NATIONAL RADIO ASTRONOMY OBSERVATORY

ELECTRONICS DIVISION TECHNICAL NOTE NO. 112

TITLE: VANE TYPE POLARIZATION CONVERTER

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DATE: July 28, 1982

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<u>VANE TYPE POLARIZATION CONVERTER</u>

Charles J. Brockway

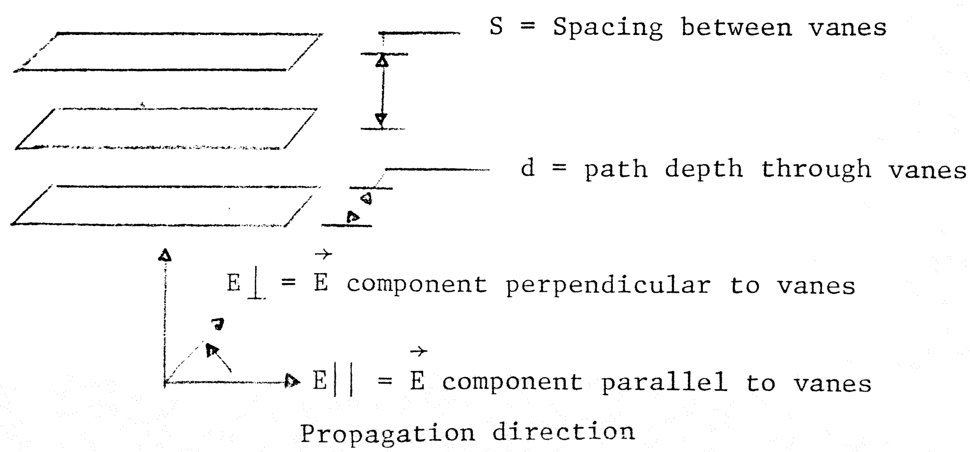
Introduction

One of the easiest and least lossy ways to convert between circular and linear polarizations is to use a set of parallel vanes placed in front of a feed aperture. The method was first described to me by Joe Carter at Haystack. The performance of an S-band polarizer of this type is briefly given in JPL Space Programs Summary 37-63, Volume II, by H. R. Buchanan. Several stations in the USA VLBI Network use the polarizers at various frequency bands, including the 140-ft Cassegrain system.

Several considerations come into mind when first using these devices. First, to fully understand how they work, the design equations should be able to be derived. Second, in making specifications for shop fabrication and feed mounting, an idea of the allowable uncertainties in vane dimensions and orientation is required. Third, the bandwidth and isolation between polarization senses should be investigated.

Looking through the literature indicates that these type polarizers have apparently not been widely used and no explicit design or analysis information could be found. This note is an attempt to provide at least part of the needed information.

1. Geometry



Assume $E_{\perp}$ unbounded or free space behavior.

2. Phase Shift

$$\phi_\perp = \beta_\perp d = (\omega/v_c)d \text{ (radians)}$$

$$\phi_{||} = \beta_{||} d = (\omega/v_p)d \text{ (radians)}$$

where:

$\phi_\perp$ = radians phase displacement of perpendicular component passing through vanes.

$\phi_{||}$ = radians phase displacement of parallel component passing through vanes.

v_c = velocity of light ($\perp$ component unbounded).

v_p = phase velocity ($||$ component bounded by vane spacing so $v_p > v_c$).

ω = radian frequency.

β = propagation constant

$$\phi_\perp - \phi_{||} \equiv \Delta\phi = \omega d\,(1/v_c - 1/v_p).$$

$$\Delta\phi = (\omega d/v_p)(v_p/v_c - 1) \quad \underline{\text{or}} \quad \boxed{\Delta\phi = \frac{2\pi d}{\lambda}\,(1 - v_c/v_p)} \qquad (1)$$

where

λ = free space wavelength.

Hence: $\boxed{\phi_{||} \text{ advanced relative to } \phi_\perp}$

Note: $\phi_\perp > \phi_{||}$ since $v_p > v_c$.

