On the Baseline Sensitivity to Changing Cable Lengths

Roger Norrod, Galen Watts

February 11, 2003

1 Introduction

On January 22, we performed tests on the GBT Equipment Room IF electronics in an attempt to isolate the source of near sinusoidal baseline ripples that show up from time-to-time. We injected a stable noise source into Converter Modules or into Sampler/Filter Modules, and simulated position switched observations by running a series of scans using the spectrometer in 200MHz mode. During the course of these tests, we induced small changes in the length of coaxial cables connecting various pieces of equipment by inserting short coaxial adapters. This introduced dramatic ripples in the baselines (Figure 1), calculated by the usual methods. We felt that it is important to understand how the ripple is introduced. This report describes modeling of two ports connected by a transmission line, measurements taken in the system, and an investigation into the required stability of phase length that these results imply.

2 Modeling of Two-Port Networks

2.1 Equations

Figure 2 shows a signal-flow representation of two two-port networks, Net A and Net B, connected by a lossless, matched transmission line. The scattering parameters s_{11} and s_{22} are input and output reflection coefficients of the networks. Parameters s_{21} and s_{12} represent the forward and reverse transmission coefficients. The s-parameters are complex values that, in general, vary with frequency. The delay of the lossless, matched transmission line is represented by a transmission coefficient in phasor notation: $e^{j\beta l}$, where $\beta = 2\pi/\lambda$ and l is the electrical length of the transmission line. In Figure 1, complex voltage signals incident on the network ports are represented by a_i, and signals reflected from the ports by b_i.

We are interested in a_4/a_1. The square of the magnitude of this quantity is the transducer power gain of the cascaded networks. The signal-flow diagram allows us to write equations for the signals leaving and reflected from each port:

$$a_2 = a_1 s_{21A}(1 - s_{11A}) + b_2 s_{22A} \tag{1}$$

$$a_3 = a_2 e^{j\beta l} \tag{2}$$

$$a_4 = a_3 s_{21B}(1 - s_{11B}) + b_4 s_{22B} \tag{3}$$

We will assume a matched load, so $b_4 = 0$. Hence,

$$b_3 = a_3 s_{11B} \tag{4}$$

$$b_2 = b_3 e^{j\beta l} = a_3 s_{11B} e^{j\beta l} \tag{5}$$

Substituting (5) and (1) into (2),

$$a_3 = e^{j\beta l}(a_1 s_{21A}(1 - s_{11A}) + a_3 e^{j\beta l} s_{11B} s_{22A}) \tag{6}$$

Distributing,

$$a_3 = a_1 s_{21A}(1 - s_{11A}) e^{j\beta l} + a_3 s_{11B} s_{22A} e^{j2\beta l} \tag{7}$$

and solving for a_3,

$$a_3 = a_1 \left(\frac{s_{21A}(1 - s_{11A}) e^{j\beta l}}{1 - s_{11B} s_{22A} e^{j2\beta l}} \right) \tag{8}$$

Substituting (8) into (3), we obtain the quantity of interest:

$$\frac{a_4}{a_1} = \frac{s_{21A} s_{21B}(1 - s_{11A})(1 - s_{11B}) e^{j\beta l}}{1 - s_{22A} s_{11B} e^{j2\beta l}} \tag{9}$$

Equation (7) shows that a_3, the input signal to Net B, consists of the input signal a_1 modified by Net A and delayed by the transmission line, plus a_3 (from an earlier time) reflected by s_{11B} and by s_{22A} and delayed by two passes through the transmission line. As will be seen in the next section, this dual reflection sets up an interference pattern that introduces ripple in the cascaded network frequency response, a well-known result. The affect on baselines of changing cable length, as might be introduced by temperature fluctuations, is also determined.

2.2 Numerical Models

Appendices A, B, and C are Mathcad worksheets showing three numerical examples typical of GBT situations. The main points to be drawn from these illustrations are:

- The transmission line embedded between non-zero reflection coefficients introduce passband ripple with characteristic frequency of $\frac{c}{2l}$, where c is the velocity of propagation in the line. The ripple amplitude is set by the product of $s22A$ and $s11B$.

- If the line length changes, the passband ripple shifts in frequency. If this happens, say, between (or during) two position-switched scans, the frequency shift in ripple pattern introduces ripple in the baseline obtained by the ratio of the two scans. The amplitude of the baseline ripple is proportional to the length change (for small changes), as a fraction of the wavelength of the signal on the line. For example, the appendices show that to keep baseline ripple below 0.1% when the transmission line is operating between 20dB return losses, the line electrical length must be stable to better than 1.44° ($\lambda/250$) at the highest passband frequency.

