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Autocorrelator Sampler Level Setting and Transfer Function

J. R. Fisher

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Introduction

This note is a brief outline of how the input levels of the three- and nine-level samplers in the GBT spectrometer (autocorrelator) are set to near-optimum values and how the transfer functions of these samplers are linearized to produce autocorrelation function outputs that are proportional to input power.

Sampler voltage transfer functions

The GBT spectrometer has two sets of signal voltage samplers. There are 8, three-level samplers that run at sampling rates up to 1600 mega-samples per second and 32, nine-level samplers that sample up to 100 mega-samples per second. The selectable sample rates are such that the spectrometer can produce spectral bandwidths of 800, 200, 50, and 12.5 MHz. The analog voltage to digital output transfer functions of the three- and nine-level samplers are shown in Figure 1.

The input voltage scales in Figure 1 assume that the input levels have been set to their optimum values such that the first sampler threshold is 0.612 and 0.267 times the rms value of the noise voltage for the three- and nine-level samplers, respectively. (These optimum values were taken from Ray Escoffier's sampler level diagram and an internal Arecibo memo by Murray Lewis (1977). Fred Schwab will have a more complete discussion of sampler properties in connection with the revised quantization corrections now under development.) However, the samplers can operate reasonably well with input levels other than those that produce optimum sensitivity so the horizontal scales in Figure 1 will change accordingly.

Setting sampler input levels

The two monitor points available to the Monitor and Control computers for setting the input levels to the spectrometer samplers are analog power detectors at

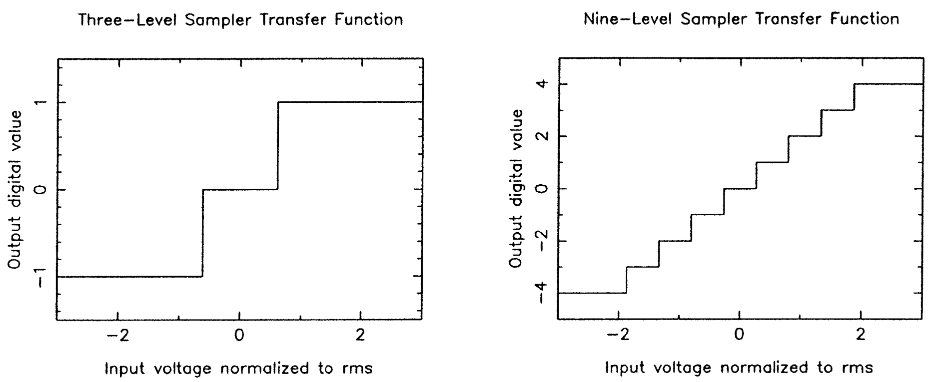

Figure 1: Analog input voltage to digital output transfer functions for the three- and nine-level samplers used in the GBT spectrometer.

the output of the Analog Filter Rack and counts of the total number of samples in the -1, 0, and $+1$ sampler output levels. For the nine-level samplers the -1, 0, and $+1$ counts correspond to aggregates of the (-4, -3, -2), (-1, 0, $+1$), and ($+2$, $+3$, $+4$) sampler output states, respectively. The analog power detectors are used to set the sampler input levels to within a few dB of the desired value, and the ratio of the counts in the zero output state to the sum of counts in the -1 and $+1$ states is used to trim the level to its final setting.

If we assume that the sampler input voltage has a normal statistical distribution of amplitudes, the probability of measuring a given instantaneous amplitude, v, is given by

$$p(v) = \frac{1}{\sqrt{2\pi}\sigma} e^{0.5v^2/\sigma^2}, \tag{1}$$

where σ is the rms value of the voltage amplitude, and the normalization factor ahead of the exponential sets the integral of probabilities over all voltages equal to unity. If we also assume that the input voltage boundaries, v_1 and v_2, between the -1 and 0 and the 0 and $+1$ output states are symmetrically placed about zero volts ($v_1 = -v_2$), as is nominally the case for the GBT spectrometer, then the ratio of 0 value counts to ± 1 counts is given by

$$R = \frac{\int_0^{v_1} e^{0.5v^2/\sigma^2} dv}{\int_{v_1}^{\infty} e^{0.5v^2/\sigma^2} dv}, \tag{2}$$

