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SIS-MIXER TO HEMT-AMPLIFIER OPTIMUM COUPLING NETWORK

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## I. Introduction

Superconductor-insulator-superconductor (SIS) mixers have become the device of choice for cryogenic, low-noise applications in the frequency range of 70 to 250 GHz [1]. The mixer is usually followed by a cryogenic low-noise, FET or HEMT amplifier and receiver single-sideband noise temperatures of under 100K have been achieved [2,3]. Theoretical minimum noise temperatures are an order-of-magnitude lower [1] and experimental laboratory results with narrow-band, low frequency I.F. amplifiers have confirmed this [4].

The general configuration discussed in this note is shown in Figure 1. The design of an I.F. amplifier driven by an SIS mixer requires special consideration for the following reasons:

- 1) The output resistance of an SIS mixer may be negative and the effect of this negative source impedance upon the I.F. amplifier noise should be understood.
- 2) The SIS mixer has a very limited output voltage and saturates at a very low input power level which can be increased if the I.F. amplifier input resistance is made very low.
- 3) The internal noise in the SIS mixer is very low and the receiver noise is critically dependent upon realizing a low noise contribution from the I.F. amplifier.
- 4) For stability reasons, an SIS mixer with a negative output resistance may require an I.F. amplifier input impedance which is low or high dependent upon the reactance vs. frequency slope in the circuit.

An important point that is often not realized by developers of SIS mixers is that the source impedance,  $Z_{opt}$ , which minimizes the I.F. amplifier noise can be chosen independently of the I.F. amplifier input impedance,  $Z_{in}$ . It has been common practice to use I.F. amplifiers which have been designed to have  $Z_{opt} = Z_{in} = 50$  ohms. This is not required;  $Z_{in}$  can be made low for reason 2) and  $Z_{opt}$  can be chosen to give best noise performance.

In this paper we will assume that a lossless coupling network exists between the FET or HEMT device and the mixer; the impedance transformation of this network will then be optimized so that the resulting  $Z_{opt}$  at the mixer plane minimizes the I.F. noise contribution. If the mixer has an output impedance,  $Z_{out}$ , with positive real-part, we simply equate  $Z_{opt} = Z_{out}$  and are finished. Lossless feedback would then be utilized to make the amplifier input impedance low; this feedback would have small effects upon  $Z_{opt}$ , but the input coupling network could be adjusted to compensate and preserve the  $Z_{opt} = Z_{out}$  condition. We will find  $Z_{opt}$  for the negative source resistance case and compute the resulting noise performance. Finally the effects of transmission lines and isolators will be considered.

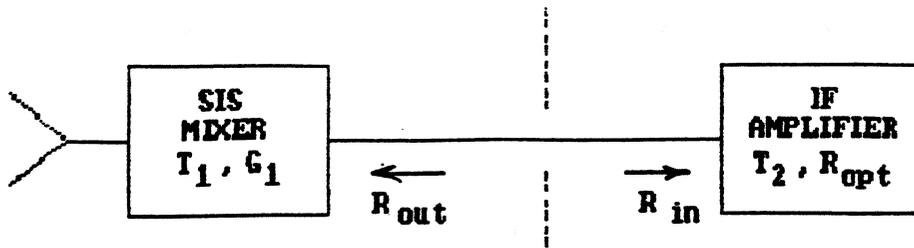


Fig. 1. General configuration discussed in this note. The mixer is described by noise temperature  $T_1$ , exchangeable gain  $G_1$ , and output resistance  $R_{out}$ . The amplifier is described by noise temperature  $T_2$ , which is a function of  $R_{out}$ , an input resistance  $R_{in}$  which has no effect upon  $G_1$  or noise performance, and an optimum generator resistance,  $R_{opt}$ . Changing the impedance transformation of the amplifier input network changes both  $R_{in}$  and  $R_{opt}$  but by use of lossless feedback,  $R_{in}$  can be changed independent of  $R_{opt}$ .