These results may be used to set specifications on the required phase stability of interconnecting cables, based on the return loss seen at each end and on the frequencies present on the cable. Changes in electrical length of cables may be induced by temperature changes, by flexing, and by poor connections. We can see that the GBT system design has made these requirements quite challenging because of the relatively high frequencies and broad bandwidths used between various subsystems. The following section discusses the types of cables used for long signal runs on the GBT.

3 GBT Cables

Three types of coaxial cables are mainly used for IF and RF signals in the GBT receiving systems.

141 Semi-rigid The solid teflon dielectric in these cables exhibits a relatively strong negative temperature coefficient, giving cable assemblies a delay coefficient of about $\rho = 90ppm/C$ near room temperature. Cables made from this material are generally less than 1 meter long. To achieve $\lambda/250$ stability at 8GHz and 1 meter length, the coefficient implies a temperature stability of just over 1C. The equation for ΔC is:

$$\Delta C \leq \frac{\Delta l}{\rho l} \tag{10}$$

where Δl is the maximum change in cable length that will achieve the required baseline stability.

Heliax FSJ1-50A The polyethylene foam dielectric in this cable has a temperature coefficient much smaller than teflon. Cable assembly temperature coefficient is typically $10ppm/C$. This type is used in the GBT Receiver Room for IF cables between the front-ends and the IF Rack, and lengths can be 4 meters. To achieve $\lambda/250$ stability at 8GHz on a 4 meter length of FSJ1-50A, temperature stability of just over 3C is required.

Belden 1673A Conformable Coax Belden does not provide a temperature coefficient, but the cable uses solid teflon dielectric, and hence likely has a coefficient similar to 141 semi-rigid. It is used for jumpers in the Equipment Room. For example, IF jumpers between the AnalogFilter Rack and the Specrometer Sampler Rack are of the 1673A type. These cables are about 6 meters long (electrical) and carry up to 1.6GHz signals. To achieve $\lambda/250$ stability at 1.6GHz on a 6 meter length of 1673A, temperature stability of 0.9C is required.

4 Summary

This report develops a means to obtain phase stability requirements for long cables used in spectroscopy. It appears that some of the baseline ripple seen in GBT position switched observations may be explained by temperature fluctuations which induce electrical length changes in long interconnecting cables.

A thorough review of each of the many cases is needed, to better pinpoint where problems may arise, and to help decide how best to make improvements. However, from the cases calculated in the previous section, it can be seen that long cables with solid teflon dielectrics are problematic.

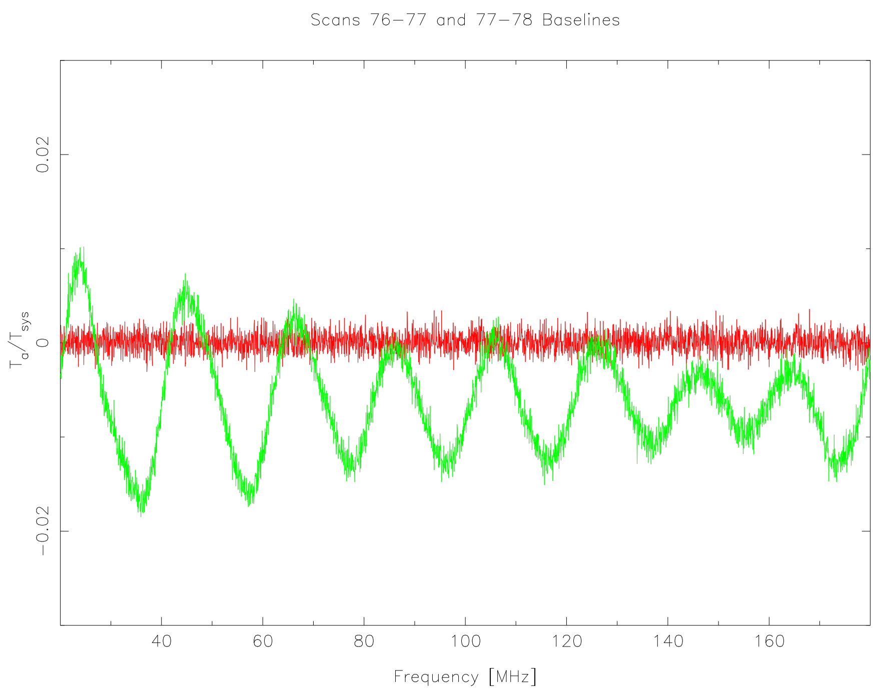


Figure 1: Data taken with the GBT Spectrometer. Baselines are shown for two pairs of scans. Between the scans used to generate the green trace, about 4cm was added to the cable between the SamplerFilter module in the AnalogFilter rack and the input to the Spectrometer sampler rack.