Figure 2 shows the results of a numerical integration of Equation 2 for a range of input levels. The ordinate of Figure 2 is the offset, in decibels, from the optimum input level given by $v_1 = 0.61\sigma$ for the three-level sampler and $v_1 = 0.801\sigma$ for the nine-level sampler. Note that the nine-level v_1 used here is three times the 0/+1 output state boundary because the measured counts ratio is for the aggregate (-4, -3, -2), (-1, 0, $+1$), and ($+2$, $+3$, $+4$) output states. The two curves in Figure 2 are closely approximated by the following

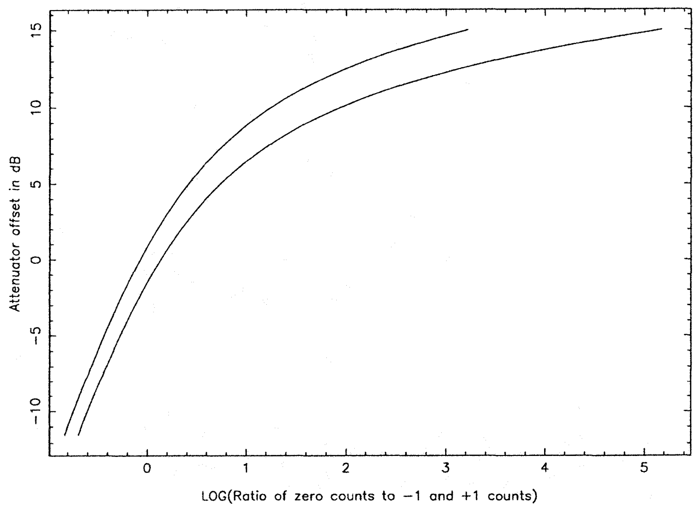

Figure 2: Input attenuation offset for optimum input level as a function of the ratio of 0 to ± 1 counts from the spectrometer sampler. The top curve is for the three-level sampler, and the bottom curve is for the nine-level sampler. More positive attenuation offset corresponds to lower input level and, hence, a higher 0 to ± 1 counts ratio.

polynomial:

$$A(R) = a_0 + a_1 Log(R) + a_2 Log(R)^2 + a_3 Log(R)^3 + a_4 Log(R)^4, \qquad (3)$$

where $A(R)$ is in dB, the coefficients for the three-level sampler are

$a_0 = 0.83464$
$a_1 = 11.38420$
$a_2 = -3.91117$
$a_3 = 0.61511$
$a_4 = -0.02205,$

and the coefficients for the nine-level sampler are

$a_0 = -1.50170$
$a_1 = 11.39038$
$a_2 = -4.09134$
$a_3 = 0.77270$
$a_4 = -0.05582.$

Equation 3 may be used by the control computer to convert measured count ratios to attenuator corrections.

Sampler power transfer functions

Digital samplers are inherently non-linear devices so we expect their power transfer functions to be non-linear. To use the outputs of the samplers to measure spectral power density this non-linearity must be corrected over the expected operating range of the devices. The uncorrected, integrated power output or zero-lag autocorrelation value from the sampler is

$$P_u = \frac{\sum_i N_i S_i^2}{\sum_i N_i}, \tag{4}$$

where N_i is the number of samples having a digital value of S_i, with i being the index of the possible values. For the nine-level sampler Equation 4 expands to

$$P_u = \frac{16(N_{-4}+N_4)+9(N_{-3}+N_3)+4(N_{-2}+N_2)+N_{-1}+N_1}{N_{-4}+N_4+N_{-3}+N_3+N_{-2}+N_2+N_{-1}+N_1+N_0} \tag{5}$$

