TO: Dana Balser, Jim Braatz, Frank Ghigo, Glen Langston, Mark McKinnon, and Richard Prestage
FROM:Ron Maddalena
SUBJECT:Continuum Calibration: an addendum

My previous memo on continuum calibration had an oversight and, as such, you should throw away the old document and replace it with the attached.

The calculation of the accuracy of an antenna temperature measurement was only appropriate when the observation had a large number of data samples. I had overlooked a correlated error term when I did the propagation of errors. The other sections of the memo have not been changed.

The Value of Tcal for Wide Bandwidths

To derive gain, system temperatures, and antenna temperatures, one must know thefrequency-averaged, gain-weighted value of the noise diode temperature **Tcal.**

$$Avrg\_Tcal := \frac{\int_0^{\infty} G(f) \cdot Tcal(f)\, df}{\int_0^{\infty} G(f)\, df}$$

Although the limits of the integral are 0 and ∞, they can be taken as $\mathbf{f_1}$ and $\mathbf{f_0}$ which define the frequency limits of the observing bandwidth.

But, note that one needs the gain **G** (in units of K/Volts or K/Counts) to calculate **Avrg_Tcal** which in turn is needed to calculate the gain. The way out of this circular dilemna is to start with a small enough bandwidth such that the change in gain across that bandwith can be assumed to be small or linear. For a small enough bandwidth then:

$$Avrg\_Tcal := \frac{\int_{f_0}^{f_1} Tcal(f)\, df}{f_1 - f_0}$$

Or, if Tcal is measured uniformally with frequency:

$$Avrg\_Tcal := \frac{\sum_{i=1}^{Num} Tcal(f_i)}{Num}$$

$$G := 1$$

Corrections to Raw Counts

The design of the GBT's DCR backend (like many others) produces data that is not only proportional to the input power but also to the sample time. Doubling the sample time but keeping the input power levels alone doubles the raw counts. This tends not to be a problem if the data are taken in the "Total power without Cal" mode or, for most other modes, if equal time is spent on each phase. [The term "phase" refers to any change in state -- "Total Power with Cal" observations have two states (Cal on and off) while "Switched Power with Cal" have four (signal-cal off; signal-cal on; reference-cal off; reference-cal on)].

For these special backends, one has to correct the raw counts for each phase by the phase duration or the time spent on that phase.

$$phase\_duration := cycle\_time \cdot (phase\_end - phase\_start) - phase\_blanking$$

where **cycle_time** is in seconds and is the time for a complete cycle of phases. **phase_end** and

phase_start, given in percentages of a cycle, define the start and end of a phase, and **phase_blanking** is the amount of blanking in seconds for that phase.

For each sample, **i**, in an observation:

$$Counts\_phase_i := \frac{Raw\_Counts\_phase_i}{phase\_duration}$$

NOTE: For those backends whose output counts are not dependent on the sampling period:

$$Counts\_phase_i := Raw\_Counts\_phase_i$$

Calculation of Gain

In general, for any time sample **i**, the measured antenna temperature can be derived from the measured counts or voltages:

$$Ta_i := G \cdot Counts_i$$

where **G** is the gain averaged across the duration of an observation and averaged across the observing bandwidth. If the noise diode is firing on and off, then:

$$Ton_i := G \cdot Counts\_on_i$$

$$Toff_i := G \cdot Counts\_off_i$$

But the difference between **Ton** and **Toff** is defined to be **Tcal**:

$$Avrg\_Tcal := Ton_i - Toff_i$$

Or:

$$Avrg\_Tcal := G \cdot Counts\_on_i - G \cdot Counts\_off_i$$

Solving for **G** and averaging over the **N** samples within an observation gives:

$$G\_Avrg := \frac{\sum_{i=1}^{N} \frac{Avrg\_Tcal}{Counts\_on_i - Counts\_off_i}}{N}$$

For switched power observations, especially if the receiver frequency is different between the signal and reference phases, one needs to calculate the above average gain for the signal and reference phases using an **Avrg_Tcal** appropriate for the phase's frequency range.

For each phase we will also be interested in the accuracy in the derived value of **G_Avrg**. This depends upon the accuracy with which one can measure an average count which, in turn, is derived from the radiometer equation.

$$\sigma\_Counts_i := \frac{Counts_i}{\sqrt{BW \cdot phase\_duration}}$$

where **BW** is the equivalent bandwidth of the observation. Then, after some substitution and propogation of errors, for each phase we get:

$$\sigma G\_Avrg := \frac{Avrg\_Tcal}{N\cdot\sqrt{BW\cdot phase\_duration}}\cdot\sqrt{\sum_{i=1}^{N}\frac{(Counts\_on_i)^2+(Counts\_off_i)^2}{(Counts\_on_i-Counts\_off_i)^4}}$$

