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Title: **COMMENTS ON EUL AND SCHIEK, 1991**

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Comments on Eul and Schiek, 1991

John Granlund

Over the past score of years, the microwave literature, particularly the *IEEE Transactions on Microwave Theory and Techniques*, has carried many articles on network analyzer calibration, indicating an extensive and continuing interest in this process. As seems to be more the rule than the exception these days, many of these articles are too cryptic to be understood by someone who has not followed the subject from the very beginning. I have selected Eul and Schiek* as a recent and quite complete paper on network analyzer calibration, and I offer the following comments as answers to the questions that struck me as I first read their article.

1. 4-Port to 2-Port Reduction

Do Eul and Schiek's (1), (2) and (3) really lead to (4), or is this just wishful thinking? They really do lead to (4), as follows: With unnecessary subscripts removed, (1) can be written

$$b_1 = S_{11}\,a_1 + S_{12}\,a_2 + S_{13}\,a_3 + S_{14}\,a_4 \qquad 1>$$

$$b_2 = S_{21}\,a_1 + S_{22}\,a_2 + S_{23}\,a_3 + S_{24}\,a_4 \qquad 2>$$

$$b_3 = S_{31}\,a_1 + S_{32}\,a_2 + S_{33}\,a_3 + S_{34}\,a_4 \qquad 3>$$

$$b_4 = S_{41}\,a_1 + S_{42}\,a_2 + S_{43}\,a_3 + S_{44}\,a_4 \qquad 4>$$

Let the boundary conditions and measurement channel gain expressions from (2) and (3) be written

$$b_2 = \frac{m_1}{\eta_1} \qquad b_4 = \frac{m_2}{\eta_2} \qquad a_2 = \frac{r_1 m_1}{\eta_1} \qquad a_4 = \frac{r_2 m_2}{\eta_2}$$

and used as written to replace b_2, b_4, a_2 and a_4 in 1> through 4>. Discard 1>. There remains

$$m_1 = \eta_1\,S_{21}\,a_1 + r_1\,S_{22}\,m_1 + \eta_1\,S_{23}\,a_3 + \frac{\eta_1 r_2}{\eta_2}\,S_{24}\,m_2 \qquad 2>$$

$$b_3 = S_{31}\,a_1 + \frac{r_1}{\eta_1}\,S_{32}\,m_1 + S_{33}\,a_3 + \frac{r_2}{\eta_2}\,S_{34}\,m_2 \qquad 3>$$

$$m_2 = \eta_2\,S_{41}\,a_1 + \frac{\eta_2 r_1}{\eta_1}\,S_{42}\,m_1 + \eta_2\,S_{43}\,a_3 + r_2\,S_{44}\,m_2 \qquad 4>$$

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*H.-J. Eul and B. Schiek, "A Generalized Theory and New Calibration Procedures for Network Analyzer Self-Calibration," *IEEE Transactions on Microwave Theory and Techniques*, vol. 39, no. 4, April 1991.

Solve 3> for a_1 and substitute in 2> and 4>. After collecting terms, this yields

$$\left[1 + r_1\left(\frac{S_{21}S_{32}}{S_{31}} - S_{22}\right)\right] m_1 + \frac{\eta_1 r_2}{\eta_2}\left(\frac{S_{21}S_{34}}{S_{31}}] - S_{24}\right) m_2$$

$$= \eta_1\left(S_{23} - \frac{S_{21}S_{33}}{S_{31}}\right) a_3 + \eta_1 \frac{S_{21}}{S_{31}} b_3 \qquad 2>$$

$$\frac{\eta_2 r_1}{\eta_1}\left(\frac{S_{32}S_{41}}{S_{31}} - S_{42}\right) m_1 + \left[1 + r_2\left(\frac{S_{34}S_{41}}{S_{31}} - S_{44}\right)\right] m_2$$

$$= \eta_2\left(S_{43} - \frac{S_{33}S_{41}}{S_{31}}\right) a_3 + \eta_2 \frac{S_{41}}{S_{31}} b_3 \qquad 4>$$

Now clearly, the result of solving 2> and 4> for m_1 and m_2 has the same form as Eul and Schiek's (4), which was to be shown. Exact expressions for the A_{ij} of (4) in terms of the S_{ij} and the η_i and r_i are not wanted. These calibration constants will be determined to within a common multiplier by using three of the eight equations that can be written from (14) and (21).

