High Precision Calibration of Wide Bandwidth Observations with the GBT

Ronald J Maddalena
NRAO, Green Bank
July 26 2007

Shelly Hynes (Louisiana School for Math, Science and the Arts)
Charles Figura (Wartburg College)
Chelen Johnson (Breck School)
NRAO-GB Scientific and Engineering staff
Typical Position-Switched Calibration Equation

\[ S(\nu) = \left( \frac{2k}{\eta_A(\nu, \text{Elev}) \cdot \text{Area}_p} \right) \cdot T_A(\nu) \cdot e^{\tau(\nu, t) \cdot A(\text{Elev}, t)} \]

\[ T_A(\nu) = \left( \frac{\text{Sig}(\nu) - \text{Ref}(\nu)}{\text{Ref}(\nu)} \right) \cdot T_{\text{Sys}}^{\text{Ref}} \]

\[ T_{\text{Sys}}^{\text{Ref}} = \left( \frac{\text{Ref}(\nu)}{\text{Ref}_{\text{On}}(\nu) - \text{Ref}_{\text{Off}}(\nu)} \cdot T_{\text{Cal}}(\nu) \right)_{\text{BW}} \]

\[ A(\text{Elev}, t) = \text{Air Mass} \]
\[ \tau(\nu, t) = \text{Atmospheric Zenith Opacity} \]
\[ T_{\text{cal}} = \text{Noise Diode Temperature} \]
\[ \text{Area} = \text{Physical area of the telescope} \]
\[ \eta_A(\nu, \text{Elev}) = \text{Aperture efficiency (point sources)} \]
\[ T_A(\nu) = \text{Source Antenna Temperature} \]

\[ S(\nu) = \text{Source Flux Density} \]
\[ \text{Sig}(\nu), \text{Ref}(\nu) = \text{Data taken on source and on blank sky (in units backend counts)} \]
\[ \text{On,Off} = \text{Data taken with the noise diode on and off} \]
\[ T_{\text{sys}} = \text{System Temperature averaged over bandwidth} \]
Position-Switched Calibration Equation

\[ S(\nu) = \left( \frac{2k}{\eta_s(\nu, \text{Elev}) \cdot \text{Area}_p} \right) \left( \frac{\text{Sig}(\nu) - \text{Ref}(\nu)}{\text{Ref}(\nu)} \right) \left( \frac{\text{Ref}(\nu)}{\text{Ref}_\text{on}(\nu) - \text{Ref}_\text{off}(\nu)} \right) T_{\text{cal}}(\nu) \cdot e^{T(\nu) \cdot A(\text{Elev})} \]

Sources of uncertainties

\[ \left( \frac{\Delta S}{S} \right)^2 = \left( \tau \cdot \Delta A \right)^2 + \left( A \cdot \Delta \tau \right)^2 + \left( \frac{\Delta T_{\text{cal}}}{T_{\text{cal}}} \right)^2 + \left( \frac{\Delta \eta}{\eta} \right)^2 \]

• 10-15% accuracy have been the ‘standard’
• Usually, errors in \( T_{\text{cal}} \) dominate
• Goal: To achieve 5% calibration accuracy without a significant observing overhead.
Air Mass Estimates

Depends upon density and index of refraction as a function of height

But, how can one get this information?
Vertical Weather Data

- Provided by the national weather services via FTP
- 60 hr forecasts (ETA model), updated every 12 hrs
- For each hour, provides as a function of height above the ground:
  - Temperature, Pressure, Dew Point, Cloud Cover, …
- ~40 heights that extend well into the stratosphere
- One can derive as a function of height:
  - Density
  - Index of refraction
  - Absorption coefficient (dry air, water – continuum & line, oxygen line, hydrosols) (Liebe model)
Vertical Weather Data
Air Mass Estimates

- Air Mass derived from the vertical values of density & index of refraction.
- For 1% calibration error, require A to ~0.1

![Graph showing Air Mass for 5/1/2004 to 5/1/2005 at 5° Elevation]

- Probably should use weather dependent Air Mass for elevations below 5 deg
- Probably can ignore weather dependency above 5 deg.
Air Mass Estimate

- Air Mass traditionally modeled as $1/\sin(\text{Elev})$
- For 1% calibration accuracy, must use a better model below 15 deg.

$$A = -0.0234 + \frac{1.014}{\sin\left(\text{Elev} + \frac{5.18}{\text{Elev} + 3.35}\right)}$$

- Good to 1 deg
- Use $1/\sin(\text{Elev})$ above 60 deg
- Coefficients are site specific, at some low level
Air Mass Estimates

100 m

-5 to +20 arcsec
Opacity Estimates

- Vertical weather data provides absorption as a function of height

\[ \tau(\nu,t) = \int_0^\infty \left( \kappa_{\text{Dry}}(\nu,t) + \kappa_{O_2}(\nu,t) + \kappa_{\text{Water\_cont}}(\nu,t) + \kappa_{\text{Water\_line}}(\nu,t) + \kappa_{\text{hydrosols}}(\nu,t) \right) dH \]

\[ T_{\text{Sys}}(\text{Elev,}\nu,t) \cong T_{\text{rcvr}}(\nu) + T_{\text{spill}}(\text{Elev}) + T_{\text{cmb}} e^{-\tau(\nu,t)A(\text{Elev})} + \int_0^\tau T(H,t) \cdot e^{\tau(h,\nu,t)} d\tau \]
Opacity Estimates