Derivation of Antenna Temperatures

i) Total Power without Cal

One cannot derive gains from a "Total Power without Cal" observation. Instead, one must use a gain derived from a previous calibration observation. In this case:

$$Ta_i := G\cdot Counts_i$$

And the accuracy of this measurements is:

$$\sigma Ta_i := Ta_i\cdot\sqrt{\frac{\sigma G^2}{G^2}+\frac{1}{BW\cdot phase\_duration}}$$

ii) Total Power with Cal

For "Total Power with Cal" observations, one can determine a $\mathbf{Ta_i}$ for both the on and off cal phases.

$$Ta\_on_i := G\cdot Counts\_on_i$$

$$Ta\_off_i := G\cdot Counts\_off_i$$

The accuracy of an individual sample is:

$$\sigma Ta\_on_i := \frac{Avrg\_Tcal\cdot Counts\_on_i}{N\cdot\sqrt{BW\cdot phase\_duration}}\cdot\left[\sum_{k=1}^{N}\frac{(Counts\_on_k)^2+(Counts\_off_k)^2}{(Counts\_on_k-Counts\_off_k)^4}\ldots\right.$$
$$+\left(\sum_{k=1}^{N}\frac{1}{Counts\_on_k-Counts\_off_k}\right)^2\ldots$$
$$\left.+\frac{-2\cdot Counts\_on_i}{(Counts\_on_i-Counts\_off_i)^2}\cdot\sum_{k=1}^{N}\frac{1}{Counts\_on_k-Counts\_off_k}\right]^{\frac{1}{2}}$$

$$\sigma Ta\_off_i := \frac{Avrg\_Tcal \cdot Counts\_off_i}{N \cdot \sqrt{BW \cdot phase\_duration}} \cdot \left[\sum_{k=1}^{N} \frac{(Counts\_on_k)^2 + (Counts\_off_k)^2}{(Counts\_on_k - Counts\_off_k)^4} \ldots + \left(\sum_{k=1}^{N} \frac{1}{Counts\_on_k - Counts\_off_k} \right)^2 \ldots + \frac{-2 \cdot Counts\_off_i}{(Counts\_on_i - Counts\_off_i)^2} \cdot \sum_{k=1}^{N} \frac{1}{Counts\_on_k - Counts\_off_k} \right]^{\frac{1}{2}}$$

You can average the on and off phase data to reduce the noise. Since the diode is on for part of an observation, the measured antenna temperature is augmented by the value of Avrg_Tcal. One should also weight the average.

$$Ta_i := G \cdot \frac{Counts\_on_i \cdot (\sigma Ta\_off_i)^2 + Counts\_off_i \cdot (\sigma Ta\_on_i)^2}{(\sigma Ta\_off_i)^2 + (\sigma Ta\_on_i)^2} - \frac{Avrg\_Tcal}{2}$$

The accuracy of the average is:

$$\sigma Ta_i := \frac{\sigma Ta\_on_i \cdot \sigma Ta\_off_i}{\sqrt{(\sigma Ta\_off_i)^2 + (\sigma Ta\_on_i)^2}}$$

Note that for observations where Tcal is much less than the system temperature, one can use equal weights in the averaging.

III) Switched Power with and without Cal

"Switched Power" observations are handled in the same way as "Total Power" except one calculates the signal and reference phases separately and, thus, derives a Ta_signal_i and a $Ta\_reference_i$.

Derivation of System Temperatures

An estimate of the system temperature is the weighted average of $\mathbf{Ta_i}$ over the duration of an observation.

i) Total Power with and without Cal

For "Total Power" observations, the average value of $\mathbf{Ta_i}$ and its accuracy are:

$$Tsystem := \frac{\sum_{i=1}^{N} \frac{Ta_i}{\left(\sigma Ta_i\right)^2}}{\sum_{i=1}^{N} \frac{1}{\left(\sigma Ta_i\right)^2}}$$

$$\sigma Tsystem := \frac{1}{\sum_{i=1}^{N} \frac{1}{\left(\sigma Ta_i\right)^2}}$$

ii) Switched Power with and without Cal

"Switched Power" observations are handled in the same way as "Total Power" except one calculates the signal and reference phases separately and, thus, derives a Tsystem_signal and a Tsystem_reference.

Derivation of Source Temperatures

i) Total Power

One cannot derive a source temperature from a total power observation directly. Instead, one usually fits a baseline to non-source areas of a data set. Alternatively, one subtracts off an independent data set that models the total power observation but without the source of interest.

ii) Switched Power

In switched power observing modes, one usually sets up the experiment so that during the signal phases the measurements are of the system plus the source. During the reference phase, the measurements are of the system only. Thus, a simple subtraction of the Ta_signal and Ta_reference data should recover the source temperature.

$$Tsrc_i := Ta\_signal_i - Ta\_reference_i$$

with an accuracy of:

$$\sigma Tsrc_i := \sqrt{\left(\sigma Ta\_signal_i\right)^2 + \left(\sigma Ta\_reference_i\right)^2}$$