3. Phase Velocity of Parallel Component

From Wave Equation for $TE_{M,0}$ modes in rectangular waveguide:

$$\beta_g = \sqrt{\frac{\omega^2}{v_c{}^2} - \frac{m^2\pi^2}{s^2}} \qquad (\mu = \varepsilon = 1)$$

where β_g = propagation constant for $\vec{E}$ bounded by vanes.

$$\beta_g = \pi\sqrt{\frac{4}{\lambda^2} - \frac{m^2}{s^2}}$$

But $v_p = \omega/\beta_g$ so:

$$\boxed{v_p = \frac{2\,v_c}{\lambda\,\sqrt{4/\lambda^2 - m^2/s^2}}} \tag{2}$$

4. <u>$\Delta\phi$ in Terms of d and s</u>

Rearranging Equation (2):

$$v_c/v_p = \sqrt{1 - \left(\frac{m\lambda}{2s}\right)^2}$$

Substituting into Equation (1):

$$\boxed{\Delta\phi = \frac{2\pi d}{\lambda}\left(1 - \sqrt{1 - \left(\frac{m\lambda}{2s}\right)^2}\right)} \tag{3}$$

Equation (3) gives the phase lead of the parallel component relative to the perpendicular component for $TE_{M,0}$ mode.

5. <u>Design Equations for Polarizer</u>

For M = 1 ($TE_{1,0}$ mode)

$$\Delta\phi = \pi/2 \text{ radians}$$

Equation (3) becomes:

$$\boxed{d = \frac{\lambda/4}{1 - \sqrt{1 - (\lambda/2s)^2}}} \tag{4}$$

Note $d = \dfrac{\lambda/4}{1 - v_c/v_p}$

Transforming Equation (4):

$$\boxed{s = d\sqrt{\frac{\lambda}{2\,d - \lambda/4}}} \qquad (5)$$

6. Constraints

a. Want $s < \lambda$ to prevent higher modes. (Note higher modes give incorrect phase shift.)

b. Must have $s > \lambda/2$ for transmission of dominant mode.

c. Note smaller s results in increased feed aperture blockage with reduction in system efficiency.

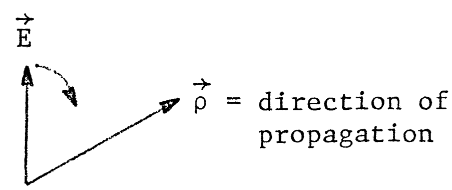
The optimum spacing is taken to be $0.8\ \lambda$.

7. Circular Polarization Sense

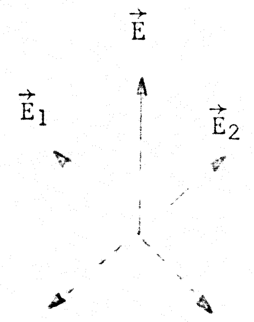
The IEEE convention for a circularly polarized wave receding from an observer is:

RCP - $\vec{E}$ rotation clockwise.

LCP - $\vec{E}$ rotation counter clockwise.



The rotating $\vec{E}$ vector can be represented by two linearly polarized correlated components equal in amplitude and in time and space quadrature.



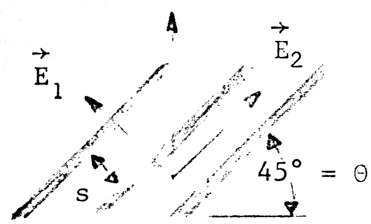
RCP - $\vec{E}$ rotating clockwise. $\vec{E}_1$ leads $\vec{E}_2$ by $\pi/2$.

LCP - $\vec{E}$ rotating counter-clockwise. $\vec{E}_1$ lags $\vec{E}_2$ by $\pi/2$.

When $\vec{E}_1$ and $\vec{E}_2$ are brought into time phase, $\vec{E}$ ceases to rotate, i.e., $\vec{E}$ becomes linearly polarized by $\pi/4$ to $\vec{E}_1$ and $\vec{E}_2$.

