Modeling the Elevation and Frequency Dependence of Aperture Efficiency for the GBT's Pipeline

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The calibration of data from any radio telescope typically requires knowing the telescope's aperture efficiency, η_A, as a function of observing wavelength and, in the case of the GBT, the observing elevation. In contrast to the representation of η_A used by the GBT analysis pipeline, this memo provides a much more traditional representation which should be much more convenient for the GBT's pipeline as it requires far fewer parameters. The pipeline's current representation requires substantial observations in order to provide the necessary modeling while the proposed representation requires observations at just a single frequency, thereby saving us telescope time. I provide a modified version of the model of Ghigo (2009b) that better matches the expected and measured performance. I also suggest we continue to use the 40-52 GHz receiver, due to the reduction in calibration uncertainty provided by the receiver's chopper wheel, to best determine the required elevation parameters.

Background

The definition of aperture efficiency can be found in many classic references (e.g., Baars, 1973; Goldsmith, 2002; Rohlfs and Wilson, 2006). As an example of a typical usage, the flux density of a point source, S, is related to the measured antenna temperature, T_A, by: $T_A = \eta_A\, A_g e^{-\tau A}\, S/2k$, where k=Boltzman's constant, τ=the zenith atmospheric opacity at the observing frequency, A = the air mass at the observing elevation, and A_g is the geometrical area of the antenna ($\pi D^2/4$ for a circular aperture of projected diameter D).

The equivalent blackbody temperature of an extended source, T_{Src}, is related to T_A by: $T_A = \left(\eta_A\, A_g e^{-\tau A}\, /\lambda^2\right) \cdot \iint_{\mathrm{Src}} T_{Src}(\theta,\varphi) P(\theta,\varphi)\, d\Omega$ where the integral is over the solid angle of the source, P is the normalized power pattern for the telescope, and λ is the observing wavelength. As a convenience, astronomers often use the assumption the observations are of a disc source with uniform temperature, T_R^*, where: $T_A = \left(\eta_A\, A_g e^{-\tau A}\, /\lambda^2\right) \cdot T_R^* \iint_{\mathrm{Src}} P(\theta,\varphi)\, d\Omega$.

Also as a convenience, radio observations often report temperatures in units of T_{MB} for extended sources, as if it were a disc sources with uniform brightness and with a size that extends to the first null in the main beam. Here, $T_A = \left(\eta_A\, A_g e^{-\tau A}\, /\lambda^2\right) \cdot T_{MB} \iint_{\mathrm{MainBeam}} P(\theta,\varphi)\, d\Omega$ where the integral is to the first nulls of the beam pattern and easily determined from astronomical observations. Alternatively, $T_A = \eta_R\, \eta_{MB} e^{-\tau A} T_{MB}$ where η_R corrects for the ohmic losses in the antenna (and is typically very close to unity) and η_{MB} is the main beam efficiency. Maddalena (2010a) gives a theoretical relationship between η_{MB} and η_A, as well as ways in which one can measure directly η_{MB}.

In each of these examples, one requires knowing $\eta_{A.}$, and therefore, in the case of the GBT, the dependence of η_A on the observing elevation, E, and wavelength.

Current Pipeline Parameterization of η_A

By comparing eq. 16 of Langston (2011) to the above equation for the relationship between flux density and T_A, one can summarize the algorithmic model for η_A that is currently used by the GBT pipeline is of the form:

$$\eta_A(\lambda, E) = \eta_{A,\infty}(Te) \cdot e^{-(4\pi\varepsilon/\lambda)^2} \cdot \sum_{i=0}^{n} A_i(\lambda) \cdot (90 - E)^i$$

To produce this equation, I have extended Langston's equation 16 to include some missing details and repaired a couple of mistakes in the original equation.

As shown above, the relationship between T_{Src} and S cannot be as simple as Langston's equation 16. Rather, T_A, not T_{Src} was probably the intended unit. In any case, the inclusion of η_l, the rear spillover efficiency, is also not correct. I have removed the *fbeam;pol:(band)* factor as this is actually a correction that should to be applied when deriving T_A, not in the algorithm which converts T_A to other units. I also substituted the Ruze equation. The equation also didn't include the important dependence on the feed illumination, *Te*. I also eliminated the use of the term 'gain' as the use in Langston is atypical. Typically 'gain' is defined either as $G = 4\pi\eta_A A_g/\lambda^2$ or as the ratio T_A/S.

