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SHORT-TERM PHASE STABILITY REQUIREMENTS FOR INTERFEROMETER COHERENCE

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I. Introduction

Rogers and Moran [1] have defined an interferometer coherence function, C, which can be expressed in terms of the Allan variances of the receiver phase references. The coherence function is defined so that it is a factor, ≤ 1, which multiplies the correlator output and thus is a system efficiency factor similar to the product of the two telescope efficiencies. The Allan variance and its relation to many other measures of frequency stability are very well described in a comprehensive review article by Rutman [2].

The results expressed in [1] are adequate and very useful for describing the degradation in system performance for local oscillator phase fluctuations at time scales > 1 second. For short times the situation changes for two reasons:

1) It becomes much easier to measure the phase fluctuations with a true rms voltmeter connected to a phase detector output as shown in Figure 1 rather than compute the Allan variance at fast (> 100 kHz) sampling rates.

2) For times < 1 second oscillators and synthesizers are more commonly specified by their single-sideband phase noise spectrum. It is possible to compute the Allan variance from this spectrum but this is an unnecessary complication.

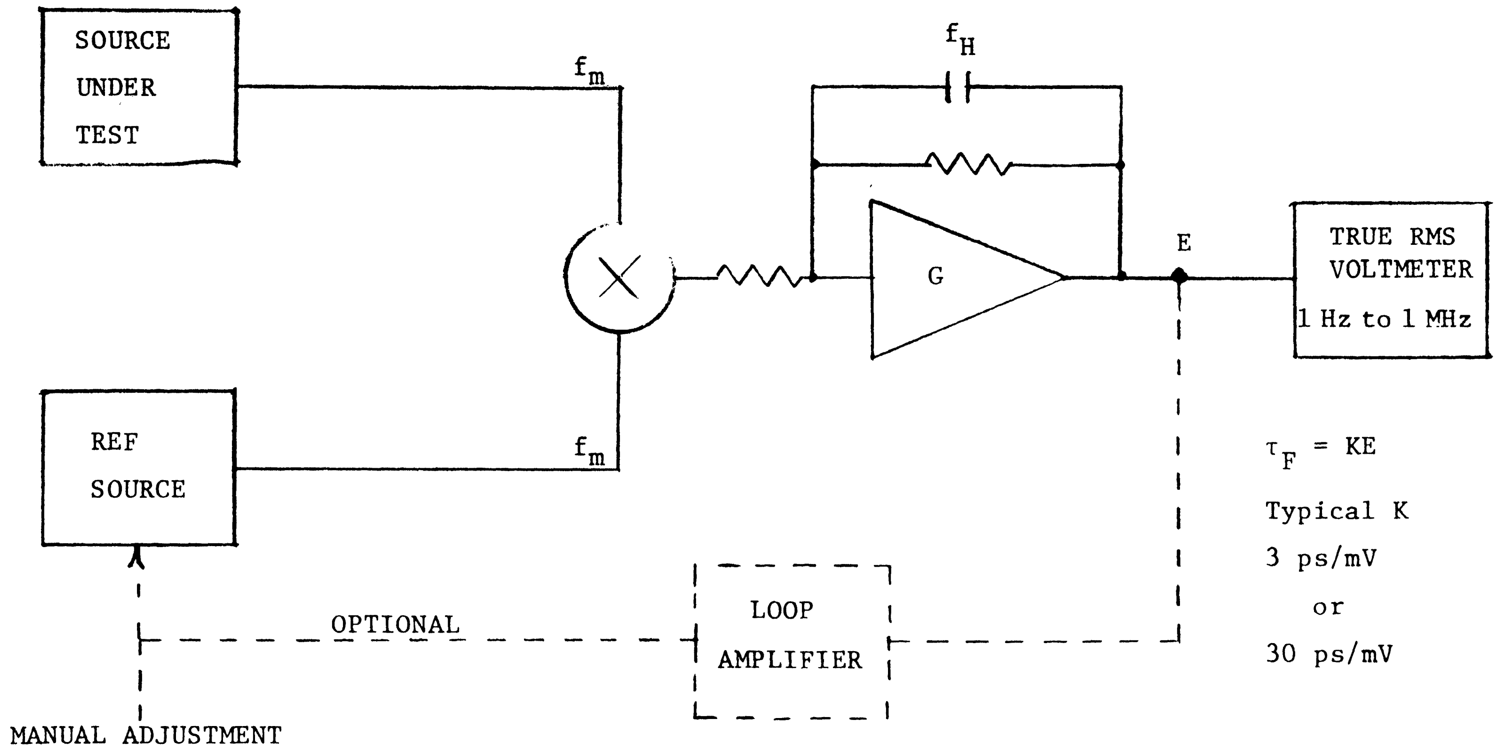

Fig. 1. Typical measurement configuration for direct measurement of wideband phase noise. The phase detector is held near quadrature by either manual adjustment of the reference source frequency or a phase-lock loop with a corner frequency of < 1 Hz. The amplifier gain is typically 10 to 100 to give an output voltage E of 10 volts for $\pi/2$ phase error; this provides a slope of 3.2 ps/mV for a sinusoidal response phase detector near quadrature and a measurement frequency, f_m, of 5 MHz.