For a noise signal with a normally distributed amplitude probability the count N_i is the product of the total number of samples times the integral of Equation 1 between the input voltage boundaries of sampler interval i. If we compute the values for Equations 4 and 5 when the noise inputs are set to their optimum levels ($v_1 = \pm 0.61\sigma$ for three-level sampling and $v_n = \pm 0.267\sigma$, $\pm 0.801\sigma$, $\pm 1.335\sigma$, $\pm 1.868\sigma$ for nine-level sampling), we get the values

$$P_u(opt, 3level) = 0.5405 \tag{6}$$

$$P_u(opt, 9level) = 3.401 \tag{7}$$

These are the values that one expects in the zero-lag output of the spectrometer when the input level is optimized, and they can be used as a check on the correct setting of the input attenuators given by Equation 3. Note that the GBT spectrometer's nine-level autocorrelation output values are scaled by a factor of 1/16 from the assumptions made here so the raw data, optimum, zero-lag value will be 3.401 / 16 = 0.2126.

Figure 3 shows the power transfer function computed for the three- and nine-level samplers by numerical integrations of Equations 1 and 4 for a range of input levels. The scales on both input and output axes of this figure are normalized to their values at the optimum input level, where the normalizing optimum output level values are given by Equations 6 and 7.

As one would expect, the output powers shown in Figure 3 fall away from a linear function at high input levels because the signal voltage waveform runs off the end of the sampling range. The tangent of neither curve near the optimum input levels passes near zero so a correction for the non-linear transfer function is required to be able to compute meaningful differential-to-total power ratios as is done in computing system noise temperatures from noise calibration on-off measurements. A close approximation to the nine-level power transfer curve over a roughly ± 10 dB range from optimum input level is given by a polynomial of the form

$$P_{norm} = C_0 + C_1 P_u + C_2 P_u^2 + C_3 P_u^3 + C_4 P_u^4 + C_5 P_u^5 + C_6 P_u^6 + C_7 P_u^7 \tag{8}$$

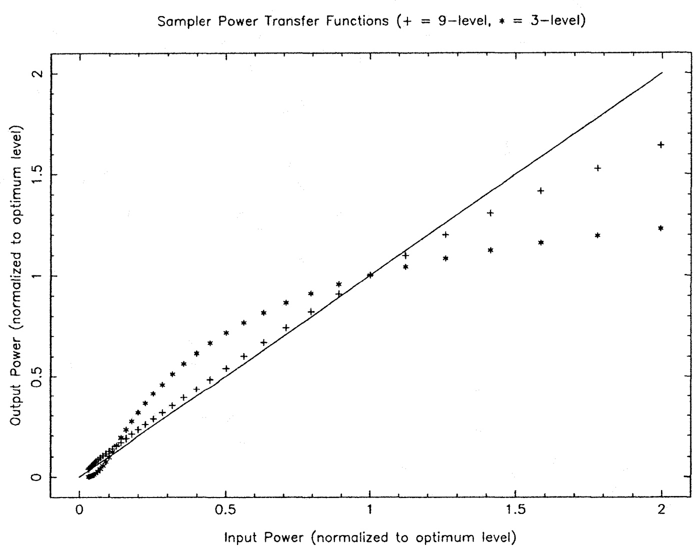

Figure 3: Power transfer functions of the three-level (asterisks) and nine-level (crosses) samplers, assuming normally distributed noise input. The input and output scales are normalized to their values at optimum input levels. The straight line is a reference showing a linear power transfer function.

where P_u is the unnormalized (zero lag) sampler output, P_{norm} is the input power normalized to the optimum value, and the coefficients are

C_0 = -0.03241744594
C_1 = 4.939640303
C_2 = -5.751574913
C_3 = 34.83143031
C_4 = -78.66637472
C_5 = 213.7108496
C_6 = -317.1011469
C_7 = 245.8618017

As noted in the "GBT Spectrometer Software Data Processing" document (April 26, 2001), there is a closed form expression for the three-level power transfer curve.