In the above equation, $\eta_{A,\infty}$, which is dependent upon depends upon Te (which in turn is dependent upon the receiver, polarization, and observing frequency) and can be thought of as the aperture efficiency the receiver would have if it were capable of observing at $\lambda=\infty$. The ε represents the rms surface errors (as defined by Ruze, 1966; see also Goldsmith, 2002; Rohls and Wilson, 2006) ***at the telescope's rigging angle***, which currently has a value of 220 μm, as determined by holographic observations (Hunter, 2010). The A_i values are polynomial coefficients that describe how the telescope's efficiency changes with elevation. Currently, values for A_i are from GBT observations whose results are stored on the web page https://safe.nrao.edu/wiki/bin/view/GB/Observing/GainPerformance that Frank Ghigo has maintained. Note that the values of A_i have been normalized so that the summation always has a maximum value of 1.0.

There are a number of issues with this formulization and the currently planned use of the web-page values. The critical issues are:

- One can only calibrate for frequencies where one has astronomical observations. Thus, currently the pipeline has no way of calibrating data from the 68-92 GHz receiver, at 48 GHz, etc.
- For all but one frequency, the table uses data that were taken when the telescope used a gravitational model for the active surface that is no longer applicable. Thus, all but one set of A_i are useless for the current observations.

- We know that the feed illumination pattern for every GBT receiver changes across each receiver's band. From optical modeling (e.g., Goldsmith 2002), changes in feed illumination will alter $\eta_{A,\infty}$. The observations that were included in the table are at most at one observing frequency per band, and, thus, the table doesn't include the frequency dependence of $\eta_{A,\infty}$ across any receiver's band.
- All of the values below 20 GHz embed the degradation in T_A from the atmosphere into the A_i coefficients. Since the opacity under which the observations were made is not included, there's no way for analysis software to remove the atmospheric attenuation under which the table values were generated. Thus, the coefficients can only be applied to days under which the atmosphere was the same as that which the table observations were made. Also, the elevation dependence of atmospheric attenuation does not follow the power law used by the table.
- Most entries have values for $\eta_{A,\infty}$ that are different for the two receiver polarizations, which is not possible for the GBT. Rather, this indicates there are issues with the calibration of the data that are then being embedded into the $\eta_{A,\infty}$ values. Thus, the values of $\eta_{A,\infty}$ are only applicable if the same calibration errors exist at the time of the real observations.

Thus, the usefulness of this table is limited at best, and could be destructive if the pipeline was implemented as outlined in Langston (2011).

Suggested Parameterization of η_A

Goldsmith (1987, 2002) provides an alternative parameterization of η_A that I believe is better suited to analysis systems like the GBT pipeline. Combining equations 30, 40, 43, and 53 of Goldsmith (2002) gives:

$$\eta_A(\lambda, E) = \eta_{\mathrm{SpillIllum}}\eta_{\mathrm{Blockage}}\eta_{\mathrm{Surface}} = \frac{2}{\alpha}(1 - e^{-\alpha})^2 \cdot \left(1 - f_b^2\right)^2 \cdot e^{-(4\pi\varepsilon(E)/\lambda)^2}$$

The first term is often considered the same as $\eta_{A,\infty}$. It is the product of the efficiencies from spillover and the feed illumination taper, both of which are parameterized by α. As Goldsmith shows, the value of α depends upon the ***effective*** Gaussian feed illumination taper, *Te*, in units of dB of power density, by α=Te(dB)/8.686. For the GBT receivers, I estimate that the typical ***effective*** Gaussian feed illumination taper is ~20 dB.

If the value for illumination didn't vary between receivers and across receiver bandpasses, the first term would have a constant value, probably the 0.71 we typically quote for the GBT. Since we know that this is not the case, for high-accuracy calibration we would need to know how α varied. An analysis system like the GBT pipeline would need to distinguish which receiver took what data, which may be problematic for those frequencies that are covered by more than one receiver.

Astronomical determinations of α are nearly impossible as it's hard to judge whether a determined value has been corrupted by uncertainties in the assumed noise diode values, atmospheric attenuation, and so on. Instead, we should use theoretical determinations of the ***effective*** Gaussian feed illumination. Since we have few theoretical values for the first suit of GBT receivers, and none for the more recent receivers, some effort should be put into theoretical determinations of α. Since we currently lack values for α, we may consider using 0.71 for the first term, which is equivalent to what Langston (2011) recommends.