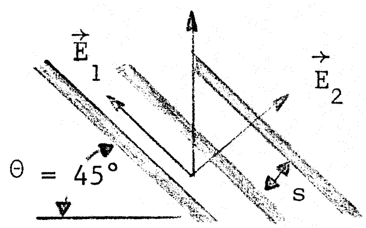
The polarizer advances the phase of the E component parallel to the vanes relative to the perpendicular component. Hence, looking into the feed aperture from the outside:

$\vec{E}$ of Feed Output



Feed	RCP
Prime Focus	LCP Sky
Cass Focus	RCP Sky

$\vec{E}$ of Feed Output



Feed	LCP
Prime Focus	RCP Sky
Cass Focus	LCP Sky

8. Vane Rotation Effect

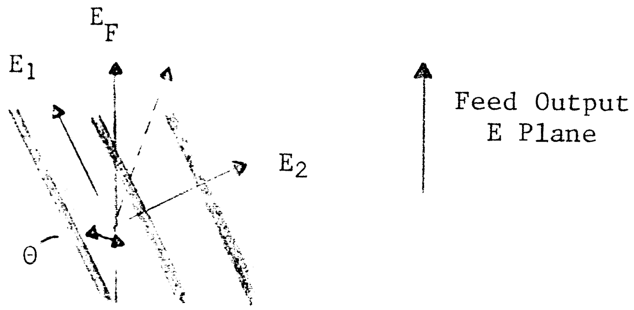
The polarizer is normally used to receive one sense of circular polarization while rejecting the opposite sense. It is desired to derive an expression showing how the wanted to unwanted polarization ratio varies with vane orientation. Let:

Θ = angle between vanes and feed output E plane.

$\vec{E}_1$, $\vec{E}_2$ = perpendicular and parallel (to the vanes) components of the circularly polarized field after passing through polarizer.

E_F = field magnitude in feed output E plane after passing through polarizer.

Assume $\Delta\phi = \pi/2$.



(a) $\vec{E}_1$ and $\vec{E}_2$ are in time phase for the desired polarization sense. Then:

$$E_F = |\vec{E}_1| \cos\Theta + |\vec{E}_2| \sin\Theta$$

but

$$|\vec{E}_1| = |\vec{E}_2| = E \quad \text{for pure circular polarization}$$

$$E_F = E\,(\cos\Theta + \sin\Theta)$$

The received power is:

$$P_R = k\,E_F^{\,2} = k\,E^2\,(\cos\Theta + \sin\Theta)^2$$

$$P_R = k\,E^2\,(1 + \sin 2\Theta)$$

The signal power is:

$$P_S = k\,|\vec{E}_1|^2 + k\,|\vec{E}_2|^2 = 2k\,E^2$$

Then:

$$\boxed{\frac{P_R}{P_S} = \frac{1 + \sin 2\Theta}{2}} \qquad (6)$$

Note if:

$\Theta = 45°$ $\quad$ $P_R/P_S = 1$ as it should for a completely polarized wave received by a matching polarized feed.

$\Theta = 0$ or $90°$ $\quad$ $P_R/P_S = 1/2$ or 3 dB down as it should for a circularly polarized wave received by a linearly polarized feed.

(b) E_1 and E_2 are 180° out of time phase for the undesired polarization sense. Then:

