because the impedance changes so rapidly with waveguide diameter, making it more sensitive to tolerance variation.

5.0 Acknowledgments

6.0 References

[1] - Hecken, Rudolf P., "A Near-Optimum Matching Section Without Discontinuities", IEEE Trans. Microwave Theory & Tech., Vol. MTT-20, No. 11, Nov. 1972

[2] - Pozar, David M., Microwave Engineering, Addison-Wesley, 1990, p. 161

[3] - Collin, Robert E., Foundations for Microwave Engineering, 2nd ed., McGraw Hill, New York, 1992, p. 371

[4] - Gould et. al., "New Non-Uniform Waveguide Taper Design Yielding Low VSWR and High Rejection", IEEE Microwave Symposium Digest, 1973, pp. 221-223
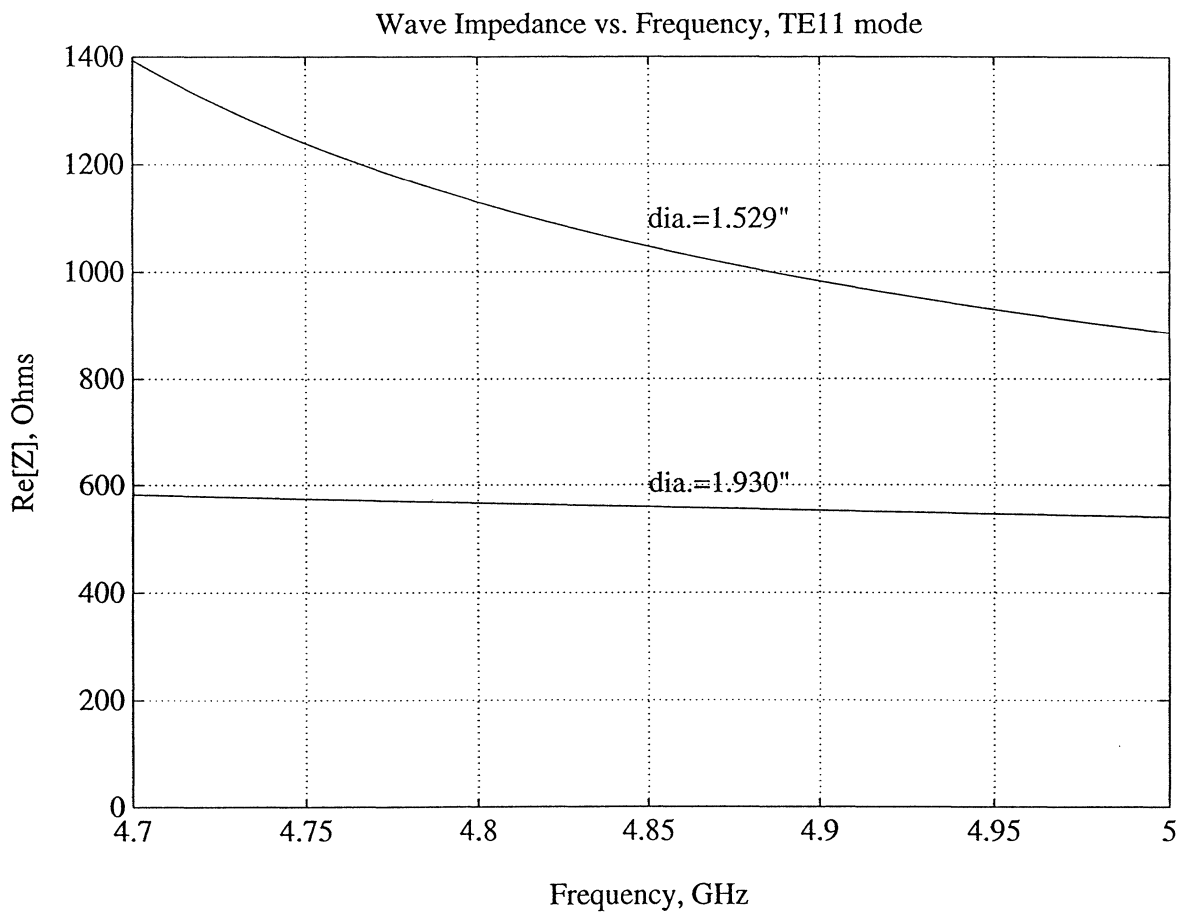
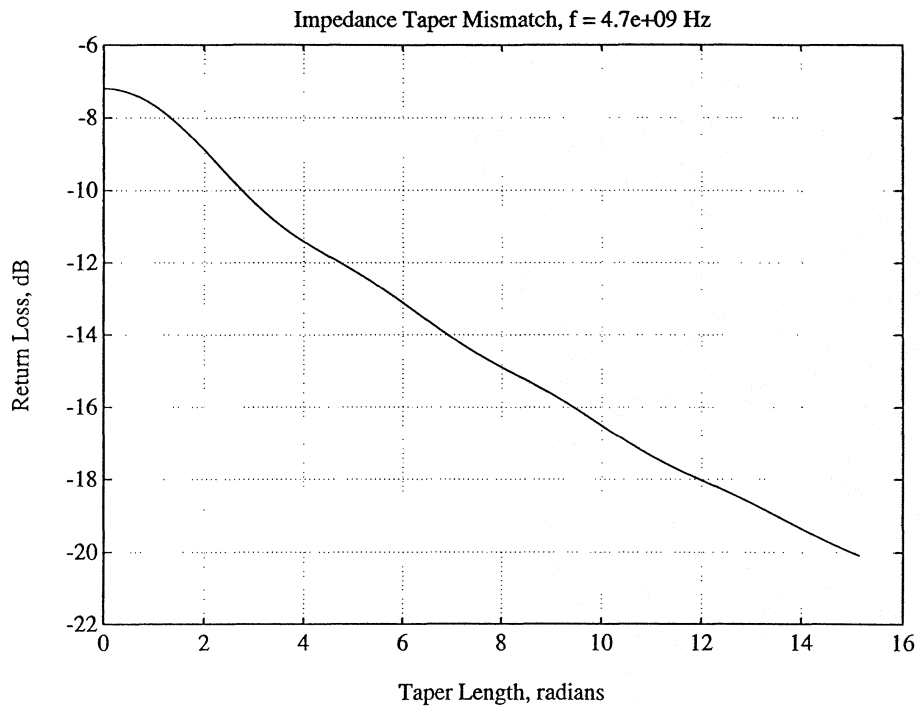Figure 1 - Dominant-mode Impedance Versus Frequency