It is proposed, then, to describe the rms value of coherence, C, as the product of two functions, C_s, for time scales > 1 second and expressed by Equation (12) of [1] in terms of Allan variance, and C_F which will be expressed in this note in terms of a wideband phase noise measurement or the single-sideband phase noise spectrum often specified by manufacturers of frequency standards and synthesizers.

II. Measurement of Fast Coherence, C_F

In Appendix I the following simple expression is derived for the rms fast coherence,

$$C_F = e^{-\psi^2/2} \tag{1}$$

where ψ is the rms phase deviation, in radians, at the interferometer RF frequency for all fluctuation frequencies greater than 1 Hz. The expression assumes Gaussian probability distribution of phase and assumes that the correlation integration interval, T, is much greater than the reciprocal of the bandwidth of the fast phase fluctuations. This latter requirement is satisfied by having T greater than a few seconds.

The direct measurement method for ψ as caused by local oscillator phase fluctuations is shown in Figure 1. The phase comparison could be made at the local oscillator frequency or can be performed at 1/N'th of the LO frequency and then multiplying the measured phases by N. In this regard, it is convenient to characterize a frequency standard or synthesizer by its fast time fluctuation, $\tau_F = \psi_m/\omega_m$ where ψ_m is the measured rms phase fluctuation in radians at measurement radian frequency ω_m. (The phase comparator output can be directly calibrated in time units by switching in a known cable time delay, typically 100 ps.) The phase deviation due to the local oscillator instability is then

$$\psi = \omega_o \tau_F \tag{2}$$

where ω_o is the radian LO frequency. A plot of C_F as a function of τ_F for various LO frequencies is shown in Figure 2. The coherence drops to 0.9 when ψ has a value of .459 radians or 26.3°; the required value of τ_F for $C_F = 0.9$ is given by

$$\tau_F(\text{ps}) = 73/f_o(\text{GHz}) \tag{3}$$

The rms phase fluctuation, ψ or τ_F, must be measured in a frequency range of 1 Hz to an upper frequency limit, f_H, which needs some discussion. Coherence is reduced by phase fluctuations of the local oscillator extending into the MHz range. As the fluctuation frequency approaches the receiver IF frequency, another harmful effect, the increase of mixer noise temperature, will occur. However, there are microwave phase-locked oscillators and filters in the local oscillator chain to act as flywheels and filter the fast fluctuations of lower frequency components. Thus, the value of f_H to be used in the measurement of τ_F for a 5 MHz frequency standard or VHF frequency synthesizer depends on the bandwidth of higher frequency phase-lock loops; values of f_H in the range of 100 kHz to a few MHz are reasonable.