The second term is the efficiency due to blockage, which depends upon the fractional of the dish radius, f_b, which is blocked. Since f_b=0 for the GBT, the second term is unity for the GBT. I have include this term here for the sake of completeness.

The third term is the surface efficiency, again the Ruze formula, but here ε is no longer the rms surface errors value at the rigging angle but are dependent upon the telescope's orientation (E for the GBT). It's important to note here that, unlike the web page table, the elevation dependence of η_A is no longer receiver or frequency dependent. Rather, since all receiver share the same surface with the same surface errors, one need only accurately derive or measure ε(E) whenever the active surface parameters are altered. Then, that determination can be applied to any other frequency.

Determining ε(E) from Existing Measurements

The usefulness of my suggested representation for η_A depends upon how one can accurately derive ε(E). Luckily, Ghigo's web page has embedded in it all that we need.

The last entry in Ghigo's table is for observations at 43.1 GHz taken on Oct 4, 2009 with the 40-52 GHz receiver. The observations (Ghigo, 2009b) used AutoOOF's for tweaking up the surface and an active surface model that is close to what we currently use. The calibration used a combination of the noise diodes, forecasted models of the atmospheric opacity, and, most importantly the receiver's 'chopper' wheel. We know that our models for atmospheric opacity have a high degree of accuracy (Maddalena 2010b). The observing frequency was high enough that the elevation dependence is easily determined and distinguishable from elevation dependence of atmospheric attenuation.

The results of the observations are presented as the variation of η_A with elevation. These can be translated into ε(E) by a simple inversion of the above equation for η_A:

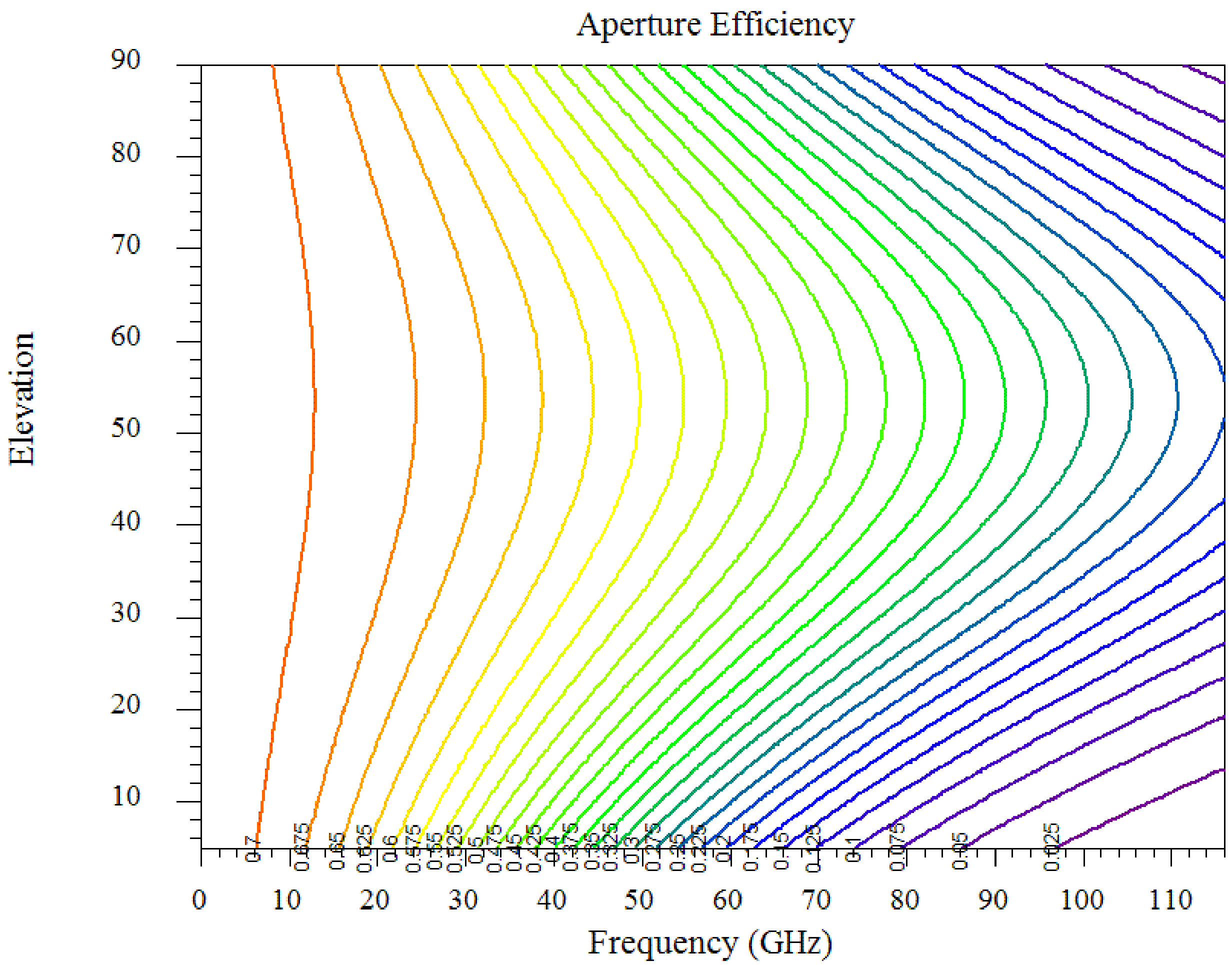
$$\varepsilon(\mathrm{E}) = \left(\frac{\lambda}{4\pi}\right) \cdot \sqrt{\ln\left[\frac{\eta_A(\mathrm{E})}{\eta_{\mathrm{SpillIllum}}} \cdot \frac{1}{\eta_{\mathrm{Blockage}}}\right]}$$