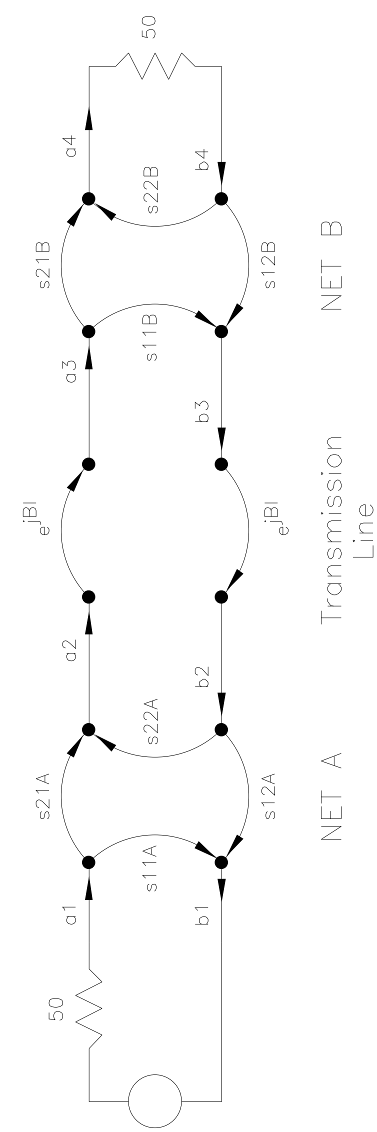


Figure 2: Schematic representation of two networks connected by a lossless, matched transmission line.

On the Baseline Sensitivity to Changing Cable Lengths
Appendix A

An example with 300cm Line at 0.8-1.0 GHz

February 2003

Mathcad File: AppendixA.mcd

$f := 0.80, 0.801 .. 1.00$ Frequency in GHz

$len := 300$ Cable length in cm.

$c := 30$ Speed of light in cm/nsec in air.

$$\beta(f) := 2\cdot\pi\cdot\frac{f}{c}$$

$$delay(\theta) := \cos(\theta) + j\cdot\sin(\theta)$$

For 2-port A:

$s11A := 0.0$ $s21A := 1.0$

$s12A := 0.0$ $s22A := 0.1$

For a specific simple example, Net A and Net B are representated by s-parameters for a lossless network with flat frequency response and reflection coefficients of 0.1 at the ends of the transmission line. That corresponds to a 20dB return loss.

For 2-port B:

$s11B := 0.1$ $s21B := 1.0$

$s12B := 0.0$ $s22B := 0.0$

The cascade transfer coefficient:

$$s21(f) := \frac{(1 - s11A)\cdot(1 - s11B)\cdot s21A\cdot s21B\cdot delay(\beta(f)\cdot len)}{1 - s22A\cdot s11B\cdot delay(2\cdot\beta(f)\cdot len)} \qquad (1)$$

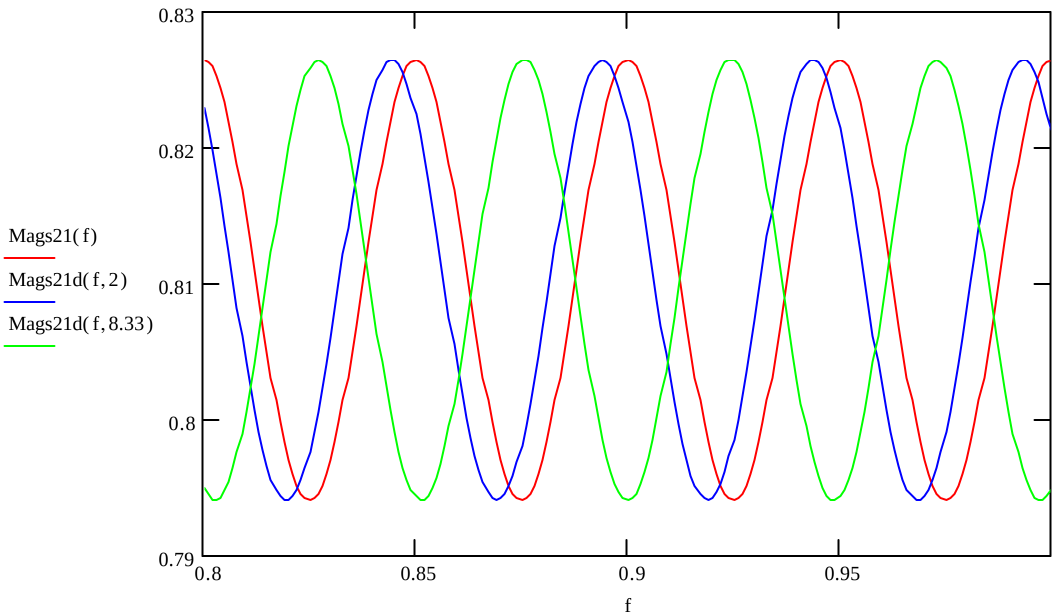
The coefficient if the cable length is increased by δ:

$$s21d(f,\delta) := \frac{(1 - s11A)\cdot(1 - s11B)\cdot s21A\cdot s21B\cdot delay(\beta(f)\cdot(len + \delta))}{1 - s22A\cdot s11B\cdot delay(2\cdot\beta(f)\cdot(len + \delta))} \qquad (2)$$