$$P_{norm} \propto 2 \times [erfc^{-1}(P_u)]^{-2}, \tag{9}$$

where $erfc^{-1}()$ is the inverse complimentary error function. This is approximated as a polynomial by the following algorithm:

$$x = (1 - P_u)^2 - 0.5625 \tag{10}$$

$$y = (1 - P_u)\frac{1.591863138 - 2.442326820x + 0.37153461x^2}{1.467751692 - 3.013136362x + x^2} \tag{11}$$

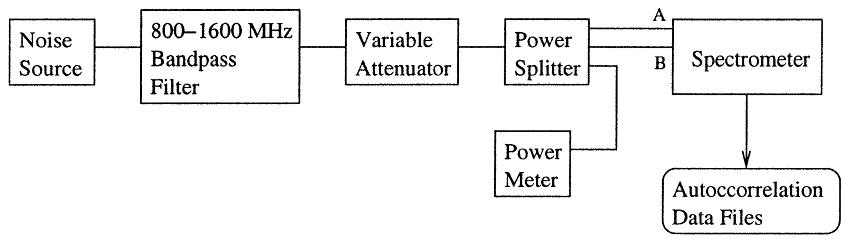

Figure 4: Setup for measuring three-level sampler power transfer characteristics.

$$P_{norm} = \frac{0.3745443672}{2y^2} \tag{12}$$

Three-level sampler measurements

To compare of the calculated curves given above to the operation of a real sampler, Rich Lacasse measured the power transfer function of two samplers in the GBT spectrometer using the experimental setup in Figure 4. The procedure was to set the input noise power to the samplers with the variable attenuator and record an autocorrelation function from each sampler. The noise power was varied from about -6 dB to +6 dB relative to the nominally optimum sampler input level. The power meter resolution was 0.01 dB.

Figure 5 shows the zero-lag autocorrelation value from both samplers plotted as a function of relative power meter reading. Since we did not have an accurate way of determining the exact rms voltage relative to the sampler quantization levels, the reference level of the input power values for each sampler has been adjusted by a constant amount to align the two data sets. The constant offsets are 0.0 and 0.3 dB relative to the power that was thought, *a priori*, to be the optimum value for samplers A and B, respectively. This adjustment is within the tolerance of the experimental setup so it seems like a reasonable correction to make. Both data sets are normalized so that zero dB input and output correspond to the zero-lag autocorrelation value given in Equation 6. The line curve in Figure 5 is the computed power transfer curve for a three-level sampler as shown in Figure 3. The measured data for the two samplers agree very well, but they differ from the computed curve by a significant amount.

The computed and measured data in Figure 5 can be aligned to within the measurement errors and a constant offset by simply dividing the expected nominal zero-lag value given in Equation 6 by 1.053. The difference in slope between the calculated and measure power transfer functions near the nominal operating point is about 0.1 dB per dB, which is a linear slope error of about 11%. In other words, a system temperatured derived by taking the ratio of the total power to the difference between the total powers measured with a calibration noise source on and off will be 11% higher than its true value.

Figure 6 shows the difference between the input power values for the two

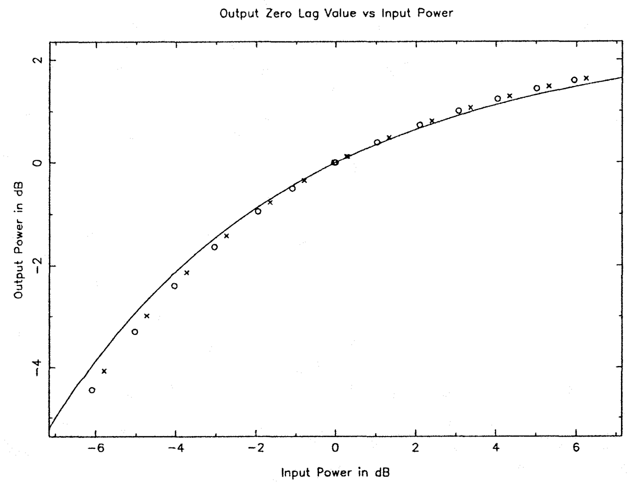

Figure 5: Computed (line) and measured power transfer curves for two of the GBT spectrometer three-level samplers. The circles are for sampler A and the crosses are for sampler B. The B input values were shifted with respect to A by 0.3 dB, and the measured and computed values are normalized to a zero-lag autocorrelation value given by Equation 6.