- Are derived opacities accurate? Comparisons using tipping radiometers have difficulties
  - Must do multiple tips for wideband observations
  - Tips take up telescope time
  - Requires knowing $T_{\text{cal}}$ to high accuracy, which requires knowing $\tau$.
  - Some dedicated tippers do not provide enough information to estimate $\tau$ near the 22 GHz water line
  - Requires a representative $T_{\text{Atm}}$ that is good to ~5 K

\[ T_{\text{Atm}} \approx \frac{\int \kappa(H) \cdot T(H) \cdot dH}{\int \kappa(H) \cdot dH} \]
Comparison of measured and estimated 22 GHz $T_{sys}$

- For 1% calibration accuracy, requires $\tau$ to 0.01
- Implies forecasted and actual $T_{sys}$ should be within 3 K
- Current model sufficient at $\tau < 0.1$
- Overestimates contribution from hydrosols. Not unexpected.
Noise Diode Estimates

- Traditionally used hot-cold load measurements
  - Provide ~10% accuracy
  - Frequency resolution sometimes wider than frequency structure in $T_{rcvr}$ or $T_{cal}$
  - Time consuming
  - Systematics/Difficulties
    - Loads must be opaque
    - Frost forming on LN$_2$ loads
    - Linearity ($T_{Hot} \gg T_{Cold}$)
  - Observers can’t do their own Hot-Cold tests
Noise Diode Estimates

- Instead, we recommend an On-Off observation
  - Use a point source with known flux -- polarization should be low or understood
  - Use the same exact hardware, exact setup as your observation. (i.e., don’t use your continuum pointing data to calibrate your line observations.)
  - Observations take ~5 minutes per observing run
  - Staff take about 2 hrs to measure the complete band of a high-frequency, multi-beam receiver.
  - Resolution sufficient: 1 MHz, sometimes better
  - Accuracy of ~ 1%, mostly systematics.
Noise Diode Estimates

\[ S(\nu) = \left( \frac{2k}{\eta_A(\nu, \text{Elev}) \cdot A_p} \right) \cdot \left( \frac{\text{Sig}(\nu) - \text{Ref}(\nu)}{\text{Ref}(\nu)} \right) \cdot \left( \frac{\text{Ref}(\nu)}{\text{Ref}_{\text{On}}(\nu) - \text{Ref}_{\text{Off}}(\nu)} \right) \cdot \text{T}_{\text{Cal}}(\nu) \cdot e^{\tau(\nu) \cdot A} \]

Remove Averaging, Solve for T_{cal}

\[ T_{\text{Cal}}(\nu) = \frac{\eta_A(\nu, \text{Elev}) \cdot \text{Area}_p}{2k \cdot e^{\tau(\nu) \cdot A}} \cdot \left( \frac{\text{Ref}_{\text{On}}(\nu) - \text{Ref}_{\text{Off}}(\nu)}{\text{Sig}(\nu) - \text{Ref}(\nu)} \right) \cdot S(\nu) \]
Noise Diode Estimates

X-band Low Cals
Right-Circular Polarization

S-band Low Cals
Y-Linear Polarization
Position-Switched Calibration Equation

\[
S(\nu) = \left( \frac{2k}{\nu_A(\nu, \text{Elev}) \cdot \text{Area}_p} \right) \cdot \left( \frac{\text{Sig}(\nu) - \text{Ref}(\nu)}{\text{Ref}(\nu)} \right) \cdot \left( \frac{\text{Ref}(\nu)}{\text{Ref}_{\text{on}}(\nu) - \text{Ref}_{\text{off}}(\nu)} \right) \cdot T_{\text{cal}}(\nu) \cdot e^{\nu(\nu) - A(\text{Elev})}
\]

Baseline structure

\[
\text{Baselines}(\nu) = \frac{T_A + T_{\text{atm}}^{\text{Sig}} - T_{\text{atm}}^{\text{Ref}}}{\langle T_{\text{sys}} \rangle_{\text{BW}}} \cdot \left\{ \left( T_{\text{recvr}} + \frac{T_{\text{cal}}}{2} \right)_{\text{BW}} - T_{\text{recvr}}(\nu) - \frac{T_{\text{cal}}(\nu)}{2} \right\}
\]

Assumption of linearity

\[
S \propto \text{Sig} - \text{Ref}
\]
Baseline Structure

![Graph showing antenna temperature versus frequency with various markers and annotations.](image-url)
Baseline Shapes

\[ S(\nu) = \left( \frac{2k}{\eta_A(\nu, \text{Elev}) \cdot A_p} \right) \cdot \left( \frac{\text{Sig}(\nu) - \text{Ref}(\nu)}{\text{Ref}(\nu)} \right) \cdot \frac{\text{Ref}(\nu)}{\text{Ref}_{\text{on}}(\nu) - \text{Ref}_{\text{off}}(\nu)} \cdot T_{\text{Cal}}(\nu) \cdot e^{\tau(\nu) \cdot A} \]