$$E_F = |\vec{E}_1| \cos \Theta - |\vec{E}_2| \sin \Theta$$

$$E_F = E (\cos \Theta - \sin \Theta)$$

$$P_R = k E^2 (1 - \sin 2\Theta)$$

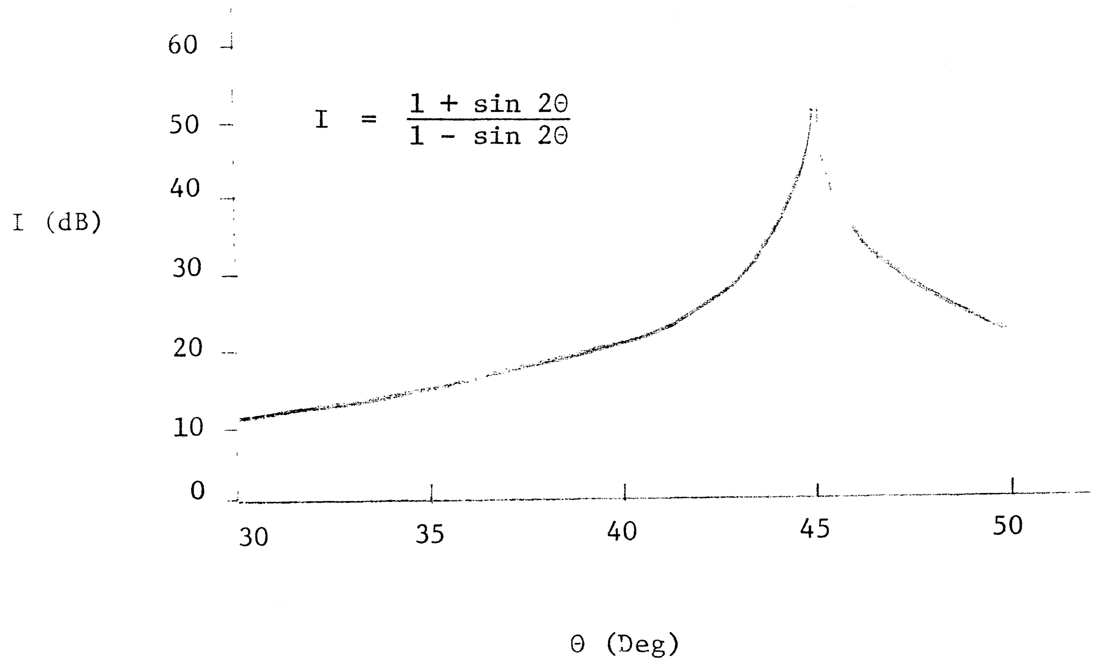
$$\boxed{P_R/P_S = \frac{1 - \sin 2\Theta}{2}}$$

(c) It follows that the ratio of desired to undesired polarization (isolation) is given by:

$$\boxed{\frac{\text{Desired}}{\text{Undesired}} = I = \frac{1 + \sin 2\Theta}{1 - \sin 2\Theta}} \qquad (8)$$

$$\boxed{\Theta = \frac{1}{2} \sin^{-1} \frac{I - 1}{I + 1}} \qquad (8a)$$

(d) A plot of Equation (8) shows that the isolation with Θ is more critical than might be expected from the absolute dependence of received power with Θ.



9. Bandwidth

It is possible to derive an expression for obtaining the ideal polarization bandwidth by differentiating Equation (3) with respect to wavelength.

$$\frac{d\,(\Delta\phi)}{d\,\lambda} = \frac{2\pi d}{\lambda^2}\left[1 - \frac{\sqrt{1-(\lambda/2s)^2}}{\sqrt{1-(\lambda/2s)^2}}\right] \quad \text{for the dominant mode}$$

Substituting Equation (4):

$$\frac{d\,(\Delta\phi)}{d\lambda} = \frac{\pi}{2\lambda}\,\frac{1}{\sqrt{1-(\lambda/2s)^2}}$$

Transposing:

$$\boxed{\frac{d\lambda}{\lambda} = \frac{2}{\pi} \sqrt{1 - (\lambda/2s)^2} \quad d(\Delta\phi) = \text{Bandwidth Factor.}} \tag{9}$$

It is noted that the bandwidth factor is a function of vane spacing, s, and phase shift differential, $d(\Delta\phi)$. The dependence on $d(\Delta\phi)$ is expected since frequencies away from center undergo a different phase shift in passing through the polarizer.

It follows that the polarizer bandwidth can only be stated for a given phase shift differential. In turn, the phase shift differential fixes the polarizer circularity isolation.

What is needed is a relation showing isolation and $d(\Delta\phi)$. This can be obtained by extending the derivation of Equations (6) and (7) to include the phasor (time) sum in addition to the vector (space) sum of the field components.

The result is:

$$\boxed{I = \left(\frac{\cos\Theta + \sin\Theta \cos d(\Delta\phi) + j \sin\Theta \sin d(\Delta\phi)}{\cos\Theta - \sin\Theta \cos d(\Delta\phi) - j \sin\Theta \sin d(\Delta\phi)} \right)^2} \tag{10}$$

where

Θ = angle between vanes and feed output E plane.