Impedance Taper Mismatch, f = 4.7e+09 Hz

Figure 2 - Predicted Taper Return Loss Versus Taper Length, 4.7 GHz

Phase Constant, TE11 vs. TEM modes, 4.7 GHz
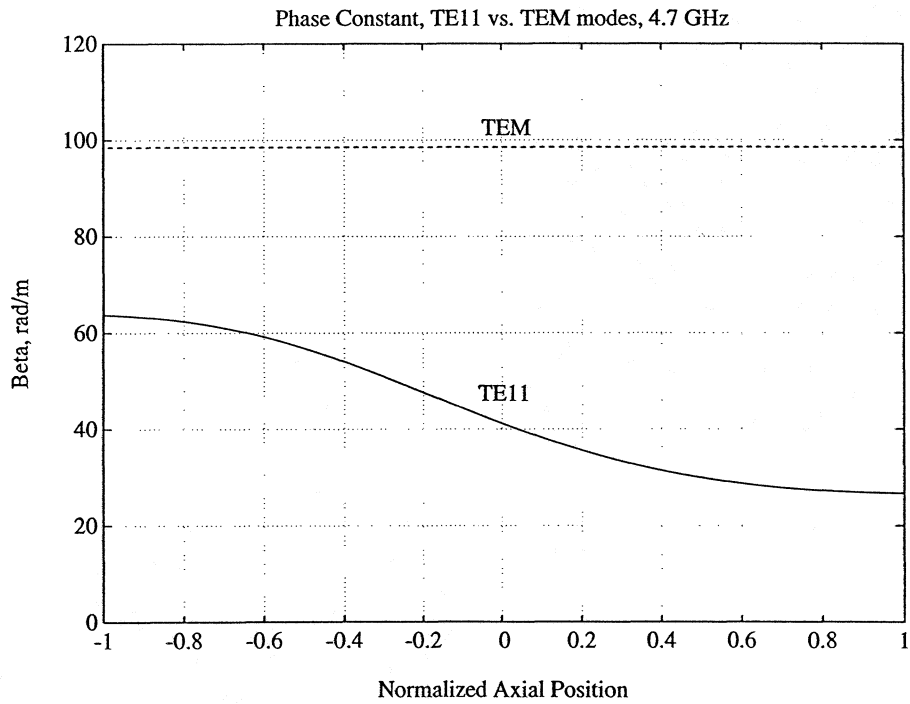
Figure 3 - Phase Constant β Versus Normalized Axial Position, 4.7 GHz