samplers and the output zero-lag values corrected by the normalizing function given in Equations 10 through 12. The points at the top of the diagram use Equations 10 through 12 as given, and the lower points are normalized with a value for P_u in Equation 10 which has been divided by 1.053. The offset of about -0.6 dB in the lower data points is due to the fact that the corrected values have not been rescaled to reflect the multiplication factor applied to the uncorrected values.

To determine whether the difference between the computed and measured power transfer functions was due to a distortion in the noise voltage waveform, Rich changed the noise source, amplifiers, and gain distribution in the test setup shown in Figure 4. He then measured all eight of the spectrometer samplers, and the results are shown in Figure 7. This figure is the same as Figure 6 except that all eight sampler results are plotted with the new noise source configuration.

The main difference between the data in Figures 6 and 7 is that the slope of the difference between the input and output powers is about 0.07 dB per dB in the second data set (Figure 7) as compared to 0.1 dB per dB in the first two sampler measurements (Figure 6). Evidently, the different noise sources had slightly different voltage statistics, but the change accounts for only about 30% of the difference between the calculated and measured power transfer functions. All of the new measured transfer functions can be closely approximated by dividing the value of P_u in Equation 10 by a factor of 1.038. The difference in measured slopes near the nominal operating points of the eight samplers is

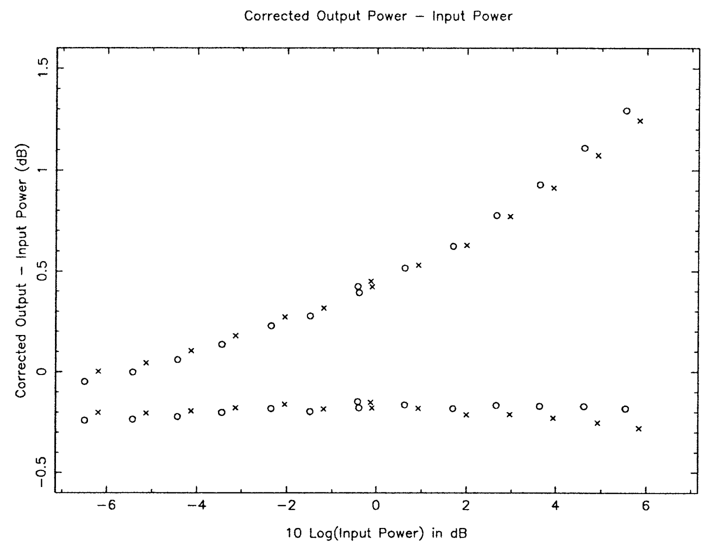

Figure 6: Difference between corrected sampler output power and the input power. The circles are for sampler A and the crosses are for sampler B. The upper data points use Equations 10 through 12 as given for the correction. The lower points use the same equations except that P_u has been divided by a factor of 1.053.