The power gain of the cascaded network is the square of the magnitude of s21:

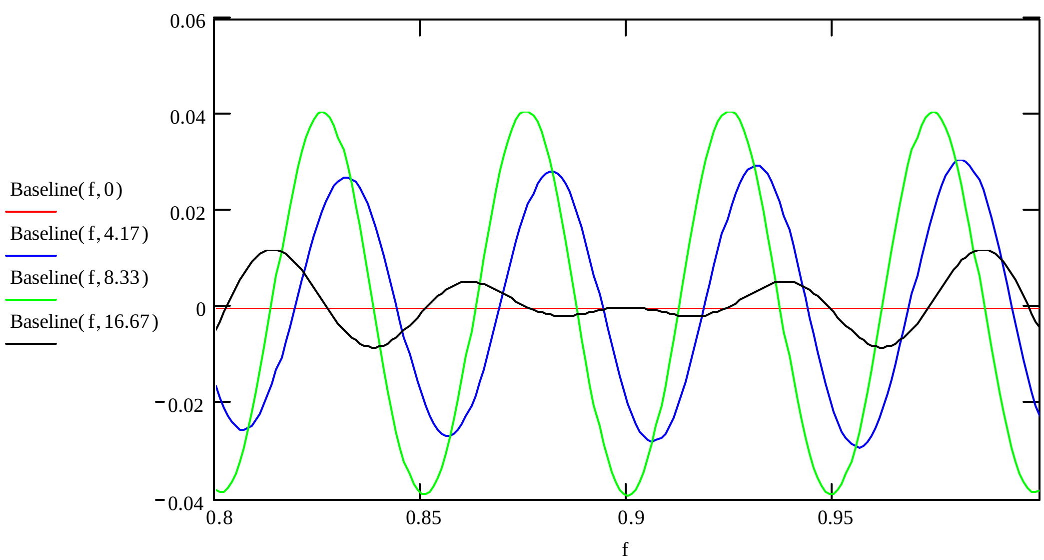
$$Mags21(f) := (|s21(f)|)^2 \qquad Mags21d(f,\delta) := (|s21d(f,\delta)|)^2$$

The following Graph plots the cascaded network frequency response with three cable lengths. The peak-peak ripple is about 0.17dB for the specific reflection coefficients in this example, and the characteristic ripple frequency is given by c/(2len). The ripple pattern shifts in frequency as the cable length changes.



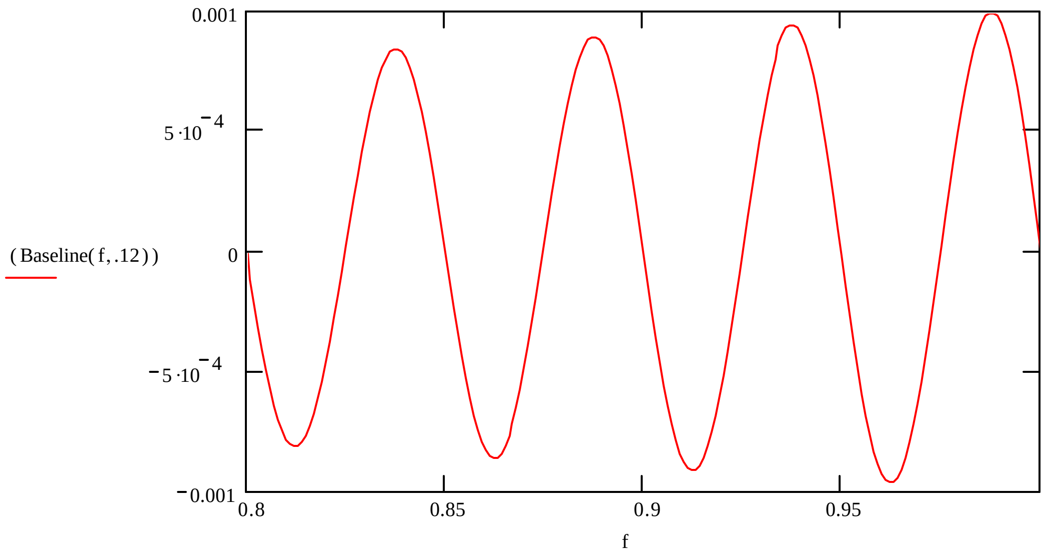
If the response with zero δ is considered as a Reference spectrum, and with non-zero δ Signal spectra, then the graph below plots the (Sig-Ref)/Ref baselines, for δ equal to zero, 1/8. 1/4, and 1/2 times the mid-band wavelength (which is 33.3cm in this example).

$$\mathrm{Baseline}(f,\delta) := \frac{\mathrm{Mags21d}(f,\delta)}{\mathrm{Mags21}(f)} - 1$$



Note that the baseline ripple amplitude peaks at δ=1/4th the mid-band wavelength.