Impedance Taper Mismatch, f = 4.7e+09 Hz
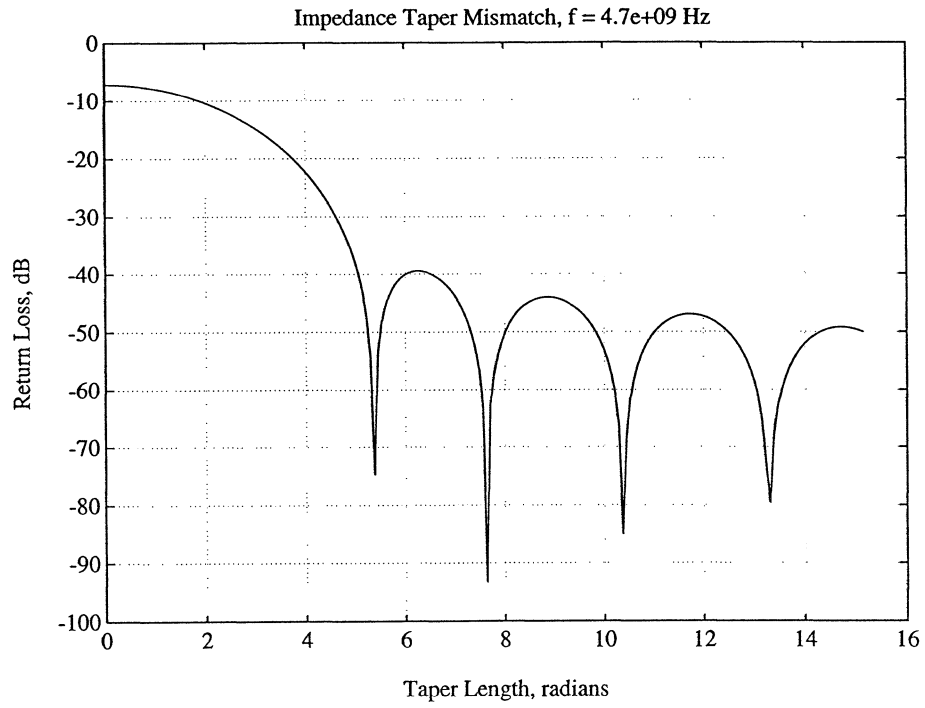
Figure 4 - Predicted Taper Return Loss vs. Taper Length, With Non-Uniform Spacing