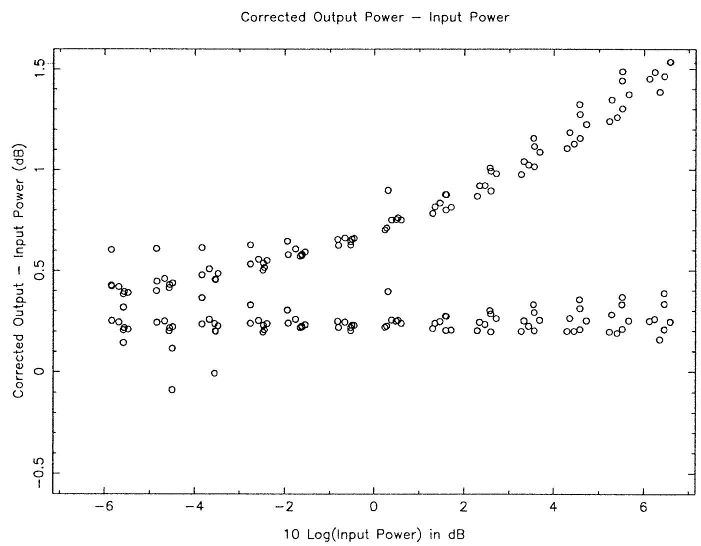

Figure 7: Difference between corrected sampler output power and the input power. The upper cluster of data points use Equations 10 through 12 as given for the correction. The lower points use the same equations except that P_u has been divided by a factor of 1.038.

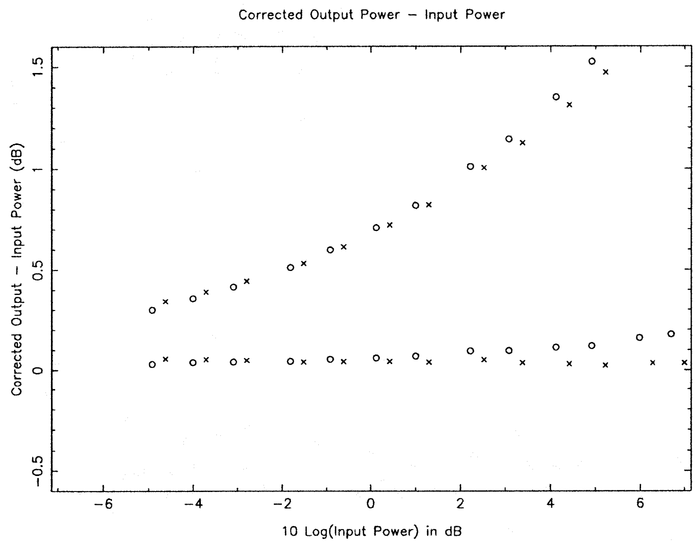

Figure 8: Difference between corrected sampler output power and the input power. The upper cluster of data points use Equations 10 through 12 as given for the correction. The lower points use the same equations except that P_u has been divided by a factor of 1.053.

less than 2% so any non-linearities in the samplers themselves are consistent throughout the devices.

We made a third set of power transfer function measurements on three-level sampler numbers 1 and 3 using the IF amplifier components in the GBT system from the internal IF noise source in the "IF Rack" through the "Analog Filter Rack," which drives the spectrometer samplers. The same IF channel 1 was used to drive both samplers for this test with the output of the Analog Filter Rack split four ways to drive the two samplers and an RF power meter simultaneously. Hence, the IF amplifiers were running at about 6 dB above their normal operating point for a given sampler input level. The results are shown in Figure 8, where the differences between corrected output power and input power are plotted in a similar fashion to Figures 6 and 7.

The sampler power transfer functions measured in the GBT system configuration agree closely with the first test setup results (Figure 6). This suggests that whatever amplitude distortions are causing the deviations from the calculated transfer function are present at a similar magnitude in the first test setup and in the GBT IF system. This seems reasonable since the power handling capacity of the amplifiers used in the two configurations are similar.

At this point it does not seem worth further effort to determine exactly where the deviation from the calculated transfer function is occurring in the sampler and amplifier electronics. The measurements appear to be quite consistent from one sampler to the next. The only difference between measurements is due to the use of a more robust RF noise amplifier chain ahead of the samplers, which does not apply to the GBT system. I suggest that we adopt an empirical correction to the zero-lag scale factor of 1.053 before applying the output power correction algorithm. The internal consistency of the measurements shown in Figures 6 and 8 indicates that the slope of the corrected output power vs input power should be accurate to better than 1% for input powers within about 4 dB of the nominal operating point of these samplers.

Reference

Lewis, B. M. 1997, "r-to-rho translations for the nine-level correlator," Arecibo internal memo, 14 February 1997, http://www.naic.edu/techinfo/teltech/upgrade/correlator/nine.htm.