2. Calibration Constants

Up to this point, the matrix equations have mostly involved 2×2 matrices, but now a set of 4 equations and 4 unknowns, described by the 4×4 matrix $\hat{C}$, is sought for evaluation of the calibration constants. To reach (14), it is convenient to start with (13), rewritten as

$$A = Q \times A \times P^{-1} = Q \times A \times R_t\ , \qquad 5>$$

where

$$R = P_t^{-1}$$

is one of the ingredients of $\hat{C}$, as is expressed by (15). Next, 5> is expanded to read

$$\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} \\ Q_{21} & Q_{22} \end{bmatrix} \times \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \times \begin{bmatrix} R_{11} & R_{21} \\ R_{12} & R_{22} \end{bmatrix}$$

$$= \begin{bmatrix} Q_{11}(A_{11}R_{11} + A_{12}R_{12}) + Q_{12}(A_{21}R_{11} + A_{22}R_{12}) & Q_{11}(A_{11}R_{21} + A_{12}R_{22}) + Q_{12}(A_{21}R_{21} + A_{22}R_{22}) \\ Q_{21}(A_{11}R_{11} + A_{12}R_{12}) + Q_{22}(A_{21}R_{11} + A_{22}R_{12}) & Q_{21}(A_{11}R_{21} + A_{12}R_{22}) + Q_{22}(A_{21}R_{21} + A_{22}R_{22}) \end{bmatrix}$$

The four equations for the A_{ij} are now in evidence. If A is subtracted from both sides first, the four equations read

$$0 = (Q_{11}R_{11} - 1)\, A_{11} + Q_{11}R_{12}A_{12} + Q_{12}R_{11}A_{21} + Q_{12}R_{12}A_{22}$$

$$0 = Q_{11}R_{21}A_{11} + (Q_{11}R_{22} - 1)\, A_{12} + Q_{12}R_{21}A_{21} + Q_{12}R_{22}A_{22}$$

$$0 = Q_{21}R_{11}A_{11} + Q_{21}R_{12}A_{12} + (Q_{22}R_{11} - 1)\, A_{21} + Q_{22}R_{12}A_{22}$$

$$0 = Q_{21}R_{21}A_{11} + Q_{21}R_{22}A_{12} + Q_{22}R_{21}A_{21} + (Q_{22}R_{22} - 1)\, A_{22} \; .$$

Notice that matrix $\hat{C}$ of (14) is correctly summarized by (15).

3. The Rank of $\hat{C}$

This question is raised following (15). Comparing (19) with (13), it is seen that

$$A = Y \times X^{-1}$$

is a potential solution of (13). To investigate this solution, I'll write (18) as

$$X = \begin{bmatrix} \alpha_1 x_{11} & \alpha_2 x_{12} \\ \alpha_1 x_{21} & \alpha_2 x_{22} \end{bmatrix} \quad \text{and} \quad Y = \begin{bmatrix} \alpha_3 y_{11} & \alpha_4 y_{12} \\ \alpha_3 y_{21} & \alpha_4 y_{22} \end{bmatrix} \; ,$$

where the α's are free parameters. Then

$$Y \times X^{-1} = \begin{bmatrix} \alpha_3 y_{11} & \alpha_4 y_{12} \\ \alpha_3 y_{21} & \alpha_4 y_{22} \end{bmatrix} \times \frac{1}{\alpha_1\alpha_2(x_{11}x_{22} - x_{12}x_{21})} \begin{bmatrix} \alpha_2 x_{22} & -\alpha_2 x_{12} \\ -\alpha_1 x_{21} & \alpha_1 x_{11} \end{bmatrix}$$

$$= \frac{1}{\alpha_1\alpha_2(x_{11}x_{22} - x_{12}x_{21})} \begin{bmatrix} \alpha_2\alpha_3 x_{22}y_{11} - \alpha_1\alpha_4 x_{21}y_{12} & -\alpha_2\alpha_3 x_{12}y_{11} + \alpha_1\alpha_4 x_{11}y_{12} \\ \alpha_2\alpha_3 x_{22}y_{21} - \alpha_1\alpha_4 x_{21}y_{22} & -\alpha_2\alpha_3 x_{12}y_{21} + \alpha_1\alpha_4 x_{11}y_{22} \end{bmatrix} \; .$$