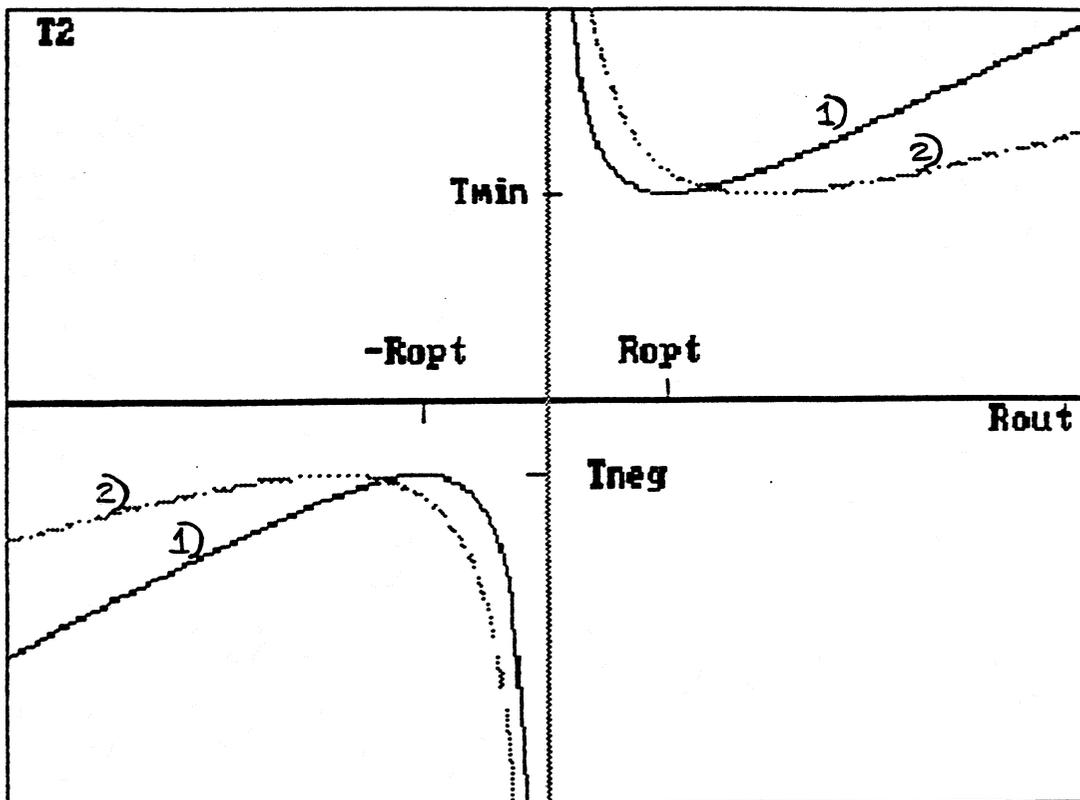


Fig. 2. The general form of the noise temperature of an amplifier as a function of generator resistance,  $R_{out}$ , is shown above. Adjustment of the impedance transformation of the amplifier input network can change the location of  $R_{opt}$ , as from curve 1) to 2), but not the value of  $T_{min}$ . For negative generator resistance, the noise temperature is negative and has a maximum value of  $T_{neg}$  at a generator resistance of  $-R_{opt}$ . The temperature,  $T_{neg}$ , is related to the noise wave coming out of the input port of the amplifier and may have absolute value higher or lower than  $T_{min}$  dependent upon noise properties of non-reciprocal or active elements in the amplifier.

SIS mixers can be realized with positive output resistance, input match, and a small amount of gain (a few dB, SSB) [5]. The output resistance may be high and an I.F. amplifier with high  $R_{opt}$  and low  $R_{in}$  appears to be desirable. If an isolator is inserted between the mixer and I.F. amplifier, then  $R_{opt} = R_{in}$  at the mixer-isolator interface [6]. Good results may be obtained, but the optimization which may result from  $R_{opt} \neq R_{in}$  is not realized.

It is important to consider the effect of a transmission line of characteristic impedance,  $Z_0$ , between the mixer and I.F. amplifier in the case  $R_{opt} \neq R_{in}$ . If  $Z_0 = R_{opt} \neq R_{in}$ , the noise performance will be independent of I.F. frequency, but the gain will depend upon the frequency-dependent transformation of  $R_{in}$  caused by the transmission line. On the other hand, if  $Z_0 = R_{in} \neq R_{opt}$ , the gain will be independent of I.F. frequency, but the noise will not. The required solution is either to make the transmission line short (for wide bandwidth) and integrate the I.F. amplifier with the mixer, or to make the transmission line one-half wavelength long at the I.F. center frequency (with of the order of 30% bandwidth).

There may be no advantage to operating the SIS mixer with negative output resistance but should this occur, there are large effects which can increase or decrease the overall noise dependent upon the noise properties of I.F. amplifier active device; this topic is discussed in the next section.