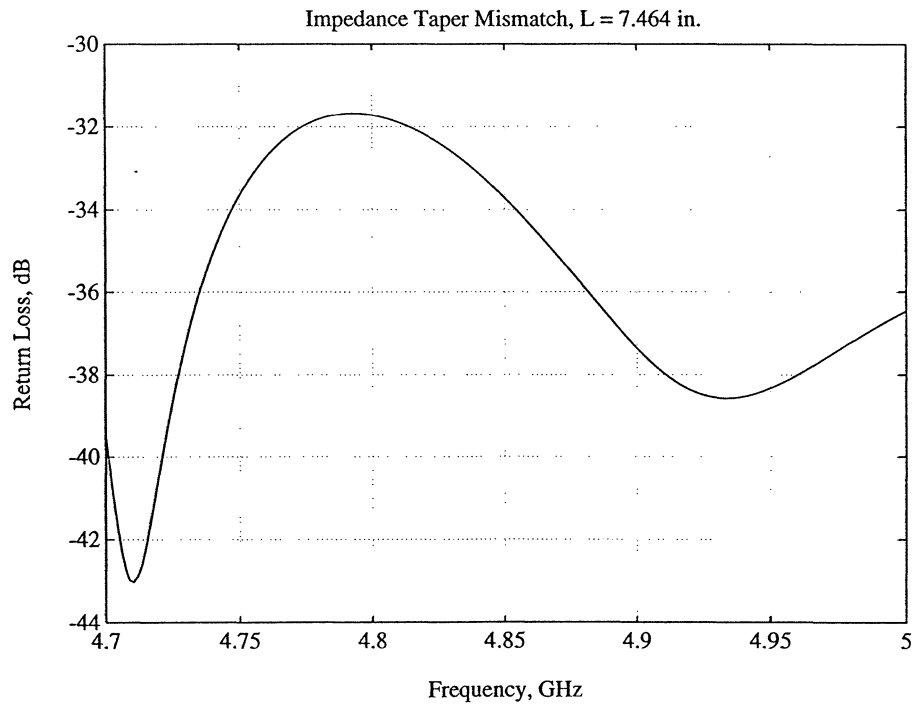
Impedance Taper Mismatch, L = 7.464 in.
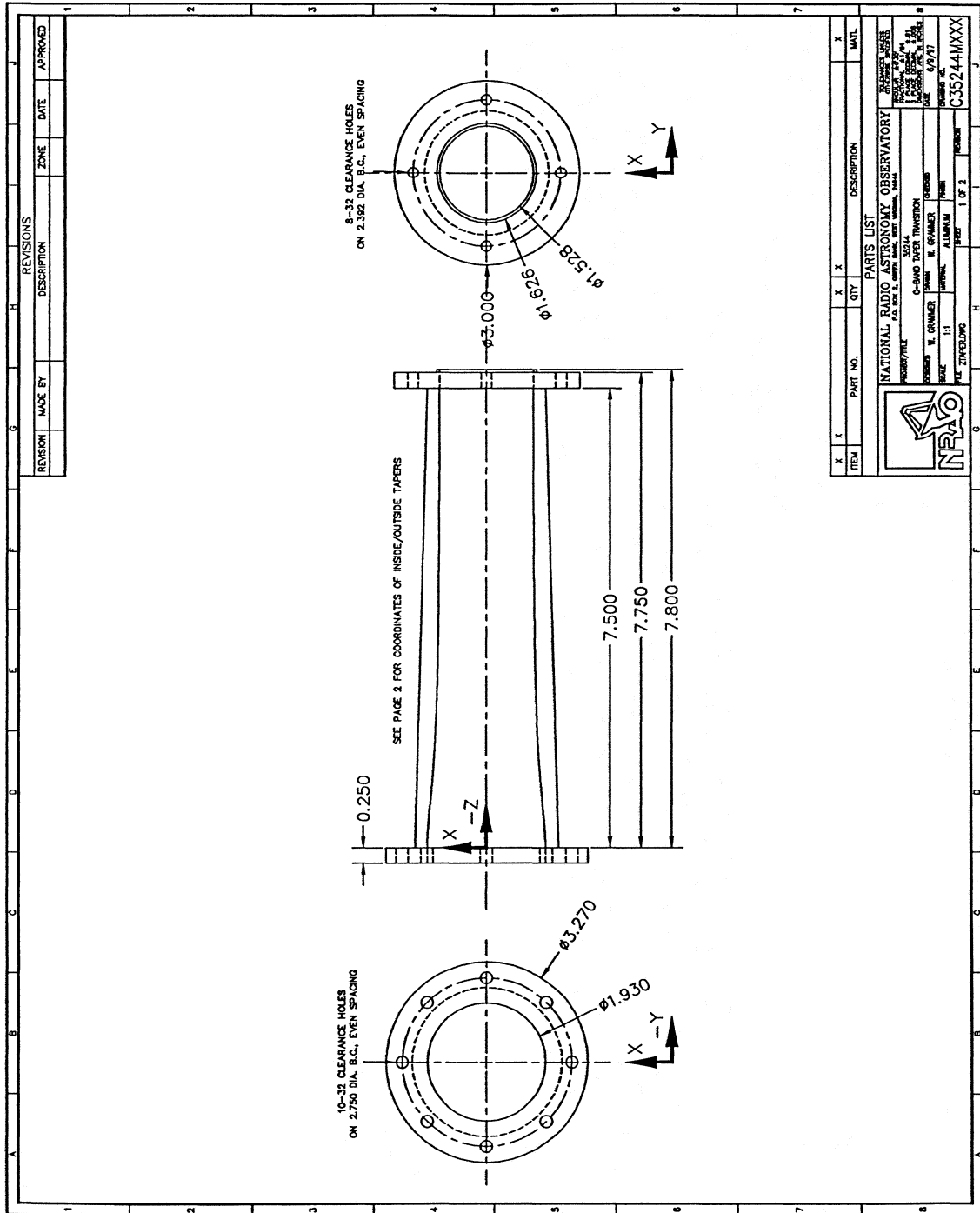
Figure 5 - Predicted Taper Return Loss Versus Frequency