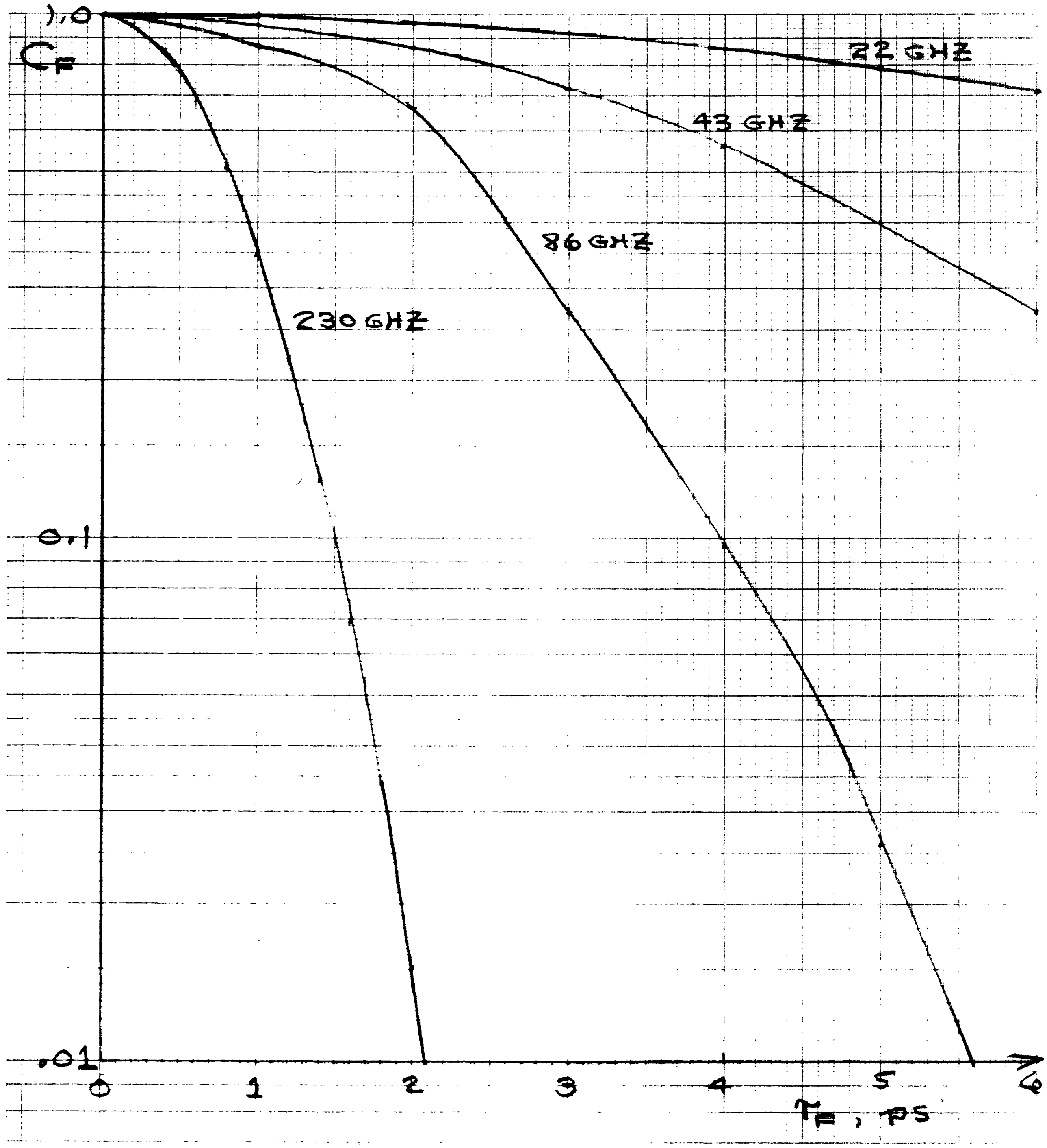

Fig. 2. Fast coherence factor, C_F, as a function of wideband rms phase noise expressed in time units, τ_F, for four interferometer RF frequencies.

The choice of 1 Hz as the lower frequency limit for measurement or estimation of C_F is rather arbitrary, but it is convenient for computation and realistic in that the hydrogen maser frequency standards commonly used for interferometry have a control loop bandwidth of about 1 Hz. Thus C_F for a maser standard should largely be determined by the phase noise characteristics of its crystal oscillator "flywheel," while C_S should be determined by the stability of the 1420 MHz maser oscillation. Other precision frequency standards are also dependent on crystal oscillators for short-term phase stability; the control loop bandwidths range from about 0.01 Hz for cesium beam standards to 100 Hz for superconducting cavity stabilized oscillators.

The values of ψ or τ_F to be used in Equation 1 or Figure 2 must include the fluctuations in both receivers of the interferometer; i.e., τ_F is the square root of the sum of the squares of the τ_F values for each receiver. Similarly, any measurement of τ_F must account for noise in the reference system; i.e., divide the measured τ_F by $\sqrt{2}$ if two identical sources are used.

III. Coherence as a Function of the Phase Spectrum

The wideband phase noise, ψ, or expressed in units of time, τ_F, can be obtained by an integration of the single-sideband phase noise spectrum, L(f), from 1 Hz to the upper limit, f_H, discussed in the previous section,

$$\tau_F^{\,2} = \frac{2}{\omega_m^{\,2}} \int_1^{f_H} L(f)\, df \qquad (4)$$

where ω_m is the measurement radian frequency. The spectrum, L(f), expressed in dB below the carrier, dBC, as a function of fluctuation frequency, typically 1 Hz to 100 kHz, is the quantity usually specified by manufacturers of frequency standards or synthesizers.

The integration of L(f) is not trivial because it usually varies over several orders of magnitude and must be plotted with log - log scales. When viewing such a plot, it is usually not obvious whether low frequencies or high frequencies will dominate the area. A short Apple BASIC program to perform this integration is listed in Appendix II. The program takes as input the phase measurement frequency, f_m, and L(f) at 8 values of f in decade steps from 1 Hz to 10 MHz. A power-law function is fit to each decade, integration is performed, and the areas of the decades are summed up to frequency, f_H. The output is expressed as τ_F as a function of f_H from 10 Hz to 10 MHz.