Say we want max baseline ripple of 0.1%. How much δ is allowed?



The answer is: A change of cable length of 0.12cm induces baseline ripple of 0.1%. That is 1.44 degrees (or λ/250) at 1 GHz, the high end of the band. In this regime of small changes, the baseline ripple amplitude scales almost linearly by cable length changes. For example, δ=0.012cm yields baseline ripple of 0.01%.

On the Baseline Sensitivity to Changing Cable Lengths
Appendix B

An example with 300cm Line at 0.8-1.6 GHz

February 2003

Mathcad File: AppendixB.mcd

$f := 0.80, 0.801 .. 1.60$ Frequency in GHz

$len := 300$ Cable length in cm.

$c := 30$ Speed of light in cm/nsec in air.

$$\beta(f) := 2 \cdot \pi \cdot \frac{f}{c}$$

$$delay(\theta) := \cos(\theta) + j \cdot \sin(\theta)$$

For 2-port A:

$s11A := 0.0$ $s21A := 1.0$

$s12A := 0.0$ $s22A := 0.1$

For a specific simple example, Net A and Net B are representated by s-parameters for a lossless network with flat frequency response and reflection coefficients of 0.1 at the ends of the transmission line. That corresponds to a 20dB return loss.

For 2-port B:

$s11B := 0.1$ $s21B := 1.0$

$s12B := 0.0$ $s22B := 0.0$

The cascade transfer coefficient:

$$s21(f) := \frac{(1 - s11A) \cdot (1 - s11B) \cdot s21A \cdot s21B \cdot delay(\beta(f) \cdot len)}{1 - s22A \cdot s11B \cdot delay(2 \cdot \beta(f) \cdot len)} \qquad (1)$$

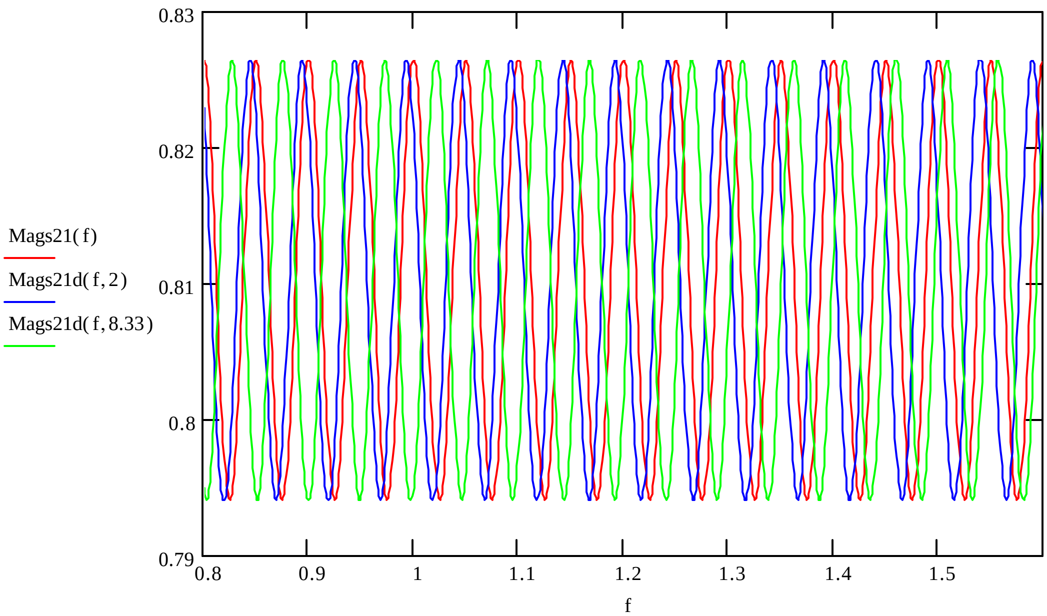
The coefficient if the cable length is increased by δ:

$$s21d(f, \delta) := \frac{(1 - s11A) \cdot (1 - s11B) \cdot s21A \cdot s21B \cdot delay(\beta(f) \cdot (len + \delta))}{1 - s22A \cdot s11B \cdot delay(2 \cdot \beta(f) \cdot (len + \delta))} \qquad (2)$$