Figure 6 - Mechanical Drawing of Waveguide Taper Transition

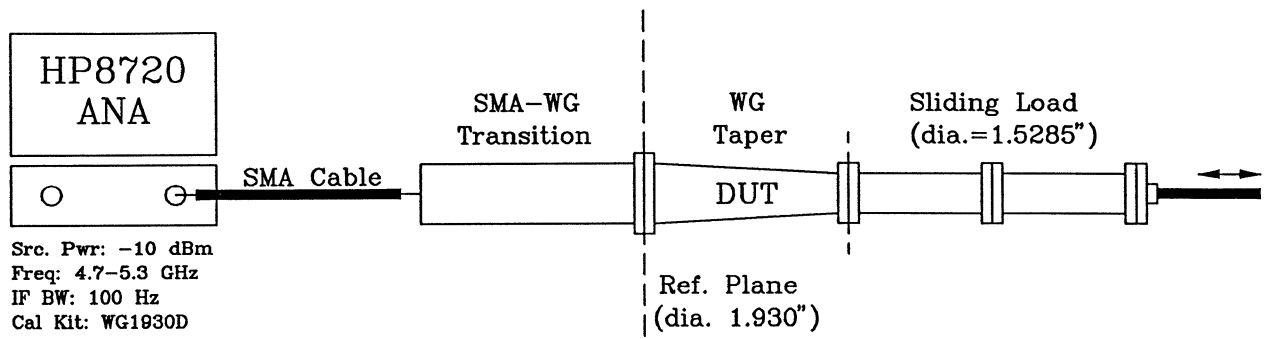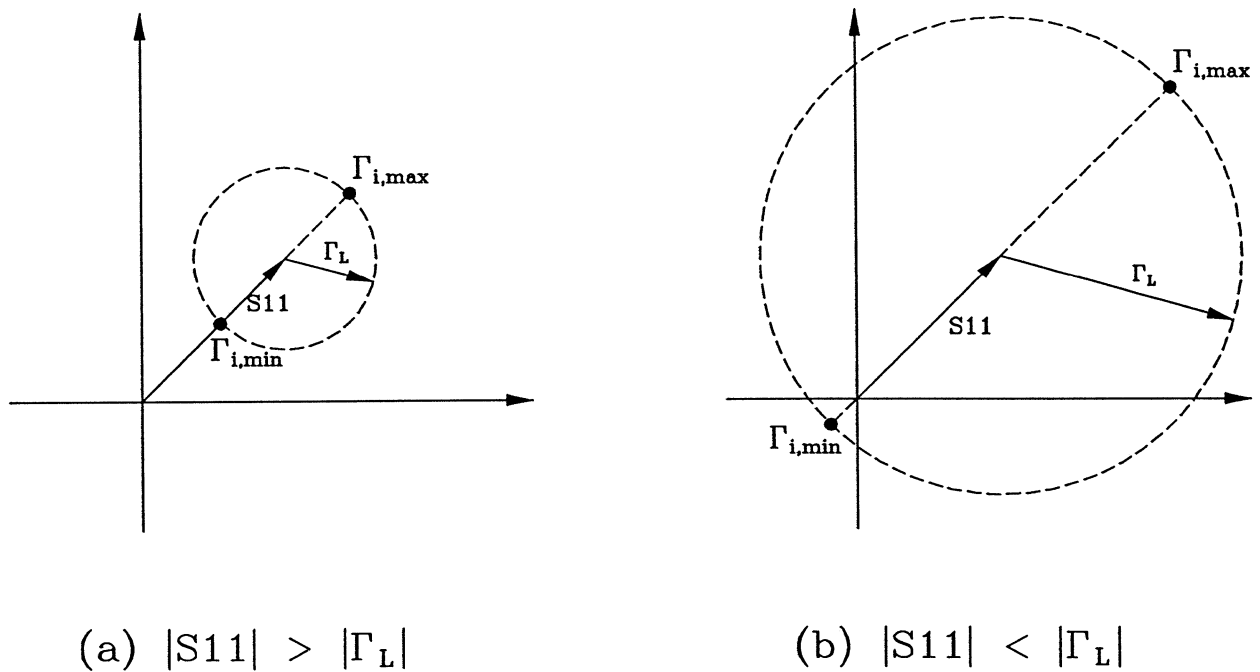Figure 7 - One-Port Measurement Setup, Waveguide Taper Transition