With $a \equiv \alpha_3/\alpha_1$ and $b = \alpha_4/\alpha_2$,

$$Y \times X^{-1} = \frac{1}{x_{11}x_{22} - x_{12}x_{21}} \begin{bmatrix} a x_{22}y_{11} - b x_{21}y_{12} & -a x_{12}y_{11} + b x_{11}y_{12} \\ a x_{22}y_{21} - b x_{21}y_{22} & -a x_{12}y_{21} + b x_{11}y_{22} \end{bmatrix} \; ,$$

which carries the two free parameters a and b. Thus the rank of $\hat{C}$ must be ≤ 2.

4. Equation (23)

It would have been helpful if, in introducing (23), Eul and Schiek had said "Substituting B -- from (12a) -- in (11) . . . ."

5. Only Seven Unknowns in A and B?

Yes. Because of the form of

$$N_x = A^{-1} \times M_x \times B, \qquad (11)$$

which will be used to evaluate the parameters N_x of a DUT, it is clear that A and B could both be multiplied by the same number without affecting parameter determination.

6. $n \in \mathbf{N}$

This means that n belongs to the set N. Mathematicians sometimes use N to mean the set of positive integers.

7. Equations (30b) and (29b) Provide the Same Information?

Yes. Consider (27), (29b), and (30b), which can be variously written as

$$\left.\begin{array}{c}\det P = \det Q \\ \dfrac{\det N_2}{\det N_1} = \dfrac{\det M_2}{\det M_1}\end{array}\right\} (27) \qquad \left.\begin{array}{c}\det U = \det V \\ \dfrac{\det N_3}{\det N_1} = \dfrac{\det M_3}{\det M_1}\end{array}\right\} (29b) \qquad \left.\begin{array}{c}\det R = \det W \\ \dfrac{\det N_3}{\det N_2} = \dfrac{\det M_3}{\det M_2}\end{array}\right\} (30b)$$

From the second forms, it is clear that (30b) is a direct consequence of (27) and (29b).

8. Potential Difficulties through Section II.C

I find two. First, for good reason, Eul and Schiek describe their two-ports using transmission parameters. (They have used one of several prevalent definitions of these parameters.) But the transmission matrix containing these parameters for a two-port does not exist -- has infinite elements -- unless the two-port has some transmission both ways. I find that if the through transmission is cut back to zero in a limiting process, both the trace and the determinant of the matrix survive in the limit, but matrix elements do not. The DUT presumably has some transmission both ways, so its transmission matrix should not create any difficulties, but when certain simple calibration standards -- open, short, or match at each port with no through transmission -- are used, what transmission matrix should be used in (12) to describe the standards? Eul and Schiek have been careful to answer these questions. In Section II.E they discuss calibration procedures that use a third standard without transmission and in II.F, procedures in which only the first standard has transmission.

Second, equation (17). If the eigenvalues of (16) are distinct, the eigenvectors appearing in the columns of X and Y are independent. This allows X and Y to be inverted and the analysis to proceed. But if the two eigenvalues are identical and the original matrix -- P or Q -- is not Hermitian, then its eigenvectors are not independent and the invertible X and Y of (17) do not exist. Doesn't this block further analysis in these special cases? Fortunately not! The work of Eul and Schiek does not require that P and Q be similar to the same <u>diagonal</u> matrix as in (17), only that they be similar to the same matrix. In the present special cases with both eigenvalues equal to λ, the Jordan canonical form

$$J = \begin{bmatrix} \lambda & 1 \\ 0 & \lambda \end{bmatrix}$$

is a good candidate. As a good book on linear algebra will explain, every matrix is similar -- as in (17) -- to a matrix in Jordan canonical form. I had better say that the diagonal matrix Λ of (16) is also a Jordan canonical form. The 1 in J above conveys the fact that the eigenvectors of P and Q are not independent.