The results of integrating the phase noise spectrum of several commercial oscillators and synthesizers is given in Table I. In cases where L(f) is not specified at low or high frequencies, an extrapolation of adjacent points has been made.

OSCILLOQUARTZ BVA-8600 SPEC

MEASUREMENT FREQUENCY=5 MHZ

F,HZ	DBC	TAU,PS
10^0	-115	-
10^1	-140	.06
10^2	-148	.07
10^3	-148	.09
10^4	-148	.19
10^5	-148	.57
10^6	-148	1.79
10^7	-148	5.67

HP 10544B/C SPECIFICATION

MEASUREMENT FREQUENCY=10 MHZ

F,HZ	DBC	TAU,PS
10^0	-85	-
10^1	-120	.8
10^2	-140	.8
10^3	-150	.8
10^4	-150	.81
10^5	-150	.83
10^6	-150	1.07
10^7	-150	2.39

OSCILLOQUARTZ B-8400 SPEC

MEASUREMENT FREQUENCY=5 MHZ

F,HZ	DBC	TAU,PS
10^0	-112	-
10^1	-135	.1
10^2	-140	.11
10^3	-140	.17
10^4	-140	.46
10^5	-140	1.43
10^6	-140	4.5
10^7	-140	14.24

HP 10811A/B SPECIFICATION

MEASUREMENT FREQUENCY=10 MHZ

F,HZ	DBC	TAU,PS
10^0	-90	-
10^1	-120	.5
10^2	-140	.51
10^3	-157	.51
10^4	-160	.51
10^5	-160	.51
10^6	-160	.55
10^7	-160	.87

HP5105A TYPICAL

MEASUREMENT FREQUENCY=500 MHZ

F,HZ	DBC	TAU,PS
10^0	-62	-
10^1	-73	.51
10^2	-84	.69
10^3	-97	.78
10^4	-103	.87
10^5	-108	1.09
10^6	-112	1.67
10^7	-116	3.02

HP 8662A TYPICAL

MEASUREMENT FREQUENCY=639 MHZ

F,HZ	DBC	TAU,PS
10^0	-97	-
10^1	-108	.01
10^2	-122	.01
10^3	-132	.01
10^4	-135	.01
10^5	-134	.02
10^6	-145	.04
10^7	-147	.06

FLUKE 6071A TYPICAL

MEASUREMENT FREQUENCY=500 MHZ

F,HZ	DBC	TAU,PS
10^0	-72	-
10^1	-83	.16
10^2	-86	.29
10^3	-93	.5
10^4	-127	.54
10^5	-145	.54
10^6	-145	.54
10^7	-145	.55

FLUKE 6160B TYPICAL

MEASUREMENT FREQUENCY=160 MHZ

F,HZ	DBC	TAU,PS
10^0	-90	-
10^1	-100	.07
10^2	-110	.1
10^3	-120	.12
10^4	-124	.15
10^5	-127	.26
10^6	-147	.32
10^7	-154	.34

Table I. Phase noise spectrum, dBC per Hz, as specified by the manufacturer, and noise τ_F in pS units integrated over the frequency range of 1 Hz to the indicated f for four crystal oscillators (top) and four VHF synthesizers (bottom).

REFERENCES

[1] A. E. Rogers and J. M. Moran, "Coherence Limits for Very Long Baseline Interferometry," IEEE Trans. on Instr. and Meas., vol. IM-30, no. 4, pp. 283-286, Dec. 1981.

[2] J. Rutman, "Characterization of Phase and Frequency Instabilities in Precision Frequency Sources: Fifteen Years of Progress," Proc. IEEE, vol. 66, pp. 1048-1075, Sept. 1978.

APPENDIX I

Rogers and Moran [1], Equations (5) and (6) find the following expression for C^2, the mean square coherence, as a function of correlator averaging time, T, and autocorrelation function of phase, $R(\tau)$,

$$C^2 = \frac{2}{T} \int_o^T (1 - \frac{\tau}{T})\, e^{R(\tau)-R(o)}\, d\tau \tag{5}$$

It is now assumed that the phase consists of a fast portion with frequency components > 1 Hz having autocorrelation function, $R_F(\tau)$, and a slow portion with frequency components < 1 Hz with autocorrelation function, $R_s(\tau)$, where $R(\tau) = R_F(\tau) + R_s(\tau)$. Equation (5) can then be expressed as,

$$C^2 = e^{-R_F(o)} \times \frac{2}{T} \int_o^T (1 - \frac{\tau}{T})\, e^{R_S(\tau)-R_S(o)}\, e^{R_F(\tau)}\, d\tau \tag{6}$$