(a) $|S11| > |\Gamma_L|$

(b) $|S11| < |\Gamma_L|$

Figure 8 - Graphical Representation, One-Port Measurement of Taper Transition

Freq. (GHz)	Max. |Γi|, dB	∠Γi, deg.	Min. |Γi|, dB	∠Γi, deg.	|S11|, dB
4.70	-15.63	132.0	*	*	**-33.41**
4.75	-17.97	93.9	-20.93	-85.7	**-34.78**
4.80	-20.28	49.7	-23.17	-132.2	**-37.26**
4.85	-22.79	17.7	-24.76	165.0	**-42.66**
4.90	-25.4	-72.5	-26.24	106.2	**-52.13**
4.95	-26.89	-161.1	-28.58	16.8	**-47.96**
5.00	-27.72	155.0	-30.52	-28.6	**-44.99**
5.05	-29.32	104.4	-33.36	-76.1	**-43.93**
5.10	-30.45	74.1	-35.54	-103.1	**-43.53**
5.15	-31.70	45.6	-36.1	-136.5	**-45.74**
5.20	-33.37	1.8	-38.35	-178.3	**-46.59**
5.25	-34.47	-26.6	-40.05	152.2	**-46.98**
5.30	-36.3	-60.0	-39.9	127.0	**-51.71**

* Two other data points were taken in lieu of the minimum, given below:
$\|S_{11,B}\|$ = -17.14 dB, $\angle S_{11,B}$ = -126.4° ; $\|S_{11,C}\|$ = -17.82 dB, $\angle S_{11,C}$ = 0.62°

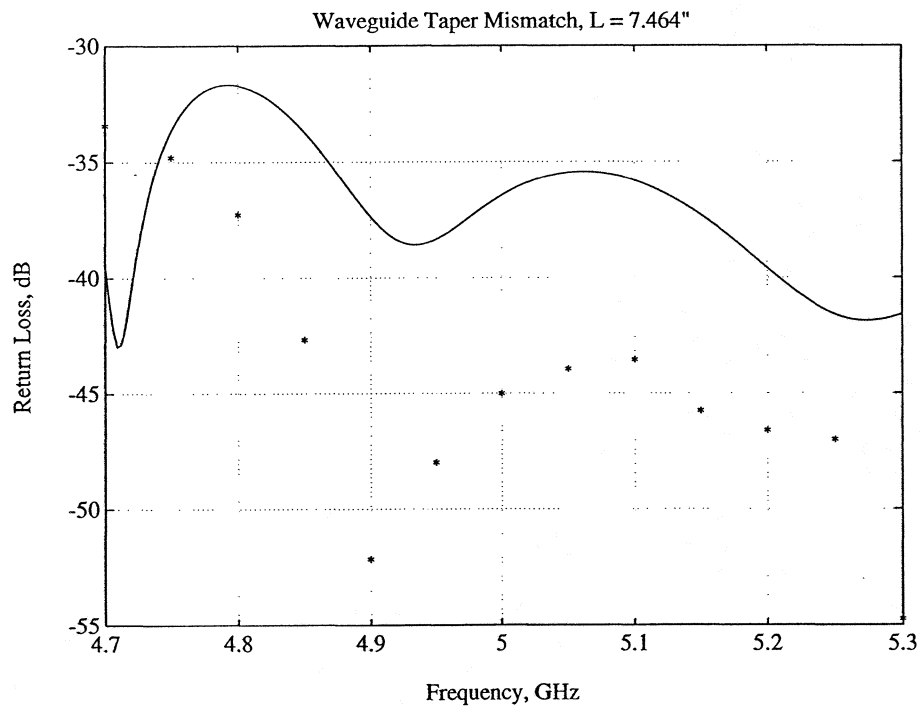Table 1 - Measured Data, Waveguide Taper Transition