Using a value of $\eta_{SpillIllum}$=0.71, and assuming $\eta_{Blockage}$ = 1, Ghigo (2009b) derived an elevation model for ε(E) that had the same shape for the two receiver polarizations, but different scaling factors. The scaling factors determined by Ghigo for the two polarizations (182 and 174 μm), and defined to be equivalent to the surface rms at the rigging angle, are substantially better than the expected 220 μm. Since ε(E) cannot be different for the two polarizations, the difference in scaling is an artifact of unresolved, small (1%) relative calibration uncertainties between the two polarizations. The low value for the derived rms of the surface implies that the absolute calibration may be off by 6%, which is actually rather good.

To determine a value for ε(E) that is more consistent with what we expect, I simply scaled Ghigo's estimate for ε(E) to the expected 220 μm. I also simplified the curve to use elevation, instead of zenith distance:

$$\varepsilon(E) = 500.954 - 10.4728 \cdot E + 0.09766 \cdot E^2$$

Using this model for ε(E), assuming $\eta_{SpillIllum}$=0.71, and $\eta_{Blockage}$ = 1, for the first time for the GBT one can now provide an estimate of $\eta_A(\lambda,E)$ that is depicted in the figure.



Summary

The definition of η_A used by the pipeline has some problems as well as a non-traditional representation. Here, I have provided an alternative, traditional representation. I have removed those entities that traditionally should not be included in the definition of η_A as well as introduced others factors that were missing. The proposed parameterization has the additional benefit in that it should be simpler to maintain as the significant contributors to the elevation and frequency dependence of η_A are now encoded into a single model for ε(E). The pipeline parameterization requires astronomical measurements at all frequencies at which data are to be calibrated. The proposed parameterization requires observations at just a single frequency, thereby saving us significant telescope time. Using the previous work of Ghigo (2009b), I provide a polynomial model of ε(E) that matches the expected and measured performance of the telescope.

From Ghigo's memos (2009a, 2009b), one can see that observations with the 40-52 GHz receiver have a significant advantage over observations made with any other GBT receiver. In particular, one can make use of the receiver's calibration wheel to reduce calibration uncertainties far beyond what can be done with other receivers. The observing frequency is high enough that surface errors produce a substantial difference in η_A from its long-wavelength value. And, it's at a frequency high enough that the elevation dependency of η_A is easily distinguished from atmospheric attenuation. The atmospheric modeling of opacity is also well established at 43 GHz (Maddalena 2010b). (If the calibration problems with the 68-92 GHz are solved, and if we can determine whether our atmospheric opacity models work in this frequency range, this receiver might end up be better suited than the 40-52 GHZ receiver is). Thus, I suggest we continue to use the 40-52 GHZ receiver after every update to the active surface models in order to update the model used by the pipeline for ε(E), and thereby $\eta_A(\lambda,E)$.

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