$d(\Delta\phi)$ = differential phase shift = $|\pi/2 - \Delta\phi|$

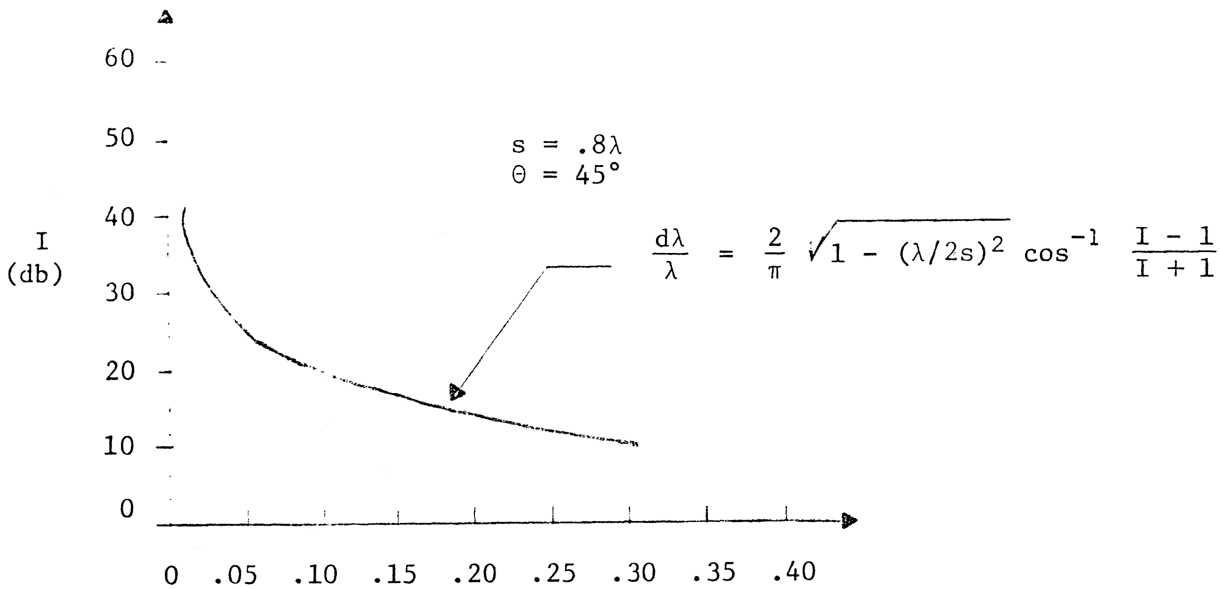
If $d(\Delta\phi) = 0$, Equation (10) becomes Equation (8) as it must since $\Delta\phi = \pi/2$ exactly was assumed for the Equation (8) derivation.

If Θ is set to its normally used position of 45° Equation (10) becomes:

$$\boxed{I = \left| \frac{1 + \cos d(\Delta\phi)}{1 - \cos d(\Delta\phi)} \right|} \tag{11}$$

$$\boxed{d(\Delta\phi) = \cos^{-1} \frac{I - 1}{I + 1}} \tag{11a}$$

Using Equation (11) to find I with $d(\Delta\phi)$ and then returning to Equation (9) and taking $s = 0.8\lambda$ (recall constraints), a plot can be made showing the ideal behavior of isolation with bandwidth factor.



Bandwidth Factor $d\lambda/\lambda$

It is seen that the bandwidth factor is about 3% for 30 dB circularity isolation indicating an inherently narrow band characteristic for the polarizer.

Conclusions

1. Normal design procedure is to chose vane spacing so $s = 0.80\lambda$. Depth is then calculated using Equation (4).
2. Vane thickness should be minimized consistent with sufficient rigidity to maintain spacing.
3. Isolation between circular polarization senses is moderately dependent on vane orientation to the feed output E plane. The angle should be $45 \pm 2°$ for 30 dB isolation.
4. Bandwidth is about $\pm$ 3% for 30 dB isolation and $\pm$ 10% for 20 dB isolation for $s = 0.80\lambda$. The 30 dB isolation bandwidth decreases to $\pm$ 2.2% for $s = 0.60\lambda$ and increases to $\pm$ 3.5% for $s = \lambda$.
5. Tolerances for vane spacing and depth are somewhat critical for good isolation. Calculations show spacing is 2 to 3 times more critical than depth. Spacing and depth tolerances of 1% and 2%, respectively, from design center values, are necessary for about 30 dB isolation. For 20 dB isolation, the respective tolerances can be relaxed to 2.5% and 5%.