Figure 9 - Theoretical vs. Measured Response, Waveguide Taper Transition

APPENDIX

MATLAB Script Files for Taper Transition Design & Analysis

```
%
% ztaper.m - Computes Hecken impedance taper profile
%
% Ref: "A Near-Optimum Matching Section Without Discontinuities",
%       Rudolf P. Hecken, IEEE Trans. Microwave Theory and Tech.,
%       Vol. MTT-20, No.11, November 1972, PP. 734-739
%
% MATLAB .m file, 6/4/97, W. Grammer
%
% Inputs:
%     z1 = input line impedance, ohms
%     z2 = output line impedance, ohms
%            ** OR **
%     d1 = input waveguide diameter, inches
%     d2 = output waveguide diameter, inches
%
%  fmin = minimum passband frequency, GHz
%     rl = worst-case taper return loss, dB
%      n = number of points in taper
%
% Outputs:
%      z = impedance vs. taper position
%   diam = waveguide diameter vs. taper position (if requested)
%  betal = minimum taper length, in radians
%   xpos = relative taper position, inches (if requested)
%----------------------------------------------------------------
%
% Enter inputs
fprintf ('\n Select type of program input:\n');
fprintf (' 1 = Input/Output impedances\n  2 = Circular waveguide input/output diameters\n');
input_type = input ('? ');
fmin = input ('Enter minimum passband frequency, GHz: ');
fmin = 1.0e9*fmin;              % convert to Hz
%
if input_type~=1             % dimensional input
   d1 = input ('Enter input waveguide diameter, inches: ');
   d2 = input ('Enter output waveguide diameter, inches: ');
   %
   % Scale the input values
   a1 = 2.54e-2*d1/2;          % convert to a radius, in meters
   a2 = 2.54e-2*d2/2;          % convert to a radius, in meters
   %
   % Compute TE11 mode impedances
   z1 = mode_z(a1,fmin);  % compute input impedance @fmin
   z2 = mode_z(a2,fmin);   % compute input impedance @fmin
%
else                            % impedance inputs
   z1 = input ('Enter input line impedance, ohms: ');
   z2 = input ('Enter output line impedance, ohms: ');
end
%
rl = input ('Enter desired minimum return loss, dB: ');
n = input ('Enter desired number of points in taper: ');
%
% Calculate required improvement factor
g0=abs(z1-z2)/(z1+z2);          % mismatch without taper
g1=10^(-rl/20);                 % mismatch with taper
```

```
emax = g1/g0;                        % improvement factor
if emax>=1
    error('Min. return loss requirement is already met - no taper needed');
end
global emax;
%
% Solve trancendental eqn. for b_x: emax=(b_x/sinh(b_x))*0.21723
b0 = 0.75*(rl-15)/5+1.2;    % estimate for root finder
b_x = fzero('bterm',b0);
global b_x;
%
% Calculate minimum taper length
betal = (b_x^2+6.523)^0.5;
fprintf(' Minimum taper length = %g radians\n',betal);
%
% Calculate impedance taper function
xi = -1:2/(n-1):1;          % array for variable 'xi'
for cnt=1:n
    gf(cnt) = g(xi(cnt));            % compute G(B,xi)
end
z = exp(0.5*log(z2*z1)+0.5*log(z2/z1).*gf);
%
% Compute physical taper profile, if possible
% Set up fixed constants
xp11 = 1.841;                        % zero of Bessel func. derivative
u0 = 4*pi*1e-7;                      % free-space permeability
e0 = 8.854e-12;                      % free-space permittivity
nu = (u0/e0)^0.5;            % free-space impedance
w = 2*pi*fmin;                       % radian frequency
k = w*(u0*e0)^0.5;                   % wave number
%
% Compute y-coordinate dimension from impedance taper
if input_type~=1                     % dimensional input
    y = mode_a(z,fmin);                  % waveguide radius vs. impedance
    diam = 2*(100/2.54)*y;      % convert to diameter, in inches
    kc=xp11*(diam*1.27e-2).^(-1);   % compute cutoff wave number vs. radius
    beta=(ones(z)*k^2-kc.^2).^0.5;  % compute phase const. vs. length
else                                 % impedance input
    fprintf(' * Physical y-coord. for impedance-only taper is undefined *\n');
    beta=ones(z)*k;             % compute phase const. vs. length
end
%
% Compute x-coordinates of taper profile
beta_min=min(beta);                  % find minimum value of prop. const.
xlen = betal/beta_min;               % required minimum taper length
x = 0:xlen/(n-1):xlen;               % set up array of X-coord. points
xcorr=beta_min*beta.^(-1);           % compute x-coord. weighting function
x=x.*xcorr;                          % apply weighting to axial dimension
xpos=(100/2.54)*x;                   % convert to inches