## II. Negative Resistance Cascading Theory

The conventional noise figure cascading formula of Friis (7) uses concepts of available power and available gain. In the case of a negative output resistance, both of these quantities and the second stage noise figure become infinite and the first stage noise figure becomes indeterminate. Fortunately, this situation was recognized by Haus and Adler 30 years ago, and they made modifications to the theory [8,9]. The modifications are somewhat strange and involve the concepts of exchangeable power and exchangeable gain which are understandable mathematically but are somewhat obscure from a physical point of view. The important physical concept is that the conventional minimum noise temperature,  $T_{min}$ , of an amplifier is the minimum with respect to variation of the source impedance within the positive resistance plane. When the source impedance is allowed to vary into the negative resistance plane, a new noise minimum noise temperature which we will call  $T_{neg}$  is revealed. In the formalism of Haus and Adler,  $T_{neg}$  is also negative but is divided by a negative exchangeable gain to provide a positive contribution of the second stage to the overall noise temperature. For minimum noise,  $|T_{neg}|$  should be minimized, and this is an entirely different case than the minimization of  $T_{min}$ . In terms of the correlated noise voltage-and current-source model of a noisy amplifier [10], the negative resistance allows complete cancellation of the correlated noise components from the two sources. In terms of the noise wave model of a noisy amplifier [11], the source reflection coefficient  $> 1$  produced by a negative resistance allows the noise wave coming out of the amplifier input to more completely cancel the correlated portion of the ingoing noise wave. The minimum noise temperature,  $T_{neg}$ , does not occur when the cancellation is complete because the magnitude of the uncorrelated noise must also be considered.

A formal application of the Haus/Adler noise theory to the case of an SIS mixer is as follows. The exchangeable power,  $P_e$ , of a current source,  $I_s$ , with internal shunt resistance,  $R_s$ , is given by,

$$P_e = |I_s|^2 \cdot R_s/4 \quad (1)$$

and the exchangeable gain,  $G_1$ , of the mixer is the ratio of exchangeable power at the I.F. output terminals divided by the exchangeable power of the R.F. source. If  $R_s$  is negative, the exchangeable power is negative; if  $R_s$  is positive, the exchangeable power is positive and is equal to the available power of the source. For an SIS mixer with negative I.F. output resistance driven from a positive R.F. source resistance, the exchangeable gain is negative. In terms of the gain,  $G_o$ , into normalizing impedance,  $Z_o$  (typically 50 ohms),  $G_1 = G_o/(1 - |\Gamma_{out}|^2)$  where  $\Gamma_{out} = (Z_{out} - Z_o)/(Z_{out} + Z_o)$  is the output reflection coefficient of the mixer.

The noise temperature of the mixer,  $T_1$ , without I.F. amplifier is given by,

$$T_1 = N_e/(G_1 \cdot k \cdot \Delta f) \quad (2)$$

where  $N_e$  is the exchangeable noise power at the mixer I.F. output terminals due to noise sources within the mixer.

The noise temperature of the cascade of mixer and I.F. amplifier is then given by,

$$T_{12} = T_1 + T_2/G_1 \quad (3)$$

where  $T_2$  and  $G_1$  are both negative if the mixer has negative output resistance. Both  $T_1$  and  $G_1$  are properties of the mixer and are independent of the I.F. load impedance, but the I.F. noise temperature,  $T_2$ , is dependant upon the mixer output impedance,  $Z_{out}$ . For a given  $Z_{out}$ , we wish to find the lossless coupling network which minimizes  $T_2$ . For this purpose the I.F. noise temperature,  $T_2$ , is most conveniently expressed in the form,

$$T_2 = T_{min} + N T_o |Z_{out} - Z_{opt}|^2/(R_{out} R_{opt}) \quad (4)$$

where  $T_{min}$ ,  $N$ , and  $Z_{opt}$  are four noise parameters describing the I.F. amplifier and  $T_o = 290K$ . The noise parameter,  $N$ , is equal to  $g_n R_{opt}$  where  $g_n$  is the noise conductance. This form is used because  $T_{min}$  and  $N$  are invariant to variation of a lossless coupling network which can be used to transform  $Z_{opt}$  to whatever value minimizes  $T_2$ . By differentiating  $T_2$  with respect to  $R_{opt}$ , we find the optimum positive values of  $R_{opt}$  and the resulting minima of  $T_2$  as follows:

$$R_{opt} = R_{out} \quad T_2 = T_{min} \quad \text{for } R_{out} > 0 \quad (5)$$

$$R_{opt} = -R_{out} \quad T_2 = T_{neg} = T_{min} - 4 N T_o \quad \text{for } R_{out} < 0 \quad (6)$$

For all cases  $X_{opt} = X_{out}$ . It can be shown as a property of noise parameters [6] that  $T_{min} < 4 N T_o$  and thus that the optimum  $T_2$  for negative  $R_{out}$ , which will call  $T_{neg}$  is always negative. Furthermore, since  $T_{neg}$  is a maxima,  $T_2$  is always negative for negative  $R_{out}$ . It is also interesting to note that the optimum value of  $R_{opt}$  is a positive value equal to the absolute value of  $R_{out}$ , and that

$T_{neg}$  is invariant to a lossless coupling network since both  $T_{min}$  and  $N$  are invariant. Thus,  $T_{neg}$  is a property of the active device or non-reciprocal elements in the amplifier. These results are illustrated in Figure 2.