The power gain of the cascaded network is the square of the magnitude of s21:

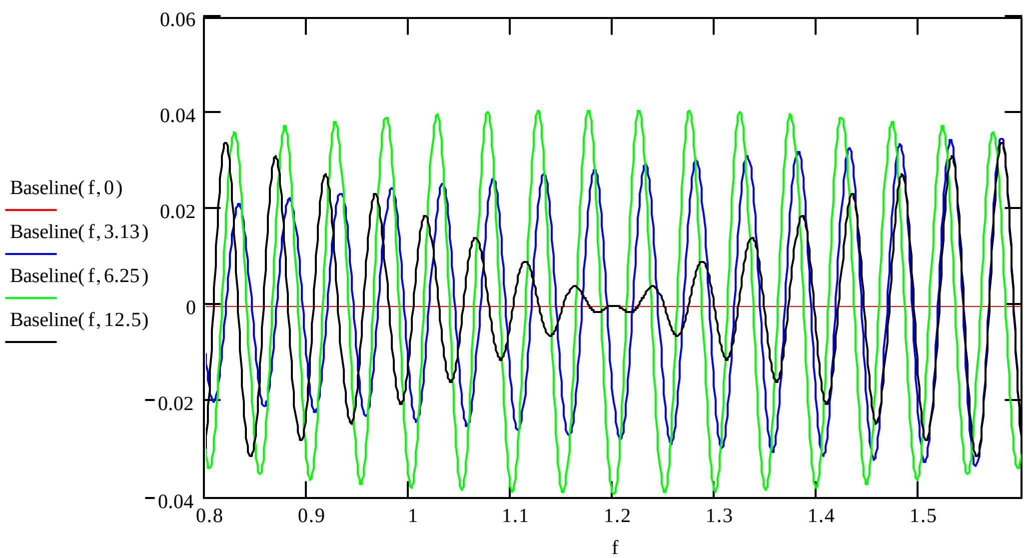
$Mags21(f) := (|s21(f)|)^2$ $\qquad$ $Mags21d(f, \delta) := (|s21d(f, \delta)|)^2$

The following Graph plots the cascaded network frequency response with three cable lengths. The peak-peak ripple is about 0.17dB for the specific reflection coefficients in this example, and the characteristic ripple frequency is given by c/(2len). The ripple pattern shifts in frequency as the cable length changes.



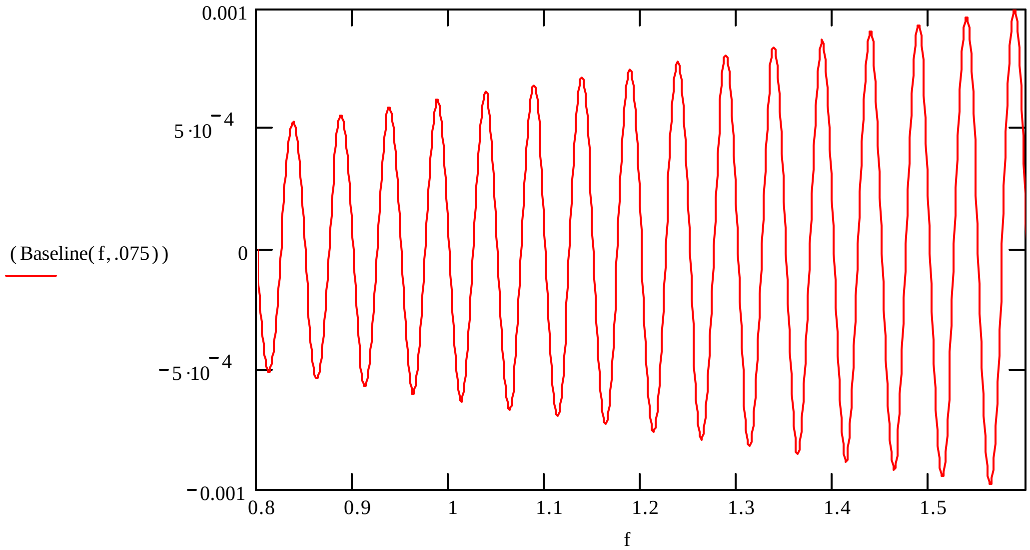
If the response with zero δ is considered as a Reference spectrum, and with non-zero δ Signal spectra, then the graph below plots the (Sig-Ref)/Ref baselines, for δ equal to zero, 1/8. 1/4, and 1/2 times the mid-band wavelength (which is 33.3cm in this example).

$$\text{Baseline}(f, \delta) := \frac{\text{Mags21d}(f, \delta)}{\text{Mags21}(f)} - 1$$



Note that the baseline ripple amplitude peaks at δ=1/4th the mid-band wavelength.