Remove Averaging – Vector Calibration

\[ S(\nu) = \left( \frac{2k}{\eta_A(\nu, \text{Elev}) \cdot A_p} \right) \cdot \left( \frac{\text{Sig}(\nu) - \text{Ref}(\nu)}{\text{Ref}_{\text{on}}(\nu) - \text{Ref}_{\text{off}}(\nu)} \right) \cdot T_{\text{Cal}}(\nu) \cdot e^{\tau(\nu) \cdot A} \]

- Traditional equation OK for narrow bandwidth observations
- Traditional provides good calibration only at band center
- Vector algorithm provides good calibration across wide bandwidths
- Vector algorithm is substantially nosier when \( T_A \neq 0 \)

\[ \sigma^2 \approx \frac{1}{BW \cdot t} \left( \frac{T_A^2}{2} + \frac{T_{\text{sys}}^2}{2} + \frac{T_{\text{sys}}^2 T_A}{T_{\text{Cal}}} \right) \]

- Smooth \( \text{Ref}_{\text{on}} - \text{Ref}_{\text{off}} \) – use Savitzky-Golay smoothing
Non-linearity

- If system is linear,
  - $P_{out} = B \times P_{in}$
  - $(\text{Sig}_{on} - \text{Sig}_{off}) - (\text{Ref}_{on} - \text{Ref}_{off}) = 0$

- Model the response curve to 2nd order:
  - $P_{out} = B \times P_{in} + C \times P_{in}^2$

- Our ‘On-Off’ observations of a calibrator provide:
  - Four measured quantities: $\text{Ref}_{off}$, $\text{Ref}_{on}$, $\text{Sig}_{off}$, $\text{Sig}_{on}$
  - $T_A$ From catalog
  - Four desired quantities: $B$, $C$, $T_{cal}$, $T_{sys}$

- It’s easy to show that:
  - $C = \left[ (\text{Sig}_{on} - \text{Sig}_{off}) - (\text{Ref}_{on} - \text{Ref}_{off}) \right] / (2T_A T_{cal})$

- Thus:
  - Can determine if system is sufficiently linear
  - Can correct to 2nd order if it is not
Non-linearity

$$(\text{SigOn-SigOff}) - (\text{RefOn-RefOff})$$

$\text{Power In} \rightarrow$
Summary

- To obtain few percent calibration accuracy
  - Wide bandwidths require frequency dependent opacities, efficiencies, $T_{sys}$, and $T_{cal}$.
  - New weather-independent model for air mass is usually sufficient
  - Opacities from vertical, forecasted weather data sufficient for medium to low-opacity conditions
  - Simple observation of calibrator provides high accuracy, high frequency resolution $T_{cal}$.
  - Assumptions of traditional on-off calibration algorithm introduces baseline shapes for wide bandwidth observations. Should use ‘vector’ algorithms.
  - 2nd order non-linearity -- measurable (by-product of $T_{cal}$ observation) and correctable. Might be significant.
Typical Position-Switched Calibration Equation

\[ S(\nu) = \left( \frac{2k}{\eta_A(\nu, Elev) \cdot Area_p} \right) \cdot T_A(\nu) \cdot e^{\tau(\nu, t) \cdot A(Elev, t)} \]

\[ T_A(\nu) = \left( \frac{Sig(\nu) - Ref(\nu)}{Ref(\nu)} \right) \cdot T_{Sys}^{Ref} \]

\[ T_{Sys}^{Ref} = \left( \frac{Ref(\nu)}{Ref_{On}(\nu) - Ref_{Off}(\nu)} \cdot T_{Cal}(\nu) \right)_{BW} \]

\[ T_{Sys}^{Ref}(Elev, \nu, t) \cong T_{rcvr}(\nu) + T_{spill}(Elev) + T_{cmb} e^{-\tau(\nu, t) \cdot A(Elev)} + T_{Atm}(\nu, t) \cdot (1 - e^{-\tau(\nu, t) \cdot A(Elev, t)}) \]

A(Elev, t) = Air Mass
\( \tau(\nu, t) = \) Atmospheric Zenith Opacity
Area = Physical area of the telescope
\( \eta_A(\nu, Elev) = \) Aperture efficiency (point sources)
\( T_A(\nu) = \) Source Antenna Temperature
S(\nu) = Source Flux Density
Sig(\nu), Ref(\nu) = Data taken on source and on blank sky (in units backend counts)

On, Off = Data taken with the noise diode on and off
\( T_{sys}(\nu, Elev, t) = \) System Temperature
\( T_{CMB} = \) Cosmic Microwave Background
\( T_{rcvr}(\nu) = \) Receiver Temperature
\( T_{spill}(Elev) = \) Antenna Spillover
\( T_{Atm}(\nu, t) = \) Representative temperature of the atmosphere