A further physical interpretation can be given to  $T_{neg}$  by examination of the noise wave model of a noisy amplifier [11]. This model represents the noise in the amplifier by two correlated noise waves having temperatures,  $T_a$  and  $T_b$ , coupled in and out of the amplifier input terminals, respectively; close inspection reveals that  $T_b = -T_{neg}$ . (Note that in [11] the quantity  $T_d = 4 N T_o$ .) The total noise coming of the amplifier input port is not just  $T_b$ , but also contains a contribution of  $T_a$  reflected from the amplifier input terminals. However, in the case of an amplifier with an ideal input isolator with termination temperature,  $T_i$ , then  $T_b = -T_{neg} = T_i$ . In this case we see that  $T_{neg}$  is independent of  $T_{min}$ . A very noisy amplifier with a cold input isolator could produce very little noise when driven by a negative resistance with magnitude close to the isolator characteristic impedance. This case can be understood as a negative resistance amplifier which precedes the amplifier. Another viewpoint is to consider the isolator as part of the mixer with impedances chosen for high gain. In practice the only problem with this case is that the isolator forces the input resistance,  $R_{in}$ , to be equal to  $R_{opt}$  and as  $R_{out}$  becomes close to  $-R_{opt}$  for low noise, it will also become close to  $-R_{in}$  and produce high and unstable gain. A better approach may be to design an amplifier with low  $R_{in}$  for stable gain and low  $T_b = -T_{neg}$  and  $R_{opt} = -R_{out}$  (instead of low  $T_{min}$  and  $R_{in} = R_{opt}$  as is normal practice).

Some examples of measured values of noise parameters at cryogenic temperatures are given in the table below:

Transistor	Mitsubishi MGF1412 GASFET	Fujitsu FHR01FH HEMT
Reference	[12]	[13]
Temperature	15K	12.5K
Frequency	1.6GHz	8.4GHz
$T_{min}$	7.4K	10.3K
$4 N T_o$	16.0K	13.9K
$-T_{neg}$	8.6K	3.6K
$R_{opt}$	50	4.6
$X_{opt}$	0	17.0

The MGF1412 values are for a complete three-stage amplifier with an input coupling network while the FHR01FH values refer to the transistor alone. The low value of  $-T_{neg}$  at 8.4GHz for the HEMT is encouraging.

## REFERENCES

- [1] John R. Tucker and Marc J. Feldman, "Quantum Detection at Millimeter Wavelengths," Reviews of Modern Physics, vol. 57, no. 4, October 1985.
- [2] Larry R. D'Addario, "An SIS Mixer for 90-120 GHz with Gain and Wide Bandwidth," Int. J. of Infrared and Millimeter Waves, vol. 5, pp. 1419-1422, 1984.
- [3] S.-K. Pan, M. J. Feldman, A. R. Kerr and P. Timbie, "Low-Noise, 115 GHz Receiver Using Superconducting Tunnel Junctions," Appl. Phys. Lett., **43**(8), pp. 786-788, October 15, 1983.
- [4] D. W. Face, D. E. Prober, W. R. McGrath and P. L. Richards, "High-Quality Tantalum Superconducting Tunnel Junctions for Microwave Mixing in the Quantum Limit," Appl. Phys. Lett., **48**(16), pp. 1098-1100, April 21, 1986.
- [5] A. R. Kerr, private communication, February 17, 1987.
- [6] Marian W. Pospieszalski, "On the Noise Parameters of Isolator and Receiver with Isolator at the Input," IEEE Trans. Microwave Theory and Tech., vol. MTT-34, no. 4, April 1986.
- [7] H. T. Friis, "Noise Figures of Radio Receivers," Proc. IRE, vol. 32, pp. 419-422, July 1944.
- [8] H. A. Haus, and R. B. Adler, "An Extension of the Noise Figure Definition," Proc. IRE, 45, 690-691, May 1957.
- [9] H. A. Haus and R. B. Adler, "Optimum Noise Performance of Linear Amplifiers," Proc. IRE, 46, 1517-1533, August 1958.
- [10] H. Rothe and W. Dahlke, "Theory of Noisy Four-Poles," Proc. IRE, vol. 44, pp. 811-818, June 1956.
- [11] R. P. Meys, "A Wave Approach to the Noise Properties of Linear Microwave Devices," IEEE Trans. on Microwave Theory and Tech., vol. MTT-26, pp. 34-37, January 1978.
- [12] S. Weinreb, "Noise Parameters of NRAO 1.5 GHz GASFET Amplifiers," Electronics Division Internal Report No. 231, National Radio Astronomy Observatory, Charlottesville, Virginia, December 1982.
- [13] M. Pospieszalski and S. Weinreb, "FET's and HEMT's at Cryogenic Temperatures - Their Properties and Use in Low-Noise Amplifiers," to be presented at the 1987 IEEE MTT-S International Microwave Symposium, Las Vegas, Nevada, June 1987.