Say we want max baseline ripple of 0.1%. How much δ is allowed?



The answer is: A change of cable length of 0.075cm induces baseline ripple of 0.1%. That is 1.44 degrees at 1.6GHz. The broad fractional bandwidth of this example exhibits variation in ripple amplitude across the band. In this regime of small changes, the baseline ripple amplitude scales almost linearly by cable length changes. For example, δ=0.0075cm yields baseline ripple of 0.01%.

On the Baseline Sensitivity to Changing Cable Lengths
Appendix C

An example with 300cm Line at 5.90-6.10 GHz

February 2003

Mathcad File: AppendixC.mcd

$f := 5.90, 5.901 .. 6.10$ Frequency in GHz

$len := 300$ $\qquad$ $c := 30$ speed of light in cm/nsec

$\beta(f) := 2 \cdot \pi \cdot \frac{f}{c}$

$delay(\theta) := \cos(\theta) + j \cdot \sin(\theta)$

For 2-port A:

$s11A := 0.0$ $\qquad$ $s21A := 1.0$

$s12A := 0.0$ $\qquad$ $s22A := 0.1$

For a specific simple example, Net A and Net B are representated by s-parameters for a lossless network with flat frequency response and reflection coefficients of 0.1 at the ends of the transmission line. That corresponds to a 20dB return loss.

For 2-port B:

$s11B := 0.1$ $\qquad$ $s21B := 1.0$

$s12B := 0.0$ $\qquad$ $s22B := 0.0$

The cascade transfer coefficient:

$$s21(f) := \frac{(1 - s11A) \cdot (1 - s11B) \cdot s21A \cdot s21B \cdot delay(\beta(f) \cdot len)}{1 - s22A \cdot s11B \cdot delay(2 \cdot \beta(f) \cdot len)} \qquad (1)$$

The coefficient if the cable length is increased by δ:

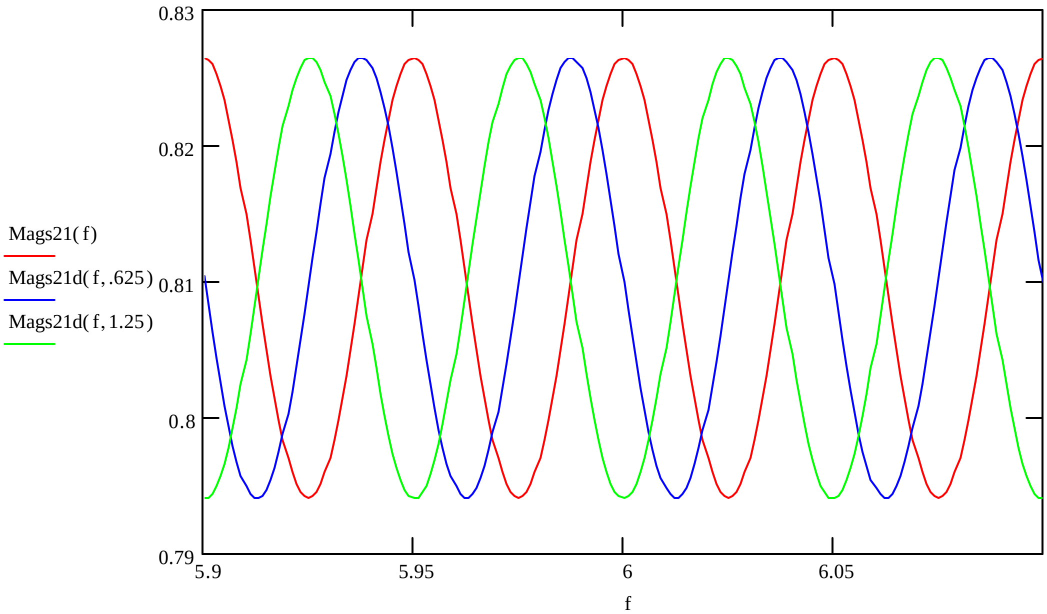
$$s21d(f, \delta) := \frac{(1 - s11A) \cdot (1 - s11B) \cdot s21A \cdot s21B \cdot delay(\beta(f) \cdot (len + \delta))}{1 - s22A \cdot s11B \cdot delay(2 \cdot \beta(f) \cdot (len + \delta))} \qquad (2)$$

The power gain of the cascaded network is the square of the magnitude of s21:

$Mags21(f) := (|s21(f)|)^2$ $\qquad$ $Mags21d(f, \delta) := (|s21d(f, \delta)|)^2$