The quantity $R_F(o)$ is the mean square of the phase variation expressed in radians. If the rms value of the phase is ψ, then

$$R_F(o) = \psi^2 \tag{7}$$

The fast fluctuations are dominated by components with periods short compared to T; the quantity $e^{R_F(\tau)}$ under the integral is then ~ 1 over most of the range of integration since $R_F(\tau) \sim 0$ for τ large compared to the noise periods. For τ small, $e^{R_F(\tau)}$ does not become very large since $R_F(\tau) \leq R_F(o) \leq 1$ for a reasonable frequency standard. Making these approximations, the mean square coherence can be expressed as

$$C^2 = C_F^{\,2} \times C_s^{\,2} \tag{8}$$

where

$$C_F^{\,2} = e^{-\psi^2} \tag{9}$$

$$C_s^{\,2} = \frac{2}{T}\int_0^T \left(1 - \frac{\tau}{T}\right) e^{R_s(\tau) - R_s(0)}\, d\tau \tag{10}$$

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APPENDIX II

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100  REM  PHASE NOISE INTEGRATION PROGRAM, S. WEINREB, MAY
 16, 1983.
110 C1 = 2.30258:C2 = 1.4142:C4 = 6.283
120  TEXT : HOME : PRINT "  PHASE NOISE INTEGRATION PROGRA
M": PRINT : PRINT "ENTER TITLE, MEASUREMENT FREQUENCY IN
 MHZ FOLLOWED BY  8 VALUES OF SSB PHASE  SPECTRUM AT F=1HZ
 TO 10MHZ IN UNITS OF  -DBC IN 1HZ BANDWIDTH. TYPE RETURN
AFTEREACH ENTRY"
130  PRINT : INPUT "TITLE? ";NM$
140  PRINT : INPUT "FM=";FM
150  FOR K = 0 TO 7: PRINT "10^";K;: PRINT " HZ ?  ";: INPUT
"-";L(K): NEXT
160  PRINT : PRINT "PRESS SPACE IF CORRECT;TYPE X IF NOT":
 GET Z$: IF Z$ = "X" THEN 120
170  FOR K = 1 TO 7:B(K) = (L(K - 1) - L(K)) / 10:A(K) =  EXP
( - C1 * L(K) / 10 - C1 * K * B(K)): NEXT K
180  REM  PRINT D$;"PR#1": PRINT P$;"8L";P$;"72N": REM   F
OR CITOH PRINTER ON
190  TEXT : HOME : PRINT NM$
200  PRINT : PRINT "MEASUREMENT FREQUENCY=";FM;" MHZ"
210  PRINT : PRINT "F,HZ       DBC      TAU,PS "
220 C3 = 10 ^  + 6 * C2 / (C4 * FM)
230 S = 0
240  FOR K = 1 TO 7
250  IF  - 1.1 < B(K) AND B(K) <  - 0.9 THEN C(K) = C1 * A
(K): GOTO 270
260 C(K) = A(K) / (B(K) + 1):C(K) = C(K) * 10 ^ (K * B(K) +
K):C(K) = C(K) * (1 - 10 ^  - (B(K) + 1))
270 S = S + C(K):T(K) = C3 *  SQR (S)
280  PRINT "10^";K; TAB( 10); - L(K); TAB( 20); INT (100 *
T(K) + .5) / 100
290  NEXT
300  PRINT : PRINT "TYPE SPACE FOR HARD COPY OR RETURN TO
QUIT"
310  GET Z$: IF Z$ <  >  CHR$ (32) THEN  END
320 D$ =  CHR$ (13) +  CHR$ (4):P$ =  CHR$ (9)
330  PRINT D$;"PR#1": PRINT P$;"8L";P$;"72N": REM    TURN
ON CITOH PRINTER AND SET MARGINS
350  PRINT : PRINT NM$: PRINT : PRINT "MEASUREMENT FREQUEN
CY=";FM;" MHZ"
360  PRINT : PRINT "F,HZ       DBC      TAU,PS "
370  FOR K = 0 TO 7
380 T$ =  STR$ ( INT (100 * T(K) + .5) / 100): IF K = 0 THEN
T$ = "  -  "
390  PRINT "10^";K;: HTAB 18: PRINT  - L(K);: HTAB 28: PRINT
T$
400  NEXT K
410  CALL  - 26868: REM   PRINTER OFF
420  GOTO 120
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