%
% Compute and plot return loss for the taper
mg=gam_in(x,z,beta,fmin);
```
●●●●●●●●●●●●●●●●●●●●●●●●●●●●●●●●●●

```
%
%  mode_z.m - computes circular waveguide TE11 mode impedance
%          for a given radius 'a', frequency 'f'
%
function z = mode_z(a,f)
%
%  Set up fixed constants
xp11 = 1.841;                  % zero of Bessel func. derivative
u0 = 4*pi*1e-7;                % free-space permeability
e0 = 8.854e-12;                % free-space permittivity
nu = (u0/e0)^0.5;              % free-space impedance
w = 2*pi*f;                    % radian frequency
k = w*(u0*e0)^0.5;             % wave number
denom = 2*pi*a*(u0*e0)^0.5;    % computed constant
fc_te11 = xp11/denom;          % TE11 mode cutoff frequency
%
%  Compute mode impedance
ff = 1-(fc_te11^2)*(f^(-2));
z = nu*ff^(-0.5);              % TE11 mode wave impedance



%
%  mode_a.m - computes circular waveguide radii 'a' for a given
%          TE11 mode impedance array 'z', frequency 'f'
%
function a = mode_a(z,f)
%
%  Set up fixed constants
xp11 = 1.841;                  % zero of Bessel func. derivative
u0 = 4*pi*1e-7;                % free-space permeability
e0 = 8.854e-12;                % free-space permittivity
nu = (u0/e0)^0.5;              % free-space impedance
w = 2*pi*f;                    % radian frequency
k = w*(u0*e0)^0.5;             % wave number
%
%  Compute waveguide radius, in meters
kc = k*(1-(z/nu).^(-2)).^0.5;  % cutoff wave number
a = xp11*kc.^(-1);             % waveguide radius
function y = bterm(x)
y = emax-(x/sinh(x))*0.21723;



%
%  g.m - function to compute G(B,xi), for taper transition design
%
%  Inputs: xi (-1 <= xi <= 1);  b_x is global
%
function y = g(xi)
y = (b_x/sinh(b_x))*quad8('fun1',0,xi);



%
%  fun1.m - defines a function within the integral defining G(B,xi)
%
%  Inputs: xi (-1 <= xi <= 1);  b_x is a global (external) variable
%
function y = fun1(xi)
a = j*b_x*(1-xi.^2).^(0.5);
y = bessel(0,a);
```

```
%
%  gam_in.m - Computes input reflection coef. of continuous
%            impedance taper transition, perfectly terminated
%
%  Inputs:
%        x = taper x-coordinate array
%        z = taper impedance array
%        beta = phase constant array
%        fmin = minimum frequency in Hz
%  Output:
%        mg = input return loss of terminated taper
%
function mg = gam_in(x,z,beta,fmin)
%
npts=201;                       % number of taper lengths to compute
smax=3;                         % maximum no. of taper lengths
sfac=0:smax/(npts-1):smax;      % taper length scaling factors
beta_min=min(beta);             % find minimum value of prop. const.
%
for ii=1:npts                   % calc. refl. coef. for range of taper lengths
   x1=x*sfac(ii);               % scale the taper length first
   bl(ii)=beta_min*max(x1);
   g(ii)=gamma_in(x1,z,beta);   % compute reflection coefficient
end
mg=20*log10(abs(g));            % compute return loss vs. frequency
%
plot(bl,mg);                    % plot results
grid;
xlabel('Taper Length, radians');
ylabel('Return Loss, dB');
title(['Impedance Taper Mismatch, f = ',num2str(fmin),' Hz']);


%
function y = gamma_in(x,z,beta)          % compute refl. coef.
%
n=length(z);
beta2=(beta(1:(n-1))+beta(2:n))/2;  % interpolate between points
x2=(x(1:(n-1))+x(2:n))/2;           % interpolate between points
z2=(z(1:(n-1))+z(2:n))/2;           % interpolate between points
y=0.5*sum(exp(-j*2*beta2.*x2).*diff(z)./z2);
```

```
%
% fswp.m - Computes swept frequency response of continuous
%          impedance taper transition, perfectly terminated
%
% Note: This script requires values calculated by ZTAPER.M
%
npts=201;                        % number of frequencies to compute
fmax=5.0e9;
fdelt=(fmax-fmin)/(npts-1);
freq=fmin:fdelt:fmax;            % set up frequency array
kc=xp11*y.^(-1);         % compute cutoff wavenumber vs. radius
k=2*pi*freq*(u0*e0)^0.5; % compute wavenumber vs. frequency
%
for ii=1:npts                    % calc. refl. coef. for range of frequencies
   beta=(k(ii)^2-kc.^2).^0.5;    % compute phase constant
   g(ii)=gamma_in(x,z,beta);     % compute reflection coefficient
end
mg=20*log10(abs(g));             % compute return loss vs. frequency
%
plot((freq/1.0e9),mg);           % plot results
grid;
xlabel('Frequency, GHz');
ylabel('Return Loss, dB');
title(['Impedance Taper Mismatch, L = ',num2str(100*xlen/2.54),' in.']);
```
•••••••••••••••