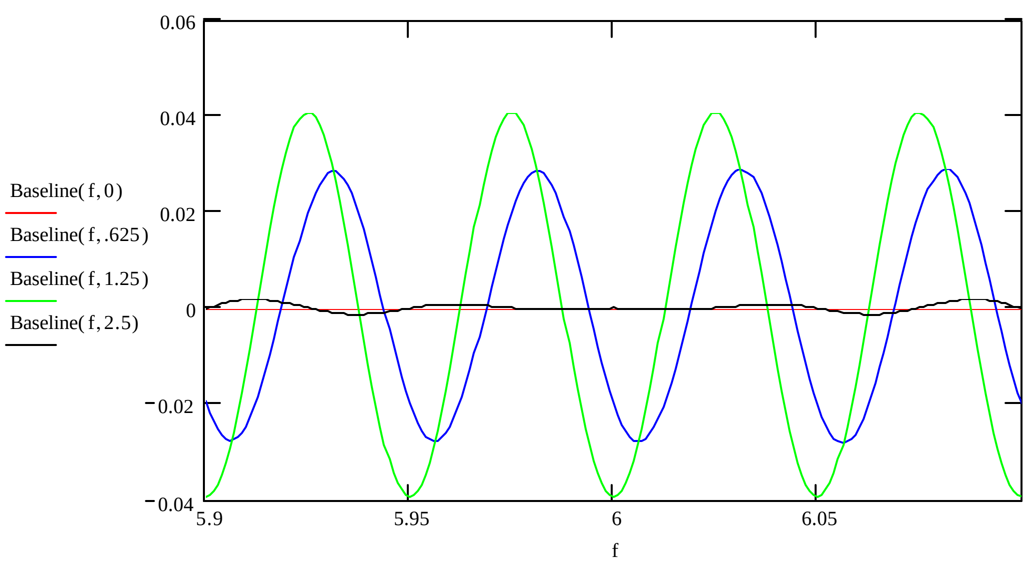
The following Graph plots the cascaded network frequency response with three cable lengths. The peak-peak ripple is about 0.17dB for the specific reflection coefficients in this example. The ripple pattern shifts in frequency as the cable length changes.

ripple pattern shifts in frequency as the cable length changes.



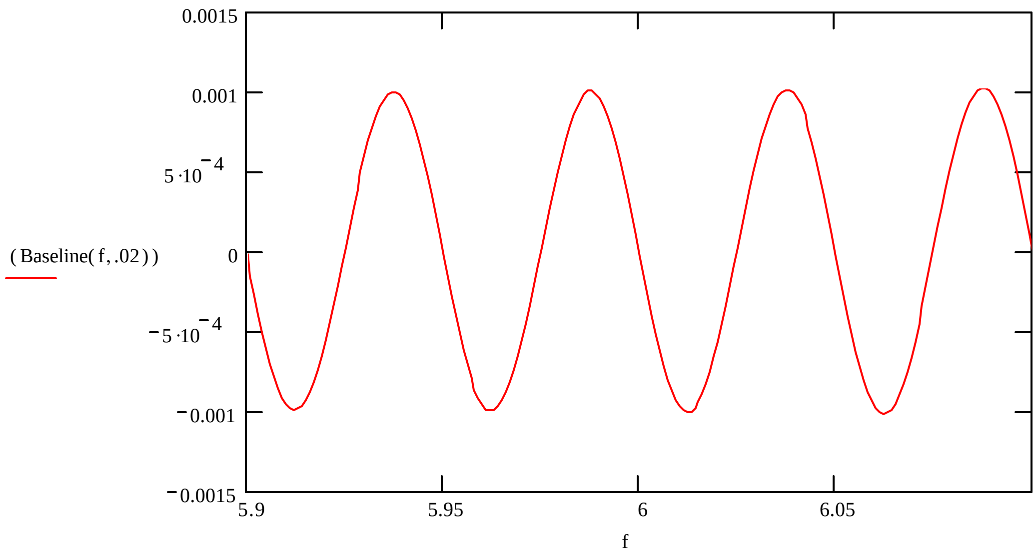
If the response with zero δ is considered as a Reference spectrum, and with non-zero δ Signal spectra, then the graph below plots the (Sig-Ref)/Ref baselines, for δ equal to zero, 1/8. 1/4, and 1/2 times the mid-band wavelength (which is 5cm in this example).

$$\text{Baseline}(f,\delta) := \frac{\text{Mags21d}(f,\delta)}{\text{Mags21}(f)} - 1$$



Note that the baseline ripple amplitude peaks at δ=1/4th the mid-band wavelength.

Say we want max baseline ripple of 0.001. How much δ is allowed?



The answer is: A change of cable length of 0.02cm induces baseline ripple of 0.1%. That is 1.44 degrees at 6GHz.